The Geometrical Scalar Gravity [GSG] versus Old days of Einstein-Grossmann proposal

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It is a metric theory of gravity with the following properties:

- The gravitational interaction is described by a scalar field \( \phi \);
- The field \( \phi \) satisfies a nonlinear dynamics;
- The theory satisfies the principle of general covariance.

In other words, this is not a theory restricted to the realm of special relativity; the theory was constructed following the geometrical scalar gravity [GSG].


The motivation to GSG could be understood as proceeding from the lemma:

**Theorem**

The solution for the empty space, meaning \( p = 0 \) and \( \mu = 0 \), is given by solving the dynamic equation (II):

\[
\nabla \phi = 0
\]

where \( \phi \) is the scalar field.

**Summary of GSG’s equations**

**Metric:**

\[
q^{\mu\nu} = \alpha q^{\mu\nu} + \frac{\beta}{2} \left( q^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - 2 \partial_\mu \phi \partial_\nu \phi \right)
\]

**Equation of motion:**

\[
\frac{\partial q^{\mu\nu}}{\partial x_\nu} = 0
\]

If the Newtonian limit is extrapolated:

\[
\phi = \frac{1}{2} \left( \frac{3}{8} \left( 1 + \frac{\mu}{\rho} \right) \rho + \frac{\mu}{\rho} \right)
\]

From the analogy of the planetary orbits, GSG gives:

\[
\frac{\partial \gamma^{\mu\nu}}{\partial x_\nu} = - \frac{3}{8} \rho \phi g^{\mu\nu}
\]


**Hydrostatic equation for GSG:**

\[
d\rho = -f(r) \frac{d\rho}{d\tau}
\]

**Hydrostatic equation for GR:**

\[
d\rho = -m(r) \frac{d\rho}{d\tau}
\]

**The Newtonian limit of the equations (I) and (II) is given by:**

\[
\nabla \phi = -\frac{1}{\rho}
\]

where the function \( f(r) \) is defined as \( f(r) = \rho \phi \).

**Causality Theorem on GSG**

We are still investigating the physical interpretation of different forms of applying the Cauchy’s theorem on GSG. Nevertheless even an unpretentious approach can exemplify the kind of new features which GSG brings to gravity and to the understanding of gravitational energy.

**Example**

For example, the dynamic equation (II) can be written in its integral form as:

\[
\int _{r_1} ^{r_2} \sqrt{\frac{g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi}{\partial_\mu \phi \partial_\nu \phi}} \cdot d\tau = \int _{\gamma_1} ^{\gamma_2} \sqrt{\frac{g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi}{\partial_\mu \phi \partial_\nu \phi}} \cdot d\gamma
\]

where \( \gamma_1 \) and \( \gamma_2 \) are constants.