Cosmology with non-minimal derivative coupling

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Plan

- Scalar fields: *minimal* and *nonminimal* coupling to gravity
- Horndeski theory
- Scalar fields with *nonminimal derivative coupling*
- Cosmological models with nonminimal derivative coupling
- Perturbations
- Summary

Based on
Sushkov, PRD **80**, 103505 (2009)
Saridakis, Sushkov, PRD **81**, 083510 (2010)
Sushkov, PRD **85**, 123520 (2012)
Skugoreva, Sushkov, Toporensky, PRD **88**, 083539 (2013)

Scalar fields minimally coupled to gravity

$$S = \int d^4x\sqrt{-g}\left[L_{GR} + L_S\right]$$

L_{GR} – *gravitational Lagrangian*

general relativity; $L_{GR} = R$
square gravity; $L_{GR} = R + cR^2$
$f(R)$-theories; $L_{GR} = f(R)$
etc...

L_S – *scalar field Lagrangian*;

ordinary STT; $L_S = -\epsilon(\nabla\phi)^2 - 2V(\phi)$
$\epsilon = +1$ – **canonical** scalar field
$\epsilon = -1$ – **phantom** or **ghost** scalar field with negative kinetic energy
$V(\phi)$ – potential of self-action
K-essence; $L_s = K(X)$ $[X = (\nabla\phi)^2]$
etc...

Scalar fields nonminimally coupled to gravity

Bergmann-Wagoner-Nordtvedt scalar-tensor theories

$$S=\int d^4x\sqrt{-g}\big[f(\phi)R-h(\phi)(\nabla\phi)^2-2V(\phi)\big]$$

$f(\phi)R \Longrightarrow$ nonminimal coupling between ϕ and R

Scalar fields nonminimally coupled to gravity

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Conformal transformation to the Einstein frame *(Wagoner, 1970)*:

$$\tilde{g}_{\mu\nu}=f(\phi)g_{\mu\nu};\quad \frac{d\phi}{d\psi}=f\left|fh+\frac{3}{2}\left(\frac{df}{d\phi}\right)^2\right|^{-1/2}$$

$$S=\int d^4x\sqrt{-\tilde{g}}\left[\tilde{R}-\epsilon(\tilde{\nabla}\psi)^2-2U(\psi)\right]$$

$\psi \Longrightarrow$ new scalar field

$U(\psi) \Longrightarrow$ new effective potential

$$\epsilon=\mathrm{sign}\left[fh+\tfrac{3}{2}\left(\tfrac{df}{d\phi}\right)^2\right]$$

Scalar fields nonminimally coupled to gravity

General scalar-tensor theories

$$S = \int d^4x \sqrt{-g}\left[F(\phi, R) - (\nabla\phi)^2 - 2V(\phi)\right]$$

$F(\phi, R) \Longrightarrow$ generalized nonminimal coupling between ϕ and R

Scalar fields nonminimally coupled to gravity

General scalar-tensor theories

$$S = \int d^4x\sqrt{-g}\big[F(\phi, R) - (\nabla\phi)^2 - 2V(\phi)\big]$$

$F(\phi, R) \Longrightarrow$ generalized nonminimal coupling between ϕ and R

Conformal transformation to the Einstein frame *(Maeda, 1989)*:

$$\tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}; \qquad \frac{\Omega^2}{16\pi} \equiv \left|\frac{\partial F(\phi, R)}{\partial R}\right|$$

$$S = \int d^4x\sqrt{-\tilde{g}}\left[\frac{\tilde{R}}{16\pi} - h(\phi)\psi^{-1}(\tilde{\nabla}\phi)^2 - \frac{3}{32\pi}\psi^{-2}(\tilde{\nabla}\psi)^2 + U(\phi, \psi)\right]$$

$\psi \equiv \Omega^2 \Longrightarrow$ new (second!) scalar field
$U(\phi, \psi) \Longrightarrow$ new effective potential

Some remarks:

- A nonminimal scalar field is conformally equivalent to the minimal one possessing some effective potential $V(\phi)$
- A behavior of the scalar field is *"encoded"* in the potential $V(\phi)$
- The potential $V(\phi)$ is a very important ingredient of various cosmological models: a slowly varying potential behaves like an effective cosmological constat providing one or more than one inflationary phases.
  An appropriate choice of $V(\phi)$ is known as a problem of fine tuning of the cosmological constant.

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Nonminimal derivative coupling generalization

$$S = \int d^4x\sqrt{-g}\big[R - g^{\mu\nu}\phi_{,\mu}\phi_{,\nu} - 2V(\phi)\big]$$

$F(\phi, R, R_{\mu\nu}, \dots)$

nonminimal coupling generalization!

$K(\phi, \phi_{,\mu}, \phi_{;\mu\nu}, \dots, R, R_{\mu\nu}, \dots)$

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nonminimal derivative coupling generalization!

Horndeski (Galileon) theory

In 1973, Horndeski derived the action of the most general scalar-tensor theories with second-order equations of motion
G.Horndeski, IJTP **10**, 363 (1974)

Horndeski Lagrangian:

$$\mathcal{L} = \sum_i \mathcal{L}_i$$

$$\begin{aligned}
\mathcal{L}_2 &= K(\phi, X), \qquad X = \tfrac{1}{2}(\nabla\phi)^2 \\
\mathcal{L}_3 &= G_3(\phi, X)\Box\phi, \\
\mathcal{L}_4 &= G_4(\phi, X)R + G_{4,X}(\phi, X)\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right], \\
\mathcal{L}_5 &= G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi \\
&\quad + \tfrac{1}{6}G_{5,X}(\phi, X)\left[(\Box\phi)^3 - 3(\Box\phi)(\nabla_\mu\nabla_\nu\phi)^2 + (\nabla_\mu\nabla_\nu\phi)^3\right],
\end{aligned}$$

Notice: *Generally, there are no conformal transformations which transform the Horndeski theory to the Einstein frame!*

Theory with nonminimal kinetic coupling

Action:

$$S=\int d^4x\sqrt{-g}\left\{\frac{R}{8\pi}-[\epsilon g^{\mu\nu}+\kappa G^{\mu\nu}]\phi_{,\mu}\phi_{,\nu}-2V(\phi)\right\}$$

Field equations:

$$G_{\mu\nu}=8\pi\big[T_{\mu\nu}+\kappa\Theta_{\mu\nu}\big],$$
$$[\epsilon g^{\mu\nu}+\kappa G^{\mu\nu}]\nabla_\mu\nabla_\nu\phi=V'_\phi$$

$$T_{\mu\nu}=\epsilon\nabla_\mu\phi\nabla_\nu\phi-\tfrac{1}{2}\epsilon g_{\mu\nu}(\nabla\phi)^2-g_{\mu\nu}V(\phi),$$
$$\Theta_{\mu\nu}=-\tfrac{1}{2}\nabla_\mu\phi\,\nabla_\nu\phi\,R+2\nabla_\alpha\phi\,\nabla_{(\mu}\phi R^\alpha_{\nu)}-\tfrac{1}{2}(\nabla\phi)^2G_{\mu\nu}+\nabla^\alpha\phi\,\nabla^\beta\phi\,R_{\mu\alpha\nu\beta}$$
$$+\nabla_\mu\nabla^\alpha\phi\,\nabla_\nu\nabla_\alpha\phi-\nabla_\mu\nabla_\nu\phi\,\Box\phi+g_{\mu\nu}\big[-\tfrac{1}{2}\nabla^\alpha\nabla^\beta\phi\,\nabla_\alpha\nabla_\beta\phi+\tfrac{1}{2}(\Box\phi)^2$$
$$-\nabla_\alpha\phi\,\nabla_\beta\phi\,R^{\alpha\beta}\big]$$

Notice: *The field equations are of second order!*

Cosmological models: General formulas

Sushkov, 2009; Saridakis, Sushkov, 2010

Ansatz:

$$ds^2 = -dt^2 + a^2(t)d\mathbf{x}^2,$$
$$\phi = \phi(t)$$

$a(t)$ *cosmological factor*, $H = \dot{a}/a$ *Hubble parameter*

Field equations:

$$3H^2 = 4\pi\dot{\phi}^2\left(\epsilon - 9\kappa H^2\right) + 8\pi V(\phi),$$
$$2\dot{H} + 3H^2 = -4\pi\dot{\phi}^2\left[\epsilon + \kappa\left(2\dot{H} + 3H^2 + 4H\ddot{\phi}\dot{\phi}^{-1}\right)\right] + 8\pi V(\phi),$$
$$\left[(\epsilon - 3\kappa H^2)a^3\dot{\phi}\right]^{\cdot} = -a^3\frac{dV(\phi)}{d\phi}$$

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$$\left[(\epsilon - 3\kappa H^2)a^3\dot{\phi}\right]^{\cdot} = -a^3\frac{dV(\phi)}{d\phi}$$

$$V(\phi) \equiv const \quad \Longrightarrow \quad \dot{\phi} = \frac{C}{a^3(\epsilon - 3\kappa H^2)}$$

Trivial model without kinetic coupling ($\kappa = 0$)

$$S = \int d^4x\sqrt{-g}\left[\frac{R}{8\pi} - (\nabla\phi)^2\right]$$

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Solution:

$$a_0(t) = t^{1/3}; \quad \phi_0(t) = \frac{1}{2\sqrt{3\pi}} \ln t$$

$$ds_0^2 = -dt^2 + t^{2/3} d\mathbf{x}^2$$

$t = 0$ *is an initial singularity*

Model without free kinetic term ($\epsilon = 0$)

$$S = \int d^4x\sqrt{-g}\left[\frac{R}{8\pi} - \kappa G^{\mu\nu}\phi_{,\mu}\phi_{,\nu}\right]$$

Model without free kinetic term ($\epsilon = 0$)

$$S = \int d^4x \sqrt{-g}\left[\frac{R}{8\pi} - \kappa G^{\mu\nu}\phi_{,\mu}\phi_{,\nu}\right]$$

Solution:

$$a(t) = t^{2/3}; \quad \phi(t) = \frac{t}{2\sqrt{3\pi|\kappa|}}, \quad \kappa < 0$$

$$ds_0^2 = -dt^2 + t^{4/3}d\mathbf{x}^2$$

$t = 0$ *is an initial singularity*

Model for an ordinary scalar field ($\epsilon = 1$) without potential ($V = 0$)

$$S = \int d^4x\sqrt{-g}\left\{\frac{R}{8\pi} - (g^{\mu\nu} + \kappa G^{\mu\nu})\phi_{,\mu}\phi_{,\nu}\right\}$$

Cosmological models: III. Ordinary scalar field without potential

Model for an ordinary scalar field ($\epsilon = 1$) without potential ($V = 0$)

$$S = \int d^4x\sqrt{-g}\left\{\frac{R}{8\pi} - (g^{\mu\nu} + \kappa G^{\mu\nu})\phi_{,\mu}\phi_{,\nu}\right\}$$

Asymptotic for $t \to \infty$:

$$a(t) \sim a_0(t) = t^{1/3}; \quad \phi(t) \sim \phi_0(t) = \frac{1}{2\sqrt{3\pi}}\ln t$$

Notice: *At large times the model with $\kappa \neq 0$ has the same behavior like that with $\kappa = 0$*

Cosmological models: III. Ordinary scalar field without potential

Asymptotics for early times

The case $\kappa < 0$:

$$a_{t\to 0} \approx t^{2/3}; \quad \phi_{t\to 0} \approx \frac{t}{2\sqrt{3\pi|\kappa|}}$$

$$ds^2_{t\to 0} = -dt^2 + t^{4/3} d\mathbf{x}^2$$

$t = 0$ is an initial singularity

The case $\kappa > 0$:

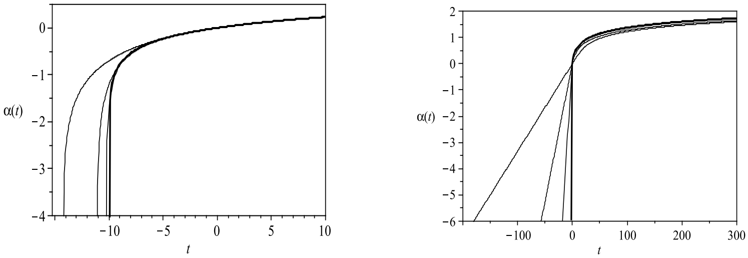
$$a_{t\to -\infty} \approx e^{H_\kappa t}; \quad \phi_{t\to -\infty} \approx C e^{-t/\sqrt{\kappa}}$$

$$ds^2_{t\to -\infty} = -dt^2 + e^{2H_\kappa t} d\mathbf{x}^2$$

de Sitter asymptotic with $H_\kappa = 1/\sqrt{9\kappa}$

Cosmological models: III. Ordinary scalar field without potential

Plots of $\alpha = \ln a$ in case $\kappa \neq 0$, $\epsilon = 1$, $V = 0$.



(a) $\kappa < 0$;
$\kappa = 0; -1; -10; -100$

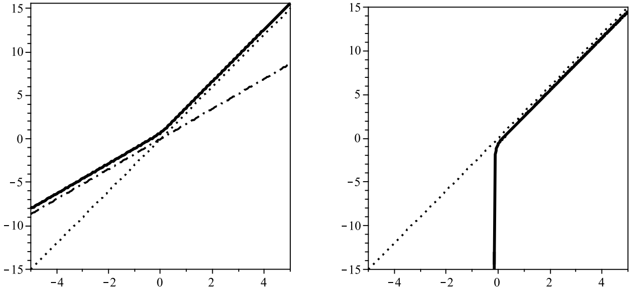
(b) $\kappa > 0$;
$\kappa = 0; 1; 10; 100$

De Sitter asymptotics: $\alpha(t) = t/3\sqrt{\kappa}$

Notice: *In the model with nonmnimal kinetic coupling one get de Sitter phase without any potential!*

Cosmological models: IV. Phantom scalar field without potential

Plots of $\alpha(t)$ in case $\kappa < 0$, $\epsilon = -1$, $V = 0$



(a) $(9k^2)^{-1} < \dot{\alpha}^2 < (3k^2)^{-1}$ (b) $\dot{\alpha}^2 > (3k^2)^{-1}$

De Sitter asymptotics:

$\alpha_1(t) = t/\sqrt{9|\kappa|}$ *(dash-dotted),*

$\alpha_2(t) = t/\sqrt{3|\kappa|}$ *(dotted).*

Models with the constant potential $V(\phi)=\Lambda/8\pi=const$

$$S=\int d^4x\sqrt{-g}\left[\frac{R-2\Lambda}{8\pi}-[\epsilon g^{\mu\nu}+\kappa G^{\mu\nu}]\phi_{,\mu}\phi_{,\nu}\right]$$

Models with the constant potential $V(\phi) = \Lambda/8\pi = const$

$$S = \int d^4x\sqrt{-g}\left[\frac{R-2\Lambda}{8\pi} - [\epsilon g^{\mu\nu} + \kappa G^{\mu\nu}]\phi_{,\mu}\phi_{,\nu}\right]$$

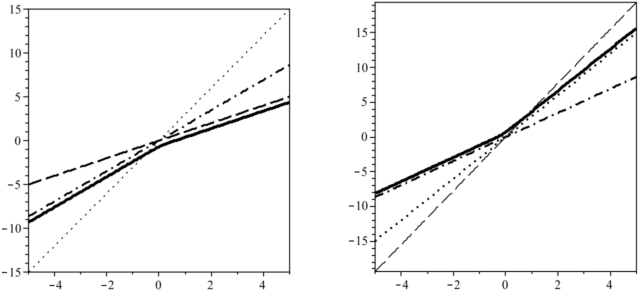
There are two exact de Sitter solutions:

I. $\alpha(t) = H_\Lambda t, \quad \phi(t) = \phi_0 = const,$

II. $\alpha(t) = \dfrac{t}{\sqrt{3|\kappa|}}, \quad \phi(t) = \left|\dfrac{3\kappa H_\Lambda^2 - 1}{8\pi\kappa}\right|^{1/2} t,$

$$H_\Lambda = \sqrt{\Lambda/3}$$

Plots of $\alpha(t)$ in case $\kappa > 0$, $\epsilon = 1$, $V = \Lambda$

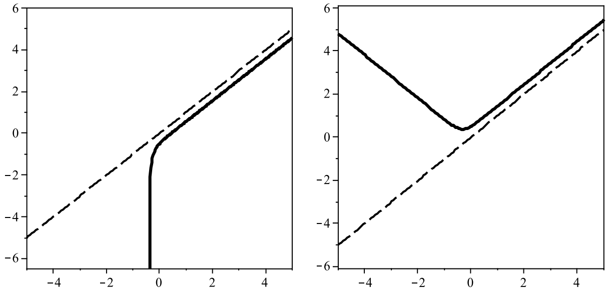


(a) $H_\Lambda^2 < \dot{\alpha}^2 < 1/9\kappa$ (b) $1/9\kappa < \dot{\alpha}^2 < 1/3\kappa < H_\Lambda^2$

De Sitter asymptotics:
$\alpha_1(t) = H_\Lambda t$ *(dashed),*
$\alpha_2(t) = t/\sqrt{9\kappa}$ *(dash-dotted),*
$\alpha_3(t) = t/\sqrt{3\kappa}$ *(dotted).*

Cosmological models: V. Constant potential

Plots of $\alpha(t)$ in cases $\kappa > 0$, $\epsilon = -1$ and $\kappa < 0$, $\epsilon = 1$

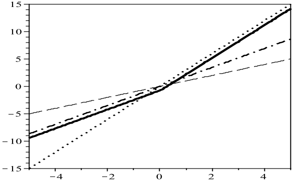


(a) $\kappa < 0$, $\epsilon = 1$ (b) $\kappa > 0$, $\epsilon = -1$

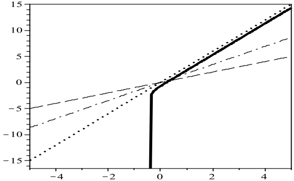
De Sitter asymptotic:
$\alpha_1(t) = H_\Lambda t$ *(dashed).*

Cosmological models: V. Constant potential

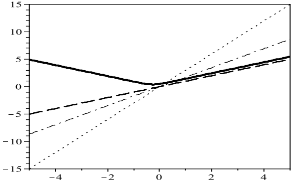
Plots of $\alpha(t)$ in case $\kappa < 0$, $\epsilon = -1$



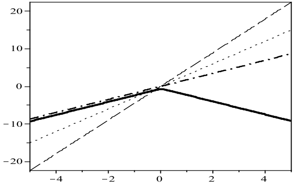
(a) $H_\Lambda^2 < 1/9|\kappa| < \dot{\alpha}^2 < 1/3|\kappa|$



(b) $H_\Lambda^2 < 1/9|\kappa| < 1/3|\kappa| < \dot{\alpha}^2$



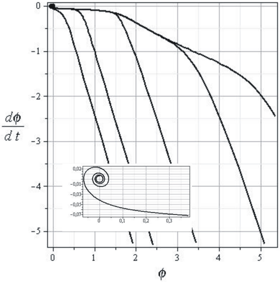
(c) $\dot{\alpha}^2 < H_\Lambda^2 < 1/9|\kappa|$

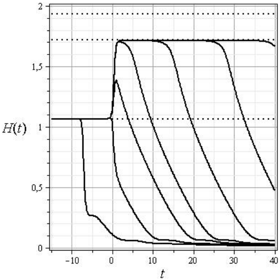


(d) $\dot{\alpha}^2 < 1/9|\kappa| < H_\Lambda^2$

De Sitter asymptotics: $\alpha_1(t) = H_\Lambda t$ *(dashed),*
$\alpha_2(t) = t/\sqrt{9\kappa}$ *(dash-dotted),* $\alpha_3(t) = t/\sqrt{3\kappa}$ *(dotted).*

$$S=\int d^4x\sqrt{-g}\left\{\frac{R}{8\pi}-\left[g^{\mu\nu}+\kappa G^{\mu\nu}\right]\phi_{,\mu}\phi_{,\nu}-m^2\phi^2\right\}$$





Asymptotics:

$H_{t\to-\infty}\approx 1/\sqrt{9\kappa}(1+\frac{1}{2}\kappa m^2)$ primary inflation

$H_{-\infty<t<\infty}\approx 1/\sqrt{3\kappa}\left(1\pm\sqrt{\frac{1}{6}\kappa m^2}\right)$ secondary inflation

$H_{t\to\infty}\approx\frac{2}{3t}\left[1-\frac{\sin 2mt}{2mt}\right]$ oscillatory asymptotic or “graceful” exit from inflation

Cosmological scenarios with nonminimal kinetic coupling and matter Sushkov, PRD 85 (2012) 123520

$$S=\int d^4x\sqrt{-g}\left\{\frac{R-2\Lambda}{8\pi}-[g^{\mu\nu}+\kappa G^{\mu\nu}]\phi_{,\mu}\phi_{,\nu}\right\}+S_{matter}$$

Cosmological scenarios with nonminimal kinetic coupling and matter Sushkov, PRD 85 (2012) 123520

$$S = \int d^4x\sqrt{-g}\left\{\frac{R-2\Lambda}{8\pi} - [g^{\mu\nu} + \kappa G^{\mu\nu}]\phi_{,\mu}\phi_{,\nu}\right\} + S_{matter}$$

Stress-energy tensor: $T_{\mu\nu} = \mathrm{diag}(\rho, p, p, p)$
Field equations:

$$3H^2 = 4\pi\dot{\phi}^2\left(1 - 9\kappa H^2\right) + \Lambda + 8\pi\rho,$$
$$2\dot{H} + 3H^2 = -4\pi\dot{\phi}^2\left[1 + \kappa\left(2\dot{H} + 3H^2 + 4H\ddot{\phi}\dot{\phi}^{-1}\right)\right] + \Lambda - 8\pi p$$

$$\left[(1 - 3\kappa H^2)a^3\dot{\phi}\right]^{\cdot} = 0$$

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$$\left[(1-3\kappa H^2)a^3\dot\phi\right]^{\cdot} = 0 \quad \Longrightarrow \quad \dot\phi = \frac{C}{a^3(1-3\kappa H^2)}$$

Realistic cosmological scenario: Master equation

Perfect fluid: $\rho_i = \dfrac{\rho_{i0}}{a^{3(1+w_i)}}, \quad p_i = w_i \rho_i$

Density parameters: $\Omega_{m_i 0} = \dfrac{\rho_{i0}}{\rho_{cr}}, \quad \Omega_{\phi 0} = \dfrac{C^2/2}{\rho_{cr}}, \quad \Omega_{\Lambda 0} = \dfrac{\Lambda}{8\pi \rho_{cr}}$

$\rho_{cr} = 3H_0^2/8\pi$

Modified Friedmann equation:

$$H^2 = H_0^2 \left[\Omega_{\Lambda 0} + \sum_i \frac{\Omega_{m_i 0}}{a^{3(1+w_i)}} + \frac{\Omega_{\phi 0}(1 - 9\kappa H^2)}{a^6 (1 - 3\kappa H^2)^2} \right]$$

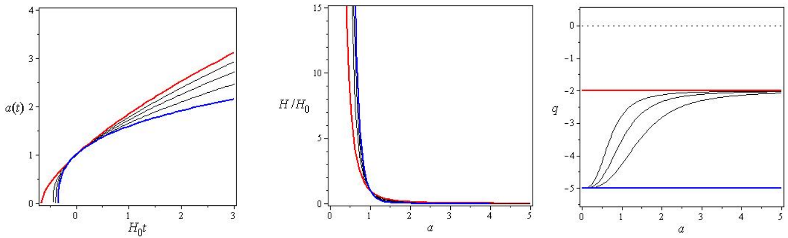
Constraint for parameters:

$$\Omega_{\Lambda 0} + \sum_i \Omega_{m_i 0} + \frac{\Omega_{\phi 0}(1 - 9\kappa H_0^2)}{(1 - 3\kappa H_0^2)^2} = 1$$

Realistic cosmological scenario: Simple model

No coupling: $\kappa = 0$; No cosmological constant: $\Lambda = 0$
Pressureless matter: $p = 0$; $\rho = \rho_0/a^3$

$$H^2 = H_0^2 \left[\frac{\Omega_{m0}}{a^3} + \frac{\Omega_{\phi 0}}{a^6} \right]; \quad \Omega_{m0} + \Omega_{\phi 0} = 1$$



Scale factor $a(t)$ | Hubble parameter $H(a)$ | Acceleration parameter $q(a)$

$a(t) \propto (t - t_*)^\delta$

$H = \dot{a}/a$

$H(a) \to \infty$ at $a \to 0$

$H(a) \to 0$ at $a \to \infty$

$q = \ddot{a}a/\dot{a}^2$

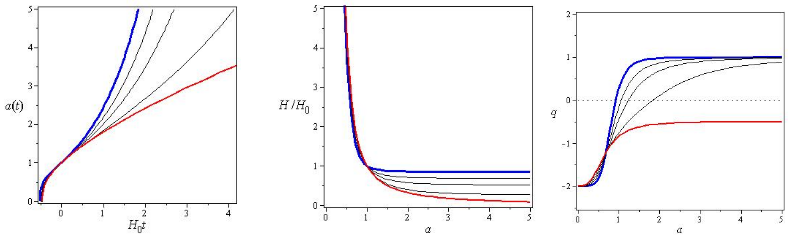
q is negative

$\Omega_{\phi 0} = 0;\ 0.25;\ 0.5;\ 0.75;\ 1$, and $\Omega_{m0} = 1 - \Omega_{\phi 0}$

Realistic cosmological scenario: Simple model with cosmological constant

No coupling: $\kappa = 0$

$$H^2 = H_0^2\left[\Omega_{\Lambda 0} + \frac{\Omega_{m0}}{a^3} + \frac{\Omega_{\phi 0}}{a^6}\right]; \quad \Omega_{\Lambda 0} + \Omega_{m0} + \Omega_{\phi 0} = 1$$



Scale factor $a(t)$ — Hubble parameter $H(a)$ — Acceleration parameter $q(a)$

$H = \dot{a}/a$ — $q = \ddot{a}a/\dot{a}^2$

$a(t) \propto (t - t_*)^{1/3}$ at $t \to t_*$ — $H(a) \to \infty$ at $a \to 0$ — q changes its sign

$a(t) \propto e^{H_\Lambda t}$ at $t \to \infty$ — $H(a) \to H_\Lambda$ at $a \to \infty$

$\Omega_{\phi 0} = 0.23$, $\Omega_{\Lambda 0} = 0;\ 0.07;\ 0.27;\ 0.47;\ 0.73$, and $\Omega_{m0} = 1 - \Omega_{\phi 0} - \Omega_{\Lambda 0}$

Realistic cosmological scenario with kinetic coupling $\kappa > 0$

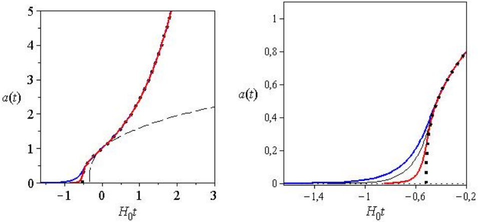
$$H^2 = H_0^2 \left[\Omega_{\Lambda 0} + \frac{\Omega_{m0}}{a^3} + \frac{\Omega_{\phi 0}(1 - 9\kappa H^2)}{a^6 (1 - 3\kappa H^2)^2} \right]$$

Universal asymptotic:

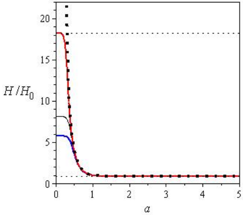
$$H \to H_\kappa = 1/\sqrt{9\kappa} \quad \text{at} \quad a \to 0$$

Notice: *The asymptotic $H \approx H_\kappa$ at early cosmological times is* only *determined by the coupling parameter κ and does* not *depend on other parameters!*

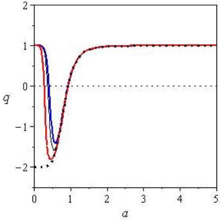
Realistic cosmological scenario: Numerical solutions



Scale factor $a(t)$



Hubble parameter $H(a)$



Acceleration parameter q

Realistic cosmological scenario: Estimations

$H_\kappa t_f \sim 60$ e-folds

$t_f \simeq 10^{-35}$ sec the end of initial inflationary stage

$\Rightarrow H_\kappa = 1/\sqrt{9\kappa} \simeq 6 \times 10^{36}$ sec^{-1}

$$\kappa \simeq 10^{-74} \text{ sec}^2 \quad \text{or} \quad l_\kappa = \kappa^{1/2} \simeq 10^{-27} \text{ cm}$$

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$H_0 \sim 70\ (\mathrm{km/sec})/\mathrm{Mpc} \sim 10^{-18}$ sec^{-1} Present Hubble parameter

$\gamma = 3\kappa H_0^2 \simeq 10^{-109}$ Extremely small!

$$H^2 = H_0^2 \left[\Omega_{\Lambda 0} + \frac{\Omega_{m0}}{a^3} + \frac{\Omega_{\phi 0}(1 - 9\kappa H^2)}{a^6(1 - 3\kappa H^2)^2} \right] \Rightarrow \Omega_{\Lambda 0} + \Omega_{m0} + \Omega_{\phi 0} \approx 1$$

$$\Omega_{\Lambda 0} = 0.73, \Omega_{\phi 0} = 0.23, \Omega_{m0} = 0.04 \quad \Rightarrow \quad q_0 = 0.25$$

Perturbed FRW metric: $g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$

background metric: $\bar{g}_{00} = -1,\ \bar{g}_{i0} = \bar{g}_{0i} = 0,\ \bar{g}_{ij} = a^2(t)\delta_{ij}$

metric perturbations (Newtonian gauge):

$$\begin{aligned} h_{00} &= -\Phi, \quad h_{0i} = h_{i0} = 0, \\ h_{ij} &= a^2\left(\Psi\delta_{ij} + \partial_i C_j + \partial_j C_i + \Theta_{ij}\right) \end{aligned}$$

Perturbed SET:

$$\begin{aligned} \delta T_{00} &= \rho\Phi + \delta\rho, \\ \delta T_{i0} &= -(\rho + p)\delta u_i, \\ \delta T_{ij} &= p h_{ij} + a^2\delta_{ij}\delta p \end{aligned}$$

Perturbed scalar field:

$$\phi\left(\mathbf{x}, t\right) = \bar{\phi}_0\left(t\right) + \delta\phi\left(\mathbf{x}, t\right)$$

Perturbations: Perturbed equations

Perturbed Einstein equations

$$
\begin{aligned}
&P_1\Phi + P_2\dot\Psi + P_3\nabla^2\Psi + P_4\delta\dot\phi + P_5\nabla^2\delta\phi + P_6\delta\rho = 0,\\
&Q_1\Phi + Q_2\dot\Phi + Q_3\nabla^2\Phi + Q_4\dot\Psi + Q_5\ddot\Psi + Q_6\nabla^2\Psi\\
&\qquad + Q_7\delta\dot\phi + Q_8\delta\ddot\phi + Q_9\nabla^2\delta\phi + Q_{10}\delta p = 0,\\
&R_1\partial_i\Phi + R_2\partial_i\dot\Psi + R_3\partial_i\delta\phi + R_4\partial_i\delta\dot\phi + R_5\nabla^2\dot C_i + R_6\delta u_i = 0,\\
&S_1\partial_i\partial_j\Phi + S_2\partial_i\partial_j\Psi + S_3\partial_i\partial_j\delta\phi\\
&\qquad + S_4\left(\partial_i\dot C_j + \partial_j\dot C_i\right) + S_5\left(\partial_i\ddot C_j + \partial_j\ddot C_i\right)\\
&\qquad + S_6\dot\theta_{ij} + S_7\ddot\theta_{ij} + S_8\nabla^2\theta_{ij} = 0
\end{aligned}
$$

P_i, Q_j, R_k, S_l – coefficients depending on unperturbed values

$$
P_1 = -8\pi\left(\rho + \tfrac{9}{2}\kappa H^2\dot\phi^2\right), \quad \text{and so on}\ldots
$$

Tensor perturbations

Two polarizations: $\theta_{ij} \longrightarrow \theta^{+}, \ \theta^{\times}$

Equation for tensor modes

$$(1+4\pi\kappa\dot{\phi}^2)\ddot{\theta}+\left(3H+4\pi\kappa(2\dot{\phi}\ddot{\phi}+3H\dot{\phi}^2)\right)\dot{\theta}+\frac{k^2}{a^2}(1-4\pi\kappa\dot{\phi}^2)\theta=0$$

The case $4\pi\kappa\dot{\phi}^2 \ll 1$*:*

$$\ddot{\theta}+3H\dot{\theta}+\frac{k^2}{a^2}\theta=0$$

The case $4\pi\kappa\dot{\phi}^2 \gg 1$*:*

$$\ddot{\theta}+\left(2\frac{\ddot{\phi}}{\dot{\phi}}+3H\right)\dot{\theta}-\frac{k^2}{a^2}\theta=0$$

Conclusions

- The non-minimal kinetic coupling provides an *essentially new* inflationary mechanism which does not need any fine-tuned potential.
- At early cosmological times the coupling κ-terms in the field equations are dominating and provide the quasi-De Sitter behavior of the scale factor: $a(t) \propto e^{H_\kappa t}$ with $H_\kappa = 1/\sqrt{9\kappa}$ and $\kappa \simeq 10^{-74}$ sec^2 (or $l_\kappa \equiv \kappa^{1/2} \simeq 10^{-27}$ cm)
- The model provides a natural mechanism of epoch change without any fine-tuned potential.

THANKS FOR YOUR ATTENTION!