Constant-roll inflation

Hayato Motohashi

Kavli Institute for Cosmological Physics

University of Chicago

Martin, HM, Suyama, 1211.0083

HM, Starobinsky, Yokoyama, 1411.5021; in prep.

2016.05.08-14 SW10: Hot topics in modern cosmology

Inflation

Canonical single field inflation

$$3H^2 = \frac{\dot{\phi}^2}{2} + V$$

$$-2\dot{H} = \dot{\phi}^2$$

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V}{\partial \phi} = 0$$

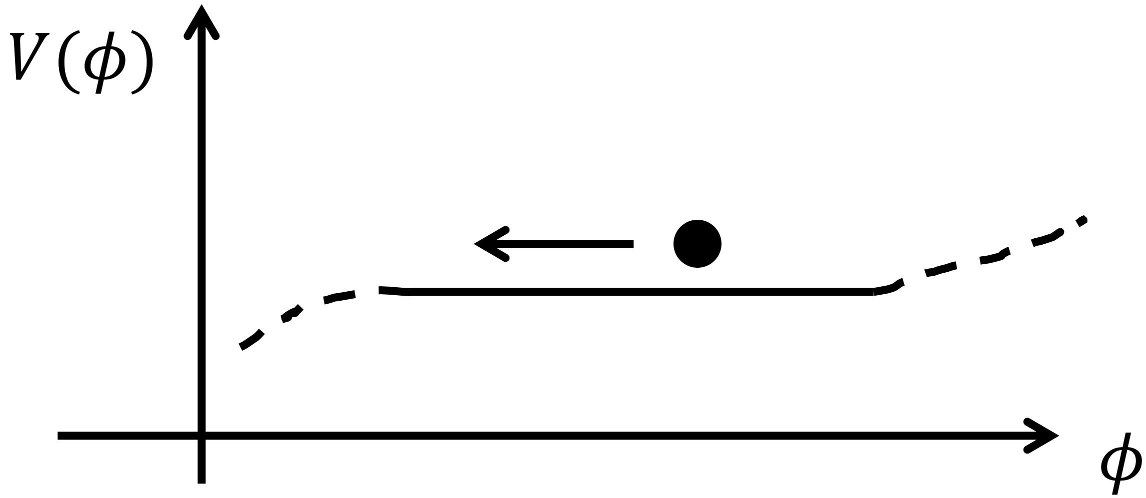
Slow-roll approximation $\ddot{\phi} \simeq 0$

Ultra slow-roll : constant potential $\ddot{\phi} = -3H\dot{\phi}$

Constant-roll generalization $\ddot{\phi} = \beta H\dot{\phi}$ (β: constant)

Ultra slow-roll inflation

Kinney, gr-qc/0503017



Constant potential $V \simeq V_0$

$$\ddot{\phi} = -3H\dot{\phi} \;\Rightarrow\; \dot{\phi} \propto a^{-3}$$

$$3H^2 = \frac{\dot{\phi}^2}{2} + V \simeq V_0$$

Slow-roll parameter

$$\epsilon_1 \equiv -\frac{\dot{H}}{H^2} = \frac{\dot{\phi}^2}{2H^2} \propto a^{-6}$$

Ultra slow-roll inflation

Curvature perturbation on superhorizon scales

$$\zeta_k = A_k + B_k \int \frac{dt}{a^3 \epsilon_1}$$

Constant mode

Slow-roll $\epsilon_1 \simeq$ const $\ll 1$: decaying mode ✓

Ultra slow-roll $\epsilon_1 \propto a^{-6}$: growing mode ✗

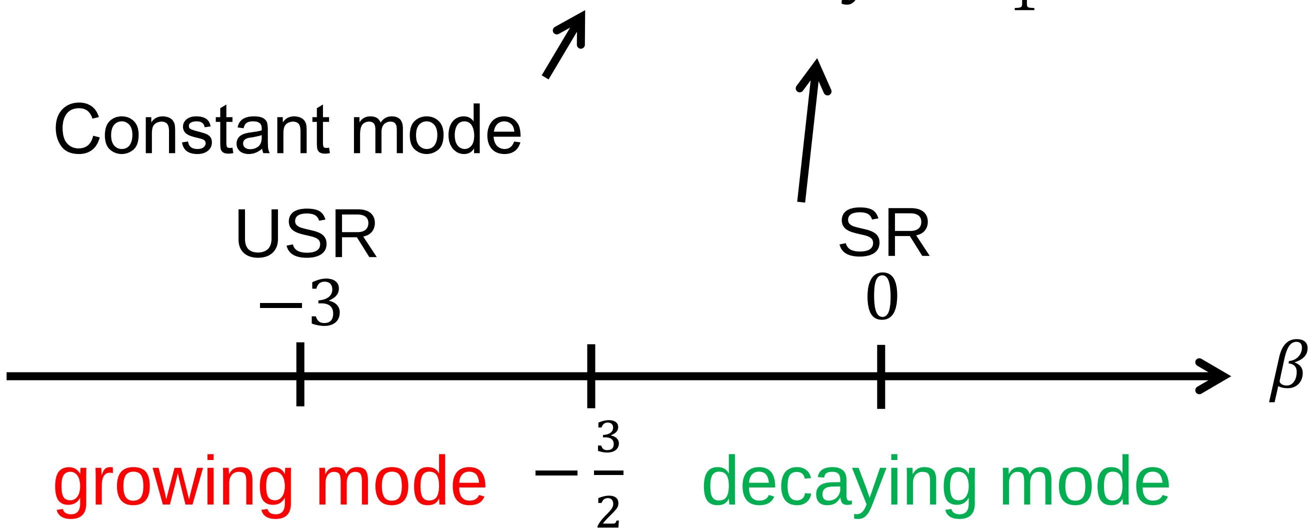
Generalization

Martin, HM, Suyama, 1211.0083

What about if $\epsilon_1 \propto a^{2\beta}$? $(a^3\epsilon_1)^{-1} \propto a^{-3-2\beta}$

Curvature perturbation on superhorizon scales

$$\zeta_k = A_k + B_k \int \frac{dt}{a^3 \epsilon_1}$$



There exists a potential with this status for which inflaton rolls with constant rate $\ddot{\phi} = \beta H \dot{\phi}$.

Constant-roll potential

HM, Starobinsky, Yokoyama, 1411.5021

With constant-roll condition

$$\ddot{\phi} = \beta H \dot{\phi}$$

the evolution equation $-2\dot{H} = \dot{\phi}^2$ or $\dot{\phi} = -2\frac{dH}{d\phi}$ implies

$$\frac{d^2 H}{d\phi^2} = -\frac{\beta}{2} H$$

Thus, H is given by linear combination of $e^{\pm\sqrt{-\beta/2}\phi}$.

The potential is then given by

$$V = 3H^2 - 2\left(\frac{dH}{d\phi}\right)^2$$

which is linear combination of $e^{\pm\sqrt{-2\beta}\phi}$.

For each $V(\phi)$, we can get $\phi(t), H(t), a(t)$ analytically.

Constant-roll potential

The potential includes

Abbott, Wise, 1984
Lucchin, Matarese, 1985

a) $V \propto e^{\sqrt{-2\beta}\phi}$ with $\beta < 0$: Power-law inflation

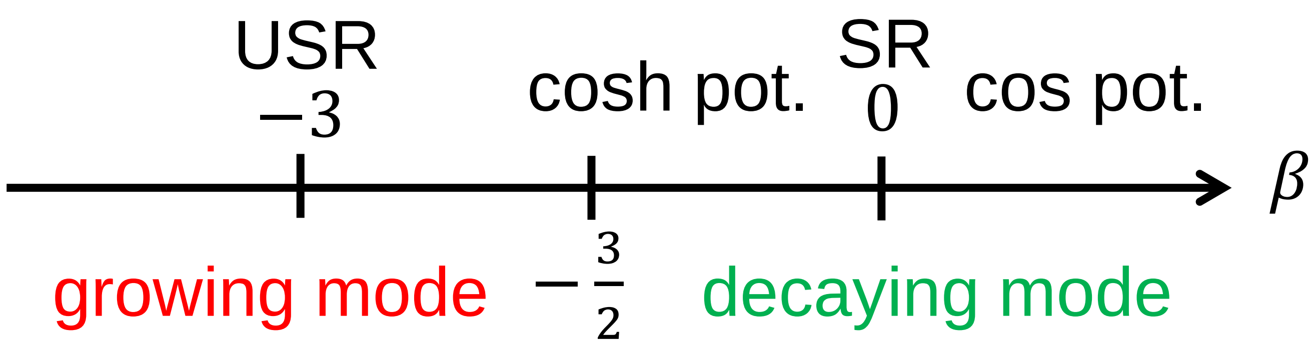
✗ but $r = 8(1 - n_s) \approx 0.28$ is too large.

Barrow, 1994

b) $V \propto \cosh(\sqrt{-2\beta}\phi)$ + const with $\beta < 0$: Known

c) $V \propto \cos(\sqrt{2\beta}\phi)$ + const with $\beta > 0$: New potential

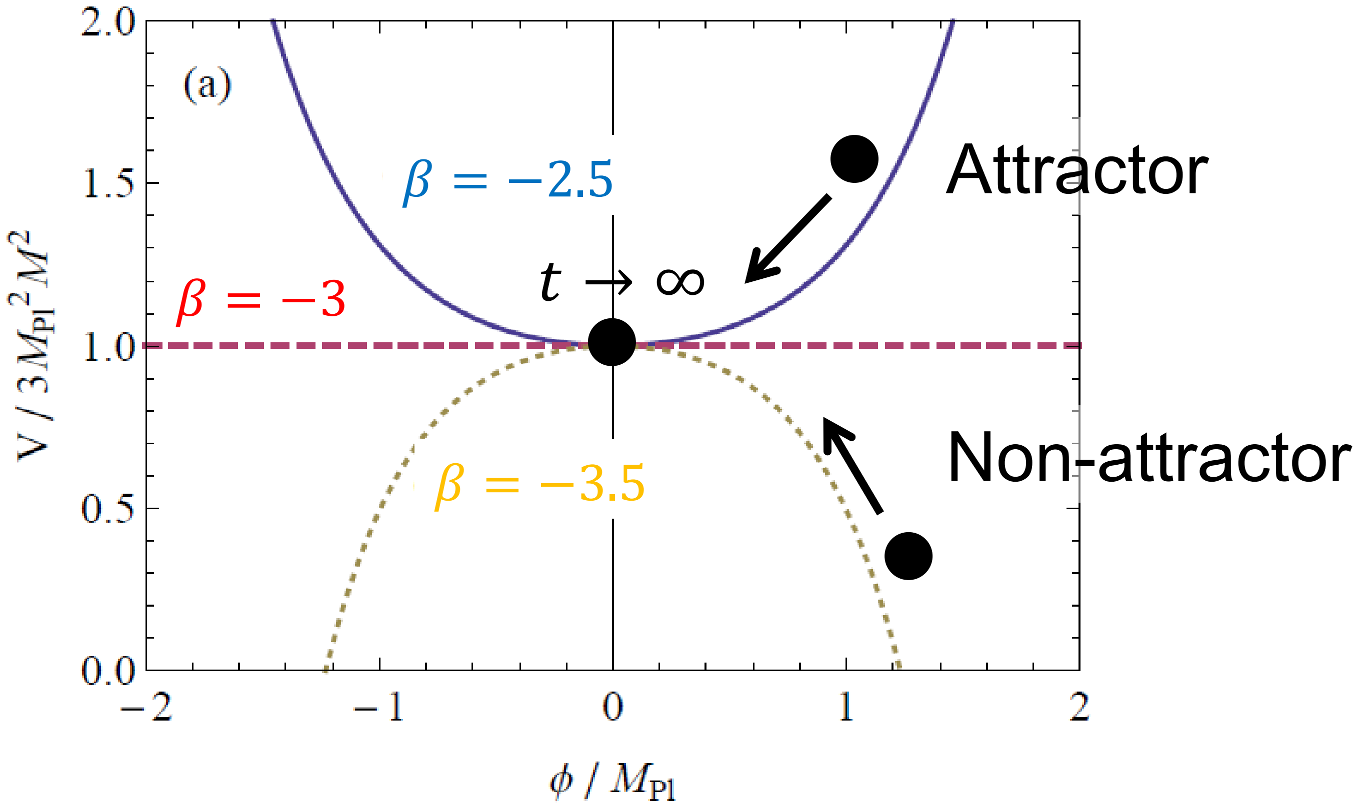
Let us further investigate b) cosh pot. and c) cos pot.



cosh potential ($\beta < 0$)

$$\frac{V(\phi)}{M^2 M_{Pl}^2} = 3\left[1 - \frac{3+\beta}{6}\left\{1 - \cosh\left(\sqrt{-2\beta}\,\frac{\phi}{M_{Pl}}\right)\right\}\right]$$

Assume inflation ends before $\phi = 0$ (e.g. waterfall).



cosh potential ($\beta < 0$)

$$\frac{V(\phi)}{M^2 M_{Pl}^2} = 3\left[1 - \frac{3+\beta}{6}\left\{1 - \cosh\left(\sqrt{-2\beta}\,\frac{\phi}{M_{Pl}}\right)\right\}\right]$$

Assume inflation ends before $\phi = 0$ (e.g. waterfall).

Analytic solution

$$\frac{\phi(t)}{M_{Pl}} = \sqrt{\frac{2}{-\beta}}\ln\left[\coth\left(\frac{-\beta}{2}Mt\right)\right] \to 0 \quad (t \to \infty)$$

$$\frac{H(t)}{M} = \coth(-\beta M t) \to 1$$

$$a(t) \propto \sinh^{-1/\beta}(-\beta M t) \to e^{Mt}$$

Slow-roll parameters ($\epsilon_1 \equiv -\dot{H}/H^2$, $\epsilon_{n+1} \equiv \dot{\epsilon}_n / H\epsilon_n$)

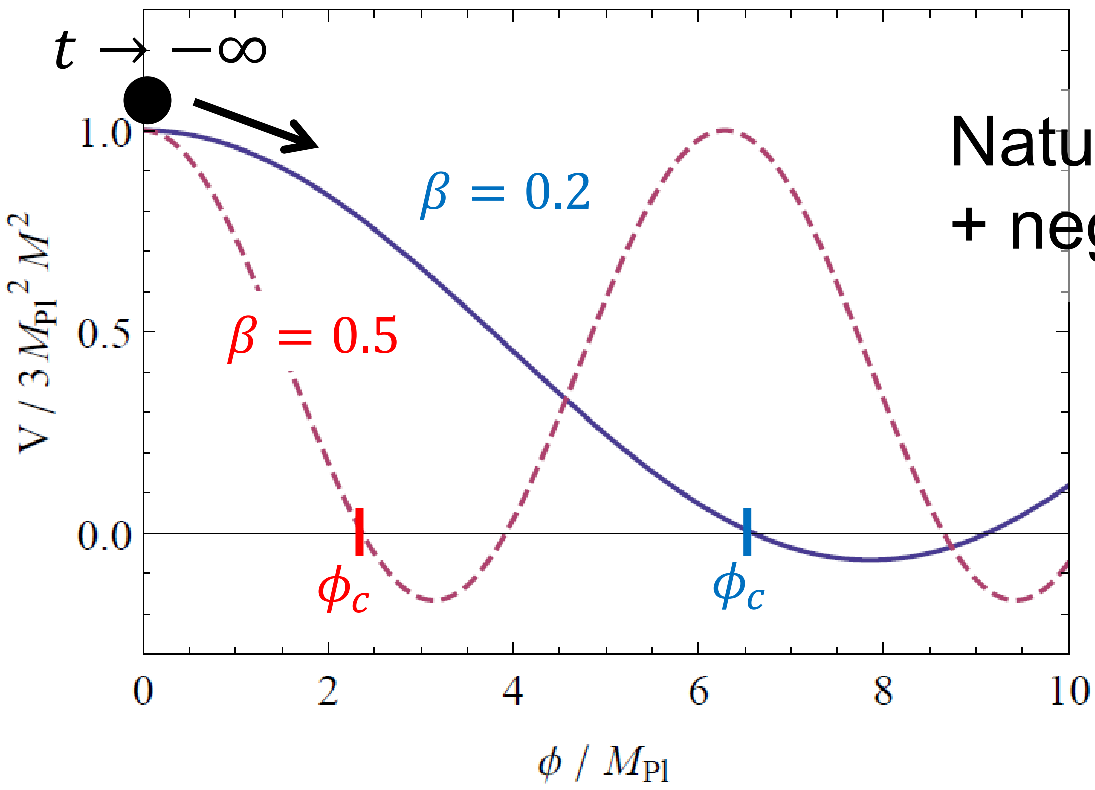
$$2\epsilon_1 = \epsilon_{2n+1} = -\beta / \cosh^2(-\beta M t) \to 0$$

$$\epsilon_{2n} = 2\beta \tanh^2(-\beta M t) \to 2\beta$$

New cos potential ($\beta > 0$)

$$\frac{V(\phi)}{M^2 M_{Pl}^2} = 3\left[1 - \frac{3+\beta}{6}\left\{1 - \cos\left(\sqrt{2\beta}\frac{\phi}{M_{Pl}}\right)\right\}\right]$$

Assume inflation ends before $\phi = \phi_c$ (e.g. waterfall).



Natural inflation
+ negative CC

New cos potential ($\beta > 0$)

$$\frac{V(\phi)}{M^2 M_{Pl}^2} = 3\left[1 - \frac{3+\beta}{6}\left\{1 - \cos\left(\sqrt{2\beta}\frac{\phi}{M_{Pl}}\right)\right\}\right]$$

Assume inflation ends before $\phi = \phi_c$ (e.g. waterfall).

Analytic solution

$$\frac{\phi(t)}{M_{Pl}} = 2\sqrt{\frac{2}{\beta}}\arctan(e^{\beta M t}) \to 0 \qquad (t \to -\infty)$$

$$\frac{H(t)}{M} = -\tanh(\beta M t) \to 1$$

$$a(t) \propto \cosh^{-1/\beta}(\beta M t) \to e^{Mt}$$

Slow-roll parameters

$$2\epsilon_1 = \epsilon_{2n+1} = 2\beta/\sinh^2(\beta M t) \to 0$$

$$\epsilon_{2n} = 2\beta\tanh^2(\beta M t) \to 2\beta$$

Same asymptotic values

Curvature perturbation

Mukhanov-Sasaki equation

$$v_k'' + \left(k^2 - \frac{z''}{z}\right)v_k = 0$$

where $v_k = \sqrt{2} z \zeta_k$ with $z = a\sqrt{\epsilon_1}$.

$$\epsilon_1 \equiv -\dot{H}/H^2$$

$$\epsilon_{n+1} \equiv \dot{\epsilon}_n / H\epsilon_n$$

Without approximation,

$$\frac{z''}{z} = a^2H^2\left(2 - \epsilon_1 + \frac{3}{2}\epsilon_2 + \frac{1}{4}\epsilon_2^2 - \frac{1}{2}\epsilon_1\epsilon_2 + \frac{1}{2}\epsilon_2\epsilon_3\right)$$

For both cosh potential and cos potential,

$$\frac{z''}{z} \to \frac{(\beta+2)(\beta+1)}{\tau^2} = \frac{\nu^2 - 1/4}{\tau^2}$$

where

$$\nu \equiv \sqrt{(\beta+2)(\beta+1) + 1/4} = |\beta + 3/2|$$

Curvature perturbation

Since spectral index is given by

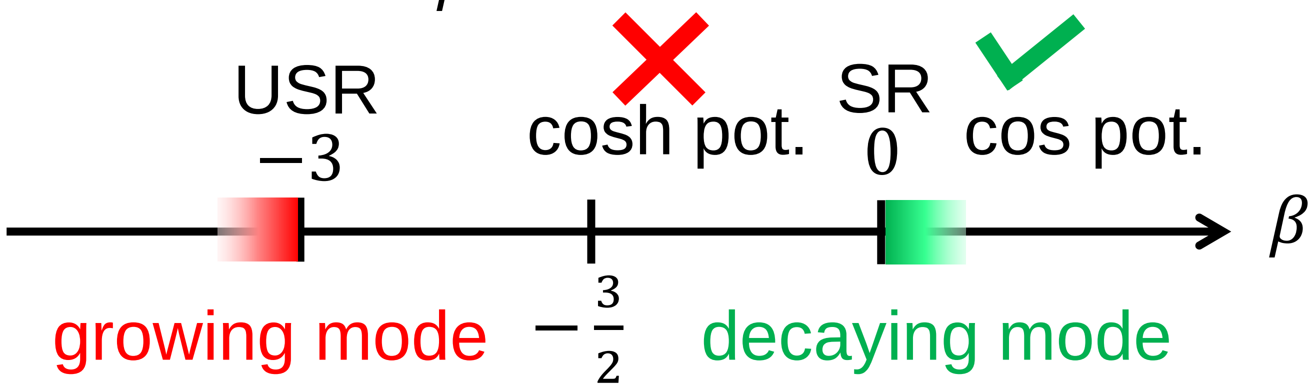
$$n_s - 1 = 3 - 2\nu$$

we obtain

$$\beta = \frac{n_s - 7}{2} \quad \text{or} \quad \frac{1 - n_s}{2}$$

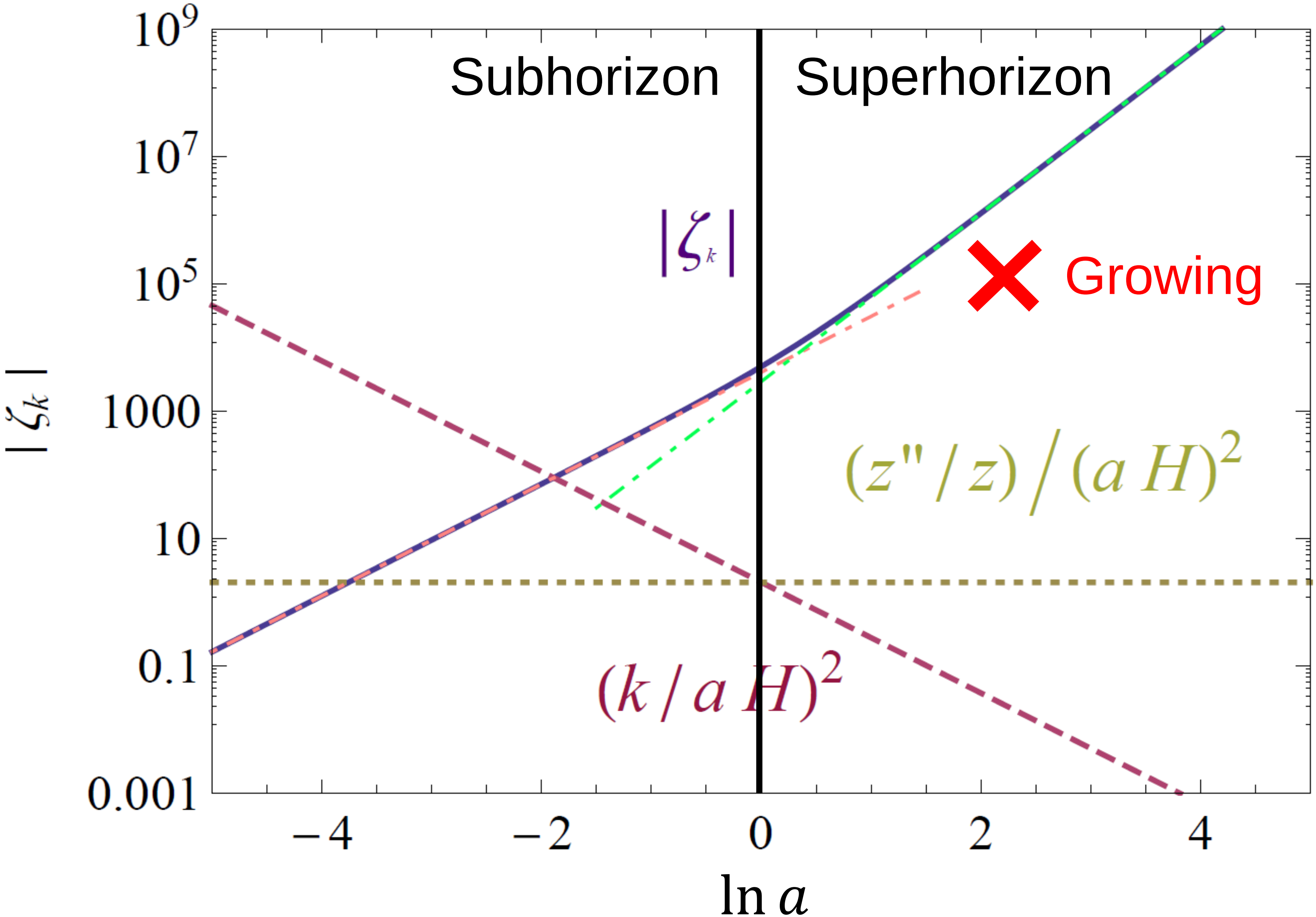
For example, for $n_s = 0.96$,

$$\beta = -3.02 \;\; \text{or} \;\; 0.02$$

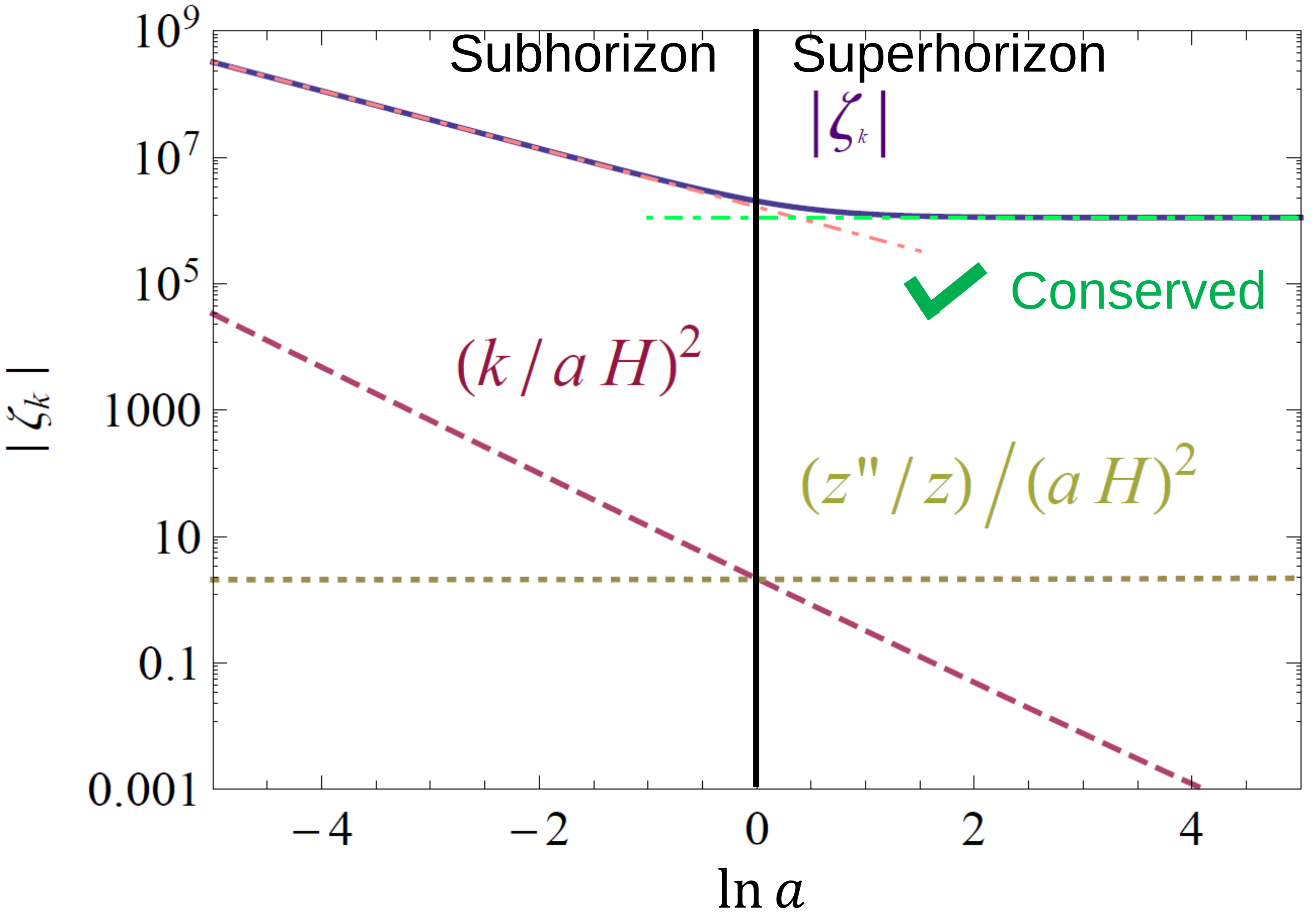


ζ_k is conserved on superhorizon scales for the cos pot.

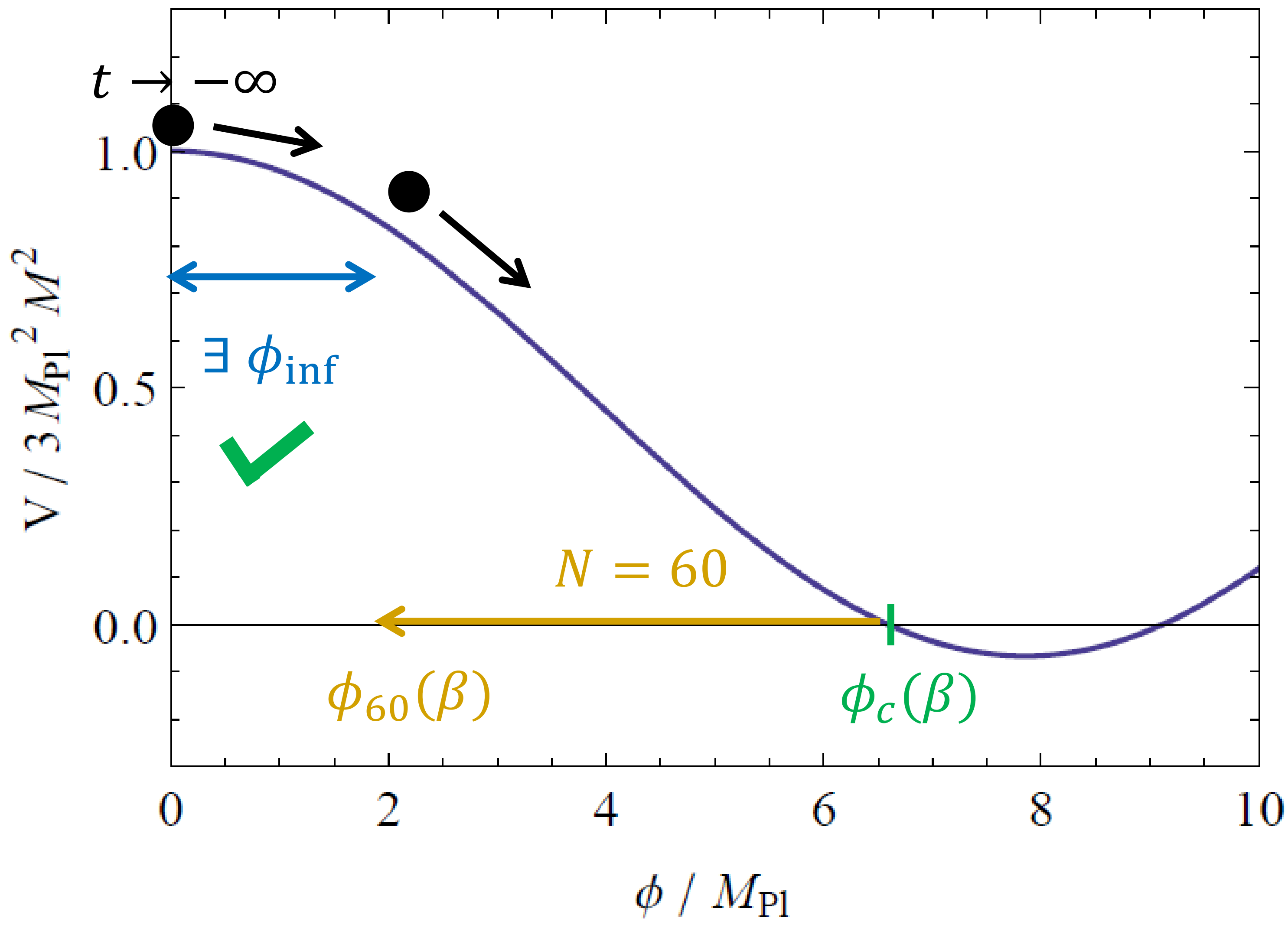
Curvature perturbation $\beta = -3.02$



Curvature perturbation $\beta = 0.02$

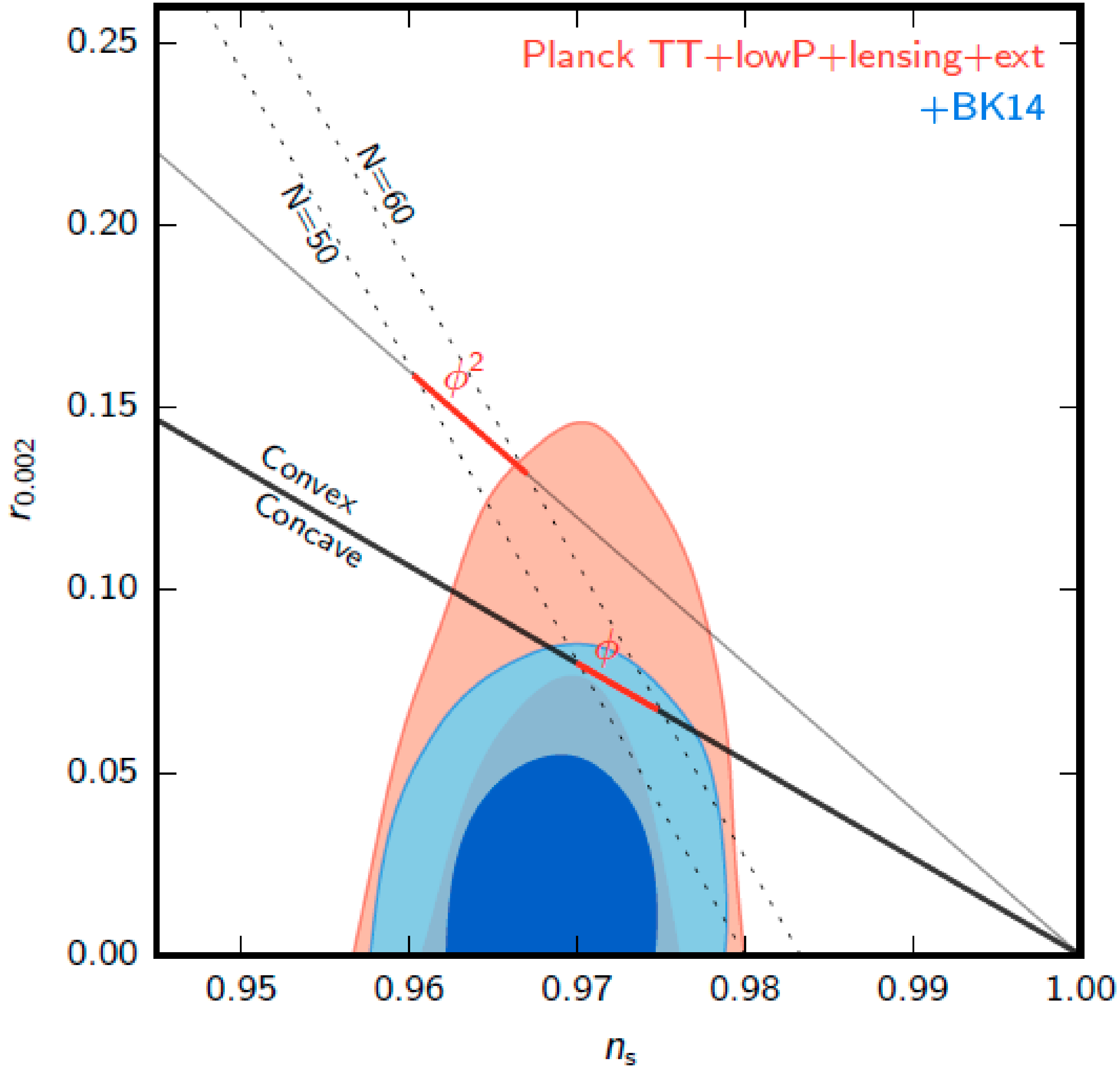


Sufficient number of efolds



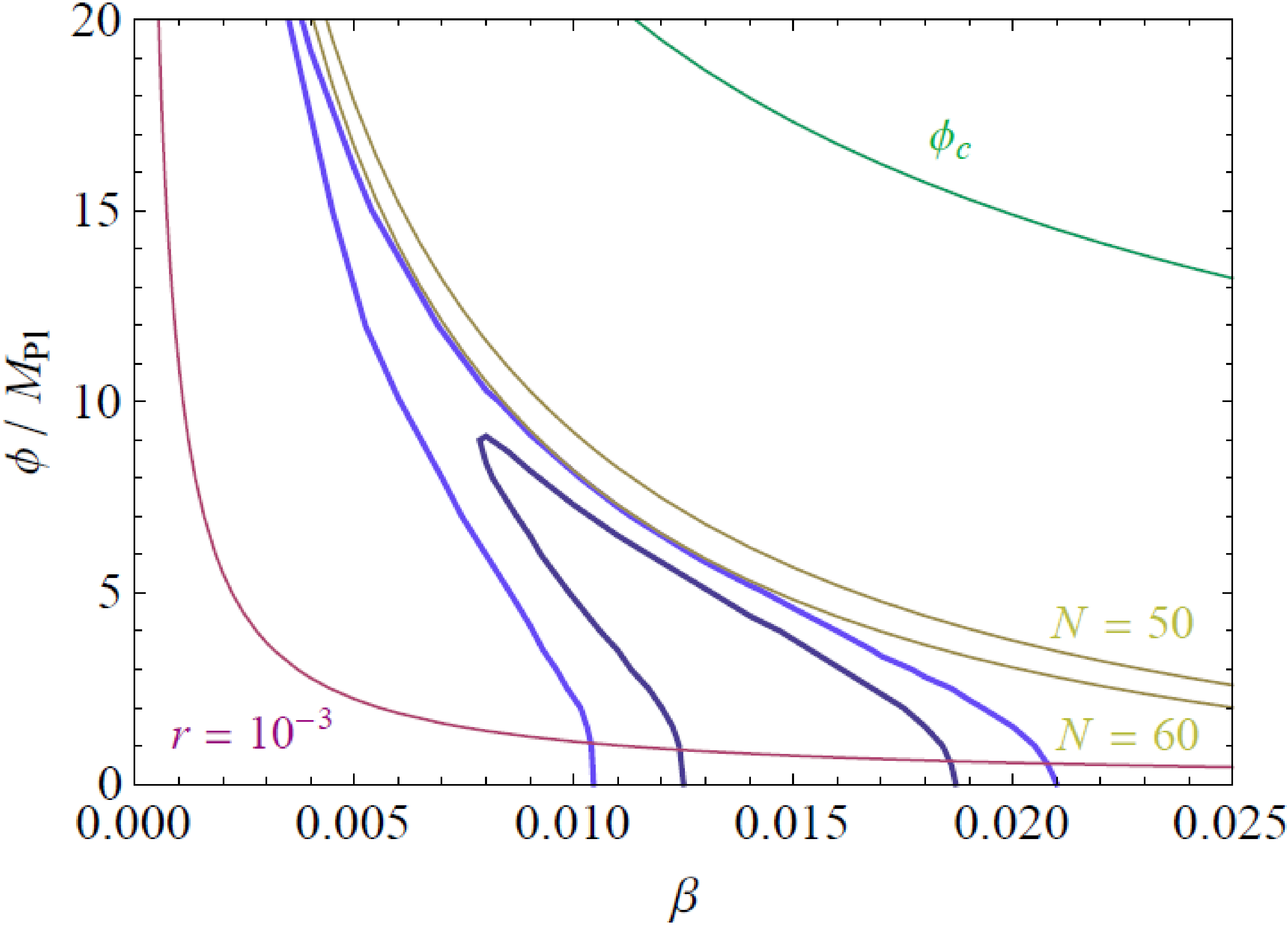
Observational constraint

Ade et al., 1510.09217



Observational constraint

HM, Starobinsky, Yokoyama, in prep.



Summary

Constant-roll condition $\ddot{\phi} = \beta H \dot{\phi}$ is a generalization of slow-roll and ultra slow-roll.

Inflationary dynamics is fully analytically solvable.

The cos potential model
(= natural inflation + small negative CC)
does not suffer from superhorizon evolution of ζ_k.

There exists parameter region which satisfies observational constraint.