Self-tuning of the cosmological constant in generalized Galileon/Horndeski theories

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What is self-tuning?

Much too large vacuum energy density $\rho_{\rm vac} \equiv M_{\rm Pl}^2\,\Lambda$

$$S = \frac{M_{\rm Pl}^2}{2}\int\sqrt{-g}\,R\,d^4x - \int\sqrt{-g}\,M_{\rm Pl}^2\,\Lambda\,d^4x + S_{\rm matter}$$

- $|\rho_{\rm naive}| \sim M_{\rm Pl}^4 \sim 10^{122}\,M_{\rm Pl}^2\,\Lambda_{\rm obs} = 10^{122}\,\rho_{\rm obs}$
- $|\rho_{\rm dimensional\ regularization}| \sim 10^8\,{\rm GeV}^4 \sim 10^{55}\,\rho_{\rm obs}$ (depends on renorm. scale)
- $|\rho_{\rm EW\ phase\ transition}| \sim 10^8\,{\rm GeV}^4 \sim 10^{55}\,\rho_{\rm obs}$
- $|\rho_{\rm QCD\ phase\ transition}| \sim 10^{-2}\,{\rm GeV}^4 \sim 10^{45}\,\rho_{\rm obs}$

"Fab Four" ($\in$ Horndeski theories, + beyond)

Large $\Lambda_{\rm bare}$ but $\exists$ cosmological solution where $T_{\mu\nu}(\varphi)$ exactly compensates $M_{\rm Pl}^2\,\Lambda_{\rm bare}\,g_{\mu\nu} \Rightarrow \Lambda_{\rm effective} = 0$ strictly

[Charmousis, Copeland, Padilla, Saffin 2012]

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What is self-tuning?

Self-tuning

Large Λ_{bare} but $T_{\mu\nu}(\varphi)$ almost compensating $M_{\text{Pl}}^2\,\Lambda_{\text{bare}}\,g_{\mu\nu}$
so that $\Lambda_{\text{effective}}$ = observed value

[Appleby, De Felice, Linder 2012; Linder 2013; Martín-Moruno, Nunes, Lobo 2015; Starobinsky, Sushkov, Volkov 2016]

But backreaction of φ on $g_{\mu\nu}$ is generically huge

Can one pass solar-system tests?

[Babichev, GEF 2013; Babichev, Charmousis 2014; Cisterna, Delsate, Rinaldi 2015; Appleby 2015]

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Plan of this talk

- Self-tuning in all shift-symmetric beyond Horndeski theories?
  (No analysis of the full cosmological history, nor study of the stability)
- Spherical body embedded in such a Universe: solar-system tests?
- Is the large cosmological constant problem solved?

Answer: Each step will reduce the space of allowed models

Generalized Horndeski theories

Notation: $\varphi_\mu \equiv \partial_\mu \varphi, \qquad \varphi_{\mu\nu} \equiv \nabla_\mu \nabla_\nu \varphi$

Generalized Galileons (in 4 dimensions)

$$L_{(2,0)} \equiv -\frac{1}{3!}\,\varepsilon^{\mu\nu\rho\sigma}\,\varepsilon^{\alpha}{}_{\nu\rho\sigma}\,\varphi_\mu\,\varphi_\alpha = \varphi_\mu^2,$$

$$L_{(3,0)} \equiv -\frac{1}{2!}\,\varepsilon^{\mu\nu\rho\sigma}\,\varepsilon^{\alpha\beta}{}_{\rho\sigma}\,\varphi_\mu\,\varphi_\alpha\,\varphi_{\nu\beta},$$

$$L_{(4,0)} \equiv -\,\varepsilon^{\mu\nu\rho\sigma}\,\varepsilon^{\alpha\beta\gamma}{}_{\sigma}\,\varphi_\mu\,\varphi_\alpha\,\varphi_{\nu\beta}\,\varphi_{\rho\gamma},$$

$$L_{(5,0)} \equiv -\,\varepsilon^{\mu\nu\rho\sigma}\,\varepsilon^{\alpha\beta\gamma\delta}\,\varphi_\mu\,\varphi_\alpha\,\varphi_{\nu\beta}\,\varphi_{\rho\gamma}\,\varphi_{\sigma\delta},$$

$$L_{(4,1)} \equiv -\,\varepsilon^{\mu\nu\rho\sigma}\,\varepsilon^{\alpha\beta\gamma}{}_{\sigma}\,\varphi_\mu\,\varphi_\alpha\,R_{\nu\rho\beta\gamma} = -4\,G^{\mu\nu}\varphi_\mu\varphi_\nu,$$

$$L_{(5,1)} \equiv -\,\varepsilon^{\mu\nu\rho\sigma}\,\varepsilon^{\alpha\beta\gamma\delta}\,\varphi_\mu\,\varphi_\alpha\,\varphi_{\nu\beta}\,R_{\rho\sigma\gamma\delta}.$$

Generalized Horndeski theories

Multiply these Lagrangians by arbitrary functions of $X \equiv -\dfrac{\varphi_\mu^2}{M^2}$ (dimensionless)

Shift-symmetric beyond Horndeski theories

$$S = \frac{M_{\rm Pl}^2}{2}\int \sqrt{-g}\,(R - 2\Lambda_{\rm bare})\, d^4x + S_{\rm matter}[\text{matter}, g_{\mu\nu}]$$
$$+ \int \sqrt{-g}\,\Big\{M^2 f_2(X) L_{(2,0)} + f_3(X) L_{(3,0)} + \frac{1}{M^2} f_4(X) L_{(4,0)}$$
$$+ \frac{1}{M^4} f_5(X) L_{(5,0)} + s_4(X) L_{(4,1)} + \frac{1}{M^2}\, s_5(X) L_{(5,1)}\Big\} d^4x$$

Equivalent to other notations used in the literature, e.g.

$$\frac{1}{M^2} f_4(X) L_{(4,0)} + s_4(X) L_{(4,1)} = G_4(\varphi_\lambda^2) R - 2G_4'(\varphi_\lambda^2)\left[(\Box\varphi)^2 - \varphi_{\mu\nu}\varphi^{\mu\nu}\right]$$
$$+ F_4(\varphi_\lambda^2)\, \varepsilon^{\mu\nu\rho\sigma}\, \varepsilon^{\alpha\beta\gamma}{}_{\sigma}\, \varphi_\mu\, \varphi_\alpha\, \varphi_{\nu\beta}\, \varphi_{\rho\gamma} + \text{tot. div.}$$

Fields equations

Two useful simplifications:

- Shift-symmetry ⇒ scalar field equation reads $\nabla_\mu J^\mu = 0$,
  with scalar current $J^\mu \equiv \dfrac{-1}{\sqrt{-g}} \dfrac{\delta S}{\delta(\partial_\mu \varphi)}$

- Diffeomorphism invariance ⇒ Einstein's equations greatly simplified by combining them with the scalar current as

$$G^{\mu\nu} + \Lambda_{\text{bare}}\, g^{\mu\nu} - \frac{T^{\mu\nu}}{M_{\text{Pl}}^2} + \frac{J^\mu \varphi^\nu}{M_{\text{Pl}}^2}$$

No longer any $f'_{2,3,4,5}(X)$ in them

Cosmological equations

Background field equations in FLRW geometry

$$
\begin{aligned}
-X f_2 + 6\left(\frac{H}{M}\right)^2 \left[X^2 f_4 + 2X s_4\right] & \\
-12\left(\frac{H}{M}\right)^3 \left[X^{5/2} f_5 + 2X^{3/2} s_5\right] &= \frac{M_{\rm Pl}^2}{M^4}\left(\Lambda_{\rm bare} - 3H^2\right),
\end{aligned}
$$

$$
\begin{aligned}
[X f_2]' - 3\frac{H}{M}\left[X^{3/2} f_3\right]' + 6\left(\frac{H}{M}\right)^2 \left[X^2 f_4 + 2X s_4\right]' & \\
-6\left(\frac{H}{M}\right)^3 \left[X^{5/2} f_5 + 2X^{3/2} s_5\right]' &= 0.
\end{aligned}
$$

- For a given theory $\Rightarrow$ predict H and $X \equiv \dot{\varphi}^2/M^2$
- For a wanted $\Lambda_{\rm effective} = 3H^2_{\rm observed}$ $\Rightarrow$ "predict" M and $X \neq 0$

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$$-Xf_2 + 6\left(\frac{H}{M}\right)^2\left[X^2 f_4 + 2Xs_4\right] - 12\left(\frac{H}{M}\right)^3\left[X^{5/2}f_5 + 2X^{3/2}s_5\right] = \frac{M_{\rm Pl}^2}{M^4}\left(\Lambda_{\rm bare} - 3H^2\right),$$

$$[Xf_2]' - 3\frac{H}{M}\left[X^{3/2}f_3\right]' + 6\left(\frac{H}{M}\right)^2\left[X^2 f_4 + 2Xs_4\right]' - 6\left(\frac{H}{M}\right)^3\left[X^{5/2}f_5 + 2X^{3/2}s_5\right]' = 0.$$

- For a given theory $\Rightarrow$ predict H and $X \equiv \dot{\varphi}^2/M^2$
- For a wanted $\Lambda_{\rm effective} = 3H^2_{\rm observed} \Rightarrow$ "predict" M and $X \neq 0$

Cosmological self-tuning

Result: Aside from singular limiting cases (that we all studied), it is always possible to obtain $\Lambda_{\text{effective}} = 3H^2_{\text{observed}}$ with an appropriate value of the mass scale M.

But

- At least two of the Lagrangians $L_{(n,p)}$ must be present.
- M cannot be of the same order of magnitude as the bare vacuum energy scale $\left(M^2_{\text{Pl}}\Lambda_{\text{bare}}\right)^{1/4}$.
  All other values (either larger or smaller) are possible, depending on the functions $f_{2,3,4,5}(X)$ and $s_{4,5}(X)$.

Example (in the Horndeski subclass)

$f_2 = -X^{-5/4}, \quad f_4 = 6\,X^{-5/2}, \quad s_4 = -X^{-3/2}, \quad$ with $\quad \Lambda_{\text{bare}} \sim M^2_{\text{Pl}}$

would need $\quad M = (32\,M^2_{\text{Pl}}\,\Lambda_{\text{bare}}\,H^2)^{1/6} \sim 100$ MeV

This corresponds to $G_2\left(\varphi^2_\lambda\right) = M^{9/2}(-\varphi^2_\lambda)^{-1/4}$, and $G_4\left(\varphi^2_\lambda\right) = 2M^3(-\varphi^2_\lambda)^{-1/2}$

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Backreaction of φ on $g_{\mu\nu}$ near a spherical body

— Write field equations for static and spherically symmetric body.

— **Simplifying hypothesis:** Assume $\varphi = \dot{\varphi}_c t + \psi(r)$, with $\dot{\varphi}_c = \text{const.}$

Can the solution be close enough to Schwarzschild, in spite of large $T_{\mu\nu}(\varphi)$?

- When different Lagrangians $L_{(n,p)}$ dominate cosmology and the local behavior of φ, then possible to have

$$T_{\mu\nu}(\varphi) \approx M_{\text{Pl}}^2 \left(\Lambda_{\text{bare}} - \Lambda_{\text{effective}}\right) g_{\mu\nu},$$

but this requires well-chosen functions $f_n(X)$ and $s_n(X)$.

- When the same Lagrangian $L_{(n,p)}$ contributes significantly both for cosmology and the local φ, then generically impossible to get a Newtonian potential $\propto 1/r$.

But particular cases can work, e.g. when $L_{(5,0)}$ and $L_{(5,1)}$ dominate locally, then the condition

$$X(2f_5 + X f_5') + 2(s_5 + X s_5') = 0$$

suffices for the local backreaction of φ to be negligible.

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Exact Schwarzschild–de Sitter solution

Extra assumption: $X \equiv -\frac{\varphi_\mu^2}{M^2} = \text{const.}$ everywhere

Necessary and sufficient conditions for exact Schwarzschild–de Sitter

$$
\begin{aligned}
-Xf_2 + 6\left(\frac{H}{M}\right)^2 \left[X^2 f_4 + 2X s_4\right] &= \frac{M_{\rm Pl}^2}{M^4}\left(\Lambda_{\rm bare} - 3H^2\right),\\
[Xf_2]' + 6\left(\frac{H}{M}\right)^2 \left[X^2 f_4 + 2X s_4\right]' &= 0,\\
Xf_5 + 2s_5 = 0 \quad \text{and} \quad [Xf_5 + 2s_5]' &= 0,\\
\left[X^{3/2} f_3\right]' &= 0.
\end{aligned}
$$

$\Rightarrow$ Effective cosmological constant

$$
\Lambda_{\rm effective} = \frac{\Lambda_{\rm bare} + \frac{M^4}{M_{\rm Pl}^2} X f_2}{1 + 2\left(\frac{M}{M_{\rm Pl}}\right)^2 \left(X^2 f_4 + 2X s_4\right)}
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⇒ Effective cosmological constant

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$$

Exact Schwarzschild–de Sitter solution (continued)

These solutions also describe regular hairy black holes

$$\varphi = \dot{\varphi}_c t + \psi(r) \quad \text{with} \quad \dot{\varphi}_c = \text{const.},$$

$$\psi'(r) = -\dot{\varphi}_c \frac{\sqrt{r_s/r + (Hr)^2}}{1 - r_s/r - (Hr)^2},$$

$$X \equiv -\varphi_\mu^2/M^2 = \dot{\varphi}_c{}^2/M^2 = \text{const.},$$

$$\text{scalar current} \quad J^\mu = 0,$$

$$T_{\mu\nu}(\varphi) = M_{\text{Pl}}^2 \left(\Lambda_{\text{bare}} - \Lambda_{\text{effective}}\right) g_{\mu\nu},$$

$$\Box\varphi = -3H\dot{\varphi}_c - (3\dot{\varphi}_c r_s^2)/(8H^3 r^6) + \mathcal{O}\left(r_s^3\right).$$

Renormalization of Newton's constant

$$G_{\text{effective}} = \frac{G_{\text{bare}}}{1 + 4\left(\frac{M}{M_{\text{Pl}}}\right)^2 X^{1/2}\left[X^{5/2} f_4 + 2X^{3/2} s_4\right]'} \Rightarrow \left(M_{\text{Pl}}^{\text{bare}}\right)^2 \Lambda_{\text{bare}} \sim \left(M_{\text{Pl}}^{\text{eff}}\right)^2 \Lambda_{\text{eff}}!$$

The huge vacuum energy density problem is **not** solved!

Necessary and sufficient conditions for exact Schwarzschild–de Sitter

$$-X f_2 + 6\left(\frac{H}{M}\right)^2\left[X^2 f_4 + 2X s_4\right] = \frac{M_{\text{Pl}}^2}{M^4}\left(\Lambda_{\text{bare}} - 3H^2\right),$$

$$\left[X f_2\right]' + 6\left(\frac{H}{M}\right)^2\left[X^2 f_4 + 2X s_4\right]' = 0,$$

$$X f_5 + 2 s_5 = 0 \quad \text{and} \quad \left[X f_5 + 2 s_5\right]' = 0,$$

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Exact Schwarzschild–de Sitter without any renormalization of G

$$-X f_2 + 6\left(\frac{H}{M}\right)^2 \left[X^2 f_4 + 2X s_4\right] = \frac{M_{\text{Pl}}^2}{M^4}\left(\Lambda_{\text{bare}} - 3H^2\right),$$

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Extra condition $\rightarrow$ $$\left[X^{5/2} f_4 + 2X^{3/2} s_4\right]' = 0,$$

$$X f_5 + 2 s_5 = 0 \quad \text{and} \quad \left[X f_5 + 2 s_5\right]' = 0,$$

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Conclusions

- Cosmological self-tuning ($\Lambda_{\text{effective}} \ll \Lambda_{\text{bare}}$) always possible if $\exists$ at least two Lagrangians $L_{(n,p)}$, and scale $M \approx \left(M_{\text{Pl}}^2 \Lambda_{\text{bare}}\right)^{1/4}$.

- Exact Schwarzschild–de Sitter solution possible if **5** conditions are imposed on the functions $f_n(X)$ and $s_n(X)$.

  $f_2(X)$, $f_4(X)$ and $s_4(X)$ *define* the solution ($\Lambda_{\text{effective}}$ and X), while $f_3(X)$, $f_5(X)$ and $s_5(X)$ are *passive* for the background.

- On can get $\left(M_{\text{Pl}}^{\text{eff}}\right)^2 \Lambda_{\text{eff}} \ll \left(M_{\text{Pl}}^{\text{bare}}\right)^2 \Lambda_{\text{bare}}$ if a **6**th condition is imposed on $f_4(X)$ and $s_4(X)$.

  But there still exists an infinity of allowed models!

- **Still to be studied**: stability, more realistic cosmology, other post-Newtonian tests of gravity (strong equivalence principle, preferred-frame effects, ...).