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鑑三

Phantom Crossing and Anomalous Growth Index of Fluctuations in f(R) gravity

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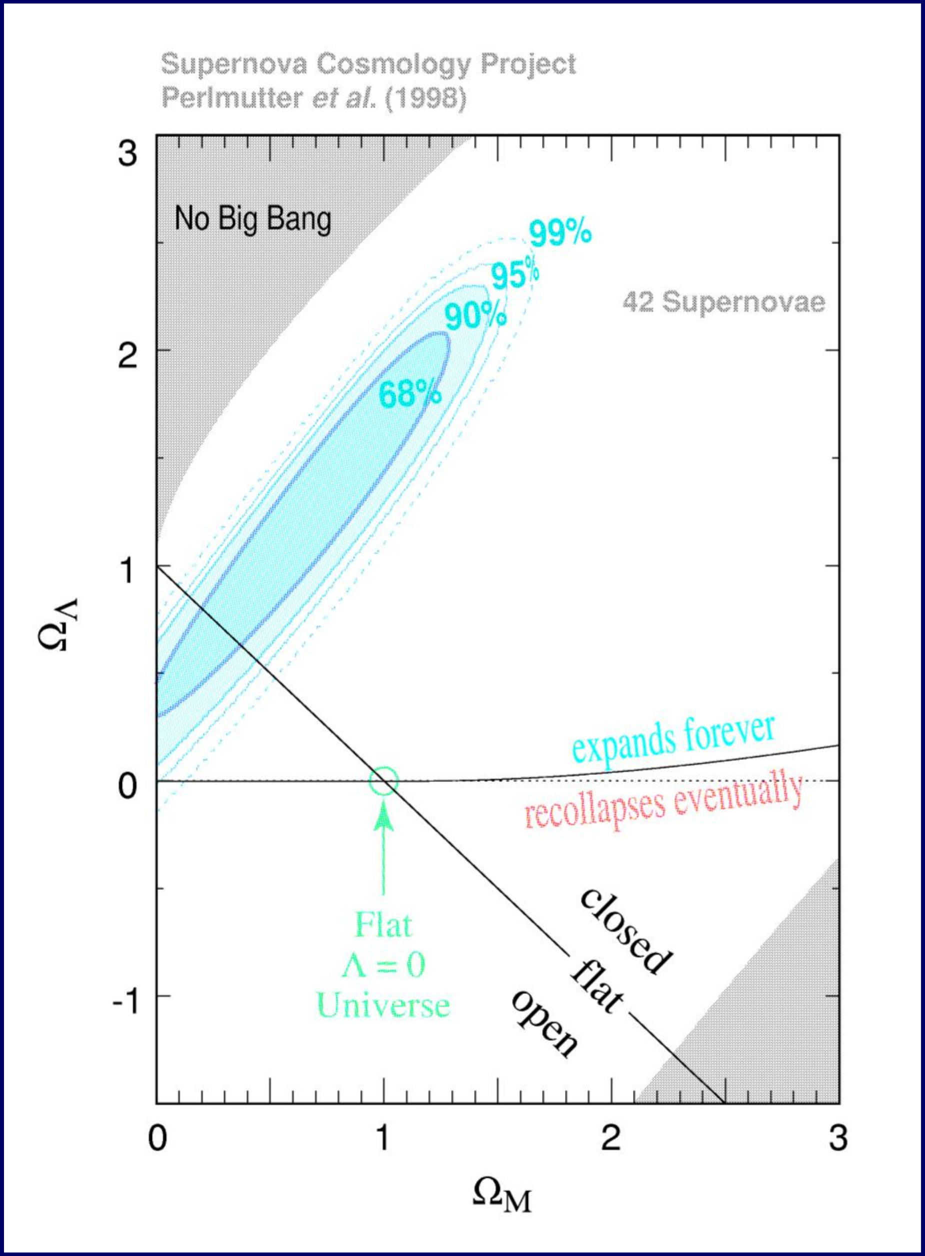
RESCEU

Accelerated Expansion of the Universe:

Dark Energy
or
Modified Gravity ?

$$\boxed{\begin{array}{c}\Lambda = 0? \\ \\ \Lambda > 0?\end{array}}$$



The Cosmological Constant Problem remains.

Cosmological constant Λ = vacuum energy density ρ_{v}

$$\boxed{\rho_{\mathrm{v}} = \frac{3M_{Pl}^2}{8\pi}\Lambda = 3M_G^2\Lambda}$$

$$M_G = M_{Pl}\Big/\sqrt{8\pi} = 2.4\times 10^{18}\,\mathrm{GeV}$$

Reduced Planck scale

Contribution of zero-point energy of a quantum field

$$\langle\rho_{\mathrm{v}}\rangle = \int_0^{k_c}\frac{d^3k}{(2\pi)^3}\frac{1}{2}\sqrt{k^2+m^2} \cong \frac{k_c^4}{16\pi^2}$$

Taking the cutoff k_c =Planck scale, we find

$$\langle\rho_{\mathrm{v}}\rangle \cong 4M_G^4$$

Observationally,

$$\rho_{\mathrm{v}} < \rho_{cr0} = 4 \times 10^{-47}\,\mathrm{GeV}^4$$

$$= 10^{-120}\, M_G^4$$

Cosmological Constant Problem I

Why is the vacuum energy density equal to zero with the accuracy of 120 digits?

The question remains whether $\Lambda = 0$ or not.

Contribution of zero-point energy of a quantum field

$$\langle \rho_v \rangle = \int_0^{M_{Pl}} \frac{d^3k}{(2\pi)^3} \frac{1}{2} \sqrt{k^2 + m^2} \approx 4M_G^2$$

This is a problem of Perturbation Theory and we do not know the answer to this issue.

Currently popular dynamical dark energy models and modified gravity models assume $\Lambda_{eff} = \Lambda_{bare} + \langle \rho_v \rangle = 0$ at the ground state.

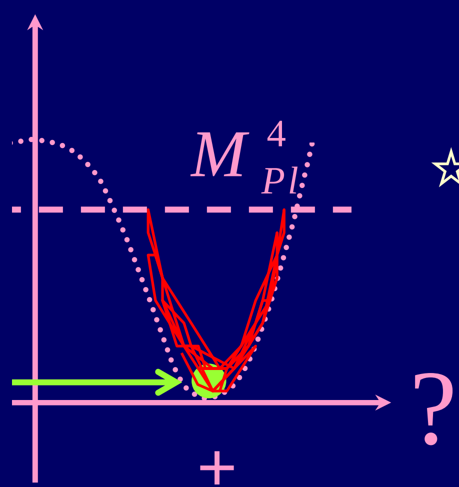
A Rule for $f(R)$ gravity models

$$L = \frac{1}{16\pi G} f(R) \to 0 \quad \text{for} \quad R \to 0$$

Before starting discussion on $f(R)$ models let me recall one of the least known dark energy model which may be related to this cosmological constant problem.

JY Phys Rev Lett 88(2002)151302

Imagine that we found a mechanism to cancel perturbative contribution and bare cosmological constant at a perturbative vacuum.



☆ Suppose also that there exist two degenerate perturbative vacua $|+\rangle,\ |-\rangle$ with vanishing vacuum energy density

☆ True energy eigen state

$$|S\rangle = \frac{1}{\sqrt{2}}\left(|+\rangle + |-\rangle\right), \quad |A\rangle = \frac{1}{\sqrt{2}}\left(|+\rangle - |-\rangle\right)$$

$\langle + | + \rangle = \langle - | - \rangle = 1,\ \langle + | - \rangle = 0$ are assumed to the lowest order.

We assume an instanton solution exists which describes quantum tunneling with the Euclidean action S_0.

☆ Calculate $\langle A|e^{-\mathcal{H}T}|A\rangle$ by summing up contributions of

$$\langle A|e^{-\mathcal{H}T}|A\rangle = \frac{1}{2}\left(\langle +|e^{-\mathcal{H}T}|+\rangle + \langle -|e^{-\mathcal{H}T}|-\rangle - \langle +|e^{-\mathcal{H}T}|-\rangle - \langle -|e^{-\mathcal{H}T}|+\rangle\right)$$

$$= \sum_{j=0}^{\infty} \frac{1}{(2j)!}\left(kVTe^{-S_0}\right)^{2j} - \sum_{j=0}^{\infty} \frac{1}{(2j+1)!}\left(kVTe^{-S_0}\right)^{2j+1}$$

$$= \exp\left(-kVTe^{-S_0}\right)$$

Contribution of each instanton or anti-instanton.
T : Euclidean time, V : Spatial volume, k : const.

☆ Degeneracy is broken by incorporating instantons. The asymmetric superposition has exponentially small vacuum energy density

$$\boxed{\rho_A = ke^{-S_0} \equiv m^4 e^{-S_0}}$$

which may be identified with the observed cosmological constant if we live in such a state.

☆ $|A\rangle$ state should be long-lived:

Tunneling rate/volume/time $\Gamma \approx m^4 e^{-2S_0}$.

Demanding that there should be no tunneling in the horizon volume in the cosmic age, we find

$$1 > \Gamma H_0^{-4} \approx \frac{9M_G^4}{m^4}, \quad \text{with } H_0^2 \approx \frac{\rho_0}{3M_G^2} = \frac{m^4 e^{-S_0}}{3M_G^2}.$$

☆ From $\rho_A = 10^{-120} M_G^4$ and $\Gamma H_0^{-4} < 1$, we find

$$m > M_G, \quad S_0 = 120 \ln 10 + 4 \ln\left(m/M_G\right) \approx 276.$$

Now we return to $f(R)$ models.

Based on

H Motohashi, A.A. Starobinsky, & JY Int J Mod Phys D 18(2009)1731.
H Motohashi, A.A. Starobinsky, & JY Prog Theor Phys 123(2010)XXX
H Motohashi, A.A. Starobinsky, & JY arXiv 1005.1171

The Action

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}f(R) + S_m,$$

$$f(R) = R + \lambda R_s\left[\left(1+\frac{R^2}{R_s^2}\right)^{-n} - 1\right]$$

Starobinsky model

Dark energy sector

Three parameters

$R_s \approx H_0^2$ sets the energy scale.

$n,\ \lambda$: model parameters.

(Starobinsky 07)

f(R) gravity is essentially a scalar-tensor theory.

A conformal transformation yields the Einstein action plus a scalar field.

$$\bar{g}_{\mu\nu} = e^{-\sqrt{\frac{2}{3}}\frac{\phi}{M_G}} g_{\mu\nu} \qquad \phi = -\sqrt{\frac{2}{3}} M_G \ln\left| f'(R) \right|$$

$$S_E = \frac{M_G^2}{2} \int \sqrt{-\bar{g}}\, d^4x \bar{R} + \int \sqrt{-\bar{g}}\, d^4x \left[-\frac{1}{2} (\partial\phi)^2 - V[\phi] \right]$$

$$V[\phi] = \frac{M_G^2}{2} \left[\frac{R}{f'(R)} - \frac{f(R)}{f'(R)^2} \right] \qquad R = R(\phi) \qquad M_G^2 = \frac{1}{8\pi G}$$

A New scalar field "Scalaron."

mass^2= $\dfrac{1}{3f''(R)} + \dfrac{Rf'(R) - f(R)}{3f'(R)^2} \cong \dfrac{1}{3f''(R)}$ should be positive.

$$f(R) = R + \lambda R_s \left[\left(1 + \frac{R^2}{R_s^2} \right)^{-n} - 1 \right]$$

This model is constructed so as to satisfy the stability conditions and viability conditions.

$$f'(R) > 0 \qquad f''(R) > 0$$

$$\left| f(R) - R \right| \ll R \qquad \left| f'(R) - 1 \right| \ll 1$$

$$R f''(R) \ll 1 \qquad \text{for } R \gg R_0$$

standard ΛCDM cosmology in the early Universe

Singularity problem in the original Starobinsky model

$$\phi = -\sqrt{\frac{2}{3}} M_G \ln\left| f'(R) \right| \qquad f'(R) = 1 - 2n\lambda \frac{R}{R_s}\left(1 + \frac{R^2}{R_s^2}\right)^{-n-1}$$

ϕ is not single valued. Both $R=0$ and $R \to \pm\infty$ correspond to $\phi = 0$.

$$V[\phi(R=\infty)] - V[\phi(R=0)] = \frac{\lambda M_G^2 R_s}{2} = O(\rho_{cr0})$$

The potential has only a tiny difference between $R=0$ and $R=\infty$, and an external force due to the interaction with matter drives to $R=\infty$.

(Frolov 08, Kobayashi&Maeda 09)

A proposed remedy is to introduce a $\frac{R^2}{6M^2}$ term to break the degeneracy,

$$\phi = -\sqrt{\frac{2}{3}} M_G \ln\left| f'(R) \right| \qquad f'(R) = 1 - 2n\lambda \frac{R}{R_s}\left(1 + \frac{R^2}{R_s^2}\right)^{-n-1} + \frac{R}{3M^2}$$

so that $R = \pm\infty$ no longer corresponds to $\phi = 0$.

$$f(R) = R + \lambda R_s \left[\left(1 + \frac{R^2}{R_s^2}\right)^{-n} - 1 \right] + \frac{R^2}{6M^2},$$

If we take $M \approx 3\times 10^{13}\,\mathrm{GeV}$ the last term can account for inflation in the early Universe.

(Appleby, Battye, Starobinsky 09)

Inflation in the early Universe and today's accelerated expansion can be unified !

The last term is unimportant for today's talk and neglected hereafter.

We work in the original frame and write the field equation in the Einsteinian form:

$$R^{\mu}_{\nu} - \frac{1}{2}\delta^{\mu}_{\nu} R = -8\pi G \left(T^{\mu}_{\nu(m)} + T^{\mu}_{\nu(\mathrm{DE})} \right) ,$$

Energy momentum tensor of nonrelativistic matter

where the effective EM tensor for “Dark Energy” reads

$$8\pi G T^{\mu}_{\nu(\mathrm{DE})} \equiv \mathcal{F}'(R) R^{\mu}_{\nu} - \frac{1}{2}\mathcal{F}(R)\delta^{\mu}_{\nu} + (\nabla^{\mu}\nabla_{\nu} - \delta^{\mu}_{\nu}\Box)\mathcal{F}'(R),$$

$$\mathcal{F}(R) \equiv f(R) - R$$

Eqs. in the spatially flat FRW background

$$3H^2 = 8\pi G\rho - 3\mathcal{F}'H^2 + \frac{1}{2}(\mathcal{F}'R - \mathcal{F}) - 3H\dot{\mathcal{F}}',$$

$$2\dot{H} = -8\pi G\rho - 2\mathcal{F}'\dot{H} - \ddot{\mathcal{F}}' + H\dot{\mathcal{F}}', \qquad H = \frac{\dot{a}}{a}$$

The effective energy density and pressure of DE read

$$8\pi G\rho_{\rm DE} = \frac{1}{2}(\mathcal{F}'R - \mathcal{F}) - 3H^2\mathcal{F}' - 3H\dot{\mathcal{F}}'$$

$$= -3H\dot{R}\mathcal{F}'' + 3(H^2 + \dot{H})\mathcal{F}' - \frac{1}{2}\mathcal{F},$$

$$8\pi G(\rho_{\rm DE} + P_{\rm DE}) = 2\dot{H}\mathcal{F}' - H\dot{\mathcal{F}}' + \ddot{\mathcal{F}}',$$

$$w_{\rm DE} \equiv P_{\rm DE}/\rho_{\rm DE}$$

$$R = 12H^2 + 6\dot{H}$$

$$\mathcal{F}(R) = \lambda R_s \left[\left(1 + \frac{R^2}{R_s^2}\right)^{-n} - 1 \right]$$

For large R, we find asymptotically $\mathcal{F} = -\lambda R_s$

So it is equivalent to ΛCDM model with $\Lambda(\infty) = \lambda R_s/2$ at high redshift.

On the other hand, it has a de Sitter solution in the late time limit $\rho \to 0$ with $R \equiv R_1 \equiv x_1 R_s = const$ where x_1 is the largest solution of

$$\lambda = \frac{x(1+x^2)^{n+1}}{2\left[(1+x^2)^{n+1} - 1 - (n+1)x^2\right]}$$

because in this limit the Einstein equation reads $f'(R)R = 2f(R)$.

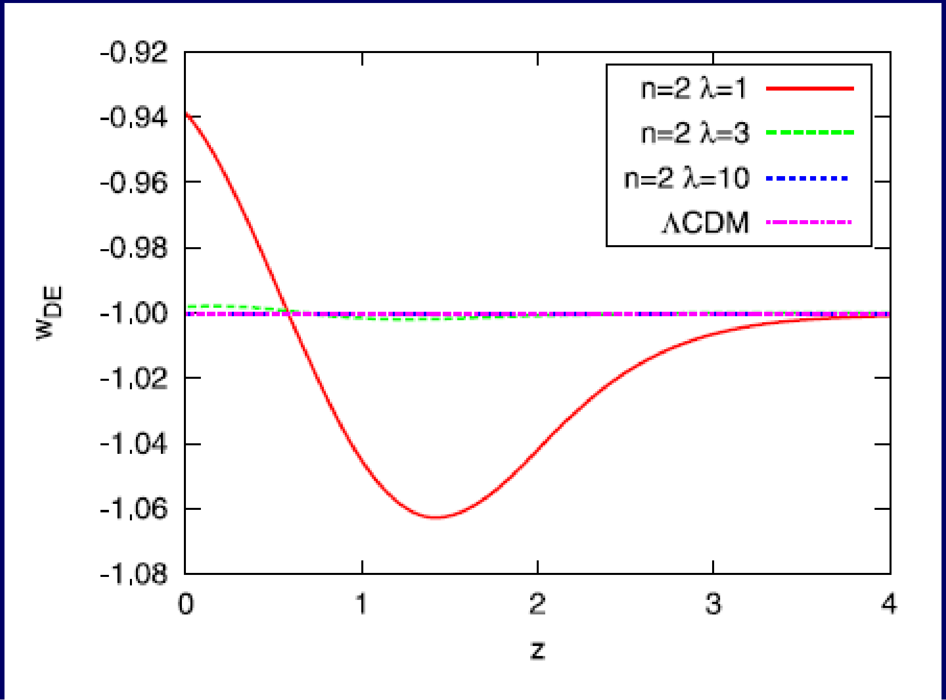
We find $x_1 < 2\lambda$, so $\Lambda(R_1) = \frac{x_1}{4} R_s < \Lambda(\infty)$

In the limit $x_1 \gg 1$ for fixed n, or $n \gg 1$ for fixed x_1 cosmic evolution is indistinguishable from ΛCDM model.

$$8\pi G\rho_{\rm DE} = -3H\dot{R}\mathcal{F}'' + 3(H^2+\dot{H})\mathcal{F}' - \frac{1}{2}\mathcal{F},$$

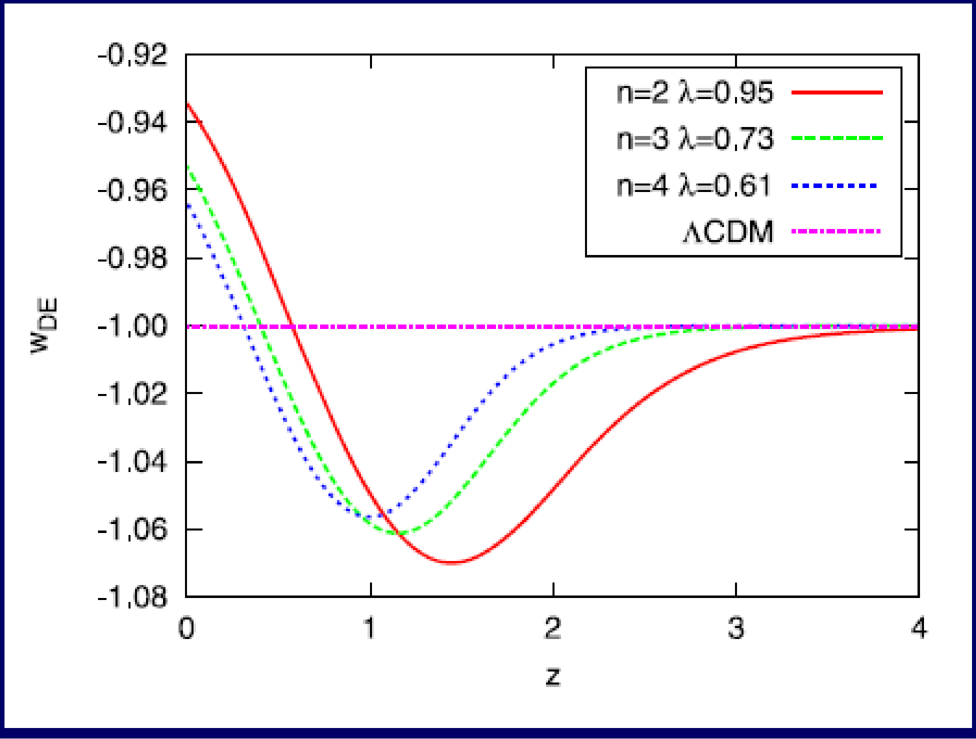
Initially ρ_{DE} is constant, and its evolution is governed by the first term. Since $\dot{R}<0$ and $\mathcal{F}''>0$ for stability, ρ_{DE} increases temporarily which means $w_{DE}<-1$ and subsequent phantom crossing.

Numerical solutions show



Stability of asymptotic de Sitter solution $f'(R_1) > R_1 f''(R_1)$ yields an lower bound on x_1 and λ (for each n).

n	$x_{1\mathrm{min}}$	λ_{min}
2	1.267	0.9440
3	1.041	0.7259
4	0.9032	0.6081



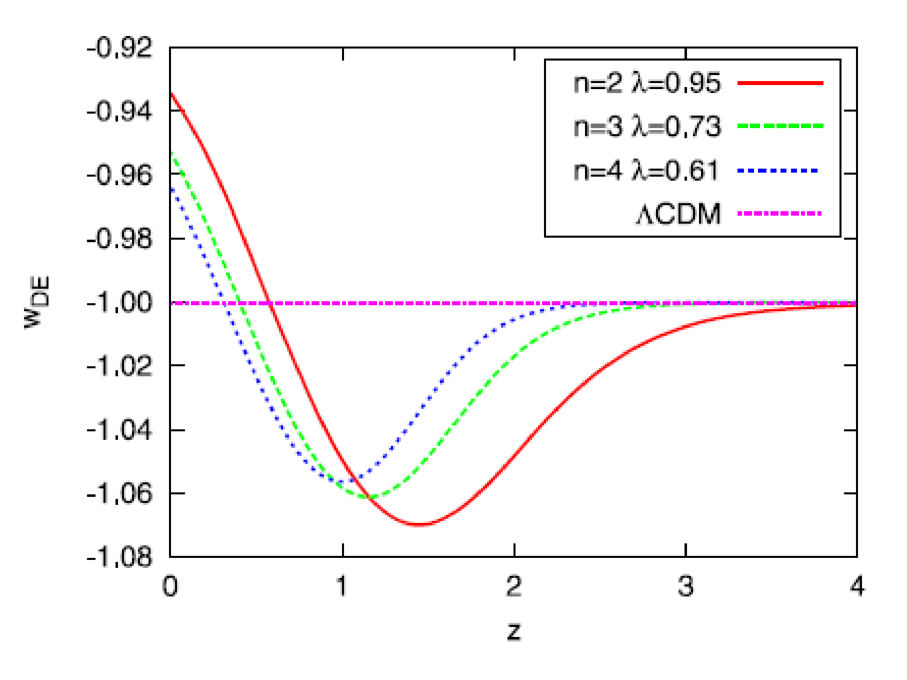
For small λ close to its lower bound, deviation from $w_{DE} = -1$ can be significant.

see also Hu & Sawicki 08

EOS parameter in the observable range 0 < z < 1 is well fit by

$$w_{DE}(z) = w_0 + w_a \frac{z}{1+z}$$

n	w_0	w_a
2	-0.92	-0.23
3	-0.94	-0.22
4	-0.96	-0.21



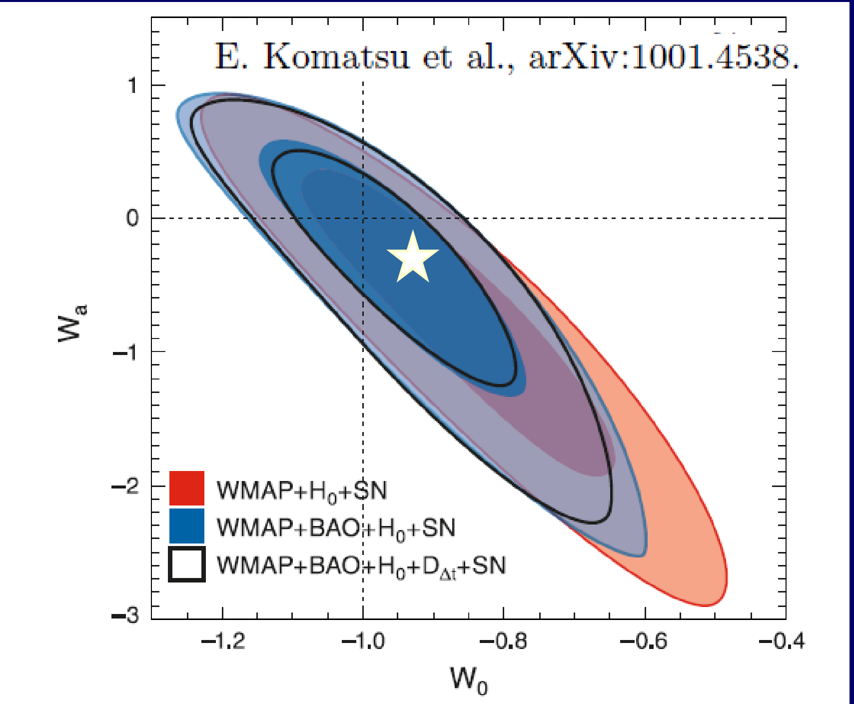


FIG. 13.— Joint two-dimensional marginalized constraint on the linear evolution model of dark energy equation of state, $w(a) = w_0 + w_a(1-a)$. The contours show the 68% and 95% CL from $WMAP$+H_0+SN (red), $WMAP$+BAO+H_0+SN (blue), and $WMAP$+BAO+H_0+$D_{\Delta t}$+SN (black), for a flat universe.

Very close to the central values of observational constraints!

§ Evolution of Density Fluctuations

Fourier mode of $\delta\rho/\rho$ satisfies

$$\boxed{\ddot{\delta}_{\boldsymbol{k}} + 2H\dot{\delta}_{\boldsymbol{k}} - 4\pi G_{\rm eff}\,\rho\delta_{\boldsymbol{k}} = 0}$$

in the subhorizon regime with

$$G_{\rm eff} = \frac{G}{F}\frac{1+4\frac{k^2}{a^2}\frac{F'}{F}}{1+3\frac{k^2}{a^2}\frac{F'}{F}}, \quad F(R) \equiv f'(R).$$

(Zhang 06, Tsujikawa 07)

The effective Gravitational constant can be rewritten as

$$G_{\rm eff} = G\left(1+\frac{1}{3}\frac{k^2/a^2m_s^2}{1+k^2/a^2m_s^2}\right)$$

with the scalaron mass

$$m_s(t) = (3F')^{-1/2}$$

In the position space, this theory has a potential

$$V(r) = -\frac{G}{r}\left(1+\frac{1}{3}e^{-m_s r}\right)$$

per unit mass. Additional force on small scale $r < 1/m_s$.

An analytic solution can be found in the matter dominated regime with the following approximation.

$$F \simeq 1 - 2n\lambda \left(\frac{R}{R_s}\right)^{-2n-1} \equiv 1 - \left(\frac{R}{R_c}\right)^{-N-1}, \qquad a(t) \propto t^{2/3}$$

$$\delta_{\boldsymbol{k}}(t) = \delta_{i\boldsymbol{k}} \left(\frac{t}{t_i}\right)^{\frac{-1\pm5}{6}} \qquad N = 2n \quad \text{and} \quad R_c = R_s(2n\lambda)^{1/(2n+1)}$$

Upper sign corresponds to the growing mode.

$$\times\ {}_2F_1\left(\frac{\pm5-\sqrt{33}}{4(3N+4)}, \frac{\pm5+\sqrt{33}}{4(3N+4)}; 1 \pm \frac{5}{2(3N+4)}; -3\frac{(N+1)k^2}{a_i^2 R_c^2}\left(\frac{t}{t_i}\right)^{2N+8/3}\right)$$

Asymptotic behaviors

$$\delta_{\boldsymbol{k}}(t) \xrightarrow{t\to 0} \delta_{i\boldsymbol{k}} \left(\frac{t}{t_i}\right)^{\frac{2}{3}} \quad \Rightarrow \quad \delta_{\boldsymbol{k}}(t) \xrightarrow{t\to\infty} \delta_{i\boldsymbol{k}} C(k) \left(\frac{t}{t_i}\right)^{\frac{-1+\sqrt{33}}{6}}$$

extra k dependence

$$C(k) = \frac{\Gamma\left(1+\frac{5}{4(3n+2)}\right)\Gamma\left(\frac{\sqrt{33}}{4(3n+2)}\right)}{\Gamma\left(1+\frac{5+\sqrt{33}}{8(3n+2)}\right)\Gamma\left(\frac{5+\sqrt{33}}{8(3n+2)}\right)}\left[\frac{6n\lambda(2n+1)k^2}{a_i^2 R_s}\left(\frac{3R_s t_i^2}{4}\right)^{2(n+1)}\right]^{\frac{-5+\sqrt{33}}{8(3n+2)}}$$

due to the emergence of scalaron force

The transition from GR regime to Scalar-Tensor regime, when scalaron force operates, occurs at time t_k determined by

$$k = a(t_k) m_s(t_k) = a(t_k) \left(\frac{R_s}{6n(2n+1)\lambda} \right)^{\frac{1}{2}} \left(\frac{R(t_k)}{R_s} \right)^{n+1}$$

This expression is proportional to $t_k^{-2n-4/3}$ at high redshift, which explains the additional power $k^{\frac{-5+\sqrt{33}}{4(3n+2)}}$.

The wavenumber crossing the scalaron radius today reads

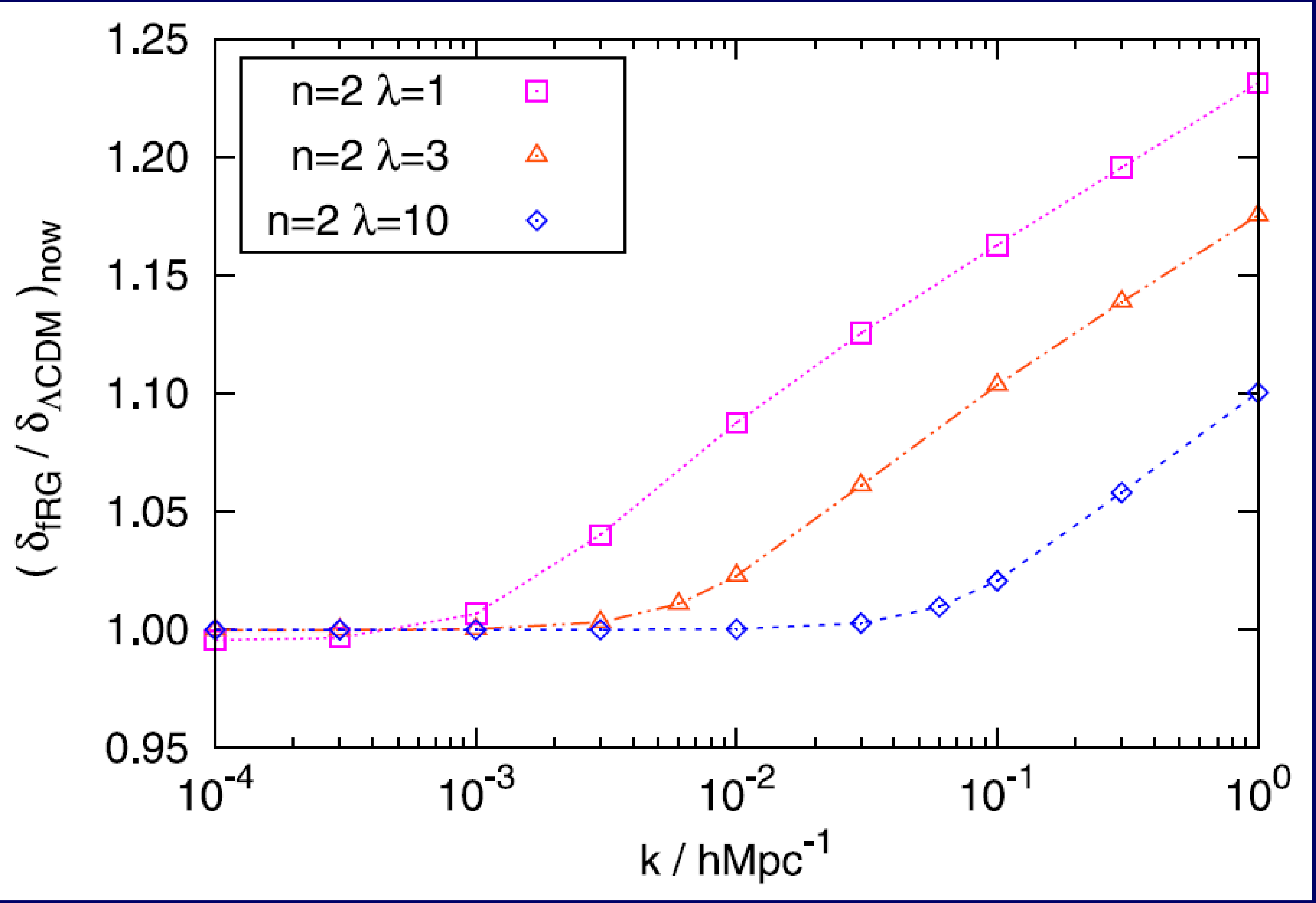
$$\frac{k_0}{a_0} = \begin{cases} 3.2\lambda^{1.88} H_0 & (n=2) \\ 5.3\lambda^{2.43} H_0 & (n=3) \\ 5.0\lambda^{2.66} H_0 & (n=4) \end{cases}$$

and

$$\frac{k_0}{a_0} = \begin{cases} 1.07 \times 10^{-3} h \text{ Mpc}^{-1} & (n=2, \lambda=1) \\ 8.44 \times 10^{-3} h \text{ Mpc}^{-1} & (n=2, \lambda=3), \\ 8.12 \times 10^{-2} h \text{ Mpc}^{-1} & (n=2, \lambda=10) \end{cases}$$

Modes below these wavenumbers evolve in the same way as in ΛCDM model.

Linear density fluctuation in f(R) gravity divided by that in ΛCDM model



$$\propto k^{\frac{-5+\sqrt{33}}{32}}$$

$$\frac{k_0}{a_0} = \begin{cases} 1.07 \times 10^{-3} h \ \mathrm{Mpc}^{-1} & (n=2, \lambda=1) \\ 8.44 \times 10^{-3} h \ \mathrm{Mpc}^{-1} & (n=2, \lambda=3), \\ 8.12 \times 10^{-2} h \ \mathrm{Mpc}^{-1} & (n=2, \lambda=10) \end{cases}$$

explains the bend scale well.

Observationally, ΛCDM model works very well.

Here we impose a simple-minded observational constraint

First we define a wavenumber to which σ_r^2 is most sensitive.

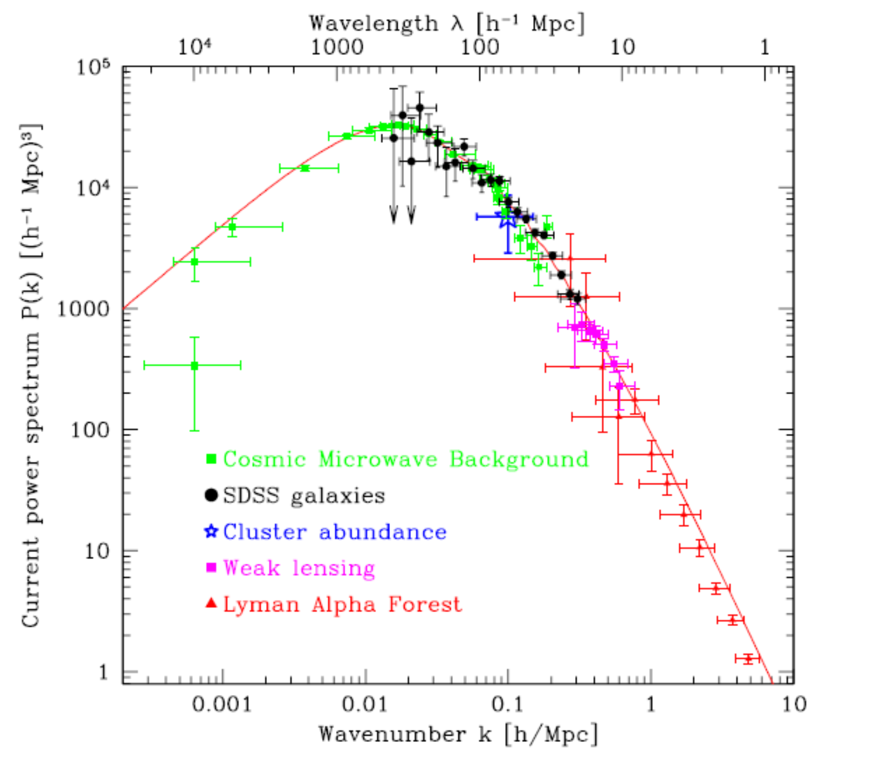
$$\sigma_r^2 = \int \frac{d^3k}{(2\pi)^3} |W(kr)|^2 P(k) \equiv \frac{4\pi k_{\rm eff}^3}{(2\pi)^3} P(k_{\rm eff}(r)), \quad W(kr) \equiv \frac{3j_1(kr)}{kr}.$$

dispersion of fluctuation on scale r.

Matter power spectrum of ΛCDM model

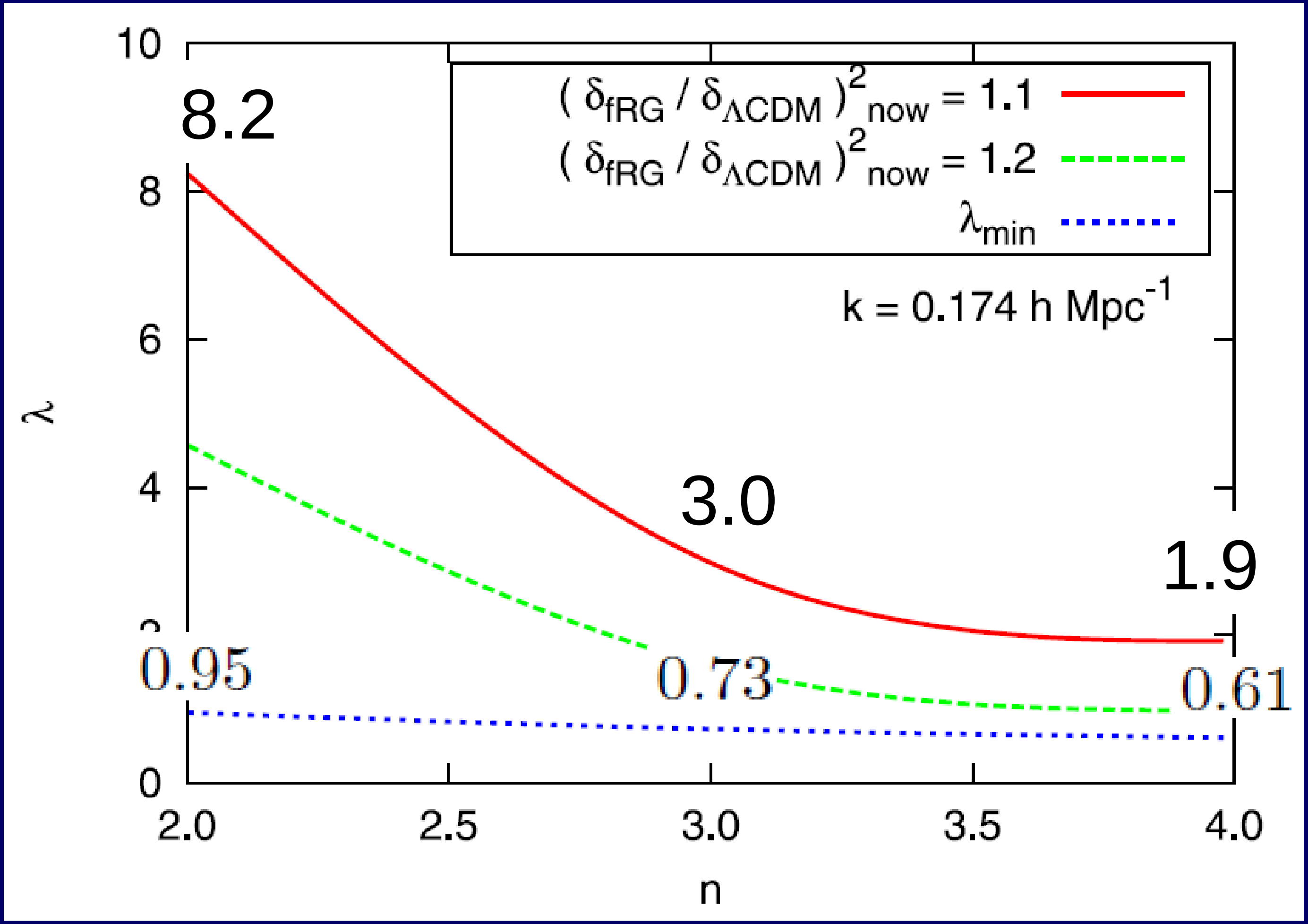
$\sigma^2_{8h^{-1}\mathrm{Mpc}}$ corresponds to

$$k_{\rm eff} = 0.174h\ \mathrm{Mpc}^{-1}$$



We consider the deviation of the power spectrum from ΛCDM model on this scale.

Requiring the deviation is small enough, we find a lower bound on λ which is much larger than $\lambda_{\rm min}$ for each n.



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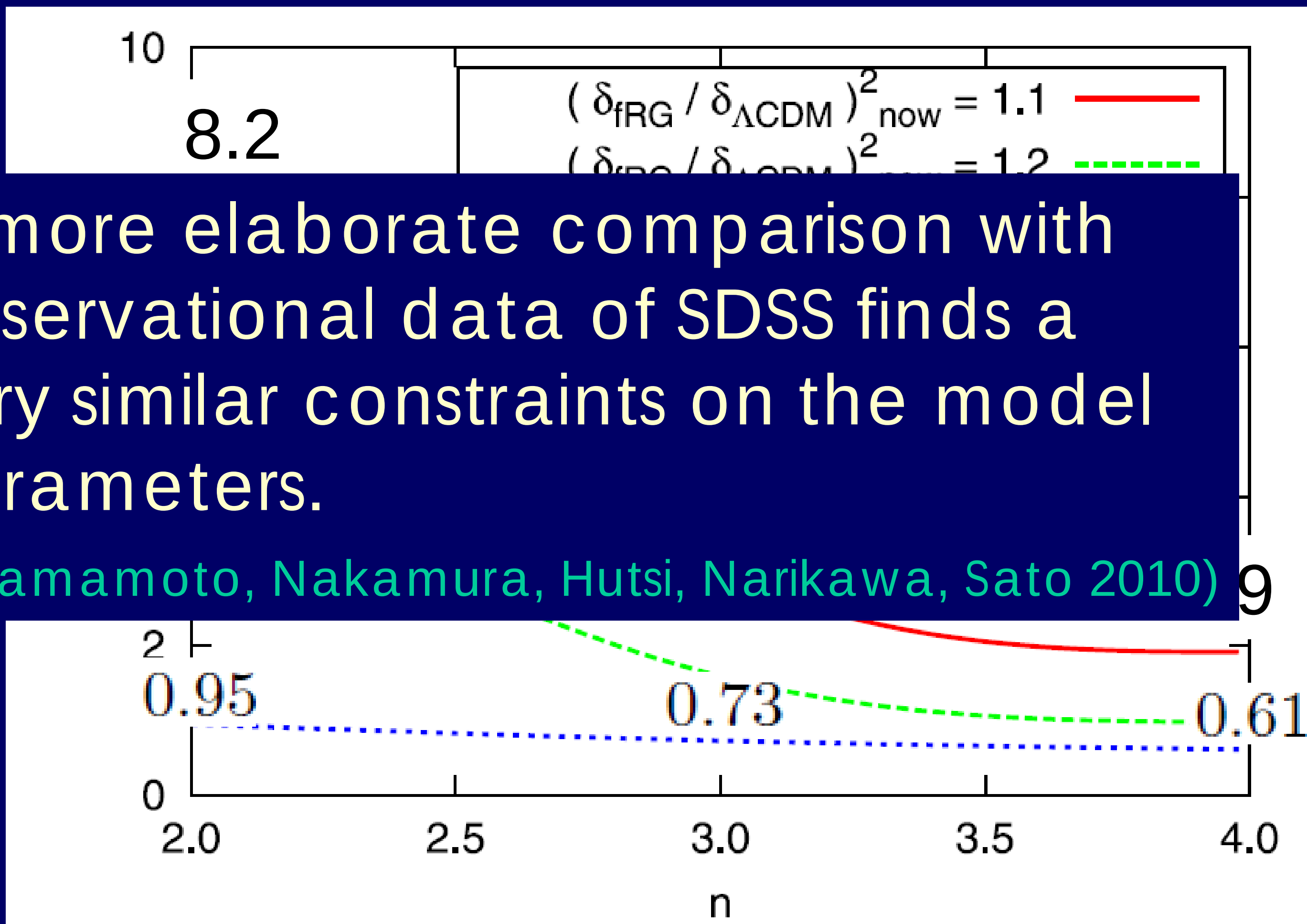


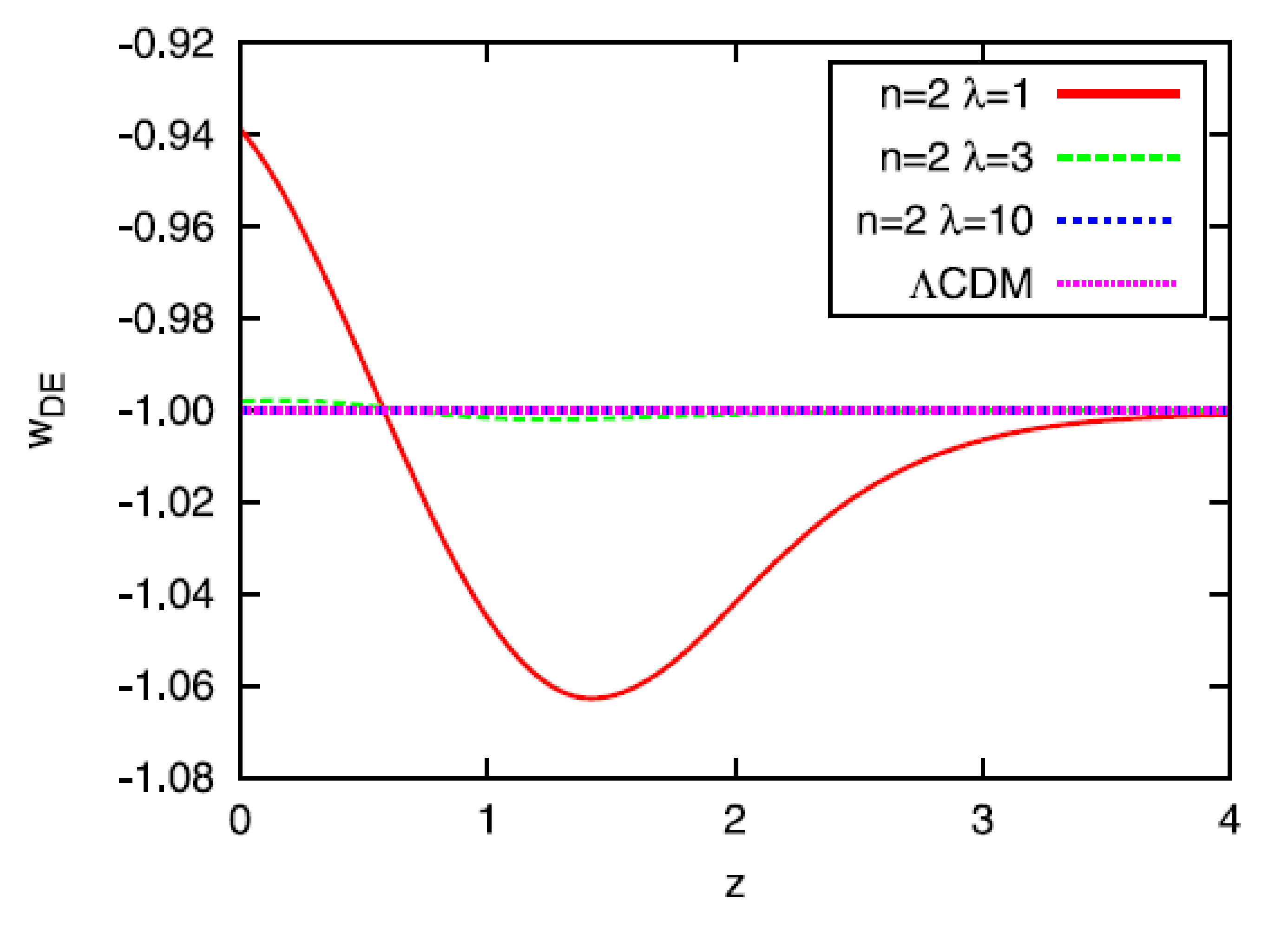
A more elaborate comparison with observational data of SDSS finds a very similar constraints on the model parameters.

(Yamamoto, Nakamura, Hutsi, Narikawa, Sato 2010)

For $n = 2$ $\lambda > 8.2$ to keep deviation < 10%.

$\lambda > 4.4$ to keep deviation < 20%.

For such large values of λ we cannot hope to observe an appreciable deviation from $w_{DE} = -1$.



§ Gravitational growth index of fluctuations: $\gamma(z)$

another observational measure to distinguish theories, defined by

$$\frac{d\ln\delta}{d\ln a} = \Omega_m(z)^{\gamma(z)}, \quad \text{or} \quad \gamma(z) = \frac{\log\left(\frac{\dot{\delta}}{H\delta}\right)}{\log\Omega_m}.$$

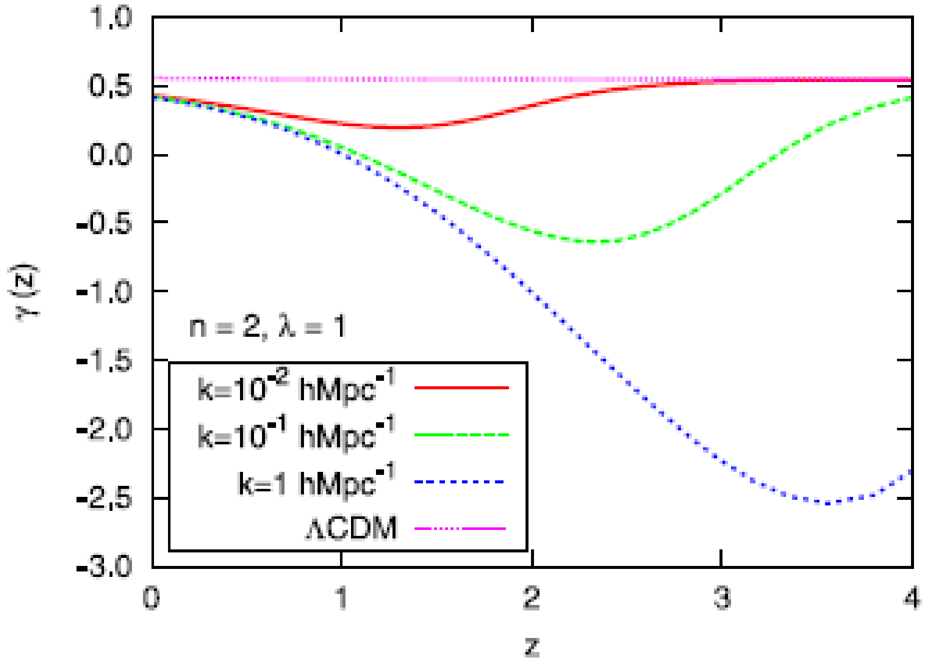
Its evolution is governed by the following equation.

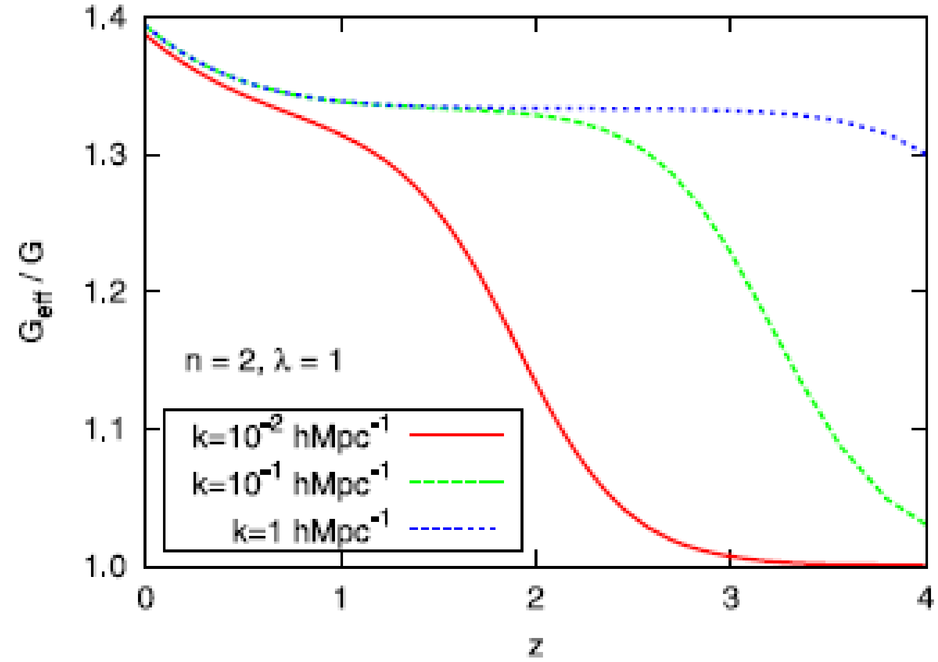
(Polarski and Gannouji 08)

$$-(1+z)\ln(1-\Omega_{\rm DE})\frac{d\gamma}{dz}$$
$$= -(1-\Omega_{\rm DE})^{\gamma} - \frac{1}{2}[1+3(2\gamma-1)w_{\rm DE}\Omega_{\rm DE}] + \frac{3}{2}\frac{G_{\rm eff}}{G}(1-\Omega_{\rm DE})^{1-\gamma},$$

In ΛCDM model, it is practically constant $\gamma(z) \simeq 0.55$

In f(R) gravity, it has a characteristic time evolution.

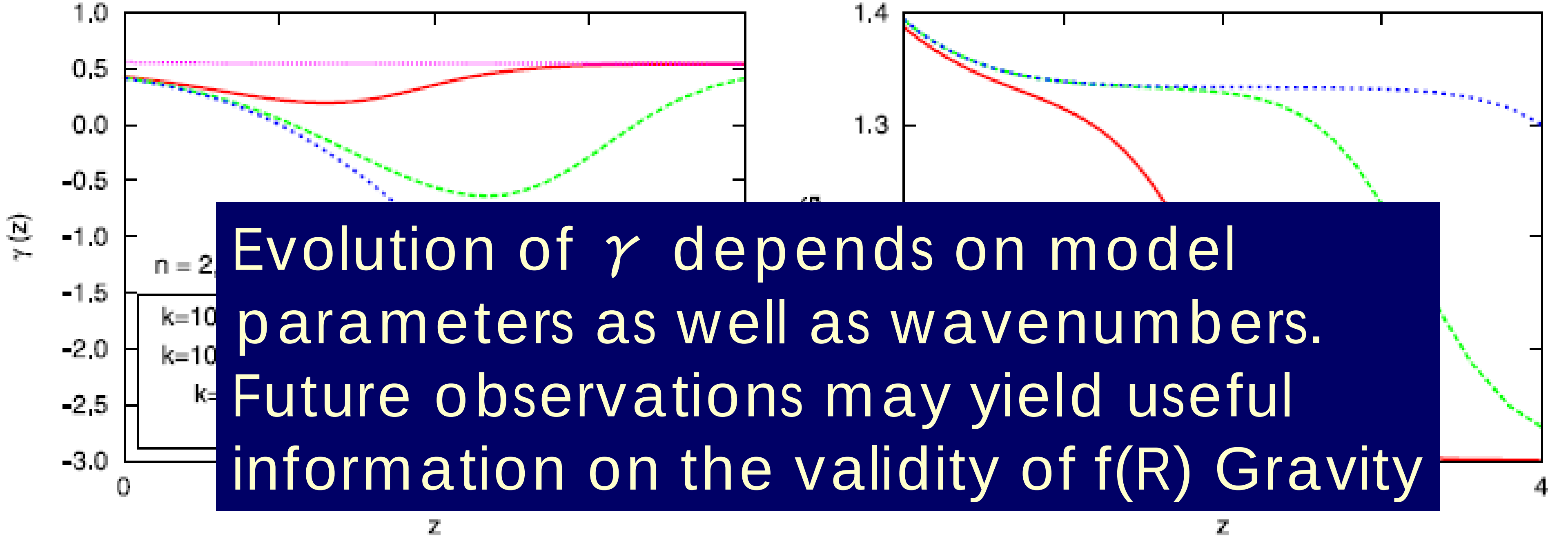




When Ω_{DE} is small, γ evolves according to

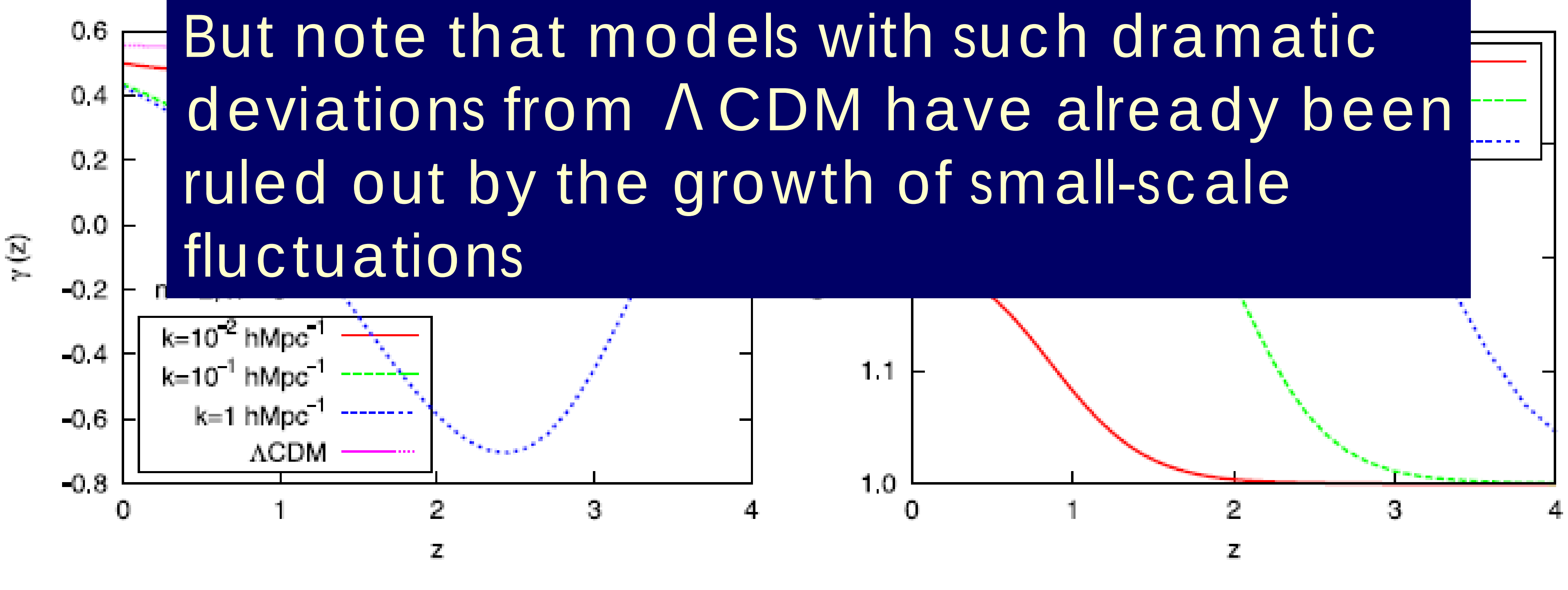
$$(1+z)\Omega_{\rm DE}\frac{d\gamma}{dz} = \boxed{\frac{3}{2}\left(\frac{G_{\rm eff}}{G}-1\right)} + \Omega_{\rm DE}\left[\frac{11}{2}\left(\gamma-\frac{6}{11}\right)-\frac{3}{2}(1-\gamma)\left(\frac{G_{\rm eff}}{G}-1\right)-\frac{3}{2}(2\gamma-1)(w_{\rm DE}+1)\right]$$

dominant at high redshift. γ starts to decrease as $G_{\rm eff}$ starts to deviate from G when k mode entered within the scalaron radius. Subsequent evolution is controlled by the second term.



Evolution of γ depends on model parameters as well as wavenumbers. Future observations may yield useful information on the validity of f(R) Gravity

But note that models with such dramatic deviations from ΛCDM have already been ruled out by the growth of small-scale fluctuations



$$m_v$$

Since f(R) gravity was introduced as an alternative to the cosmological constant, it is not interesting if deviation from ΛCDM model is manifest only in perturbed quantities.

As we have seen so far, however, anomalous growth of small-scale fluctuations imposes stringent constraints on the model parameters, allowing models with $w_{DE} = -1$ only.

Here I argue that small neutrino mass O(0.5eV) partially cancels it and models with an observable deviation from $w_{DE} = -1$ can be reconciled.

Free streaming of massive neutrinos erases small-scale fluctuations up to the scale

$$\frac{k}{a_0} \simeq 0.014 \left(\frac{\Omega_M h^2}{0.27}\right)^{1/2} \left(\frac{m_\nu}{1\mathrm{eV}}\right)^{1/2} h\mathrm{Mpc}^{-1}$$

(3 generations of degenerate mass)

This scale is to be compared with the scalaron radius today above which fluctuations grow anomalously.

$$\frac{k_0}{a_0} = \begin{cases} 3.2\lambda^{1.88} H_0 & (n=2) \\ 5.3\lambda^{2.43} H_0 & (n=3) \\ 5.0\lambda^{2.66} H_0 & (n=4) \end{cases}$$

$$\frac{k_0}{a_0} = \begin{cases} 1.07 \times 10^{-3} h\ \mathrm{Mpc}^{-1} & (n=2, \lambda=1) \\ 8.44 \times 10^{-3} h\ \mathrm{Mpc}^{-1} & (n=2, \lambda=3), \\ 8.12 \times 10^{-2} h\ \mathrm{Mpc}^{-1} & (n=2, \lambda=10) \end{cases}$$

§ Result

Relative deviation of the power spectrum from ΛCDM model remains smaller than 10% in the range

$k = 10^{-4} \sim 1h\ \mathrm{Mpc}^{-1}$

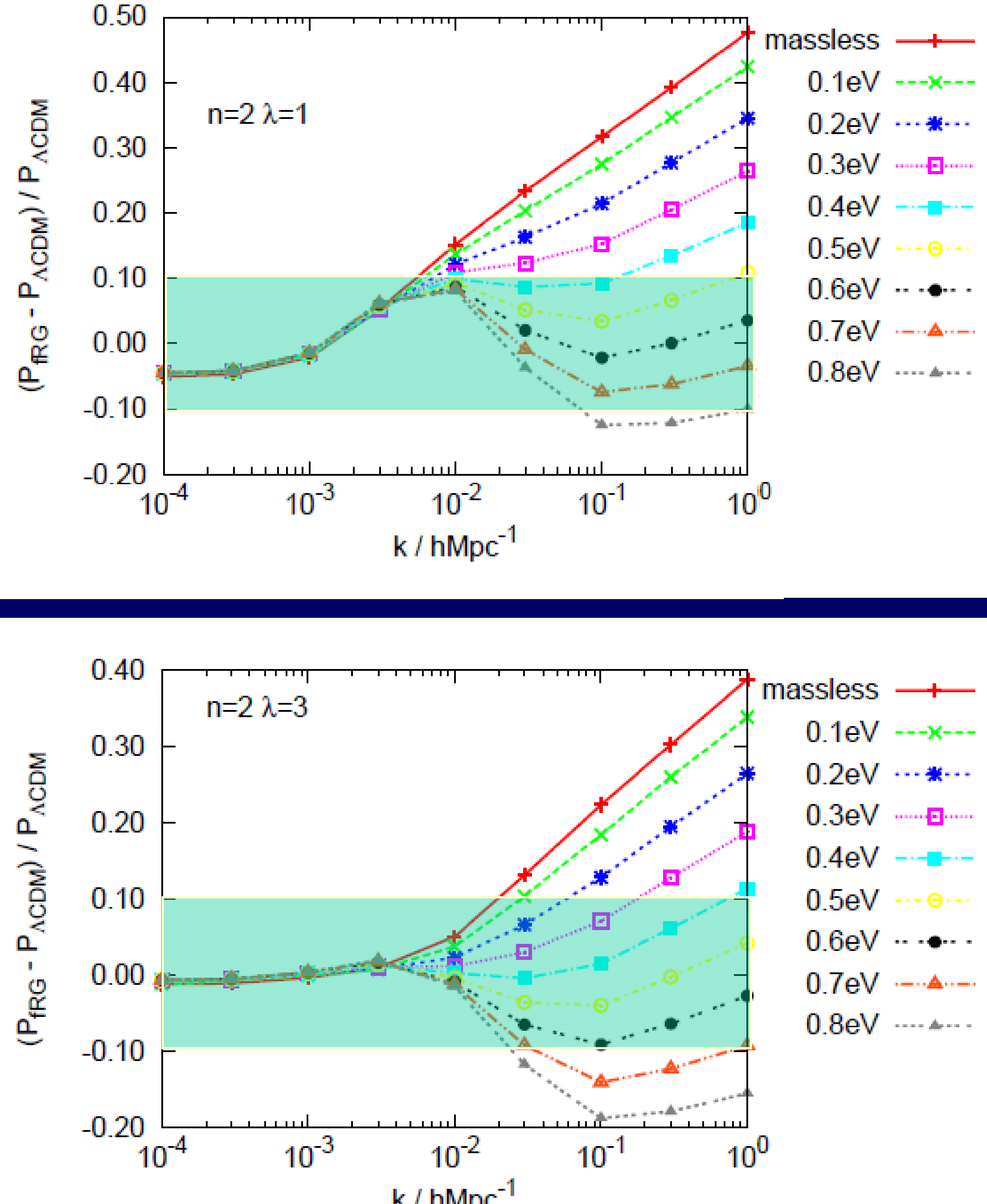
if neutrino mass is in the range

0.5—0.7eV

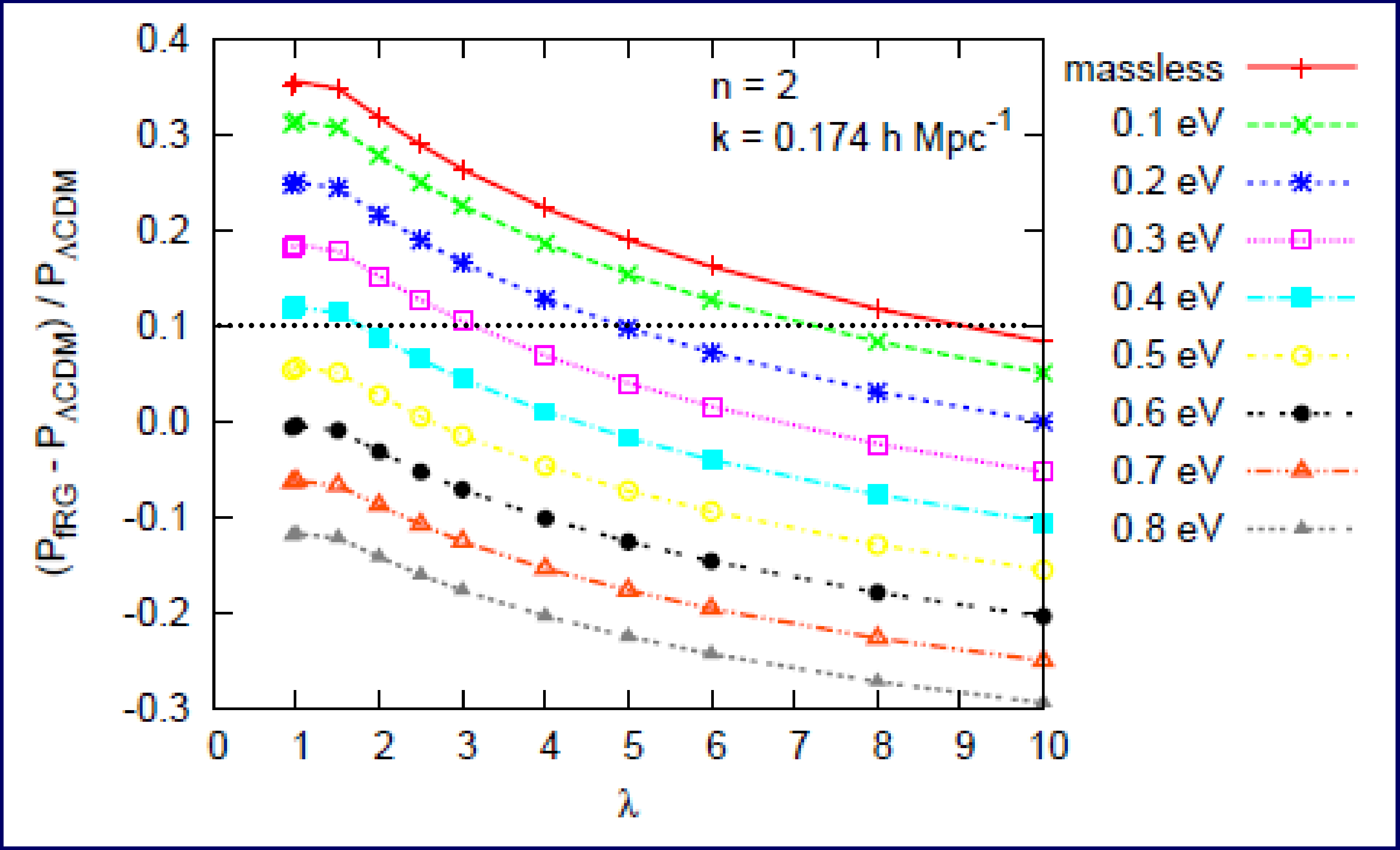
for $n=2,\ \lambda=1$ and

0.4—0.6eV

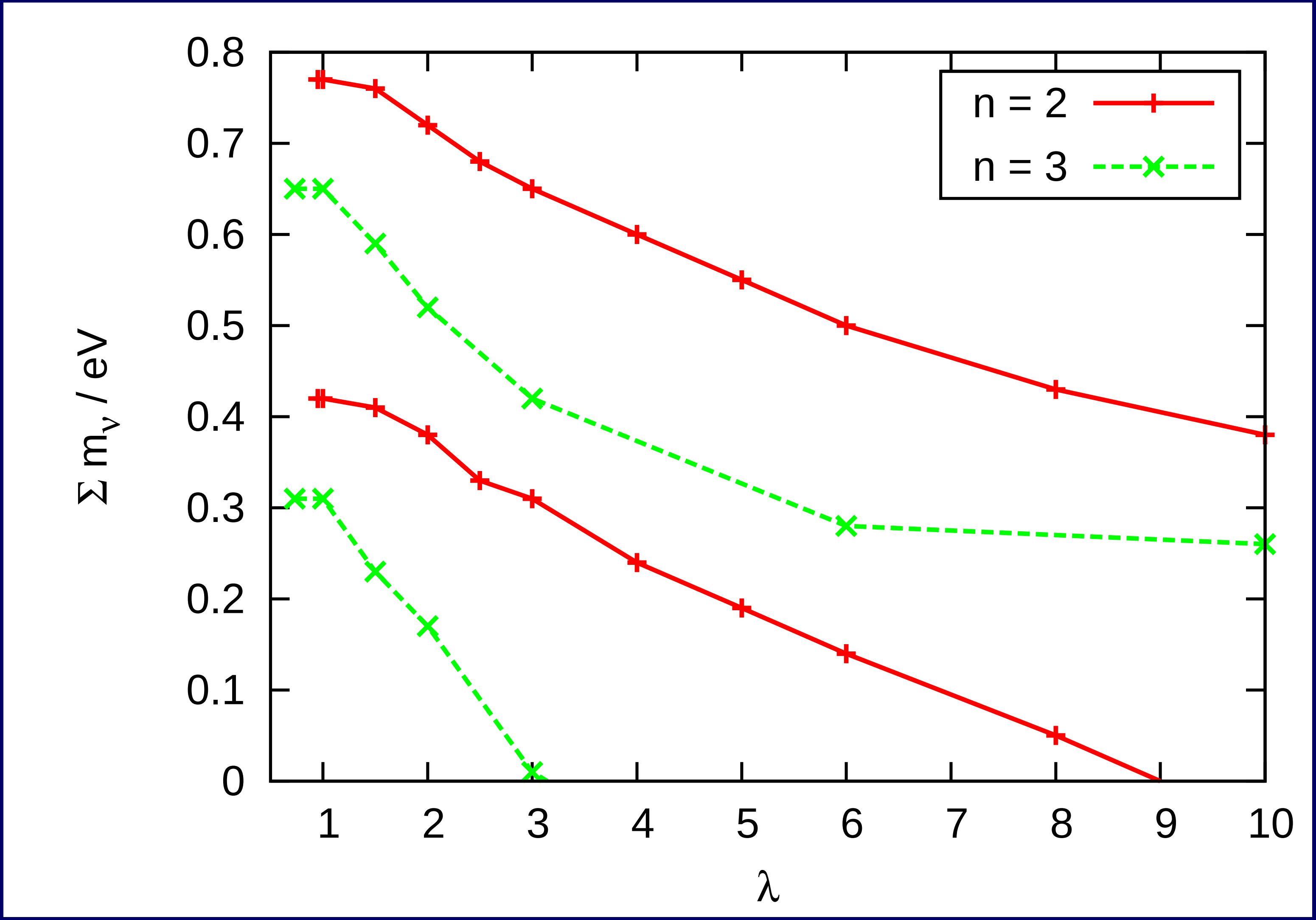
for $n=2,\ \lambda=3$.



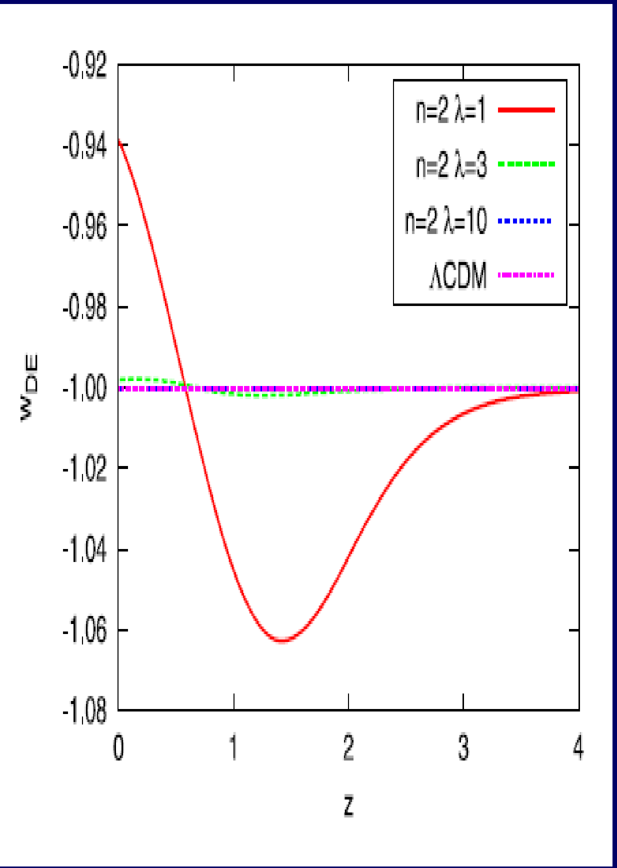
Relative deviation from ΛCDM model with massive neutrinos on scale $k = 0.174h\ \mathrm{Mpc}^{-1}$.

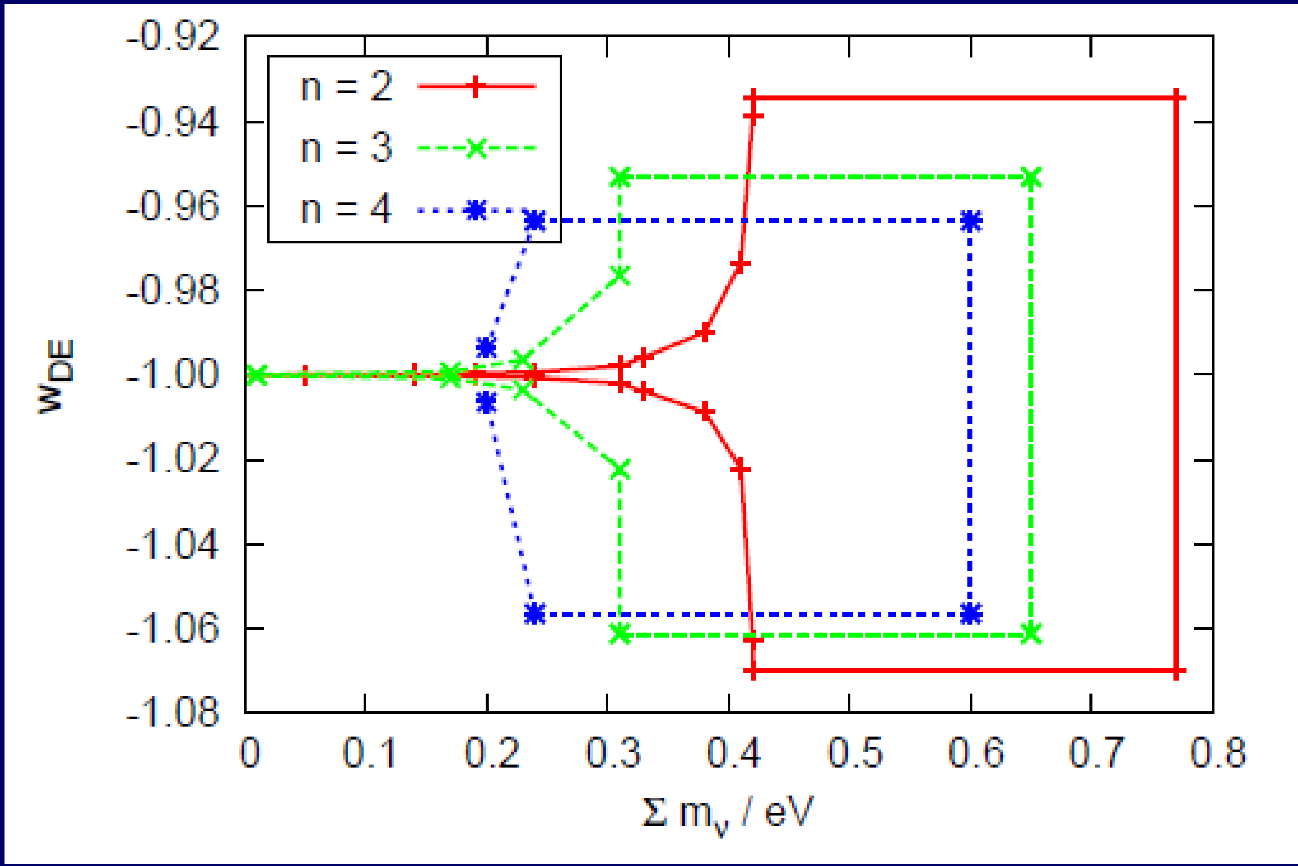


Range of the neutrino mass with which deviation of power spectrum at $k = 0.174h\ \mathrm{Mpc}^{-1}$ is smaller than 10%.

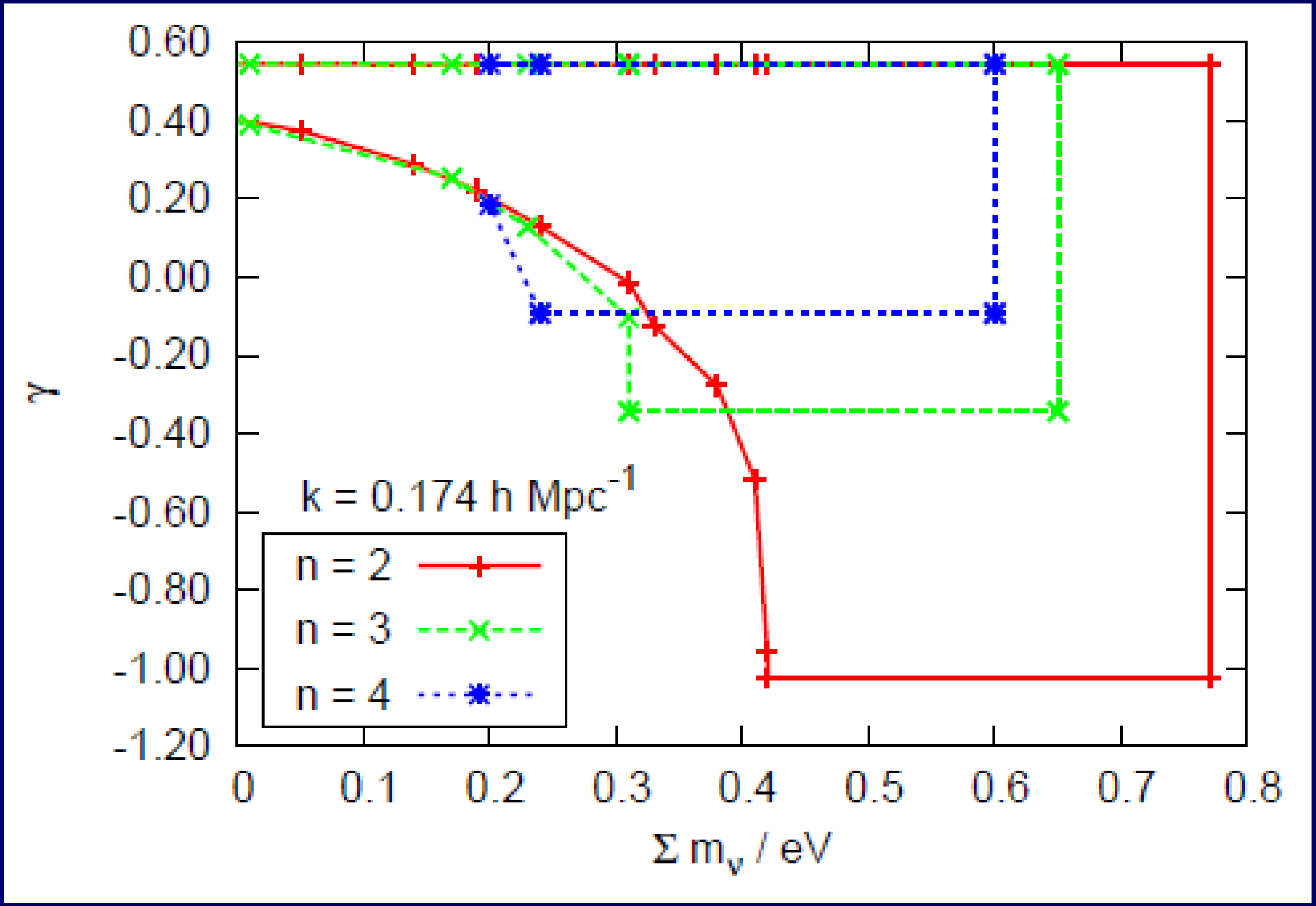


Neutrino mass makes models with smaller λ with larger deviation from $w_{DE} = -1$ viable.
Possible variation range of w_{DE} is depicted as below.





Similarly, the growth index on scale $k = 0.174h\ \mathrm{Mpc}^{-1}$ can also deviate significantly from the ΛCDM value.



Conclusion

f(R) gravity is not motivated by particle physics but can naturally unify inflation in the early Universe with current accelerated expansion.

Anomalous growth of density fluctuations on small scale constrains model parameters singnificantly, allowing only models with $w_{DE} = -1$.

However, small neutrino mass ~ 0.5 eV makes models with observable phantom crossing viable.