Quantum gravity, black holes, and the renormalisation group

Daniel Litim

Department of Physics and Astronomy

University of Sussex

New Trends in Modern Cosmology

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introduction

- **physics of classical gravity**

  Einstein's theory $G_N = 6.7 \times 10^{-11} \frac{m^3}{\mathrm{kg}\, s^2}$

  classical action

$$S_{\mathrm{EH}} = \frac{1}{16\pi G_N} \int \sqrt{\det g}(-R(g_{\mu\nu}) + 2\Lambda)$$

  valid on length scales $\sim 10^{-2} - 10^{28}\,\mathrm{cm}$

introduction

- **physics of classical gravity**

  Einstein's theory $G_N = 6.7 \times 10^{-11} \frac{m^3}{\text{kg}\, s^2}$

- **physics of quantum gravity**

  Planck length $\ell_{\text{Pl}} = \left(\frac{\hbar G_N}{c^3}\right)^{1/2} \approx 10^{-33}\,\text{cm}$

  Planck mass $M_{\text{Pl}} \approx 10^{19}\text{GeV}$

  Planck time $t_{\text{Pl}} \approx 10^{-44}\,\text{s}$

  Planck temperature $T_{\text{Pl}} \approx 10^{32}\,\text{K}$

  expect modifications at energy scales $E \approx M_{\text{Pl}}$

introduction

- **physics of classical gravity**

  Einstein's theory $\quad G_N = 6.7 \times 10^{-11} \frac{m^3}{\mathrm{kg}\, s^2}$

- **physics of quantum gravity**

  path integral approach

$$\int [dg_{\mu\nu}]_{\text{ren.}} \exp\left(-S[g_{\mu\nu}] + \text{sources}\right)$$

perturbation theory

- **structure of UV divergences**

N-loop Feynman diagram $\sim \int dp\, p^{A\,-[G]N}$

$[G] > 0:$ superrenormalisable

$[G] = 0:$ renormalisable

$[G] < 0:$ dangerous interactions

gravity: $[g_{\mu\nu}] = 0\,, \quad [\mathrm{Ricci}] = 2\,, \quad [G_N] = 2 - d$

effective expansion parameter: $G_N\, p^2 \sim \frac{p^2}{M_{\mathrm{Pl}}^2}$

perturbation theory

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  N-loop Feynman diagram $\sim \int dp\, p^{A - [G]N}$

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- **perturbative non-renormalisability**

  gravity with matter interactions

  pure gravity (Goroff-Sagnotti term)

perturbation theory

- **effective theory for gravity** (Donoghue '94)

  quantum corrections computable for energies $E^2/M_{\rm Pl}^2 \ll 1$

  knowledge of UV completion not required

perturbation theory

- **effective theory for gravity** (Donoghue '94)

  quantum corrections computable for energies $E^2/M_{\rm Pl}^2 \ll 1$

  knowledge of UV completion not required

  1-loop 'running coupling'

$$G(r) = G_0 \left(1 - \frac{167}{30\pi} \frac{G_0 \hbar}{r^2} \right)$$

perturbation theory

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- **higher derivative gravity I** (Stelle '77)

  R^2 gravity perturbatively renormalisable

  unitarity issues at high energies

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- **higher derivative gravity I** (Stelle '77)

  R^2 gravity perturbatively renormalisable

  unitarity issues at high energies

- **higher derivative gravity II** (Gomis, Weinberg '96)

  all higher derivative operators

  gravity 'weakly' perturbatively renormalisable

  no unitarity issues at high energies

UV fixed points

UV fixed points

- **asymptotic freedom**

  YM theory

UV fixed points

- **asymptotic freedom**

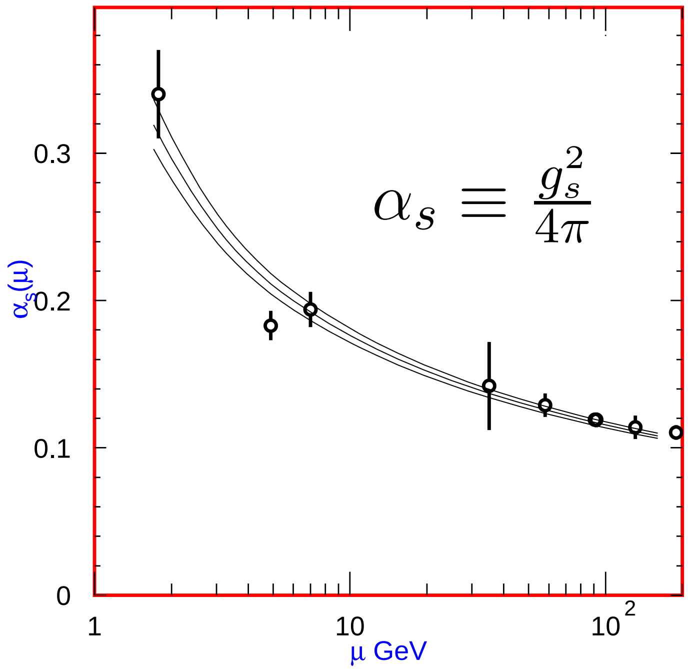
  YM theory

  running coupling

  $$\frac{dg_s}{d\ln\mu} = -\frac{7g_s^3}{16\pi^2}$$

  trivial UV fixed point

  $$g_s = 0$$



UV fixed points

- **asymptotic freedom**

  YM theory

- **asymptotic safety** (Weinberg '79)

  non-trivial UV fixed point for gravity

  well-defined continuum limit

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  critical trajectory

    stable, marginal, unstable directions

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  predictive power

  finite number of unstable directions

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- **examples**

  Gross-Neveu models $(D > 2)$

  quantum gravity in $D \approx 2$

asymptotic safety

- **RG scaling of gravitational coupling**

dimensionless coupling $$g(\mu) = Z_N(\mu)^{-1} \cdot G_N \cdot \mu^{D-2}$$

anomalous dimension $$\eta_N = -\frac{\mathrm{d}\ln Z_N}{\mathrm{d}\ln\mu}$$

RG running $$\frac{\mathrm{d}g}{\mathrm{d}\ln\mu} = (D - 2 + \eta_N)\, g$$

(DL '06,Niedermaier '06)

asymptotic safety

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- **fixed points**

  Gaussian: $g = 0$ **classical general relativity**

  non-Gaussian: $\eta_N = 2 - D$ **strong quantum effects**

asymptotic safety

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- **fixed points**

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  UV fixed point implies **weakly coupled gravity** at **high energies**

$$\mu \to \infty : \qquad G(\mu) \to g_* \mu^{2-D} \ll G_N$$

asymptotic safety

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- **fixed points**

  Gaussian: $g = 0$ **classical general relativity**

  non-Gaussian: $\eta_N = 2 - D$ **strong quantum effects**

  IR fixed point implies **strongly coupled gravity** at **low energies**

$$\mu \to 0 : \qquad G(\mu) \to g_* \mu^{2-D} \gg G_N$$

asymptotic safety

- **RG scaling of gravitational coupling**

  dimensionless coupling $$g(\mu) = Z_N(\mu)^{-1} \cdot G_N \cdot \mu^{D-2}$$

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- **fixed points**

  Gaussian: $g = 0$ **classical general relativity**

  non-Gaussian: $\eta_N = 2 - D$ **strong quantum effects**

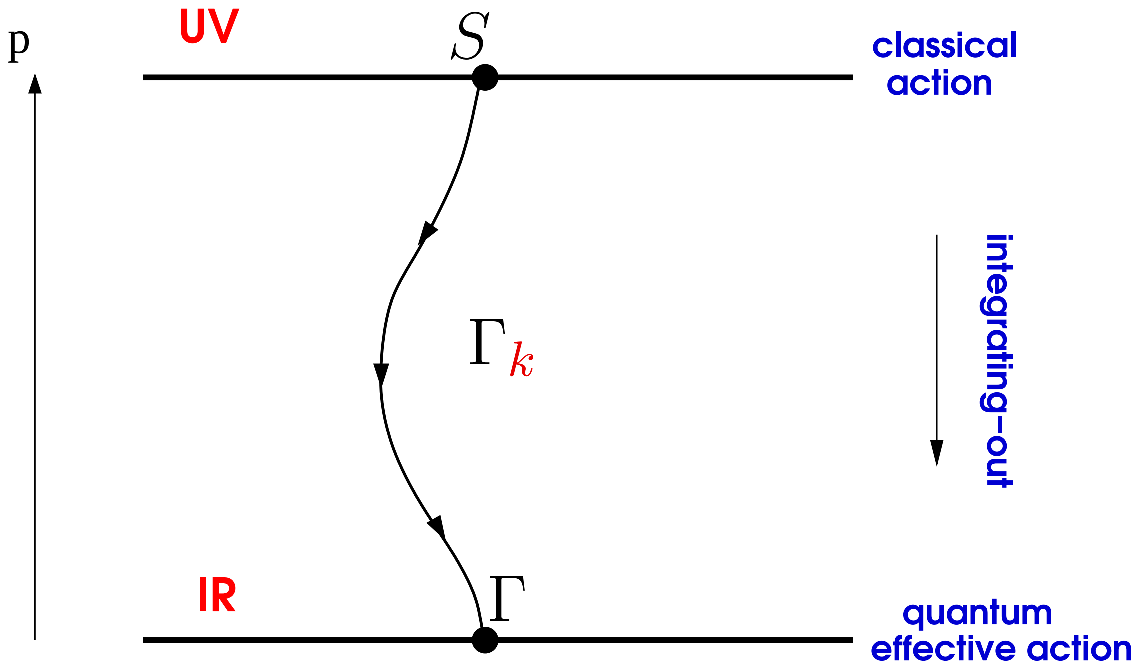
- **tools**

  discretisation: lattice technology

  continuum: **renormalisation group**

renormalisation group

- **integrating-out momentum degrees of freedom:** **“top-down”**



renormalisation group

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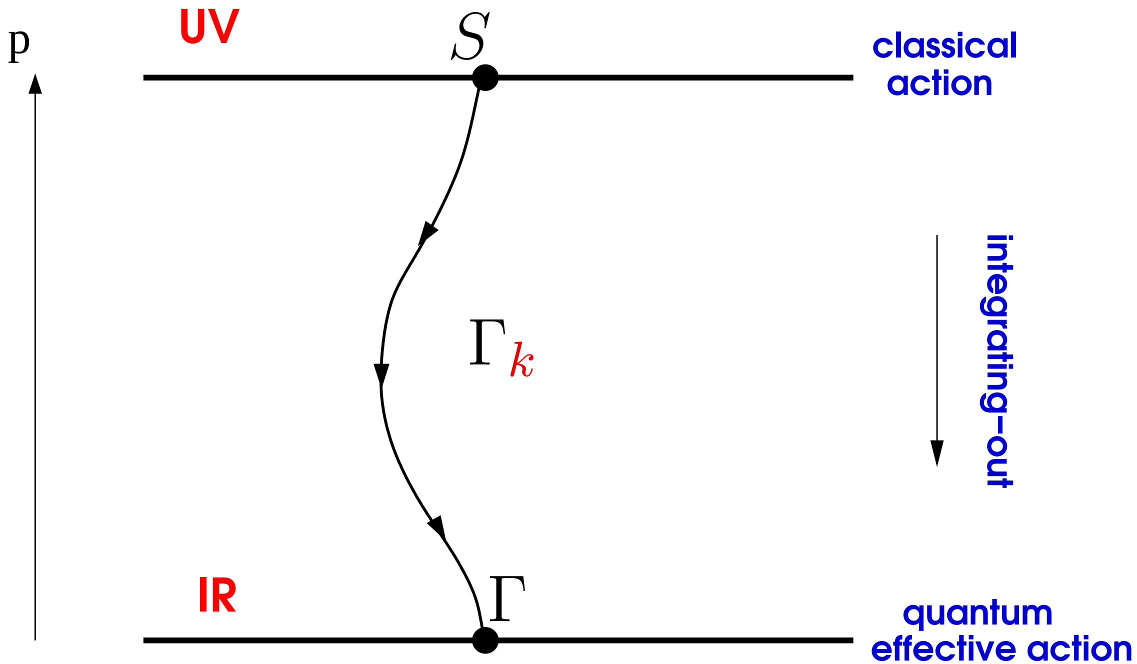
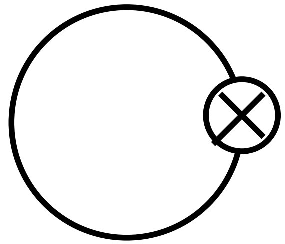


- **QCD: signatures of confinement** (Pawlowski, DL, Nedelko, Smekal ’03)

renormalisation group

- **Callan-Symanzik equation** (Callan '70, Symanzik '70)

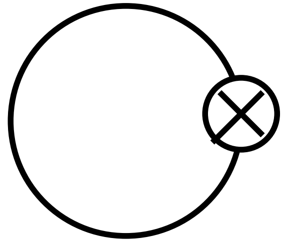
$$k\frac{\mathrm{d}\Gamma_k}{\mathrm{d}k} = \frac{1}{2}\,\mathrm{Tr}\,\left[\left(\frac{\delta^2\Gamma_k[\phi]}{\delta\phi\,\delta\phi} + k^2\right)^{-1} k\frac{\mathrm{d}k^2}{\mathrm{d}k}\right]_{\mathrm{ren.}} = \frac{1}{2}$$



renormalisation group

- **functional RG equation** (Wetterich '93)

$$k\frac{\mathrm{d}\Gamma_k}{\mathrm{d}k} = \frac{1}{2}\,\mathrm{Tr}\left[\left(\frac{\delta^2\Gamma_k[\phi]}{\delta\phi\,\delta\phi} + R_k\right)^{-1} k\frac{\mathrm{d}R_k}{\mathrm{d}k}\right] \quad = \frac{1}{2}$$



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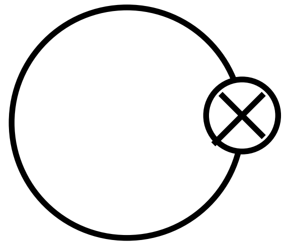
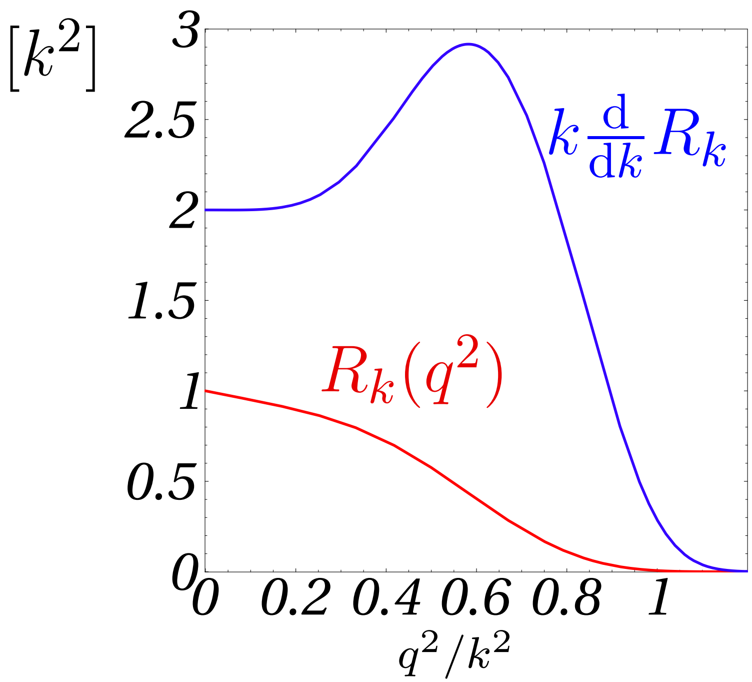


- **IR momentum cutoff**



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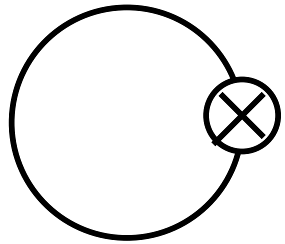

- **definition of the theory**

finite initial (boundary) condition at $k = \Lambda$: Γ_Λ ,

and finite flow equation $k\partial_k\Gamma_k$, regulator function R_k ,

altogether:

$$\Gamma = \Gamma_\Lambda + \frac{1}{2}\int_\Lambda^0 \mathrm{d}k\,\partial_k\Gamma_k[\Gamma_k^{(2)}; R_k]$$

renormalisation group

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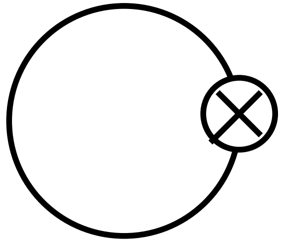
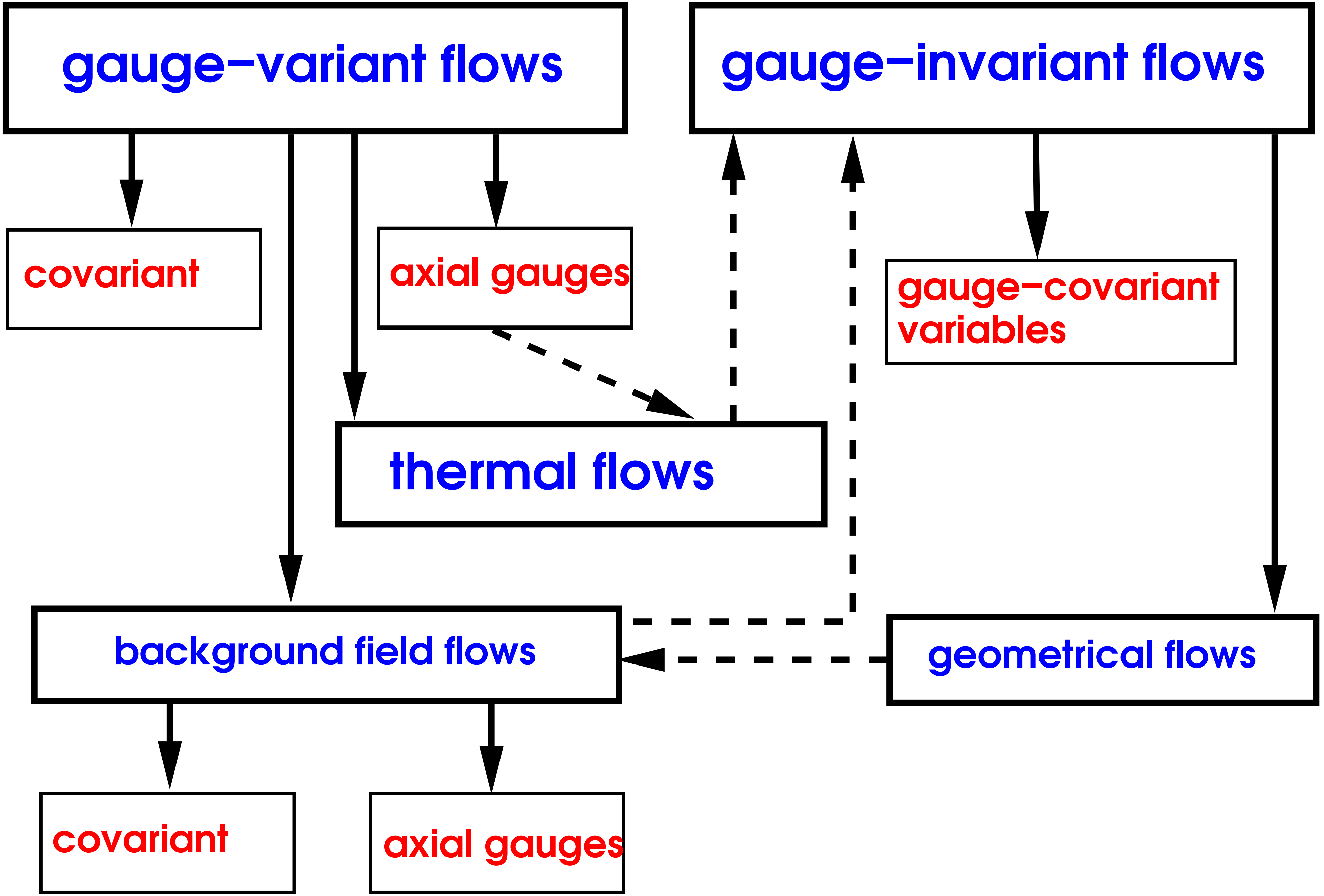


- **symmetries**

  global vs local

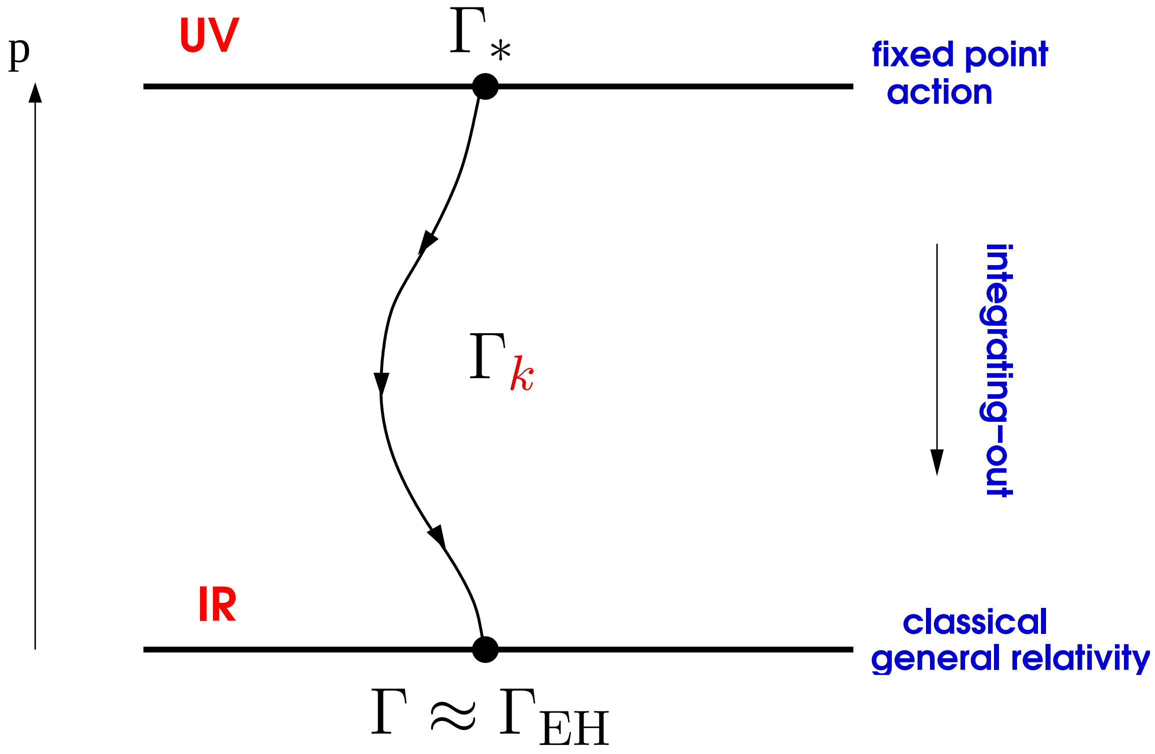
  if regulator respects symmetry: ok

  if not: **(modified) Ward identities** ensure that the physical theory $\Gamma_{k=0}$ respects the symmetry


gauge–variant flows
gauge–invariant flows
covariant
axial gauges
gauge–covariant variables
thermal flows
background field flows
geometrical flows
covariant
axial gauges

renormalisation group

- **for quantum gravity:** **“bottom-up”**



renormalisation group

- **for quantum gravity** (Reuter '96)

$$k\frac{\mathrm{d}}{\mathrm{d}k}\Gamma_k[g_{\mu\nu};\bar{g}_{\mu\nu}] = \frac{1}{2}\,\mathrm{Tr}\,\left[\left(\Gamma_k^{(2)}[g_{\mu\nu};\bar{g}_{\mu\nu}] + R_k\right)^{-1} k\frac{\mathrm{d}R_k}{\mathrm{d}k}\right]$$

renormalisation group

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- **effective action**

$$\Gamma_k = \frac{1}{16\pi G_k}\int\sqrt{g}\,(-R + 2\Lambda_k + \cdots) + S_{\mathrm{matter},k} + S_{\mathrm{gf},k} + S_{\mathrm{ghosts},k}$$

renormalisation group

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- **running couplings**

  projection of $k\partial_k\Gamma_k$ onto $\sqrt{g},\ \sqrt{g}R,\ \sqrt{g}R^2, \cdots$

  heat kernel techniques, background field method
  choice of R_k, stability (DL '01,'02)

Einstein-Hilbert theory

$$\beta_g = (D-2+\eta)\, g \qquad g_k = G_k\, k^{D-2} \qquad \eta = \frac{g\, b_1(\lambda)}{1+g\, b_2(\lambda)}$$

$$\beta_\lambda = (-2+\eta)\lambda + g(a_1 - \eta\, a_2) \qquad \lambda_k = \Lambda_k/k^2$$

$$a_1 = \frac{D(D-1)(D+2)}{2(1-2\lambda)} + \frac{D(D+2)}{1-2\alpha\lambda} - 2D(D+2)$$

$$a_2 = \frac{D(D-1)}{2(1-2\lambda)} + \frac{D}{1-2\alpha\lambda}$$

$$\begin{aligned} b_1 = & -\frac{1}{3}(1+\frac{2}{D})(D^3+6D+12) \\ & -\frac{(D+2)(D^3-4D^2+7D-8)}{(D-1)(1-2\lambda)^2} + \frac{D(D+2)(D^3-2D^2-11D-12)}{12(D-1)(1-2\lambda)} \\ & -\frac{2(D+2)(\alpha D^2-2\alpha D-D-1)}{D(1-2\alpha\lambda)^2} + \frac{(D+2)(D^2-6)}{6(1-2\alpha\lambda)} \end{aligned}$$

$$\begin{aligned} b_2 = & -\frac{D^3-4D^2+7D-8}{(D-1)(1-2\lambda)^2} + \frac{(D+2)(D^3-2D^2-11D-12)}{12(D-1)(1-2\lambda)} \\ & -\frac{2(\alpha D^2-2\alpha D-D-1)}{D(1-2\alpha\lambda)^2} + \frac{(D+2)(D^2-6)}{6D(1-2\alpha\lambda)} \end{aligned}$$

(DL′03)

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$$a_2(\lambda) = \frac{D(D+1)}{2(1-2\lambda)}$$

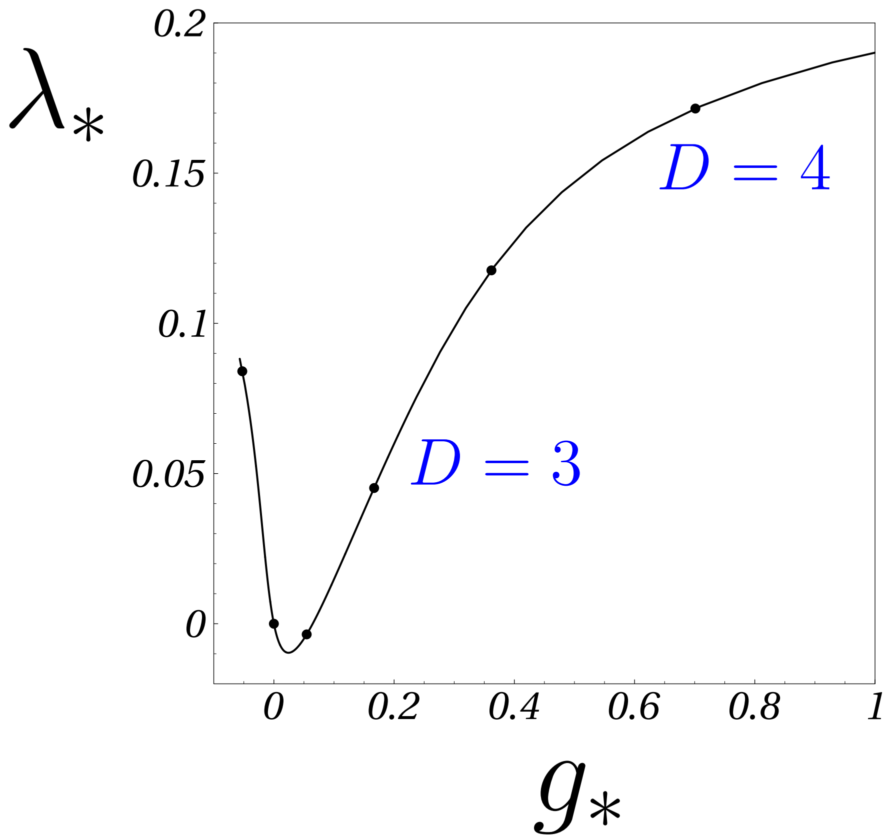
$$b_1(\lambda) = -(2+D)(4+\frac{1}{3}D^2) + \frac{D^2(D+1)(D+2)}{12(1-2\lambda)} - \frac{D(D-1)(D+2)}{(1-2\lambda)^2}$$

$$b_2(\lambda) = \frac{D(D+1)(D+2)}{12(1-2\lambda)} - \frac{D(D-1)}{(1-2\lambda)^2}$$

(DL′03)

UV fixed point

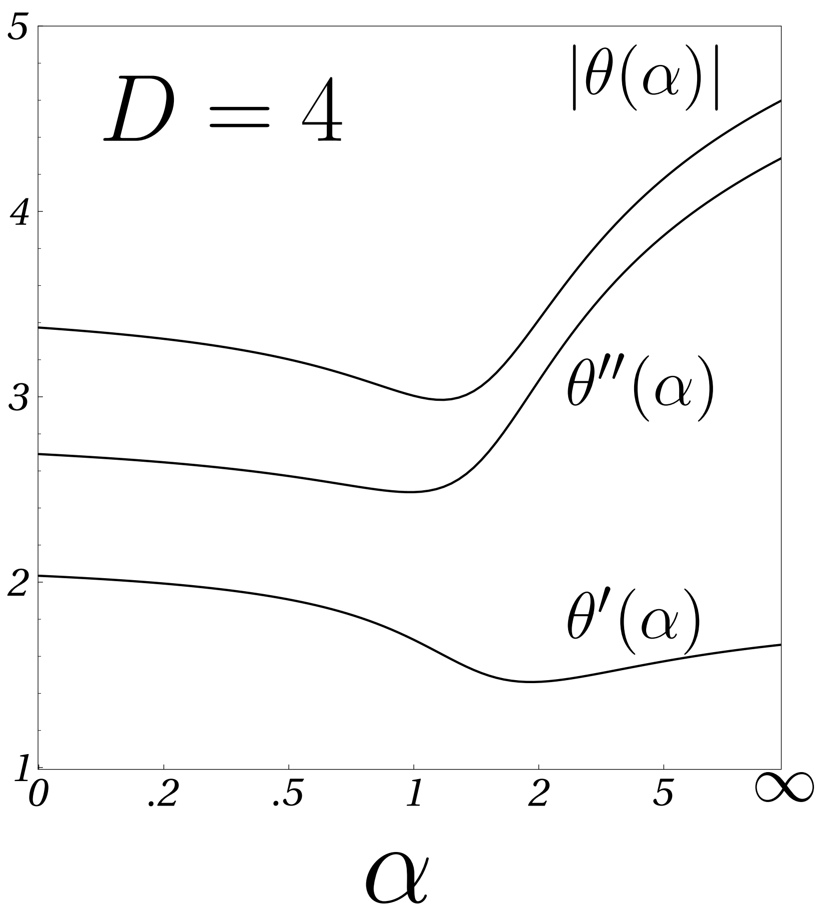
- **continuity**



- **continuous link with perturbative fixed point in** $D = 2 + \epsilon$ **dimensions**
- **real fixed point unique for any dimension**

universality

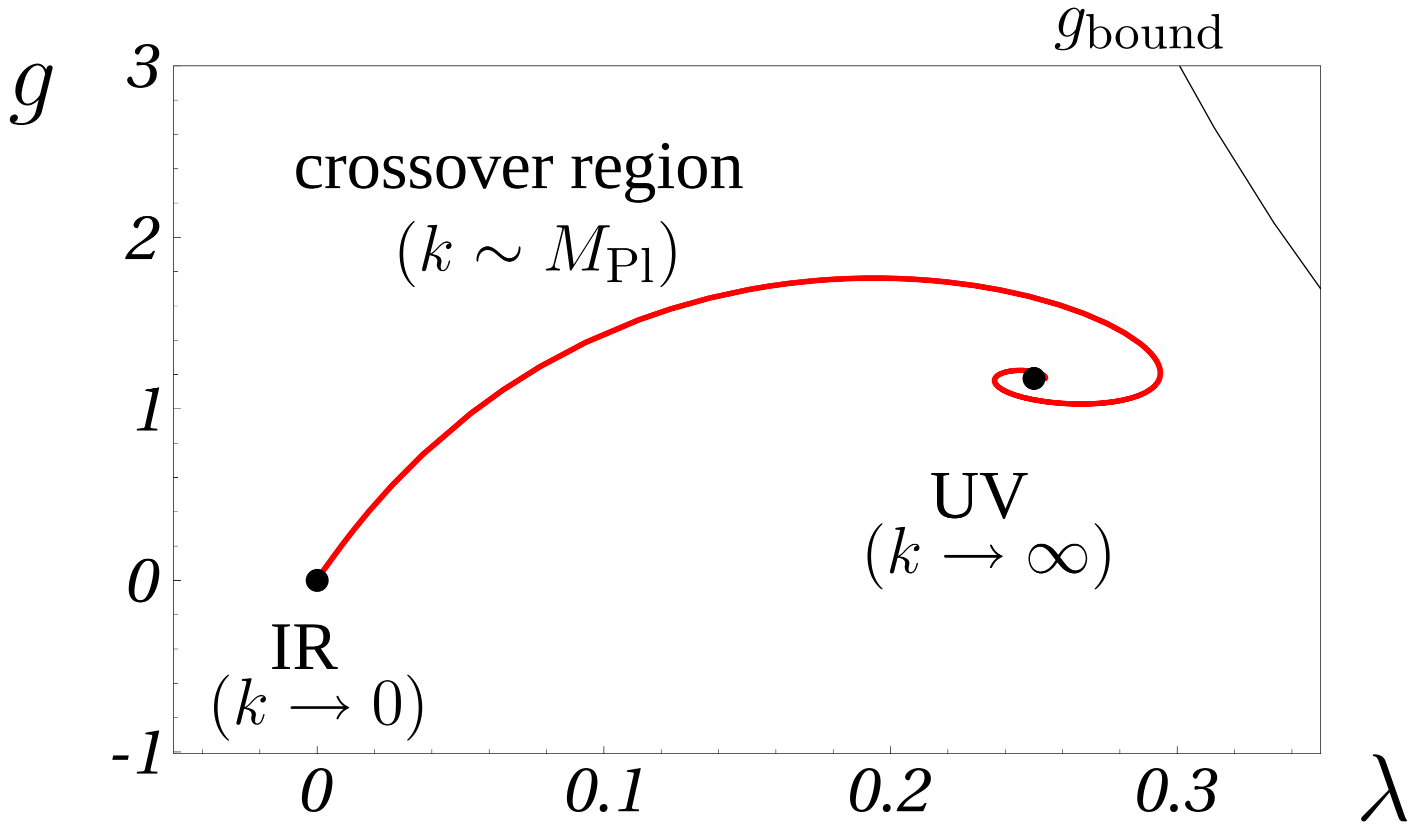
- **scaling exponents** (Lauscher, Reuter '01, DL '03, Fischer, DL '06)



- **universal eigenvalues at criticality** $\theta = \theta' + i\theta''$
- **Landau-de Witt gauge is RG fixed point** (DL, Pawlowski '98)
- **consistent for all** $\alpha \in [0, \infty]$
- **large-**α **behaviour correct**
- θ **consistent with Regge lattice simulations** (Hamber '00)

flow from UV to IR

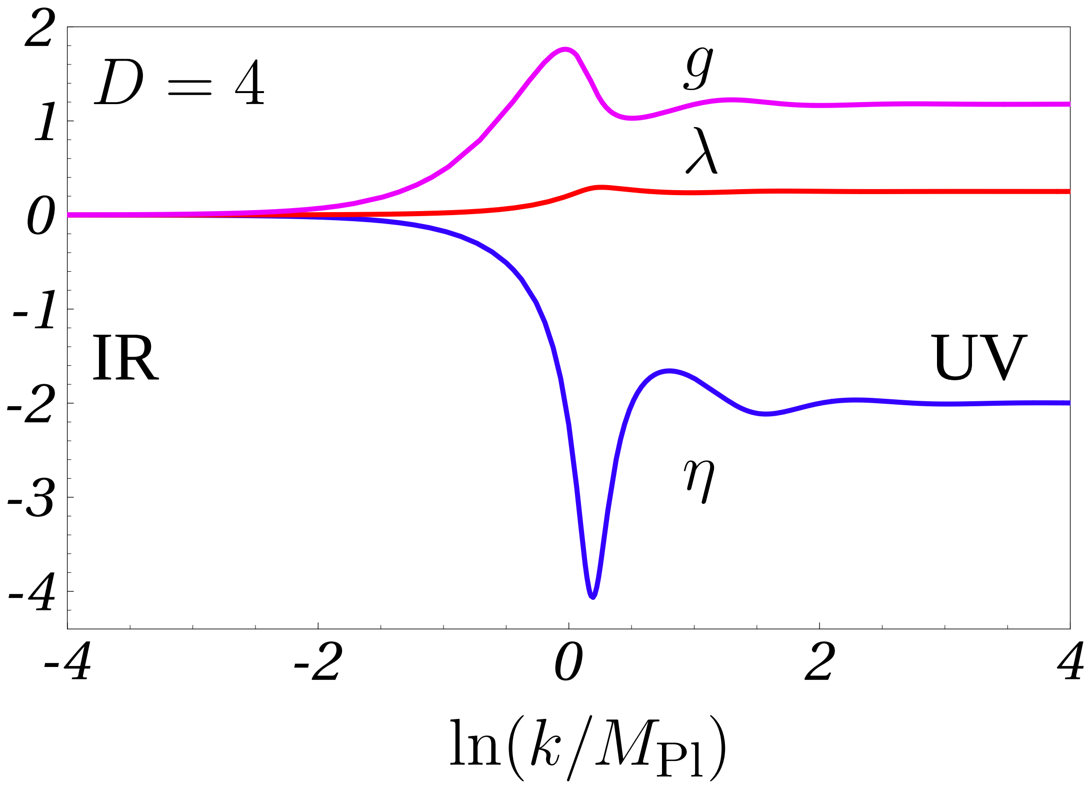
- **separatrix in four dimensions**



flow trajectories

- **cross-over behaviour**

  integrated flow with $\sqrt{g}R$, $\sqrt{g}$

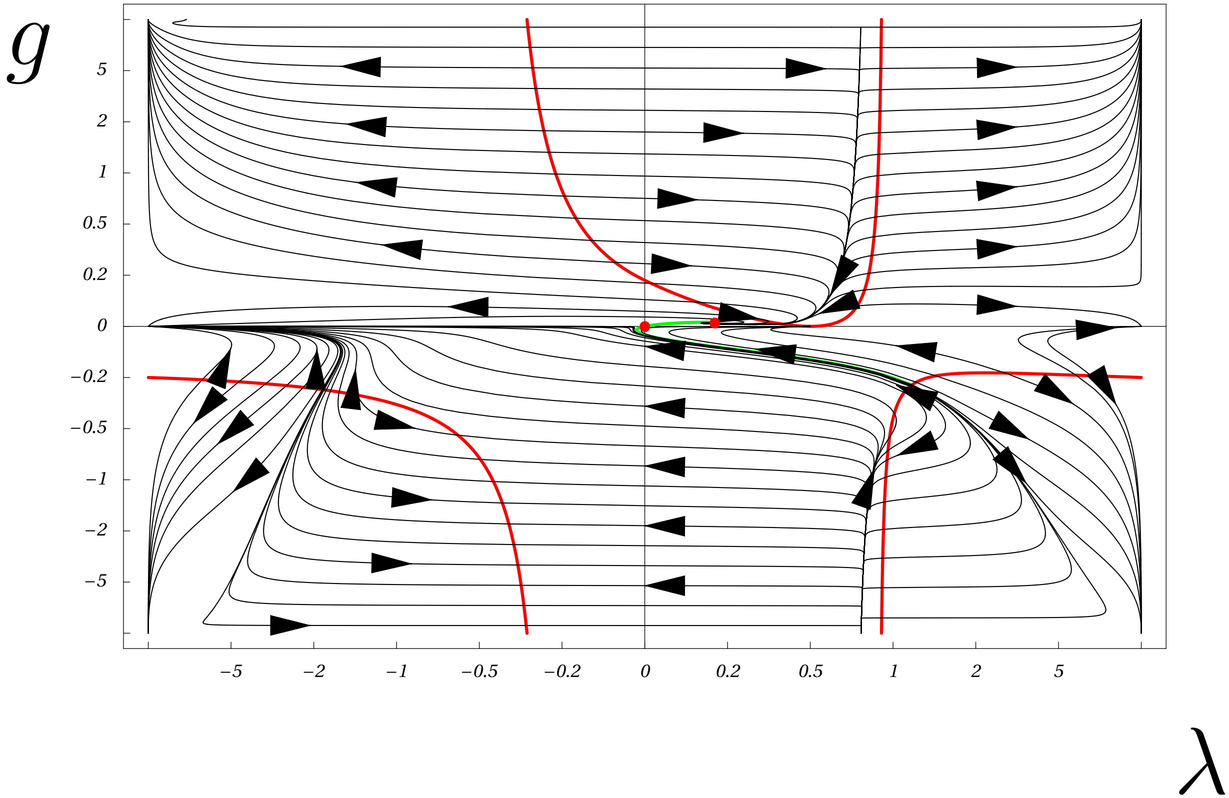


phase diagram

- **full flow**

  4D integrated flow with $\sqrt{g}R, \sqrt{g}$

Reuter, Saueressig (2001), Fischer, DL (2005)

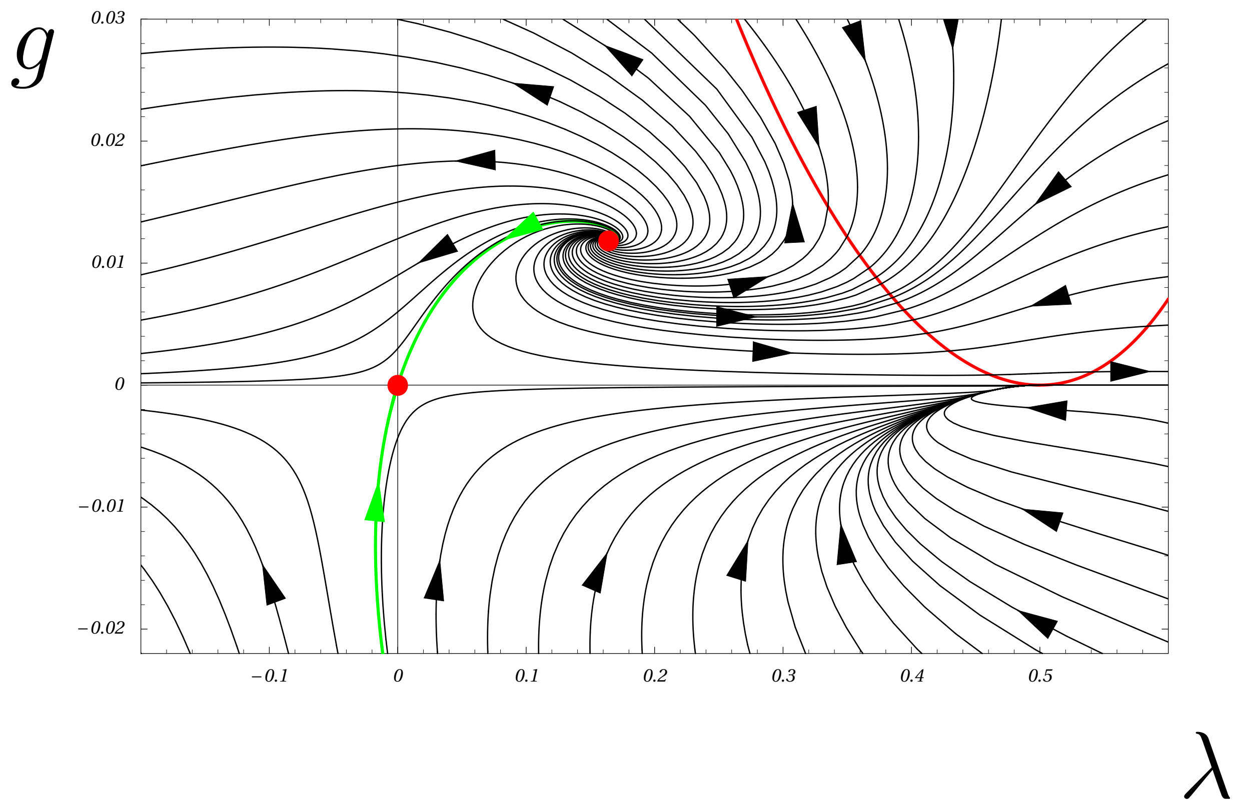


phase diagram

- **full flow – vicinity of fixed points**

  4D integrated flow with $\sqrt{g}R$, $\sqrt{g}$

Fischer, DL (2005)



more curvature invariants

- **extensions including** $\sqrt{g}$ **and** $\sqrt{g}(R)^i$, $\mathrm{i} = 1, \cdots, n$.

n	θ'	θ''	θ_2	
1	$1.1 - 2.3$	$2.5 - 7.0$	$-$	(Lauscher, Reuter '01)
1	$1.4 - 2.0$	$2.4 - 4.3$	$-$	(DL '03)
1	$1.5 - 1.7$	$3.0 - 3.2$	$-$	(Fischer, DL '06)
1	2.4	2.2	$-$	(Codello, Percacci, Rahmede '07)
2	$2.1 - 3.4$	$3.1 - 4.3$	$8.4 - 28.8$	(Lauscher, Reuter '02)
2	1.4	2.8	25.6	(DL '07)
2	1.7	3.1	3.5	(DL '07)
2	1.4	2.3	26.9	(Codello, Percacci, Rahmede '07)

more curvature invariants

- **extensions including** $\sqrt{g}$ **and** $\sqrt{g}(R)^i,\ \mathrm{i}=1,\cdots,n$.

n	θ'	θ''	θ_2	θ_3	θ_4'	θ_4''	θ_6	θ_7	θ
1	2.38	2.17							
2	1.38	2.32	26.9						
3	2.71	2.27	2.07	-4.23					
4	2.86	2.45	1.55	-3.91	-5.22				
5	2.53	2.69	1.78	-4.36	-3.76	-4.88			
6	2.41	2.42	1.50	-4.11	-4.42	-5.98	-8.58		
7	2.51	2.44	1.24	-3.97	-4.57	-4.93	-7.57	-11.1	
8	2.41	2.54	1.40	-4.17	-3.52	-5.15	-7.46	-10.2	-12.

(Codello, Percacci, Rahmede ’07, Machado, Saueressig ’07)

extensions

- **higher derivative gravity**

  1-loop (Codello, Percacci '05, Niedermaier '09)

  beyond 1-loop, matter (Benedetti, Machado, Saueressig '09, DL, Rahmede '10)

extensions

- **higher derivative gravity**
- **higher dimensions**

Einstein-Hilbert, extensions (Fischer, DL ’05)

extensions

- **higher derivative gravity**
- **higher dimensions**
- **matter fields**

large N expansion (Percacci '05)

minimally coupled (Percacci, Perini '05, Narain, Percacci '09, Narain, Rahmede '09)

extensions

- **higher derivative gravity**
- **higher dimensions**
- **matter fields**
- **Yang-Mills gravity**

  1-loop (Robinson, Wilczek ‘05, Pietrykowski ‘06, Toms ‘07, Ebert, Plefka, Rodigast ‘08)

  beyond 1-loop (Manrique, Reuter, Saueressig ’09)

  beyond 1-loop, and fully coupled system (Folkerts, DL, Pawlowski ’09)

extensions

- **higher derivative gravity**
- **higher dimensions**
- **matter fields**
- **Yang-Mills gravity**
- **dynamical ghosts**

  de Donder gauge (Groh, Saueressig '10)

  Landau-deWitt gauge (Eichhorn, Gies, '10)

extensions

- **higher derivative gravity**
- **higher dimensions**
- **matter fields**
- **Yang-Mills gravity**
- **dynamical ghosts**
- **conformally reduced gravity**

  leading order (Reuter, Weyer '08)

  next-to-leading order (Machado, Percacci '09)

extensions

- **higher derivative gravity**
- **higher dimensions**
- **matter fields**
- **Yang-Mills gravity**
- **dynamical ghosts**
- **conformally reduced gravity**
- **consistency with lattice simulations**

  simplicial gravity / Regge calculus (Hamber '00, Hamber, Williams '04)

  causal dynamical triangulations (Ambjorn, Jurkiewicz, Loll et. al. '04, '05)

extensions

- higher derivative gravity
- higher dimensions
- matter fields
- Yang-Mills gravity
- dynamical ghosts
- conformally reduced gravity
- consistency with lattice simulations
- phenomenology

cosmology, black holes

Bonanno, Reuter ’01
Weinberg ’09, Falls, DL, Raghuraman ’10

LHC phenomenology

Fischer, DL ’06
Hewett, Rizzo ’07, DL, Plehn ’07, Koch ’07, DL ’08
Falls, Hiller, DL ’10

gravitational scattering

Gerwick, DL, Plehn ’10, Brinckmann, Hiller, DL ’10

extensions

- higher derivative gravity
- higher dimensions
- matter fields
- Yang-Mills gravity
- dynamical ghosts
- conformally reduced gravity
- consistency with lattice simulations
- phenomenology

  ⋮

consistency with lattice

- **simplicial gravity formulation**

  ultraviolet fixed point in 3d and 4d (Hamber '00, Hamber, Williams '05)

  4d scaling exponents

  lattice: $\nu \approx 3$ **RG study:** $\nu = 8/3$ (DL '03)

  large-d scaling exponents

  lattice: $1/\nu \approx d - 1$ **RG study:** $1/\nu = 2d$ (DL '03)

consistency with lattice

- **simplicial gravity formulation**

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- **causal dynamical triangulation**

  dimensional crossover from 4d Monte Carlo study (Ambjorn et. al. '05)

  large distances

  lattice: $D_{\rm eff} \approx 4$ **RG studies:** $\eta \approx 0$

  short distances

  lattice: $D_{\rm eff} \approx 2$ **RG studies:** $\eta \approx -2$ (Reuter et. al. '01)

Yang-Mills at the Planck scale

- **does asymptotic freedom persist?**

  1-loop / effective theory

Robinson, WIlczek ('05)

Pietrykowski ('06)

Toms ('07)

Ebert, Plefka, Rodigast ('08)

result: asymptotic freedom persists

$$\beta_{\mathsf{YM}}\Big|_{\rm grav} = -\frac{6\,I}{\pi}\,g_{\mathsf{YM}}\,G_N\,E^2 \leq 0$$

Yang-Mills at the Planck scale

- **background field flow** S. Folkerts, DL, JM. Pawlowski ('10)

ansatz

$$\Gamma_k = \int \sqrt{g}\left[\frac{Z_{N,k}}{16\pi G_N}\left(-R(g_{\mu\nu}) + 2\bar{\Lambda}_k\right) + \frac{Z_{A,k}}{4g^2} F^a_{\mu\nu} F_a^{\mu\nu}\right]$$

$$R_k[\bar{\phi}] = \Gamma_k^{(2)}[\bar{\phi}]\, r[\bar{\phi}]$$

$$r^{gg} = r^{gg}(-\Delta_{\bar{g}}) \quad r^{\bar{\eta}\eta} = -r^{\eta\bar{\eta}} = r^{\bar{\eta}\eta}(-\Delta_{\bar{g}})$$

$$r^{AA} = r^{AA}(-\Delta_{\bar{g}}(A) \quad r^{\bar{C}C} = -r^{C\bar{C}} = r^{\bar{C}C}(-\Delta_{\bar{g}}(A))$$

Yang-Mills at the Planck scale

- **background field flow** S. Folkerts, DL, JM. Pawlowski ('10)

  flow

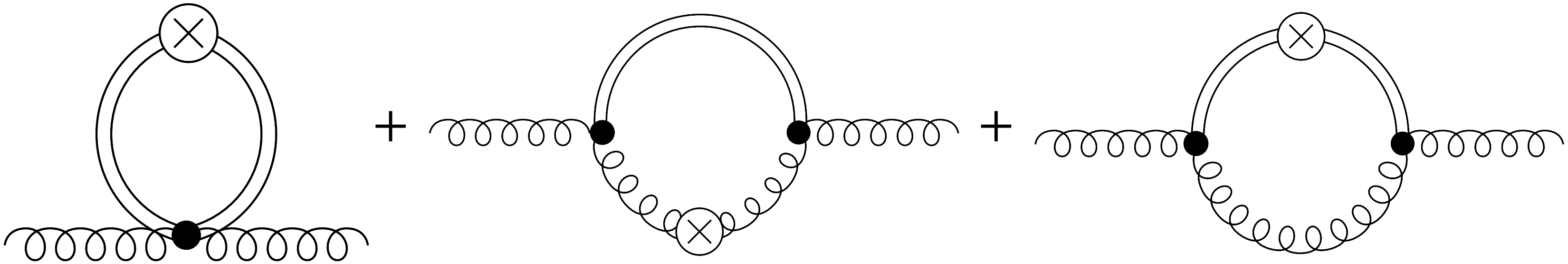
$$\partial_t \Gamma_k[\phi] = \frac{1}{2}\mathrm{Tr}\,\frac{1}{1+r[\phi]}\partial_t r[\phi] + \mathrm{Tr}\,\frac{\partial_t \Gamma_k^{(2)}[\phi,\phi]}{\Gamma_k^{(2)}[\phi,\phi]}\,\frac{r[\phi]}{1+r[\phi]}$$

  result: **no graviton contribution at one-loop**

$$\beta_g\Big|_{1-\mathrm{loop}} = \beta_{g,\mathrm{YM}}\Big|_{1-\mathrm{loop}}$$

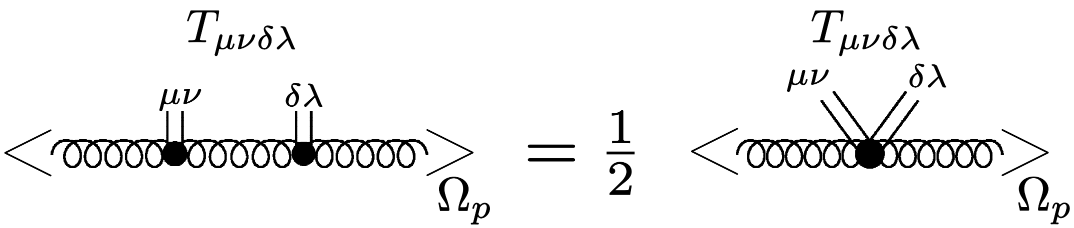
Yang-Mills at the Planck scale

- **flat background**



Yang-Mills at the Planck scale

- **kinematical identity**



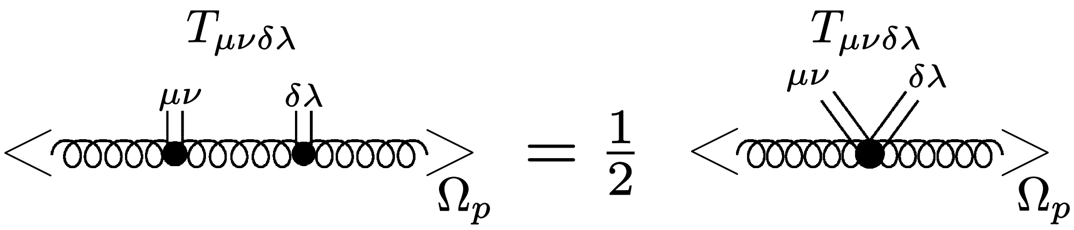
- **1-loop result**

$$\beta_{\mathsf{YM}}\Big|_{\rm grav} = -\frac{6\,I}{\pi} G_N\, g_{\mathsf{YM}}\, E^2$$

$$I = \int_0^\infty dx\, \frac{1+\alpha}{1+r_g(x)} \left(1 - \frac{1}{1+r_A(x)}\right) \geq 0$$

Yang-Mills at the Planck scale

- **kinematical identity**



- **beyond 1-loop**

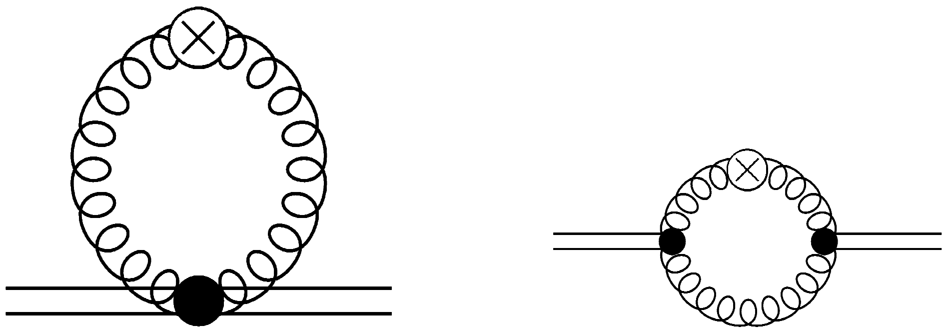
$$\beta_{\text{YM}}\Big|_{\text{grav}} \leq 0$$

asymptotic freedom persists in presence of gravity FP

quantum gravity with Yang-Mills

- **Yang-Mills contribution to gravity** S. Folkerts, DL, JM. Pawlowski ('10)

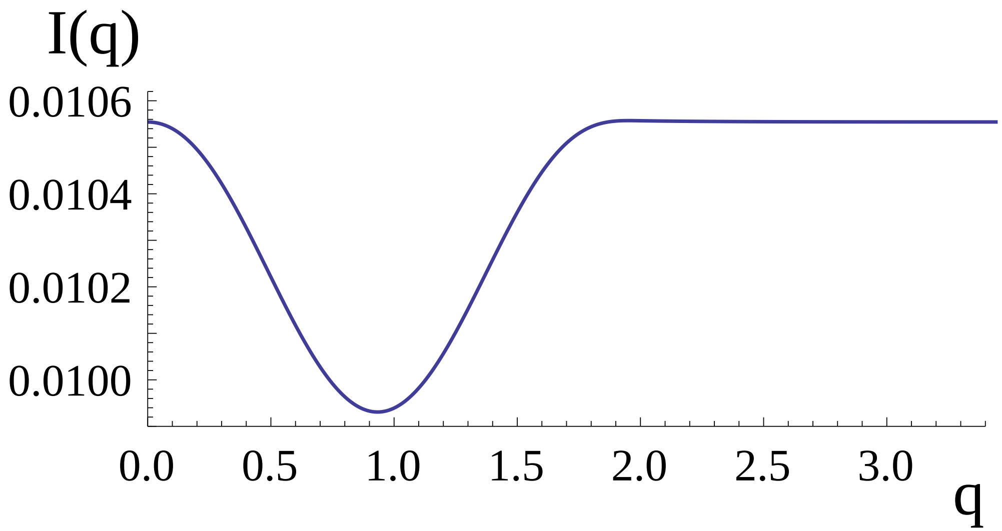
  diagrams



quantum gravity with Yang-Mills

- **Yang-Mills contribution to gravity** S. Folkerts, DL, JM. Pawlowski ('10)

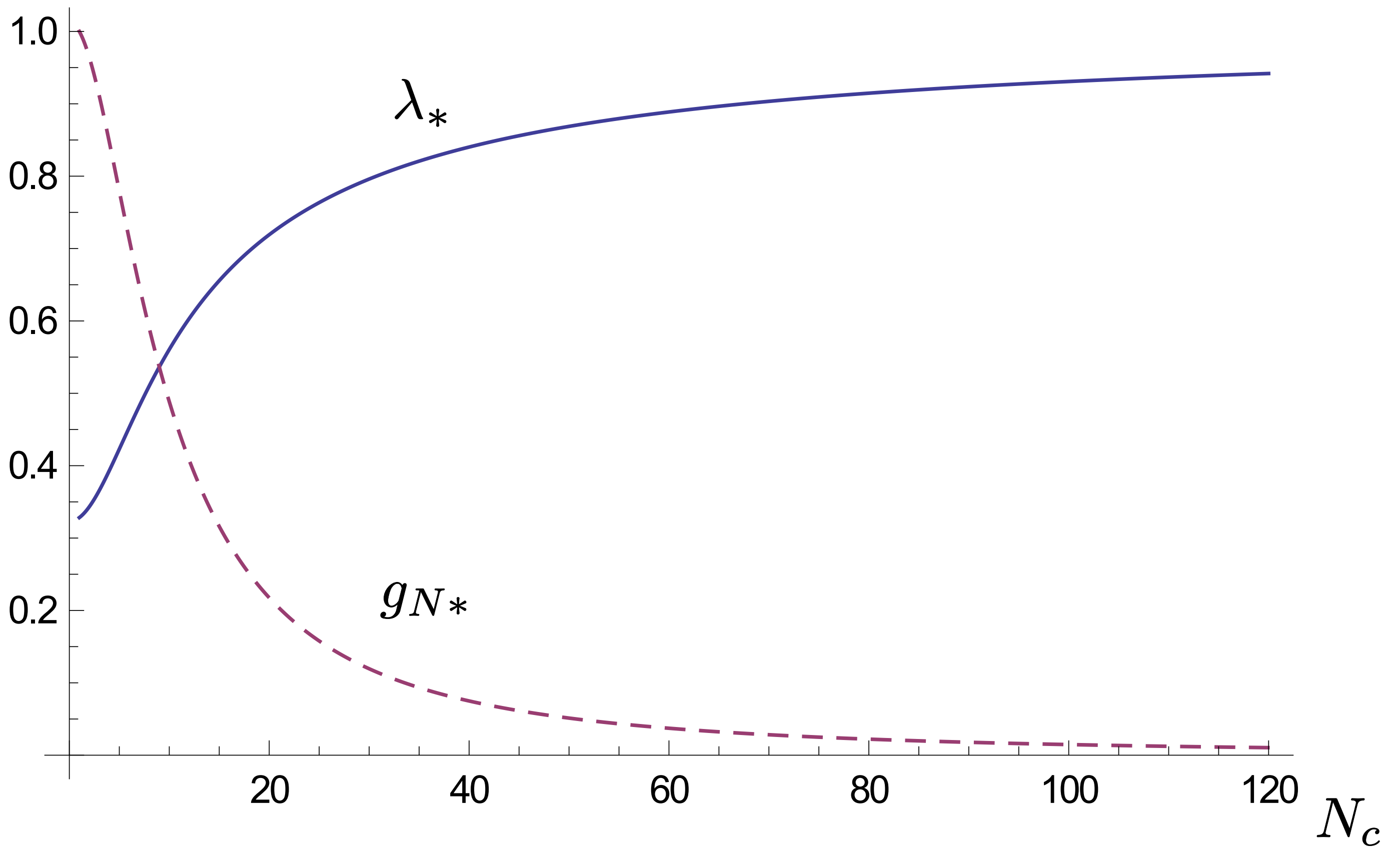
  rhs of flow equation (optimised cutoff)



quantum gravity with Yang-Mills

- **Yang-Mills contribution to gravity** S. Folkerts, DL, JM. Pawlowski ('10)

  UV fixed point of coupled system



quantum gravity and black holes

- **Schwarzschild solution**

  Schwarzschild metric

$$ds^2 = -f(r)\,dt^2 + f^{-1}(r)\,dr^2 + r^2 d\Omega^2_{d-2}$$

  classical lapse function

$$f = 1 - \frac{G_N\,M}{r^{d-3}}$$

  classical Schwarzschild radius

$$r_{\rm cl} = (G_N\,M)^{1/(d-3)}$$

quantum gravity and black holes

- **RG improved black holes** Falls, DL, Raghuraman (ERG '08, 1002.0260 [hep-th])

  running gravitational coupling

$$G_N \to G(r)\,, \quad f_{\rm cl}(r) \to f_{\rm imp}(r) = 1 - \frac{G(r)\,M}{r^{d-3}}$$

  improved Schwarzschild radius r_s from

$$f_{\rm imp}(r_s) = 0$$

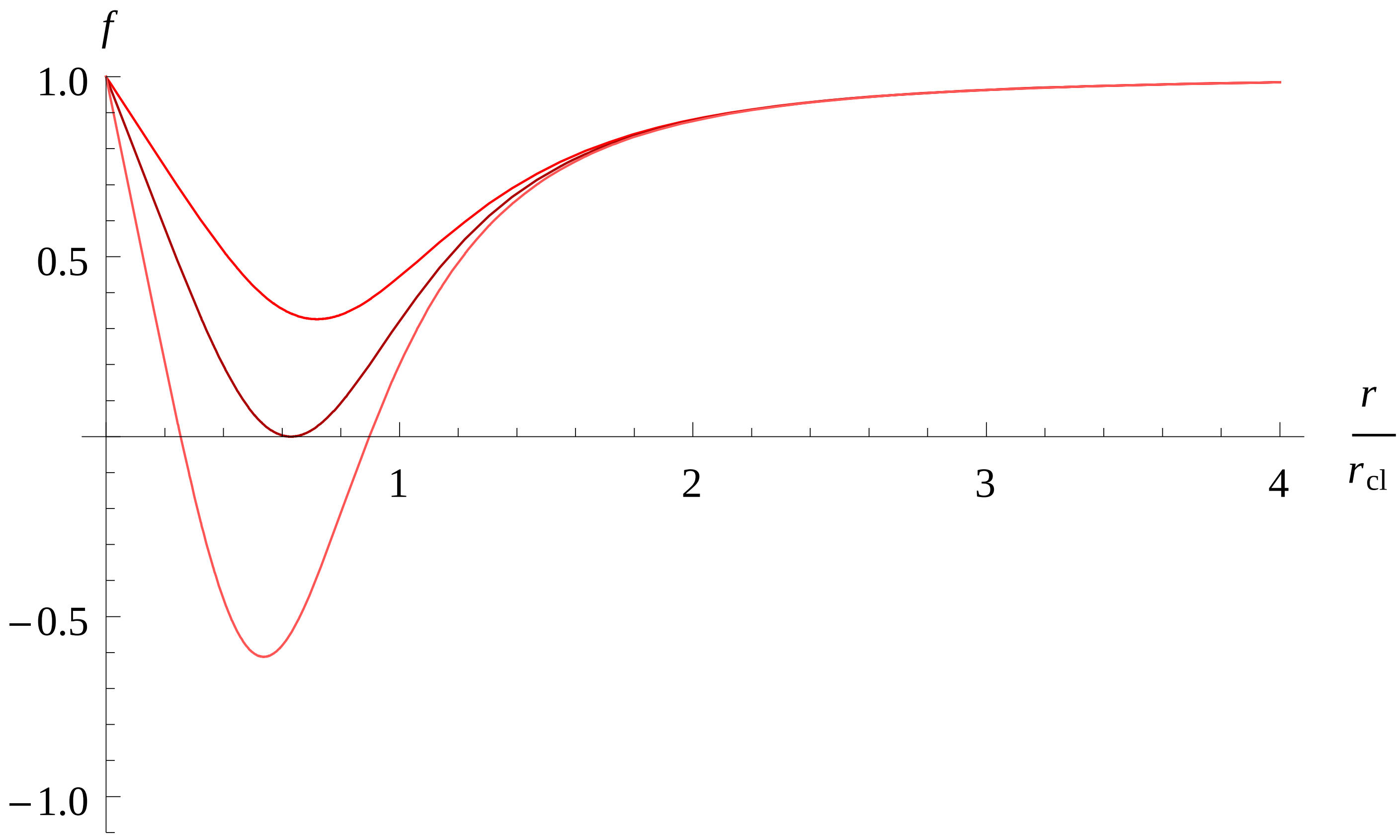
  critical black hole mass M_c from

$$d - 3 = \left.\frac{\partial \ln G(r)}{\partial \ln r}\right|_{r=r_c(M_c)}$$

quantum gravity and black holes

- **RG improved black holes** Falls, DL, Raghuraman (ERG '08, 1002.0260 [hep-th])

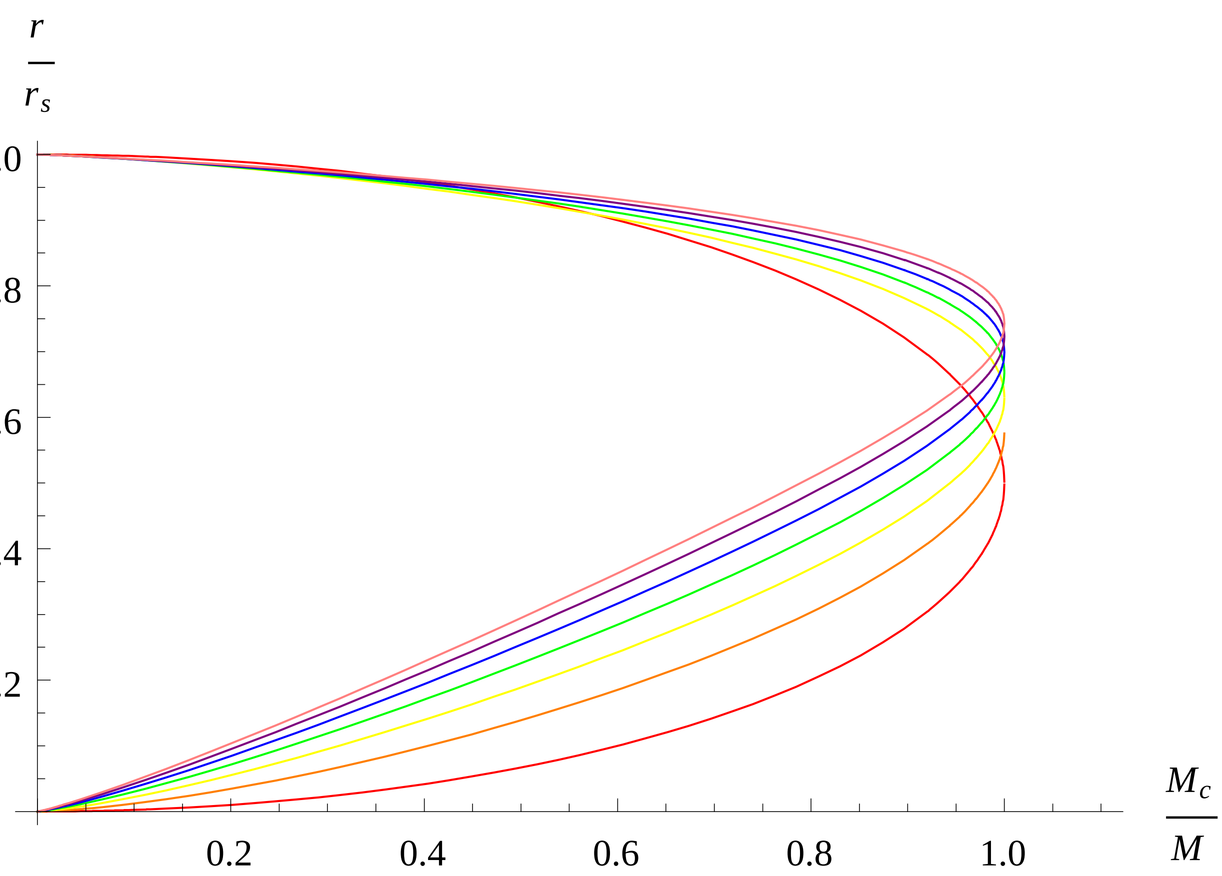
  metric, dependence on M **(D=6)**



quantum gravity and black holes

- **RG improved black holes** Falls, DL, Raghuraman (ERG '08, 1002.0260 [hep-th])

  improved Schwarzschild radii, various dimension



BH production at the LHC

- **semi-classical**

  semi-classical production cross section

$$\hat{\sigma} = \pi r_{\rm cl}^2(M = \sqrt{s}) \times \theta(\sqrt{s} - M_{\rm min})$$

  production cross section at the LHC $pp \longrightarrow$ **final state**

$$\sigma = \sum_{i,j} \int_0^1 dx_1 \int_0^1 dx_2\, f_i(x_1)\, f_j(x_2)\, \hat{\sigma}(q_i q_j \longrightarrow \textbf{final state})$$

  parton distribution functions from **CTEQ61**
  evaluated at $Q^2 = M_{\rm BH}^2$.

BH production at the LHC

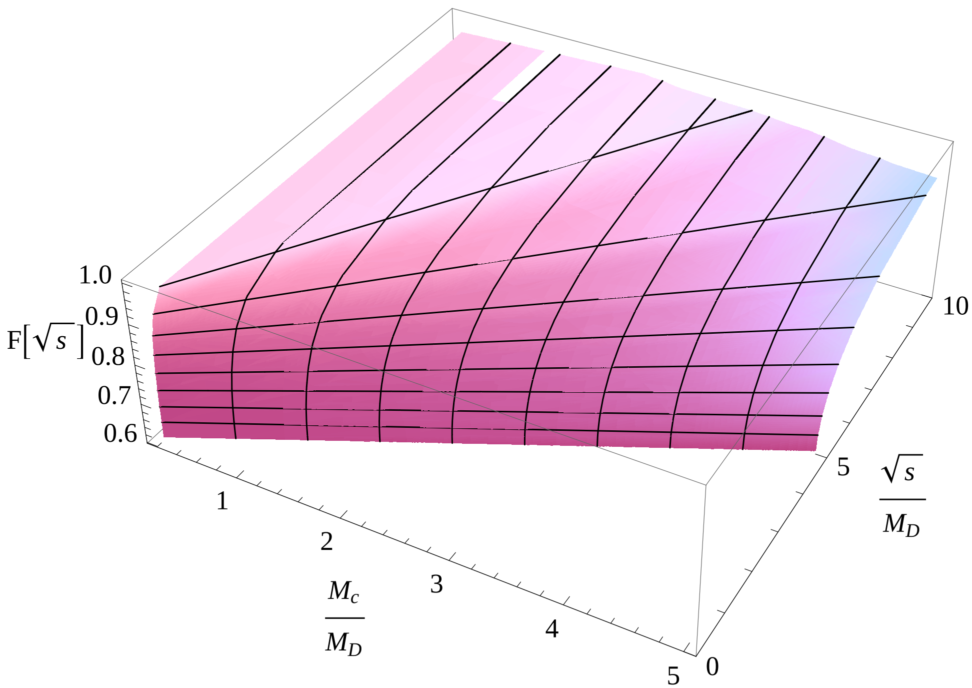
- **renormalisation group**

Falls, DL, Raghuraman 1002.0260 [hep-th]

quantum corrected production cross section

$$\hat{\sigma} \to \hat{\sigma} = F(\sqrt{s}) \times \pi r_{\rm cl}^2(M = \sqrt{s}) \times \theta(\sqrt{s} - M_c)$$

new form factor F

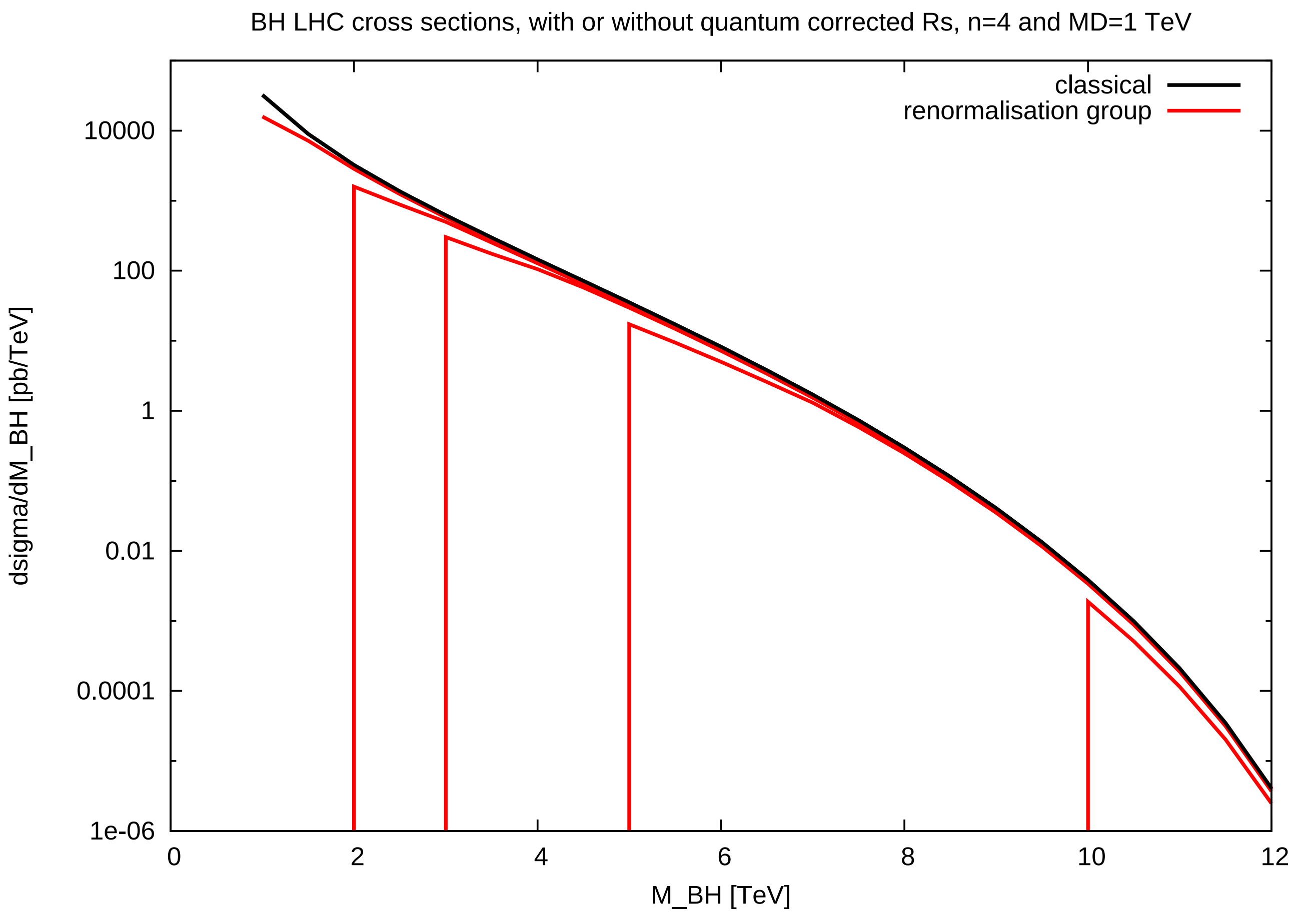


BH production at the LHC

- **semi-classical vs renormalisation group**

Falls, Hiller, DL (Pascos '09)

$n = 4$ **extra dimensions**



conclusions

- **gravity at the Planck scale**

  growing evidence for asymptotic safety

conclusions

- **gravity at the Planck scale**

  growing evidence for asymptotic safety

  various RG studies / approximations / field content

Reuter (1996), Souma (1999)
Lauscher, Reuter (2001), Reuter, Saueressig (2001)
Forgacs, Niedermayer (2002), Niedermayer (2002)
DL (2003), Percacci, Perini (2003)
Bonnano, Reuter (2004), Percacci (2004)
Bonanno (2005), Lauscher, Reuter (2005)
Percacci (2005), Fischer, DL (2006)
Codello, Percacci (2006)
Codello, Percacci, Rahmede (2007)
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conclusions

- **gravity at the Planck scale**

  growing evidence for asymptotic safety

  various RG studies / approximations / field content

  consistent with symmetry reductions

Forgacs, Niedermaier (2002)

Niedermaier (2002), (2003), (2006)

conclusions

- **gravity at the Planck scale**

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  various RG studies / approximations / field content

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  consistent with lattice studies

Hamber (2000)

Ambjorn, Jurkiewicz, Loll (2002), (2003)

conclusions

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conclusions

- **gravity at the Planck scale**

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  various RG studies / approximations / field content

  consistent with symmetry reductions

  consistent with lattice studies

- **phenomenology at the Planck scale**

  black holes, cosmology Bonanno, Reuter (2001), Falls, DL, Raghuraman)2010)

conclusions

- **gravity at the Planck scale**

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  consistent with lattice studies

- **phenomenology at the Planck scale**

  black holes, cosmology

  LHC phenomenology / low-scale QG models

DL (2003), Fischer, DL (2006)
Hewett, Rizzo (2007)
DL, Pehn (2007)
Koch (2007)
Falls, Hiller, DL (2010)

conclusions

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  consistent with lattice studies

- **phenomenology at the Planck scale**

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- **challenges**

  include more invariants, interactions, matter

  lattice $\leftrightarrow$ RG $\leftrightarrow$ loops $\leftrightarrow$ strings $\leftrightarrow$ other