Stueckelberg formalism and phenomenology of non-relativistic gravity

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+ work in progress

Plan

- Covariant form of non-relativistic gravity. Relation with Einstein-aether model
- Decoupling limit and self-interaction
- Coupling to matter. Phenomenological constraints
- Outlook

Input from Diego's talk

- In non-relativistic QG 4d Diffs are broken down to foliation preserving subgroup (FDiffs)

$$\mathbf{x} \mapsto \tilde{\mathbf{x}}(\mathbf{x}, t) \ , \qquad t \mapsto \tilde{t}(t)$$

required for renormalizability *Horava (2009)*

- This leads to a new degree of freedom -- "scalar graviton"

- A pathology-free model is given by the action

$$S = \frac{M_P^2}{2}\int d^3x dt\,\sqrt{\gamma}N\Big(K_{ij}K^{ij} - \lambda K^2 - \mathcal{V}\Big)$$

$$ds^2 = (N^2 - N_i N^i)dt^2 - 2N_i dx^i dt - \gamma_{ij}dx^i dx^j$$

$$K_{ij} = \frac{\dot{\gamma}_{ij} - \nabla_i N_j - \nabla_j N_i}{2N}$$

$$\mathcal{V} = -R - \alpha a_i a^i + \underbrace{(\text{higher order terms})}_{\text{suppressed by UV scale } M_*}$$

$$a_i = N^{-1}\partial_i N$$

Stueckelberg formalism I

To identify the effect of the new d.o.f.: restore gauge invariance by introducing Stueckelberg field

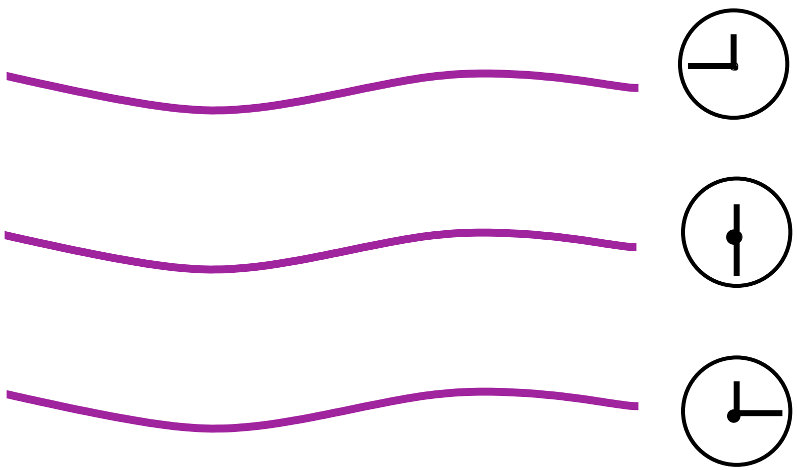
In case of gravity equivalent to covariantization

- parametrize foliation surfaces with scalar field:

$$\sigma(x) = const$$

ADM frame = gauge fixing $t = \sigma$

σ sets global time



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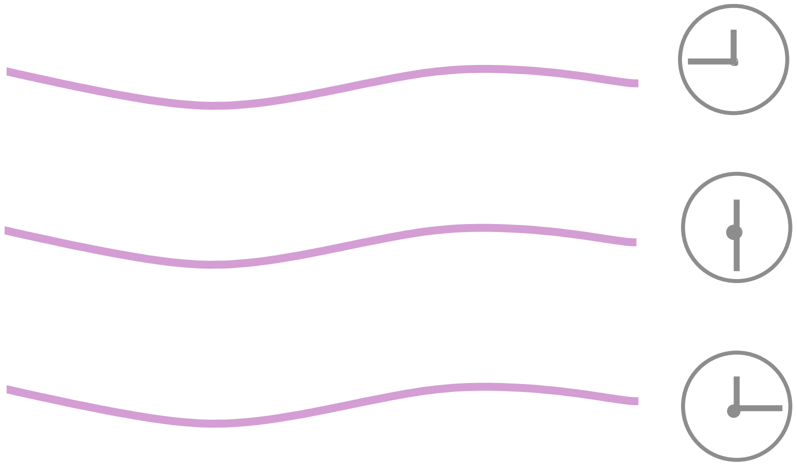

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CHRONON

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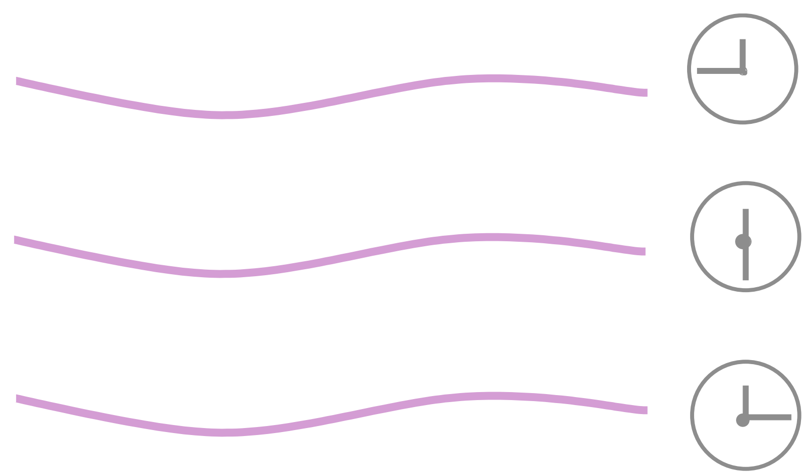

Stueckelberg formalism II

- Time reparameterizations in ADM frame

  ➡ symmetry $\sigma \mapsto \tilde{\sigma} = f(\sigma)$

  Invariant object -- unit normal to the foliation surfaces:

$$u_\mu = \frac{\partial_\mu \sigma}{\sqrt{(\partial\sigma)^2}}$$

- identify

$$\gamma_{ij} \mapsto P_{\mu\nu} = g_{\mu\nu} - u_\mu u_\nu, \quad K_{ij} \mapsto K_{\mu\nu} = P_\mu^\lambda \nabla_\lambda u_\nu$$

$a_i \mapsto a_\mu = u^\nu \nabla_\nu u_\mu$, **etc.**

Stueckelberg formalism III

- obtain the covariant action:

$$S = -\frac{M_P^2}{2}\int d^4x\sqrt{-g}\left\{{}^{(4)}R + (\lambda - 1)(\nabla_\mu u^\mu)^2 + \alpha u^\mu u^\nu \nabla_\mu u^\rho \nabla_\nu u_\rho\right\}$$

compare with Einstein-aether model

Jacobson, Mattingly (2001)

$$S_{EA} = -\frac{M_P^2}{2}\int d^4x\sqrt{-g}\left\{{}^{(4)}R + c_1 \nabla_\mu u_\nu \nabla^\mu u^\nu + c_2 \nabla_\mu u_\nu \nabla^\nu u^\mu \right.$$
$$\left. + c_3 (\nabla_\mu u^\mu)^2 + c_4 u^\mu u^\nu \nabla_\mu u^\rho \nabla_\nu u_\rho + \mu(u_\nu u^\nu - 1)\right\}$$

N.B. In our case there are no transverse vector modes

No-ghost theorem

Action contains higher derivatives

$$(\nabla_\mu u^\mu)^2 = \frac{1}{(\partial\sigma)^2}\left[\Box\sigma - \frac{\nabla^\mu\sigma\nabla^\nu\sigma}{(\partial\sigma)^2}\nabla_\mu\nabla_\nu\sigma\right]^2$$

Theorem Consider linear perturbations

$$\sigma = \bar{\sigma} + \chi$$

In the frame where background is in ADM gauge,

$$\bar{\sigma} = t$$

e.o.m. for χ is second order in time

Decoupling limit

$M_P \to \infty$

$$\left.\begin{array}{l} M_\alpha \equiv \sqrt{\alpha} M_P \\ M_\lambda \equiv \sqrt{\lambda - 1}\, M_P \end{array}\right\} \text{fixed}$$

➡ chronon perturbations decouple from the metric

$$S_\chi = \int d^4x \left[\frac{M_\alpha^2}{2} (\partial_i \dot{\chi})^2 - \frac{M_\lambda^2}{2} (\Delta \chi)^2 \right] \quad \Rightarrow \quad \Delta(\ddot{\chi} - \Delta \chi) = 0$$

single propagating mode with linear dispersion relation

$$\omega^2 = \frac{M_\lambda^2}{M_\alpha^2} p^2$$

N.B. No decoupling in UV

Chronon self-interaction

$$S_\chi = \int d^4x \Big[\frac{M_\alpha^2}{2} (\partial_i \dot{\chi})^2 - \frac{M_\lambda^2}{2} (\Delta\chi)^2 - M_\lambda^2 \, \dot{\chi} (\Delta\chi)^2$$

$$+ M_\alpha^2 \left(\dot{\chi} \partial_i \ddot{\chi} \partial_i \chi - \partial_i \dot{\chi} \partial_j \chi \partial_i \partial_j \chi \right) + \ldots \Big]$$

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- change variables: $\chi = \tilde\chi + \tilde\chi\dot{\tilde\chi}$

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$$+ M_\alpha^2 \left(\frac{1}{2}\, \dot{\tilde{\chi}}(\partial_i \dot{\tilde{\chi}})^2 - \partial_i \dot{\tilde{\chi}} \partial_j \tilde{\chi} \partial_i \partial_j \tilde{\chi} \right) + \dots \Big]$$

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- normalize canonically: $\tilde\chi = M_\alpha^{-1/2} M_\lambda^{-1/2} \hat\chi\ , \quad t = M_\alpha M_\lambda^{-1} \hat t$

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- read out would-be strong coupling scale

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$$\boxed{M_* \lesssim M_\alpha, M_\lambda}$$

Coupling to matter I

SM fields couple to u_μ

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- without derivatives

$$u_\mu \bar{\psi}\gamma^\mu \psi \qquad u^\mu u^\nu \bar{\psi}\gamma_\mu \partial_\nu \psi \qquad u^\mu u^\nu \bar{\psi}\partial_\mu \partial_\nu \psi$$

  lead to violation of Lorentz symmetry within the SM

Coupling to matter II

operators of dim > 4 $(u^\mu u^\nu \bar{\psi} \partial_\mu \partial_\nu \psi)$

UV modification of dispersion relations

$$E^2 = m^2 + p^2 + \frac{p^4}{\left(M_*^{(mat)}\right)^2} + \dots$$

timing of AGN's and GRB's

MAGIC (2008)
Fermi GMB/LAT (2009)

$$M_*^{(mat)} \gtrsim 10^{10} \div 10^{11} \mathrm{GeV}$$

N.B. $M_*^{(mat)}$ may be different from M_*

Coupling to matter III

operators of dim ≤ 4 $\quad (u_\mu \bar{\psi}\gamma^\mu\psi, \;\; u^\mu u^\nu \bar{\psi}\gamma_\mu\partial_\nu\psi)$

tightly constrained

e.g. dim 4 correct "speed of light" for different species

$$E^2 = m^2 + c^2 p^2$$

experimental bound:

$$|c_\gamma - c_{p,e}| \leq 6\times 10^{-22} \quad !$$

➡ A mechanism for suppression of Lorentz breaking at dim up to 4 is required

Universal coupling

Minimal coupling to effective metric

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} + \beta u_\mu u_\nu$$

- trade $g_{\mu\nu}$ for $\tilde{g}_{\mu\nu}$

$$S = -\frac{M_P^2}{2} \int d^4x \sqrt{-g} \Big\{ {}^{(4)}R - \beta \nabla_\mu u_\nu \nabla^\nu u^\mu$$
$$+ \lambda' (\nabla_\mu u^\mu)^2 + \alpha u^\mu u^\nu \nabla_\mu u^\rho \nabla_\nu u_\rho \Big\}$$

$$\lambda - 1 + \beta$$

- exploit connection to Einstein-aether

- Absence of gravitational Cherenkov losses by UHECR

$\Rightarrow$ $c_g \, , \; c_\chi \geq 1$ $\Rightarrow$ $\boxed{\beta \leq 1 \, , \quad \frac{\lambda' - \beta}{\alpha} \geq 1}$

- Newton law vs Friedman equation

$$G_N = \frac{1}{8\pi M_P^2 (1 - \alpha/2)} \neq G_{cosm} = \frac{1}{8\pi M_P^2 (1 + 3\lambda'/2 - \beta)}$$

$$H^2 = \frac{8\pi}{3} G_{cosm} \, \rho$$

BBN bound: $|G_{cosm}/G_N - 1| \leq 0.13$

$\Rightarrow$ $\boxed{\alpha \, , \; \beta \, , \; \lambda' \lesssim 0.1}$

PPN parameters I

Spherically symmetric solutions the same as in Einstein-aether

➡ all PPN parameters the same as in GR except $\underbrace{\alpha_1^{PPN}\ ,\ \alpha_2^{PPN}}$

measure preferred frame effects

PPN parameters II

Gravitational field of a compact object in its rest frame

$$h_{00} = -2G_N \frac{m}{r}\left(1 - \frac{\alpha_2^{PPN}}{2}\frac{(x^i v^i)^2}{r^2}\right)$$

$$h_{0i} = \frac{\alpha_1^{PPN}}{2} G_N \frac{m}{r} v^i$$

velocity with respect to preferred frame

$$h_{ij} = -2G_N \frac{m}{r}\delta_{ij}$$

Solar system bounds:

$$|\alpha_1^{PPN}| \lesssim 10^{-4}\,, \quad |\alpha_2^{PPN}| \lesssim 10^{-7}$$

PPN parameters III

$$\alpha_1^{PPN} = -4(\alpha + 2\beta)$$

$$\alpha_2^{PPN} = \frac{(\alpha + 2\beta)(\alpha - \lambda' + 3\beta)}{2(\lambda' - \beta)}$$

- vanish if $\alpha + 2\beta = 0$
- α_2^{PPN} vanishes when $\beta = 0$, $\lambda' = \alpha$ ($c_\chi = 1$)
- barring cancellations

$$\alpha\ ,\ \beta\ ,\ \lambda' \lesssim 10^{-7} \div 10^{-6}$$

+ Absence of strong coupling ➡ upper bound on the scale of quantum gravity

$$M_* \lesssim 10^{15} \div 10^{16} \mathrm{GeV}$$

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$\Rightarrow$ LV starts from dim 6

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SUSY breaking generates dim 4 LV operators suppressed by

$$\left(m_{soft}/M_*\right)^2$$

LI from SUSY: example of SQED

- SUSY algebra without boosts is closed:

$$[Q_\alpha, \bar{Q}_{\dot{\alpha}}]_+ = 2\sigma^\mu_{\alpha\dot{\alpha}} P_\mu$$

  Enough to generate superspace

- field content: $\Phi_+ \; , \; \Phi_- \; , \; V$

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superpotential: $\int d^2\theta \, \mathcal{W}_\alpha \mathcal{W}_\beta$

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$\int d^2\theta \, \Phi_+ \partial_\mu \Phi_-$ - not gauge invariant

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Outlook

- Bounds from binary pulsars
- Implications for cosmology: CMB and LSS
- Inflation
- Phenomenology of instantaneous interaction ⟺ $\Delta(\ddot{\chi} - \Delta\chi) = \partial_i J^i$