Hot topics in Modern Cosmology

10 -15 May, 2010, Cargèse

Massive gravity

Outline

- Introduction
- Generalities, $m \neq 0 \; vs \; m = 0$
- Neutral vector field
- Nonabelian vector field
- Massive gravity
    - Nondecoupling of zero-helicity modes
    - Perturbative discontinuity
    - Nonlinear corrections
    - Nonperturbative self-screening

Introduction

Theories with massless particles describe long-range forces. Seems natural to view them as a smooth limit of certain massive theories with finite-range forces.

Feynman '63

Works for spin 0 and 1/2.

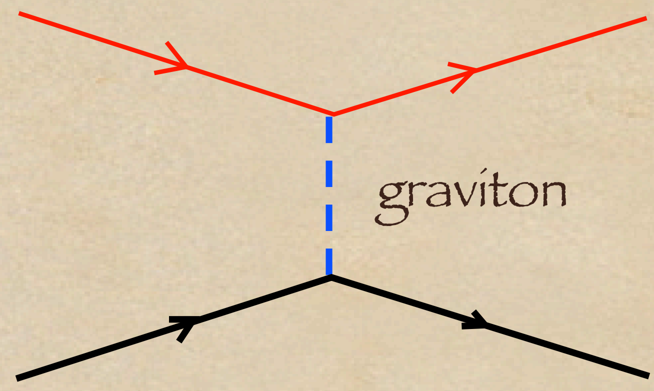
Moreover, works for spin 1 in QED.

Subtleties start at non-Abelian Yang-Mills theory.

No continuity with the hard mass.

The Higgs mechanism of mass generation leads to continuity but with extra fields in the theory.

In the most dramatical way the problem shows up in the theory of gravity.
Compare the long-range interaction in the Einstein theory (exchange of massless graviton) with the finite-range interaction due to exchange of massive spin-2 particle in the limit when the mass goes to zero.



The bending of light by the Sun is 3/4 of the Einstein theory at $m \to 0$

Three papers in 1970: Iwasaki, van Dam+Veltman, Zakharov

The limit $m \to 0$ does exist but does not coincide with $m = 0$

Generalities

Poincare group representation are different for

$m \neq 0, \; m = 0$

$m \neq 0:$ $\quad p_\mu\,,\; \mu = 0,1,2,3\,,\; p_\mu p^\mu = m^2$

spin $s\,,\; 2s+1$ polarization states

$m = 0:$ $\quad p_\mu\,,\; p_\mu p^\mu = 0$

helicity $h = \vec{s}\vec{p}/|\vec{p}|\,,$ one-dim rep

CPT relates h and $-h$

Generalities

Poincare group representation are different for

$m \neq 0, \; m = 0$

$m \neq 0 :$ $\quad p_\mu \,, \; \mu = 0, 1, 2, 3 \,, \; p_\mu p^\mu = m^2$

spin s , $2s + 1$ polarization states

$m = 0 :$ $\quad p_\mu \,, \; p_\mu p^\mu = 0$

helicity $h = \vec{s}\vec{p}/|\vec{p}|$, one-dim rep

CPT relates h and $-h$

In case of $s = 0, 1/2$ there is no discontinuity in the number of polarization states when $m \to 0$

Generalities

Poincare group representation are different for

$m \neq 0, \ m = 0$

$m \neq 0$: $p_\mu \, , \ \mu = 0, 1, 2, 3 \, , \ p_\mu p^\mu = m^2$

spin s , $2s + 1$ polarization states

$m = 0$: $p_\mu \, , \ p_\mu p^\mu = 0$

helicity $h = \vec{s}\vec{p}/|\vec{p}|$, one-dim rep

CPT relates h and $-h$

In case of $s = 0, 1/2$ there is no discontinuity in the number of polarization states when $m \to 0$

When $s = 1$ we deal with three states at $m \neq 0$ but with only two states, $h = \pm 1$, for $m = 0$

Neutral vector field

QED Lagrangian, for the massless photon

$$\mathcal{L} = -\frac{1}{4e^2} F_{\mu\nu} F^{\mu\nu} + \psi \gamma^\mu (i\partial_\mu + A_\mu)\psi - m\bar{\psi}\psi$$

Four fields A_μ but only two helicity states of the photon. The Lagrangian does not depend on all four fields due to gauge invariance

$$A_\mu \to A_\mu + \partial_\mu \lambda \,, \qquad \psi \to \mathrm{e}^{i\lambda}\psi$$

Fixing the gauge, e.g. Coulomb one, $\vec{\partial}\vec{A} = 0$, we have one field less. Besides the field A_0 is not dynamical,

$A_0 = \frac{e^2}{\Delta} \psi \gamma_0 \psi$. Two dynamical degrees of freedom, $\vec{A}^\perp$.

To switch on the mass for the vector field one adds

$$\mathcal{L}_m = \frac{m^2}{2e^2} A_\mu A^\mu$$ Proca formalism

Excluding the nondynamical A_0 we get three d.o.f.

Often is said that an introduction of the mass breaks gauge invariance. This is not correct, only the # of d.o.f. matters. The gauge invariance can be restored by an introduction of an extra field ϕ

$$\mathcal{L}_m = \frac{m^2}{2e^2}\left(A_\mu + \frac{1}{m}\,\partial_\mu \phi\right)^2$$ Stükelberg's substitution

Under the gauge transformation

$$\phi \to \phi - m\lambda$$

so $A_\mu + (1/m)\partial_\mu \phi$ is gauge invariant.

Three polarization states of the massive vector particle with the momentum k_μ are described by the polarization vectors ϵ_μ satisfying $k^\mu \epsilon_\mu = 0$. In the rest frame

$k^\mu = (m, 0, 0, 0)$

$$\epsilon^{(1)} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \qquad \epsilon^{(2)} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \qquad \epsilon^{(3)} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}.$$

After boosting along the x-axis, $k^\mu = (E, k, 0, 0)$,

$$\epsilon^{(1)} = \begin{pmatrix} \frac{k}{m} \\ \frac{E}{m} \\ 0 \\ 0 \end{pmatrix}, \qquad \epsilon^{(2)} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \qquad \epsilon^{(3)} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}.$$

the $h = 0$ polarization vector $\epsilon^{(1)}$ has kinematically large components for $E \gg m$. The $h = \pm 1$ polarizations which are linear combinations of $\epsilon^{(2,3)}$ do not grow with energy.

The kinematical growth of $\epsilon^{(1)}$ for the zero helicity state could imply a growth of interaction. However,

$$\epsilon^{\mu}_{(1)} = \frac{k^{\mu}}{m} + \frac{m}{E+k}\begin{pmatrix}1\\1\\0\\0\end{pmatrix}.$$

The first term drops from the interaction $\epsilon^{\mu} j_{\mu}$ due to the current conservation, the second vanishes at $m \to 0$.

It works for any number of the $h = 0$ quanta and shows the decoupling of the helicity zero when $m \to 0$. Note that this decoupling refers to EM interaction of the neutral vector field. The helicity zero quanta do not decouple from gravity.

Non-Abelian Vector Field

The Lagrangian of Yang-Mills theory

$$\mathcal{L} = -\frac{1}{4g^2}\Big(G^a_{\mu\nu}\Big)^2, \qquad G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + f^{abc} A^b_\mu A^c_\nu$$

The multiplet of fields A^a_μ , an adjoint representation of the group G.

Again one can introduce the "hard" mass m,

$$\mathcal{L}_m = -\frac{m^2}{2g^2}\Big(A^a_\mu\Big)^2.$$

The amplitude of the process with the n massive quanta

$$M_n = T^{a_1 \ldots a_n}_{\mu_1 \ldots \mu_n}\, \epsilon_1^{\mu_1 a_1} \ldots \epsilon_n^{\mu_n a_n}$$

Let us choose polarizations to be zero helicity, i.e.,

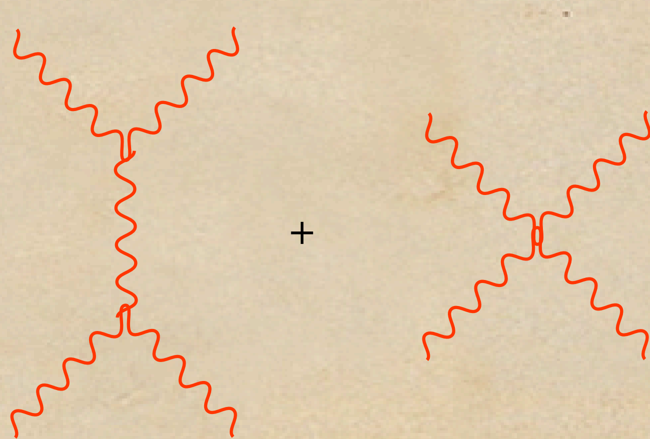
$$\epsilon^{\mu a}_{h=0} = \left(\frac{k_\mu}{m} + \frac{m}{E+|\vec{k}|}\, n^\mu\right)\chi^a, \qquad n^\mu = \left(-1, \frac{\vec{k}}{|\vec{k}|}\right)$$

If the only one polarization is longitudinal we see the same decoupling at $m \to 0$ as in the Abelian case, because $k_1^{\mu_1} T^{a_1 \dots a_n}_{\mu_1 \dots \mu_n} \, \epsilon_1^{\mu_2 a_2} \dots \epsilon_n^{\mu_n a_n} = 0$

If two quanta are longitudinal

$$M_n = \frac{m}{E_1 + |\vec{k}_1|} \, n_1^{\mu_1} \chi_1^{a_1} \frac{k_2^{\mu_2}}{m} \chi^{a_2} T^{a_1 \dots a_n}_{\mu_1 \dots \mu_n} \, \epsilon_3^{\mu_3 a_3} \dots \epsilon_n^{\mu_n a_n}$$

which does not vanish when the commutator of color generators $[t^a, t^b] \neq 0$.



Thus, for two longitudinal quanta we see a finite discontinuity at $m \to 0$. For more than two $h = 0$ quanta we get a singular behavior,

$$M_{n_{h=0}} \propto g^{n_{h=0}} \left(\frac{E}{m}\right)^{n_{h=0}-2}$$

Khriplovich, A.V. '71

It shows both, absence of the zero mass limit and nonrenormalizibility of the theory with the "hard" mass in perturbation theory.

To isolate the singular behavior it is convenient to use the Stükelberger method,

$$A_\mu = U^{-1} B_\mu U + i U^{-1} \partial_\mu U \,, \quad A_\mu = A_\mu^a t^a \,, \quad B_\mu = B_\mu^a t^a$$

$$U = \exp(i\phi^a t^a) \in G \,.$$

The substitution bring in a gauge redundancy,

$$B_\mu \to S B_\mu S^{-1} + i S \partial_\mu S^{-1}\,, \quad U \to SU\,, \quad S \in G\,,$$

where S is the gauge transformation matrix.

Fixing the gauge by putting $\vec{\partial}\vec{B} = 0$. It means that B_μ describes the transversal quanta while $U = \exp(i\phi^a t^a)$ refers to the longitudinal ones.

The leading at $E \gg m$ amplitudes for longitudinal quanta are given by the Lagrangian

$$\mathcal{L}_{h=0} = \frac{1}{4(g/m)^2}\mathrm{Tr}(\partial_\mu U^{-1}\partial^\mu U)$$

This is a chiral model with $G \otimes G$ symmetry group. The model is norenormalizable, and its coupling g/m is singular at $m \to 0$.

On the other hand it gives a hint how to overcome the strong coupling.

In the linear σ-model which for $G = \mathrm{SU}(2)$ is

$$\mathcal{L} = \frac{1}{2}\left\{\partial_\mu \Phi^\dagger \partial^\mu \Phi + m_\sigma^2 \Phi^\dagger \Phi - \frac{\lambda}{4}(\Phi^\dagger \Phi)^2\right\}, \qquad \Phi = \sigma + i\varphi^a \tau^a\,,$$

the growth will stop at energies larger than the mass of of extra σ particle, m_σ. It is just what is used for the Higgs mechanism of generation of the mass for non-Abelian vector field with σ being a physical Higgs field. Note, that in the Higgs mechanism there is no jump in the number of degrees of freedom. This provides a continuity at $m = 0$ together with renormalizability.

Massive Gravity

The Einstein theory of gravity

$$\mathcal{L} = -M_{\rm Pl}^2 \sqrt{-g} R\,, \qquad M_{\rm Pl}^2 = \frac{1}{16\pi G_N}$$

R is the scalar curvature for the metric $g_{\mu\nu}$, $g = det\|g_{\mu\nu}\|$

Near the flat vacuum $\eta_{\mu\nu} = \mathrm{Diag}(1, -1, -1, -1)$ one can expand

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

In quadratic in $h_{\mu\nu}$ approximation the Lagrangian describes the massless graviton with two polarization states, $h = \pm 2$.

One can pass to the massive $s = 2$ tensor field adding the Pauli-Fierz term

$$\mathcal{L}_m = -M_{\rm Pl}^2 \frac{m^2}{4} \left[(h_{\mu\nu})^2 - (h_\mu^\mu)^2 \right]$$

For the zero helicity state in the rest frame

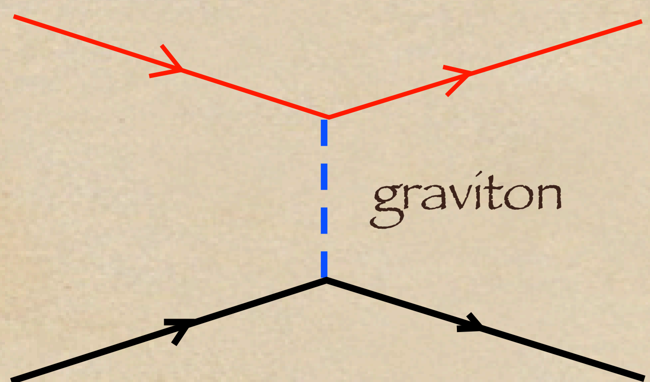
$$h_{\mu\nu} = \frac{1}{\sqrt{6}} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

After the boost along x-axis

$$h_{\mu\nu} = \frac{2}{\sqrt{6}} \begin{pmatrix} \frac{k^2}{m^2} & \frac{kE}{m^2} & 0 & 0 \\ \frac{kE}{m^2} & \frac{E^2}{m^2} & 0 & 0 \\ 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & -\frac{1}{2} \end{pmatrix} = \frac{2}{\sqrt{6}} \left[\frac{k^\mu k^\nu}{m^2} + \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & -\frac{1}{2} \end{pmatrix} \right]$$

Consider emission of the $h = 0$ quanta. The term $k^\mu k^\nu / m^2$ drops out because the interaction $h^{\mu\nu} T_{\mu\nu}$ contains the conserved energy-momentum tensor $T_{\mu\nu}$. The second term gives the finite amplitude of the emission for longitudinal quanta. This is an origin of Iwasaki-van Dam-Veltman-Zakharov discontinuity.

The discontinuity they demonstrated was actually not in the graviton emission (this was done later) but in the bending of light by the Sun.



$$A = -\frac{2}{M_{\rm Pl}^2} T_{\mu\nu} D^{\mu\nu;\alpha\beta} T'_{\alpha\beta}$$

$T_{\mu\nu}$, $T'_{\alpha\beta}$ are the energy-momentum tensor, normalized as $2p_\mu p_\nu$ at the zero momentum transfer. In the massless case the propagator

$$D_0^{\mu\nu;\alpha\beta} = \frac{1}{2k_\mu k^\mu} \left(\eta^{\mu\alpha}\eta^{\nu\beta} + \eta^{\mu\beta}\eta^{\nu\alpha} - \eta^{\mu\nu}\eta^{\alpha\beta}\right)$$

is fixed by the unitary condition for the exchange by the $h = \pm 2$ states.

$$h_{mn} \text{ with } m.n = 2, 3 \text{ and } h_{mm} = 0.$$

$$h^{(1)} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad h^{(1)} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},$$

$$\sum_{i=1,2} h^{(i)}_{mn} h^{(i)}_{kl} = \frac{1}{2} (\delta_{mk}\delta_{nl} + \delta_{ml}\delta_{nk} - \delta_{mn}\delta_{kl}) .$$

The potential for the interaction of two massive sources is

$$V_0(r) = -\int \frac{d^3k}{(2\pi)^3} \, \mathrm{e}^{i\vec{k}\vec{r}} \frac{A}{4M_1M_8} = -\frac{G_N M_1 M_2}{r}$$

In the massive case there are five states which in the rest frame are given by the traceless h_{mn} living in d=3, m,n=1,2,3

$$\sum_{i=1}^{5} h_{mn}^{(i)} h_{kl}^{(i)} = \frac{1}{2}(\delta_{mk}\delta_{nl} + \delta_{ml}\delta_{nk} - \frac{2}{3}\delta_{mn}\delta_{kl})$$

After the boost $\delta_{mn} \to -g_{\mu\nu} + (k_\mu k_\nu / m^2)$

$$D_m^{\mu\nu;\alpha\beta} = \frac{1}{2(k_\mu k^\mu - m^2)} \left(\eta^{\mu\alpha}\eta^{\nu\beta} + \eta^{\mu\beta}\eta^{\nu\alpha} - \frac{2}{3}\eta^{\mu\nu}\eta^{\alpha\beta}\right)$$

up to noncontributing terms containing k^μ.
The static potential becomes

$$V_0(r) = -\frac{4}{3}\frac{G_N M_1 M_2}{r} \mathrm{e}^{-mr}$$

The additional attraction is due to the helicity zero (graviscalar) exchange. If extra scalars are added the discrepancy increases.

Of course, one can introduce $\tilde{G}_N = \frac{4}{3} G_N$ to get the same static potential. However, when it comes to the light for which $T^\mu_\mu = 0$ we get the factor 3/4,

$$A_0 = -\frac{8\pi G_N}{k^2}\Big(T_{\mu\nu}T'^{\mu\nu} - \frac{1}{2}T^\mu_\mu T'^\nu_\nu\Big),$$

$$A_m = -\frac{3}{4}\,\frac{8\pi\tilde{G}_N}{k^2 - m^2}\Big(T_{\mu\nu}T'^{\mu\nu} - \frac{1}{3}T^\mu_\mu T'^\nu_\nu\Big)\,.$$

Nonlinear Corrections

What are corrections due to nonlinear terms in the graviton interactions?

In case of massive Yang--Mills field every extra longitudinal particle gives a factor $g \cdot \frac{E}{m}$ which implies that perturbation theory is broken energies larger than $E_{\rm cr} \sim \frac{m}{g}$.

Similarly, for the $h = 0$ particle in the massive gravity

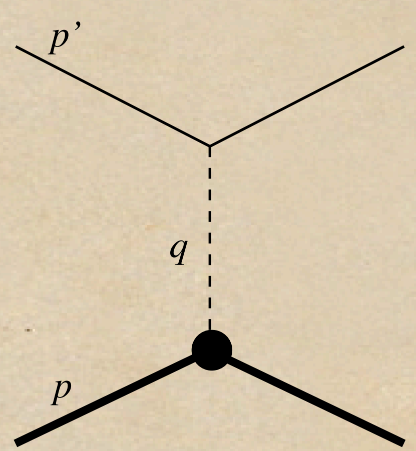
$$\frac{E}{M_{\rm Pl}} \cdot \frac{E^2}{m^2} \qquad\qquad E_{\rm cr} \sim (M_{\rm Pl} m^2)^{1/3}$$

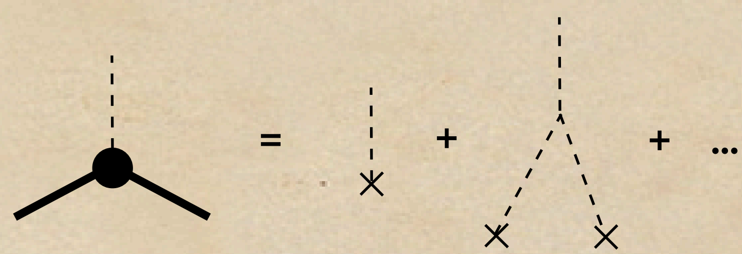
Arkani-Hamed, Georgi, Schwarz '02

The first calculation was 38 years earlier. It was in application to the field of the static source (Schwarzschild problem) where the expansion parameter was found to be

$$\frac{M_1}{M_{\rm Pl}^2\, r} \cdot \left(\frac{1}{r^2 m^2}\right)^2$$

A.V. '72





It implies that corrections are small at $r \gg r_{\rm cr}$

$$r_{\rm cr} = \left(\frac{M_1}{M_{\rm Pl}^2 m^4}\right)^{1/5} = \left(\frac{r_g}{m^4}\right)^{1/5}$$

$\sqrt{ME}/M_{\rm Pl}$ instead of $E/M_{\rm Pl}$

For the largest $m = 1/10^{25} cm$ from PDG and $r_g = 3 \cdot 10^5 cm$ for the Sun we get

$$r_{\mathrm{cr}} \sim 10^{21} cm$$

At the distance of solar system $r \sim 10^{15} cm$ the next-to-leading corrections are about 10^{32} times bigger than the leading term.

One cannot rely on weak coupling.

Nonperturbative Screening

No analog of the Higgs mechanism for the graviton mass.

What we can do about the theory with ultra-strong coupling? Classical nonlinear equations is a possible route to go beyond perturbation theory.

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - \frac{1}{2} m^2 (h_{\mu\nu} - \eta_{\mu\nu} h^{\gamma}_{\gamma}) = \frac{1}{M^2_{\rm Pl}} T_{\mu\nu}$$

Although pertubative solution generates strongly coupled zero-helicity modes nonpertubatively they can screen themselves providing a continuity at $m \to 0$ with the massless Einstein theory.

Higher powers of $h_{\mu\nu}$ in the mass term are not fixed.

Spherical symmetric metric

$$ds^2 = dx^\mu dx^\nu g_{\mu\nu} = \mathrm{e}^\nu dt^2 - \mathrm{e}^\sigma d\rho^2 - \mathrm{e}^\mu \rho^2 (d\theta^2 + \sin^2\theta d\phi^2)$$

New radius coordinate r instead of ρ and λ instead of σ

$$r = \rho\,\mathrm{e}^{\mu/2}, \qquad \mathrm{e}^\lambda = \frac{\mathrm{e}^{\sigma-\mu}}{1 - \frac{\rho}{2}\frac{d\mu}{d\rho}}$$

In massless case

$$\nu_0 = -\lambda_0 = \ln\left(1 - \frac{r_M}{r}\right) = -\frac{r_M}{r} - \frac{r_M^2}{2r^2} - \ldots, \qquad \mu_0 = 0$$

In massive case the perturbative solution at $r \gg r_M$

$$\nu_m = \frac{r_M}{r}\left(1 + \frac{7r_M}{32m^4r^2} + \ldots\right), \qquad \lambda_m = \frac{1}{2} \times \frac{r_M}{r}\left(1 - \frac{21r_M}{8m^4r^2} + \ldots\right)$$

$$\mu_m = \frac{r_M}{2m^2r^3}\left(1 + \frac{21r_M}{4m^4r^2} + \ldots\right).$$

The factor 1/2 in λ_m reflect vDVZ discontinuity.

Expansion in the mass instead of G_N leads to

$$\nu_m = -\frac{r_M}{r} + \mathcal{O}\left(m^2\sqrt{r_M r^3}\right), \qquad \lambda_m = \frac{r_M}{r} + \mathcal{O}\left(m^2\sqrt{r_M r^3}\right),$$

$$\mu_m = \sqrt{\frac{8r_M}{13r}} + \mathcal{O}\left(m^2 r^2\right).$$

This solution is clearly nonanalytic in G_N. It is valid at

$$r_c \gg r \gg r_M$$

Its asymptotics at $r \gg r_c$ is not known -- could be exponentially growing instead of decreasing. It looks as a continuous at $m \to 0$ local solution is always possible but a global, exponentially decaying solution is in doubt.

Damour, Kogan and Papazoglou '03; Deffayet et al

Unresolved Problems

Boulware and S. Deser '72 Besides five degrees of freedom for massive $s = 2$ the sixth one shows up at the nonlinear level. At the linear level

$$\partial^\mu (h_{\mu\nu} - \eta_{\mu\nu} h) = 0\,, \qquad h = h^\gamma_\gamma$$

follows from

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - \frac{1}{2} m^2 (h_{\mu\nu} - \eta_{\mu\nu} h^\gamma_\gamma) = \frac{1}{M^2_{\mathrm{Pl}}} T_{\mu\nu}$$

and R vanishes. Then $h = h^\gamma_\gamma$ is fixed

$$h = \frac{1}{3m^2 M^2_{\mathrm{Pl}}} T^\gamma_\gamma$$

Thus, $\partial^\mu h_{\mu\nu} = 0$, $h = 0$ for noninteracting field, 5 degrees of freedom. At nonlinear level R does not vanish and h becomes sixth degree of freedom. See recent works Gabadadze; De Rahm for possible resolution.

Brane Gravity

Dvali, Gabadaze and Porrati '00 One extra dimension y.

$$S = M_{\rm Pl}^2 \left\{ \int d^4x \sqrt{-g^{(4)}} R(g^{(4)}) + \frac{m_c}{2} \int d^4x dy \sqrt{g^{(5)}} R_5(g^{(5)}) \right\}$$

The parameter m_c related 5d and 4d Planck masses:

$$M_*^3 = \frac{m_c}{2} \cdot M_{\rm Pl}^2$$

Two matter sources on the brane interact by one-graviton exchange as

$$A_{1-\rm grav}(k, y = y_0) = \frac{8\pi G_N}{-k^2 + m_c\sqrt{-k^2}} \left(T^{\mu\nu} T'_{\mu\nu} - \frac{1}{3} T^\mu_\mu T'^\nu_\nu \right)$$

The coefficient 1/3 demonstrates perturbative vDVZ discontinuity. Strong coupling summation restores continuity. Deffayet, Dvali, Gabadadze and A.V. '01

Higgsization of Gravity and Breaking Lorentz Invariance

In Higgs phase of massive non-Abelian vector field the vacuum condensates are not invariant under action of group generators Q^a.

Analog for massive gravity: breaking actions of Poincare generators P_μ and $M_{\mu\nu}$, i.e. translational and Lorentz invariance.

Models with breaking Lorentz (but not translations) were suggested, Arkani-Hamed, Cheng, Luty and Mukohyama '03; Rubakov '04, Dubovsky '04 .

To illustrate the idea let's apply it to Yang-Mills theory.

Instead of the Lorentz-invariant "hard" mass term $m^2 A^a_\mu A^{a\mu}$ let us introduce

$$\frac{1}{2}(m^2)^{\mu\nu} A^a_\mu A^a_\nu \qquad\qquad (m^2)^{\mu\nu} = Diag(0, -m^2, -m^2, -m^2)$$

Equations of motion

$$D^\mu G^a_{\mu\nu} + (m^2)^\mu_\nu A^a_\mu = j^a_\nu$$

after application of D^ν give

$$D^\nu \left[(m^2)^\mu_\nu A^a_\mu\right] = 0 \quad \longrightarrow \quad \partial^n A^a_n = 0 \quad (n = 1, 2, 3)$$

It is the Coulomb gauge condition which implies two degrees of freedom for each a. The Gauss law for A_0

$$A^a_0 = \frac{1}{D^m D_m} j^a_0$$

As a result we come to two massive transversal fields plus instantaneous long-range interaction. Two currents j and J interact as

$$A = -j_0\,\frac{1}{\vec{k}^2}\,J_0 - j_i\,\frac{\delta_{ij} - (k_i k_j/\vec{k}^2)}{k^2 - m^2}\,J_j = j_\mu\,\frac{1}{k^2 - m^2}\,J^\mu + j_0\,\frac{m^2}{\vec{k}^2(k^2 - m^2)}\,J_0$$

The instantaneous term vanishes at $m = 0$.

Conclusions

- Massive modifications of gravity in the Lorentz-invariant fashion (Fierz-Pauli theory) are challenging: vDVZ discontinuity, Boulware-Deser instability, ultra-strong coupling.
- While it is possible that these theories make sense when ultra-strong coupling is fully accounted for there is no much of theoretical control.
- Lorentz-breaking condensates could produce a tractable theory in weak coupling with many interesting phenomenological consequences.