Gauge-invariant observables in perturbative quantum gravity

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Perturbative Quantum Gravity (1/2)

- Study General Relativity as effective field theory and expand action perturbatively in fluctuations around a given background, non-renormalisable but valid at scales below the Planck scale
- Infinitesimal coordinate transformation $x^\mu \to \tilde{x}^\mu = x^\mu + \xi^\mu$ ($\delta x^\mu = \xi^\mu$) gives gauge transformation $\delta A = \mathcal{L}_\xi A$
- If $A^{(0)} = 0$, then at linear order $\delta A^{(1)} = \mathcal{L}_\xi A^{(0)} = 0$, but at higher orders gauge-dependent since $A^{(1)} \neq 0$ in general
- In flat space, S-Matrix is gauge-invariant but not local $\Rightarrow$ reconstruct local observables by inverse scattering method
- One-loop quantum corrections to Newtonian potential:
$$V(r) = -\frac{Gm}{r}\left[1 + \left(\frac{41}{10\pi} + \frac{[1+\frac{5}{4}(1-6\xi)^2]N_0+6N_{1/2}+12N_1}{45\pi}\right)\frac{\ell_{\mathrm{Pl}}^2}{r^2}\right]$$
- G: Newton's constant, $\ell_{\mathrm{Pl}} = \sqrt{\hbar G/c^3}$: Planck length, N_s: number of particles of spin s, ξ: non-minimal scalar coupling

Perturbative Quantum Gravity (2/2)

- Matter corrections can also be obtained from effective action for gravitons with (spinning) point particle source, integrating out matter loops, also for curved background
- Conformal fields in de Sitter:

$$\Phi_\mathrm{A} = \frac{Gm}{\hat{r}} \left[1 + \frac{128\pi b}{3} \frac{\ell_\mathrm{Pl}^2}{\hat{r}^2} + 32\pi \ell_\mathrm{Pl}^2 H^2 \left(\beta - 4b - 3b' + 2b \ln\left(\bar{\mu} \hat{r} \right) \right) \right]$$

$$\Phi_\mathrm{H} = \frac{Gm}{\hat{r}} \left[1 + \frac{64\pi b}{3} \frac{\ell_\mathrm{Pl}^2}{\hat{r}^2} + 32\pi \ell_\mathrm{Pl}^2 H^2 \left(\beta - 2b - 3b' + 2b \ln\left(\bar{\mu} \hat{r} \right) \right) \right]$$

$$\mathbf{V} = -2G \frac{\mathbf{S} \times \hat{\mathbf{r}}}{\hat{r}^3} \left[1 + 96\pi b \frac{\ell_\mathrm{Pl}^2}{\hat{r}^2} + 32\pi \ell_\mathrm{Pl}^2 H^2 \left(\beta - 5b - b' + 2b \ln\left(\bar{\mu} \hat{r} \right) \right) \right]$$

- b, b': central charges of CFT (trace anomaly), $\hat{r} = ar$: physical distance on equal-time hypersurface, β: Coefficient of R^2, $\bar{\mu}$: renormalisation scale
- But: graviton loop corrections are gauge dependent

Relational observables (1/3)

- Let there be given 4 fields $X^{(i)}[g, \phi, \ldots]$ depending on field content, transforming under diffeo's as scalars: $\delta X^{(i)} = \xi^\mu \partial_\mu X^{(i)}$, and their background value $X_0^{(i)}$
- Expand $X^{(i)} = X_0^{(i)} + X_1^{(i)} + \ldots$ in perturbation theory and invert to obtain $X_0^{(i)}[X] \Rightarrow$ transforms inversely to a scalar
- Invariant observable $\mathcal{A}(\chi)$ is given by evaluating a field A at the position $X_0^{(i)}$, holding $X^{(i)}$ fixed
- $\Rightarrow$ Relational observables: the $X^{(i)}$ are configuration-dependent coordinates, $\mathcal{A}(\chi)$ is the value of A provided that $\chi^i = X^{(i)}$, and by evaluating at $X_0^{(i)}$ we interpret $\mathcal{A}$ as field on background
- Cosmology: only one scalar field (inflaton ϕ), but we need 4

Relational observables (2/3)

- At lowest order, we have $\delta x^\mu = \xi^\mu = \xi^\alpha \partial_\alpha x^\mu$, which thus transforms as a scalar $\Rightarrow$ generalise to higher orders
- Since coordinates are harmonic $\triangle x^i = 0$, define $X^{(i)}(x)$ as harmonic coordinates for the perturbed Laplacian $\tilde{\triangle}$ on constant-inflaton-hypersurfaces
- Since Laplacian is scalar operator, $X^{(i)}(x)$ will transform as scalars to all orders (computable to arbitrary order in perturbation theory)
- To linear order, we have decomposition of metric perturbation $h_{\mu\nu} = h^{\text{inv}}_{\mu\nu} + \mathcal{L}_Z \eta_{\mu\nu}$ (where $h^{\text{inv}}_{\mu\nu}$ contains Bardeen variables, and gauge-invariant vector and tensor) and calculate $X^{(i)}(x) = x^i + \left(\frac{1}{2} \frac{\partial^i}{\triangle} Q + Z^i \right)$ with Mukhanov-Sasaki variable Q
- Invert to obtain $x^i = X^{(i)} - \left(\frac{1}{2} \frac{\partial^i}{\triangle} Q(X) + Z^i(X) \right) + \ldots$

Relational observables (3/3)

- Invariant observable is $H_{\mu\nu}(X) = a^{-2} \frac{\partial x^\alpha}{\partial X^\mu} \frac{\partial x^\beta}{\partial X^\nu} \tilde{g}_{\alpha\beta}(x(X)) - \eta_{\mu\nu}$, holding X (and inflaton $\tilde{\phi}$) fixed
- At linear order (with $\Sigma = \frac{1}{3}\delta^{ij} h^{\text{inv}}_{ij}$):

$$H_{00} = h^{\text{inv}}_{00} - \frac{H'}{H^2 a}(Q+\Sigma) + \frac{(Q+\Sigma)'}{Ha}$$
$$H_{0k} = h^{\text{inv}}_{0k} + \frac{1}{2Ha}\partial_k (Q+\Sigma) - \frac{1}{2}\frac{\partial_k}{\triangle} Q'$$
$$H_{kl} = h^{\text{inv}}_{kl} - \delta_{kl}(Q+\Sigma) - \frac{\partial_k \partial_l}{\triangle} Q$$

- Usual invariant perturbation is recovered by choosing different time coordinate $t = \tilde{\phi} - 3/(4\left|\tilde{\nabla}\tilde{\phi}\right|\tilde{K})\tilde{\triangle}^{-1}\tilde{R}^{(3)}$, with $\tilde{K}$ extrinsic curvature of constant-$\tilde{\phi}$-hypersurfaces, $\tilde{R}^{(3)}$ induced Ricci scalar

Renormalised correlation functions and running of couplings

- Analogue in flat space: perturbed harmonic coordinates $\tilde{\Box} X^\mu = 0$
- Invariant scalar field $\Phi = \phi - X_1^\alpha \partial_\alpha \phi + \ldots$ with $X_1^\alpha(x) = \int G(x,y) \left[\partial_\mu h^{\mu\alpha} - \frac{1}{2} \partial^\alpha h \right](y) \,\mathrm{d}^4 y$ and a Green's function G fulfilling $\partial^2 G(x,y) = \delta^4(x-y)$
- 1-loop graviton corrections to invariant massless two-point function $\tilde{\mathcal{G}}_0(p) = -1/p^2 + \ell_{\mathrm{Pl}}^2 \delta_{Z_1}^{\mathrm{fin}}(\mu) - \ell_{\mathrm{Pl}}^2/(8\pi)^2 (3 - 10\xi) \ln(p^2/\mu^2)$ with $\delta_{Z_1}^{\mathrm{fin}}$: finite part of higher-derivative counterterm, μ: renormalisation scale, ξ: non-minimal scalar coupling
- Explicitly independent of gauge fixing parameters for graviton iff Green's function G is massless Feynman propagator
- Graviton corrections to $\lambda\phi^4$ coupling: $\beta = -\frac{\lambda \ell_{\mathrm{Pl}}^2}{(4\pi)^2} m^2 \left[4(2 - 21\xi + 21\xi^2) + 9\alpha(5 + 2\xi) \right]$, where $\alpha = 0$ is standard Feynman gauge and $\alpha = 1$ is from invariant four-point function $\Rightarrow$ asymptotic freedom for all ξ iff $\alpha = 1$

Open Problems

- Recover graviton corrections to Newtonian potential via effective action of gauge-invariant metric perturbation
- Calculate graviton corrections to Newtonian potential in de Sitter/inflation (possibly secular effects $\sim \ln^k a$!)
- Calculate backreaction from quantum effects in inflation; disentangle gauge effects from real physics (end inflation by secular effects?)
- Prove renormalisability to all orders
- Classify ambiguities (choice of time coordinate, ...)

Thank you for your attention

Questions?

Based on 1605.02573 (Brunetti et al.), 1601.03561 & 1701.06576 (Fröb/Verdaguer), and 1705.?????

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