Gravitational birefringence of light in Robertson-Walker cosmologies

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CPT & AMU

Hot topics in Modern Cosmology
Spontaneous Workshop XI

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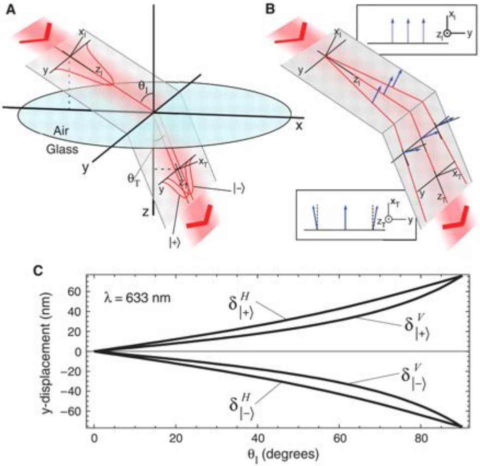
Spin Hall Effect of Light (SHEL)

in the Lab (spin-geometrical optics)

Experimental verification of SHEL ▸ SHEL

Fig. 1. The SHEL at an air-glass interface.



O Hosten, P Kwiat Science 2008;319:787-790

Science

Selected references

1. M. Onoda, S. Murakami, N. Nagaosa, "Hall Effect of Light", Phys. Rev. Lett. **93** (2004) 083901.

   K. Yu. Bliokh and Yu. P. Bliokh, "Topological spin transport of photons: the optical Magnus Effect and Berry Phase", Phys. Lett. A **333** (2004) 181–186.

2. C. Duval, Z. Horváth, P. Horváthy, "Geometrical Spinoptics and the Optical Hall Effect", *J. Geom. Phys* **57** (2007) 925–941; "Fermat Principle for spinning light and the Optical Hall effect", Phys Rev D **74** (2006) 021701 (R).

3. O. Hosten, P. Kwiat, "Observation of the Spin Hall Effect of Light via weak measurements", Science **319** (2008) 787–790.

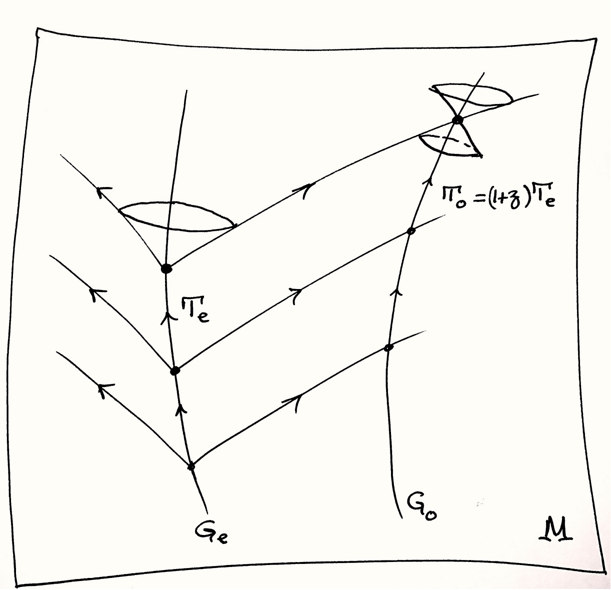
   K. Yu. Bliokh, A. Niv, V. Kleinert, E. Hasman, "Geometrodynamics of spinning light", Nature Photonics **2** (2008) 748.

How does spin affect the trajectory

of light in . . . the Cosmos?

http://lanl.arxiv.org/abs/1610.00555

The cosmological red shift



The Principle of General Covariance & Spinning particles

Matter configurations: distributions on the set of all Lorentz metrics, g, of spacetime, *M*, with signature $(-,-,-,+)$.

Spinning test particles: *first-order* tensorial distributions, $\mathcal{T}_C$, supported by a worldline *C*, viz.,

$$\langle \mathcal{T}_C, \delta \mathrm{g} \rangle = \int_C \left[\frac{1}{2} T^{\mu\nu} \, \delta \mathrm{g}_{\mu\nu} + \Theta^{\mu\nu\rho} \, \nabla_\mu \delta \mathrm{g}_{\nu\rho} \right] d\tau$$

where compactly supported variations δg serve as test-functions.

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Principle of general covariance: the value $\langle \mathcal{T}_C, \delta \mathrm{g} \rangle$ should be *independent* of the metric in the orbit of g under $\mathrm{Diff}_c(M)$; this translates as [Souriau'74]

$$\langle \mathcal{T}_C, L_Y \mathrm{g} \rangle = 0 \qquad (1)$$

for all $Y \in \mathrm{Vect}_c(M)$.

The single condition (1) yields respectively:

1. that the distribution (7) is actually of the form

$$\langle \mathcal{T}_C, \delta \mathrm{g} \rangle = \int_C \left[P^\mu \dot{X}^\nu \, \delta \mathrm{g}_{\mu\nu} + \frac{1}{2} S^{\mu\nu} \dot{X}^\rho \nabla_\mu \delta \mathrm{g}_{\nu\rho} \right] d\tau$$

where $P = (P^\mu)$ is the linear momentum , $\dot{X} = (\dot{X}^\mu)$ the velocity, and $S = (S^{\mu\nu})$ the *skew-symmetric* spin tensor at event $X \in C$,

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2. the *universal Mathisson-Papapetrou-Dixon equations*

$$\begin{aligned} \dot{P}^\mu &= -\frac{1}{2} R_{\alpha\beta}{}^\mu{}_\rho S^{\alpha\beta} \dot{X}^\rho \\ \dot{S}^{\mu\nu} &= P^\mu \dot{X}^\nu - P^\nu \dot{X}^\mu \end{aligned}$$

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3. the *first-integrals* of the Papapetrou equations

$$\Psi(Z) = P_\mu Z^\mu + \frac{1}{2} S^{\mu\nu} \nabla_\mu Z_\nu = \mathrm{const.} \qquad (2)$$

   for all Killing vector fields Z.

An equation of state

Notice that this system of ODE is **non-deterministic**: *no expression for the velocity!* Need for an equation of state! We choose [Tulczyjew'59]

$$SP = 0 \tag{3}$$

which implies $P^2 = P_\mu P^\mu = \mathrm{const}$ & $\mathrm{Tr}(S^2) = -S_{\mu\nu}S^{\mu\nu} = \mathrm{const}$. Those will be taken as constants of the system: for photons we posit

$$P^2 = 0 \qquad \& \qquad -\frac{1}{2}\mathrm{Tr}(S^2) = s^2 \tag{4}$$

with P future-pointing, and where the *scalar spin* is

$$s = \pm\hbar. \tag{5}$$

and the helicity (handedness) $\chi = \mathrm{sign}(s)$.

The Souriau-Saturnini equations

The resulting equations of motion read then intrinsically [Souriau-Saturnini'76]

$$\dot{X} = P + \frac{2}{R(S)(S)} S.R(S).P \tag{6}$$

$$\dot{P} = -s \frac{\mathrm{Pf}(R(S))}{R(S)(S)} P \tag{7}$$

$$\dot{S} = P \wedge \dot{X} \tag{8}$$

where "Pf" is the "Pfaffian",

$$\mathrm{Pf}(R(S)) = -\frac{1}{4} R(S)^{\mu\nu} \star (R(S))_{\mu\nu}$$

and

$$R(S)(S) = R_{\mu\nu\rho\sigma} S^{\mu\nu} S^{\rho\sigma}$$

is assumed to be nowhere zero to guarantee the localization of the particle on a curve. N.B. The velocity is always orthogonal to the momentum,

$$\dot{X} \perp P = 0.$$

Back

Symplectic mechanics: the free system

The system becomes singular wherever $R(S)(S) = 0$. In, e.g., flat spacetime, it reads

$$\begin{aligned} \dot{X} &\perp P \\ \dot{P} &= 0 \\ \dot{S} &= P \wedge \dot{X}. \end{aligned}$$

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- Their spaces of motions are 6-dimensional Poincaré-coadjoint orbits

$$\mathcal{O}_{\mu_0} \cong (\mathrm{Spin}(3,1) \ltimes \mathbb{R}^{3,1})/(\mathrm{SO}(2) \times \mathbb{R}^3)$$

whose origin can be freely **chosen** as $\mu_0 = (S_0, P_0) \in \mathfrak{se}(3,1)^*$ where $P_0^2 = 0$ with P_0 future-pointing, also $S_0 P_0 = 0$, and $s = \pm\sqrt{-\frac{1}{2}\mathrm{Tr}(S_0^2)}$.

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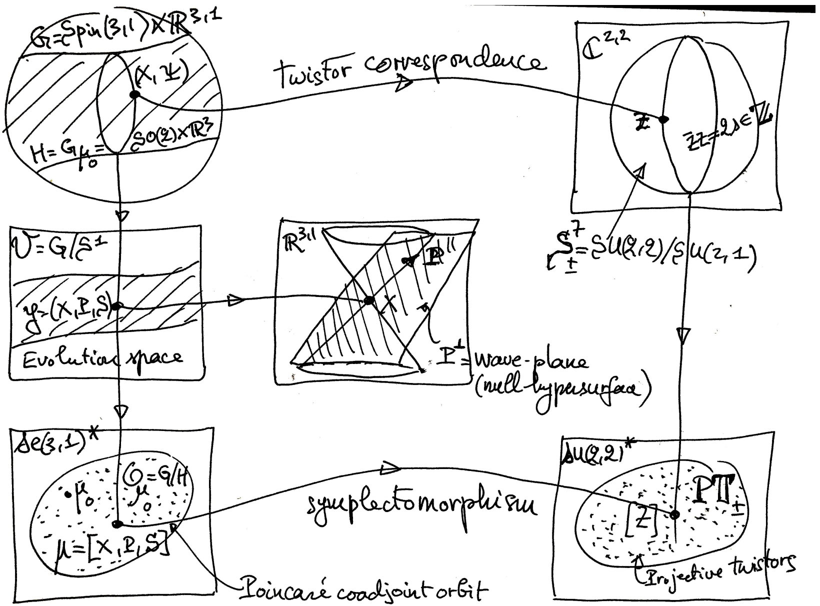
- The latter are symplectomorphic to projective twistor spaces $P\mathbb{T}^\pm$.

Free massless spinning relativistic particle



Symplectic mechanics: minimal gravitational coupling

Minimal gravitational coupling:

- Poincaré group $\rightsquigarrow$ bundle $\mathrm{Spin}(M)$ of spinor frames, or rather bundle of Lorentz frames $\mathrm{SO}_+(M)$ parametrized by (X^μ, e_a^ν) for **bosons**.
- Maurer-Cartan form $\rightsquigarrow$ Levi-Civita affine connection form $\Theta = (\omega, \theta)$.

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Using **same** origin $\mu_0 : (S_0, P_0) \in \mathfrak{se}(3,1)^*$, define the 1-form of $\mathrm{SO}_+(M)$:

$$\begin{aligned} \alpha &= \mu_0 \cdot \Theta \\ &= P_a^0\, \theta^a + \frac{1}{2} S_0^{ab} \omega_{ab}. \end{aligned} \tag{9}$$

Its exterior derivative $\Omega = d\alpha$ descends to "evolution space"

$$\mathcal{V} = \mathrm{SO}_+(M)/\mathrm{SO}(2)$$

described by X, P, S as in (3,4), namely $P^\mu = P_0^a e_a^\mu$ and $S^{\mu\nu} = S_0^{ab} e_a^\mu e_b^\nu$. This presymplectic 2-form on $\mathcal{V}$ reads

$$\Omega = d^\nabla P_\mu \wedge dX^\mu + \frac{1}{2s^2} d^\nabla S_\lambda^{\ \mu} \wedge S_\rho^{\ \lambda} d^\nabla S_\mu^{\ \rho} + \frac{1}{4} R_{\mu\nu\rho\sigma} S^{\mu\nu} dX^\rho \wedge dX^\sigma.$$

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- If Z is a Killing vector field of M, its canonical lift, $Z^\sharp$, to $\mathrm{SO}_+(M)$ verifies $L_{Z^\sharp}\alpha = 0$, and thus defines the momentum mapping Ψ, viz.,

$$\begin{aligned} \Psi(Z) &= \mu_0 \cdot \Theta(Z^\sharp) \\ &= P_a^0\,\theta^a(Z^\sharp) + \frac{1}{2} S_0^{ab}\omega_{ab}(Z^\sharp) \\ &= P_\mu Z^\mu + \frac{1}{2} S^{\mu\nu}\nabla_\mu Z_\nu. \end{aligned}$$

Specialization of the model:

Spinning light in a Roberston-Walker background

Photons in Robertson-Walker backgrounds

• The RW metric on $M = \Sigma \times \mathbb{R}$, where Σ is a connected 3-dimensional Riemannian manifold with constant curvature $R^{(3)} = 6K$, reads

$$\mathrm{g} = -a(t)^2 \frac{\|d\mathbf{x}\|^2}{b(\mathbf{x})^2} + dt^2 \quad \& \quad b(\mathbf{x}) = 1 + \frac{K}{4}\|\mathbf{x}\|^2$$

with $a > 0$ the scale factor, t the cosmic time, $\mathbf{x} \in \mathbb{R}^3$ stereographic coordinates (s.t. $b(\mathbf{x}) > 0$), and $\| \cdot \|$ the Euclidean norm.

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• The (future pointing) null linear momentum of the particle is written as

$$P = \begin{pmatrix} \frac{b}{a}\,\mathbf{p} \\ \|\mathbf{p}\| \end{pmatrix} \qquad \text{where} \qquad \mathbf{p} \in \mathbb{R}^3 \setminus \{0\}.$$

• Accordingly, the spin g-skewsymmetric $(1,1)$-tensor reads

$$S = \begin{pmatrix} j(\mathbf{s}) & -\frac{(\mathbf{s}\times\mathbf{p})}{\|\mathbf{p}\|}\frac{b}{a} \\ -\frac{(\mathbf{s}\times\mathbf{p})^T}{\|\mathbf{p}\|}\frac{a}{b} & 0 \end{pmatrix} \qquad \text{where} \qquad \mathbf{s} \in \mathbb{R}^3.$$

Photons in Robertson-Walker backgrounds II

Computing the spin-curvature coupling term, $R(S)$, we find $\mathrm{Pf}(R(S)) = 0$
$\Rightarrow$ the linear momentum, P, is parallel-transported.
The system (6)–(8) of equations of motion w.r.t. cosmic time reduces to

$$\frac{dX}{dt} = \frac{aa''b}{\|\mathbf{p}\|(K + a'^2)}\left[P - \left(1 - \frac{K + a'^2}{aa''}\right)\frac{W}{s}\right] \quad (11)$$

$$\frac{d^\nabla P}{dt} = 0 \quad (12)$$

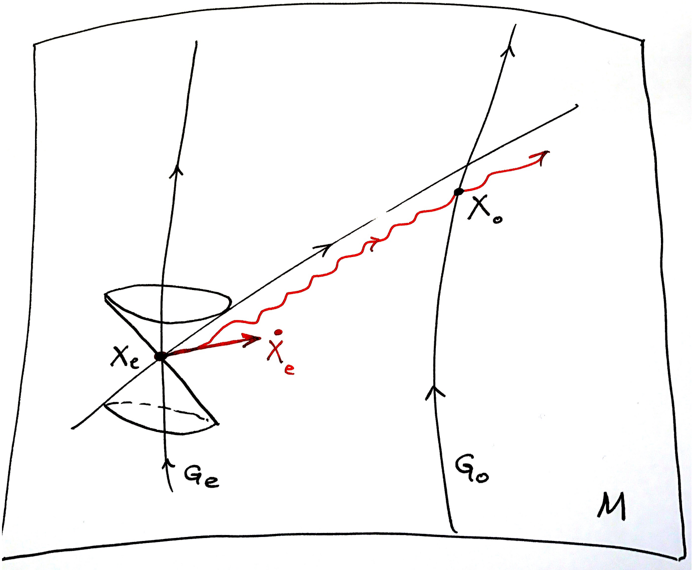
$$\frac{d^\nabla S}{dt} = P \wedge \frac{dX}{dt} \quad (13)$$

where

$$W = \|\mathbf{p}\|\begin{pmatrix} \frac{b}{a}\mathbf{s} \\ s \end{pmatrix}$$

is the polarization vector of the massless particle in the gravitational field, responsible for the anomalous velocity in (11).

Twisting photons



Photonic equations of motion

If we define the **deceleration function** $Q(t) := -a(t)a''(t)/(K + a'(t)^2)$, the previous equations of motion read in our $3+1$ decomposition:

$$\frac{d\mathbf{x}}{dt} = \frac{b}{a}\left[-Q\frac{\mathbf{p}}{||\mathbf{p}||} + (1+Q)\,\frac{\mathbf{s}}{s}\right] \tag{14}$$

$$\begin{aligned}\frac{d\mathbf{p}}{dt} &= -\frac{a'}{a}\left[-Q\,\mathbf{p} + ||\mathbf{p}||\,(1+Q)\,\frac{\mathbf{s}}{s}\right]\\ &\quad + \frac{K}{2a}\,(1+Q)\,(\mathbf{p}\cdot\mathbf{x})\frac{\mathbf{s}}{s} + \frac{K}{2a}\left[-Q\,(\mathbf{p}\cdot\mathbf{x})\,\frac{\mathbf{p}}{||\mathbf{p}||} - ||\mathbf{p}||\,\mathbf{x}\right]\end{aligned} \tag{15}$$

$$\begin{aligned}\frac{d\mathbf{s}}{dt} &= (1+Q)\,\frac{\mathbf{s}}{s}\times\mathbf{p} - \frac{a'}{a}\,\mathbf{s} + \frac{a'}{a}\left[\frac{||\mathbf{s}||^2}{s}\,(1+Q) - s\,Q\right]\frac{\mathbf{p}}{||\mathbf{p}||}\\ &\quad + \frac{K}{2a}\,(1+Q)\left[\left(\frac{\mathbf{s}}{s}\cdot\mathbf{x}\right)\mathbf{s} - \frac{||\mathbf{s}||^2}{s}\mathbf{x}\right] - \frac{K}{2a}\,Q\left[(\mathbf{s}\cdot\mathbf{x})\frac{\mathbf{p}}{||\mathbf{p}||} - s\,\mathbf{x}\right]\end{aligned} \tag{16}$$

Conservation laws: Noetherian quantities

Define $b_\pm := 1 \pm \frac{K}{4}||\mathbf{x}||^2$ ($b = b_+$), so that the generators $Z : X \mapsto \delta X$ of the 6-dimensional group of isometries of (M, g) are

$$\delta\mathbf{x} = \omega \times \mathbf{x} + \alpha\, b_- + \frac{K}{2}\mathbf{x}(\mathbf{x} \cdot \alpha), \qquad \delta t = 0$$

with $\alpha, \omega \in \mathbb{R}^3$.

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with $\alpha, \omega \in \mathbb{R}^3$.
The momenta $\mathcal{P}$ & $\mathcal{L}$ defined by $\Psi(Z) = \mathcal{P}\cdot\alpha + \mathcal{L}\cdot\omega$ (cf. (2)) read

$$\begin{aligned}
\mathcal{P} &= \frac{b_-}{b_+}\left[-a\mathbf{p} + a'\mathbf{s}\times\frac{\mathbf{p}}{\|\mathbf{p}\|}\right] \\
&\quad + \frac{K}{2b_+}\left[2\,\mathbf{x}\times\mathbf{s} + a'\,\mathbf{x}\cdot\left(\mathbf{s}\times\frac{\mathbf{p}}{\|\mathbf{p}\|}\right)\mathbf{x} - a\,(\mathbf{x}\cdot\mathbf{p})\,\mathbf{x}\right] = \mathrm{const} \\
\mathcal{L} &= \frac{1}{b_-}\left[\mathbf{x}\times\mathcal{P} + b_+\,\mathbf{s} - \frac{K}{2}(\mathbf{s}\cdot\mathbf{x})\,\mathbf{x}\right] = \mathrm{const.}
\end{aligned}$$

Conservation laws II

• The scalar spin (helicity $\times \hbar$) is indeed a first integral of the previous system (14)–(16) of ODE:

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• Define the transverse vectorial spin by

$$\mathbf{s}^{\perp} := \mathbf{s} - s\frac{\mathbf{p}}{\|\mathbf{p}\|}$$

then the following is another constant of the motion, namely

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• Since $\Theta = a(t)\partial/\partial t$ is conformal Killing v.f., the "energy" $\mathcal{E} = P_\mu\Theta^\mu$ is

$$\mathcal{E} = a\,\|\mathbf{p}\| = \mathrm{const}$$

$\Rightarrow$ The redshift formula, $z = a_0/a_e - 1$, holds in the spinning case as

$$\frac{a(t)}{\lambda(t)} = \mathrm{const.}$$

Final expression of the photonic velocity & birefingence

Our strategy: (i) express spin $\mathbf{s}(t)$ & linear momentum $\mathbf{p}(t)$ in terms of the conserved quantities $\mathcal{P}, \mathcal{L}, \mathcal{E}, \mathcal{S}$; (ii) plug those in Eq. (14) for the velocity $d\mathbf{x}/dt$; (iii) reduce hence the system (14)–(16) of equations of motion to a **non-autonomous ODE**:

$$\frac{d\mathbf{x}}{dt} = A(t)\mathbf{x} + \mathbf{B}(t) \tag{17}$$

where — in the flat case, $K = 0$, to simplify the notation:

$$\begin{aligned} A &= F\left[-\frac{a'}{\mathcal{E}}\, j(\mathcal{P})^2 + \frac{a'^2}{\mathcal{E}^2}(\mathcal{L}\cdot\mathcal{P})\, j(\mathcal{P})\right] + \frac{1}{a\,s}\left(1 - \frac{aa''}{a'^2}\right) j(\mathcal{P}) \\ \mathbf{B} &= F\left[\mathcal{P} + \frac{a'}{\mathcal{E}}\mathcal{L}\times\mathcal{P} + \frac{a'^2}{\mathcal{E}^2}(\mathcal{L}\cdot\mathcal{P})\,\mathcal{L}\right] + \frac{1}{a\,s}\left(1 - \frac{aa''}{a'^2}\right)\mathcal{L} \end{aligned}$$

highlighting the spin $s = \pm\hbar$ that induces gravitational birefringence, with

$$F(t) := -\frac{a''(t)}{a'(t)^2}\,\frac{1}{\mathcal{E}\left[1 + \dfrac{s^2}{\mathcal{E}^2}a'(t)^2 + \dfrac{\mathcal{S}^2}{\mathcal{E}^2}\right]}.$$

For the **integration** of the photonic equations of motion and the demonstration of **birefringence of light** in ΛCDM backgrounds via

1. Numerical integration
2. Perturbative solutions

Rendez-vous with Thomas, next talk . . .

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Thank you

Spin Hall Effect of Light in picture [Onoda et al., Bliokh et al.]

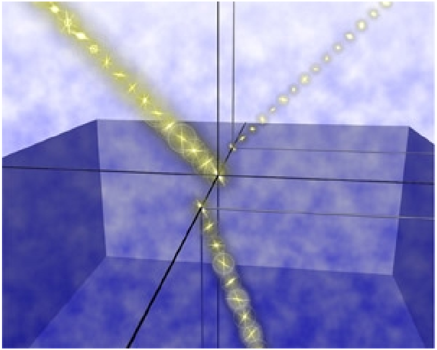

Figure: Onoda & al. (2004)

New Snel-Descartes laws

1. T(ransmission (Refraction)
   - law of sines
   - transverse shift
2. R(eflection
   - law of mirror symmetry
   - transverse shift

The optical spin Hall shift at I(incident angle θ_I reads Back

$$(\Delta y)_{R,T} = \frac{\langle\sigma_3\rangle_{R,T}\cos\theta_{R,T} - \langle\sigma_3\rangle_I \cos\theta_I}{k_I \sin\theta_I} \sim \lambda_I$$