

Infinite Derivative Theories of Gravity and potential ways of probing it

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Warren Siegel, Tirthabir Biswas, Alex Koshelev,

Sergei Vernov, Erik Gerwick, Tomi Koivisto

Aindriu Conroy, Spyridon Talaganis, Ali Teimouri, James Edholm

Phys. Rev. Lett. (2012), JCAP (2012, 2011), JCAP (2006)

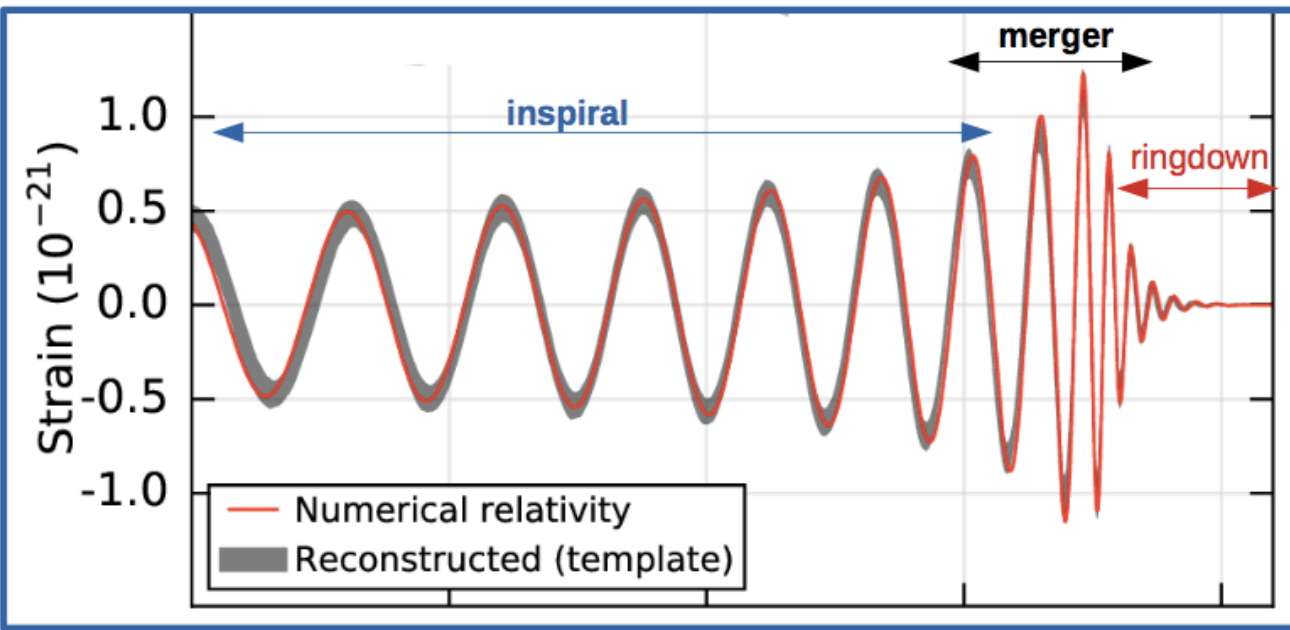
Class.Quant. Grav. (2013), Phys. Rev. D (2014), 1412.3467 (Class. Quant. Grav. 2014),

1503.05568 (Phys. Rev. Lett. 2015), 1509.01247 (Phys. Rev. D, 2015), 1602.08475,

1603.03440, 1604.01989, 1701.01009

Cargese : Hot Topics in Cosmology, 2017

Testing Einstein's Gravity in Infrared

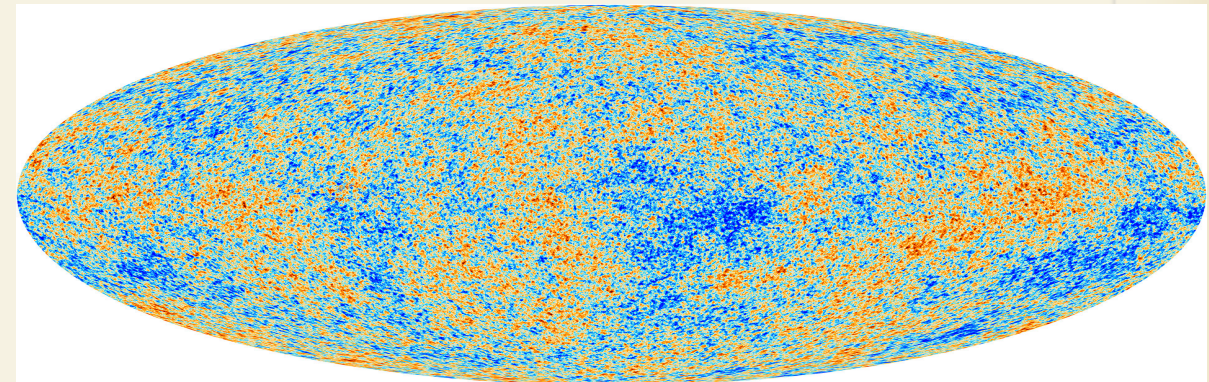


GW150914: fact sheet

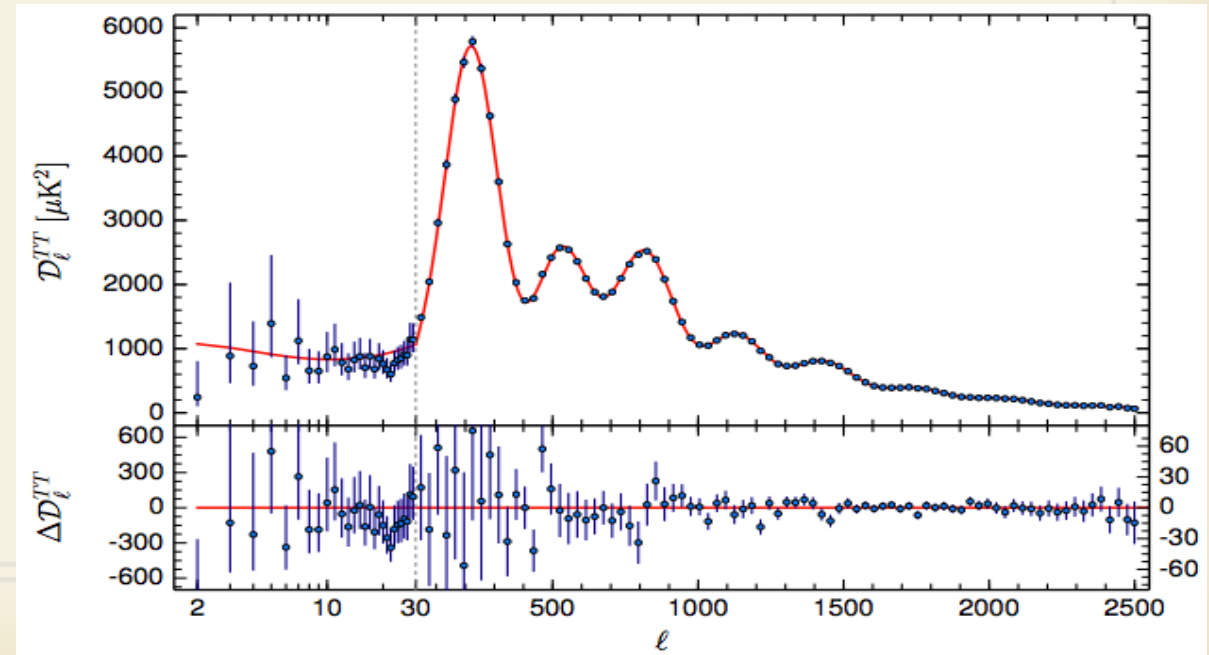
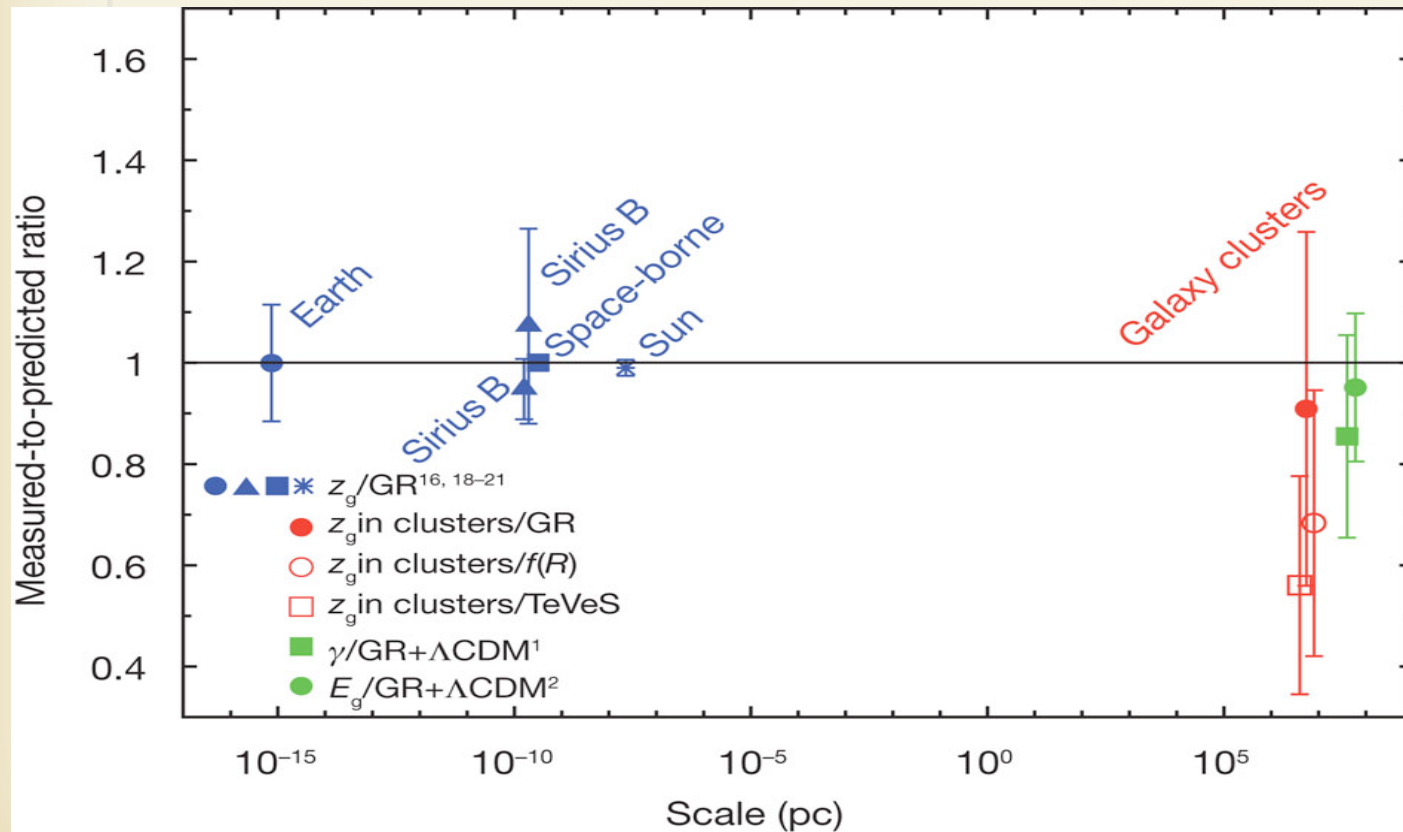
$$m_1 = 36^{+5}_{-4} M_{\odot}$$

$$m_2 = 29^{+4}_{-4} M_{\odot}$$

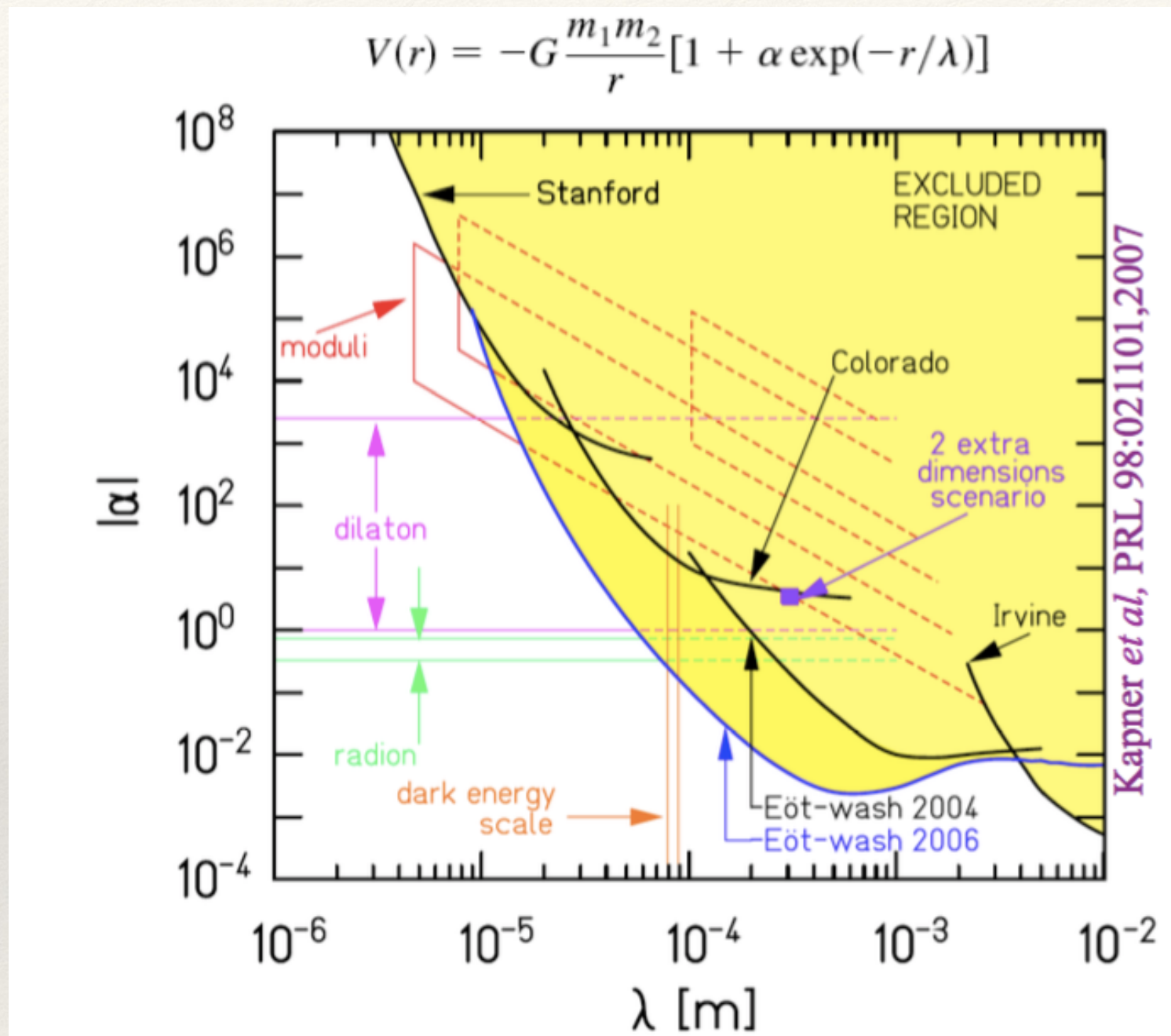
Cosmic Microwave Background Fluctuations



Large scale structure data



Regarding Scales...



No departure from Newtonian Gravity
up to

$$10^{-5} \text{ m} \sim 100 \text{ (eV)}^{-1}$$

or, $M \sim 10^{-2} \text{ eV}$

$$(10^{27} \text{ eV})^4$$

$$(10^{-2} \text{ eV})^4$$

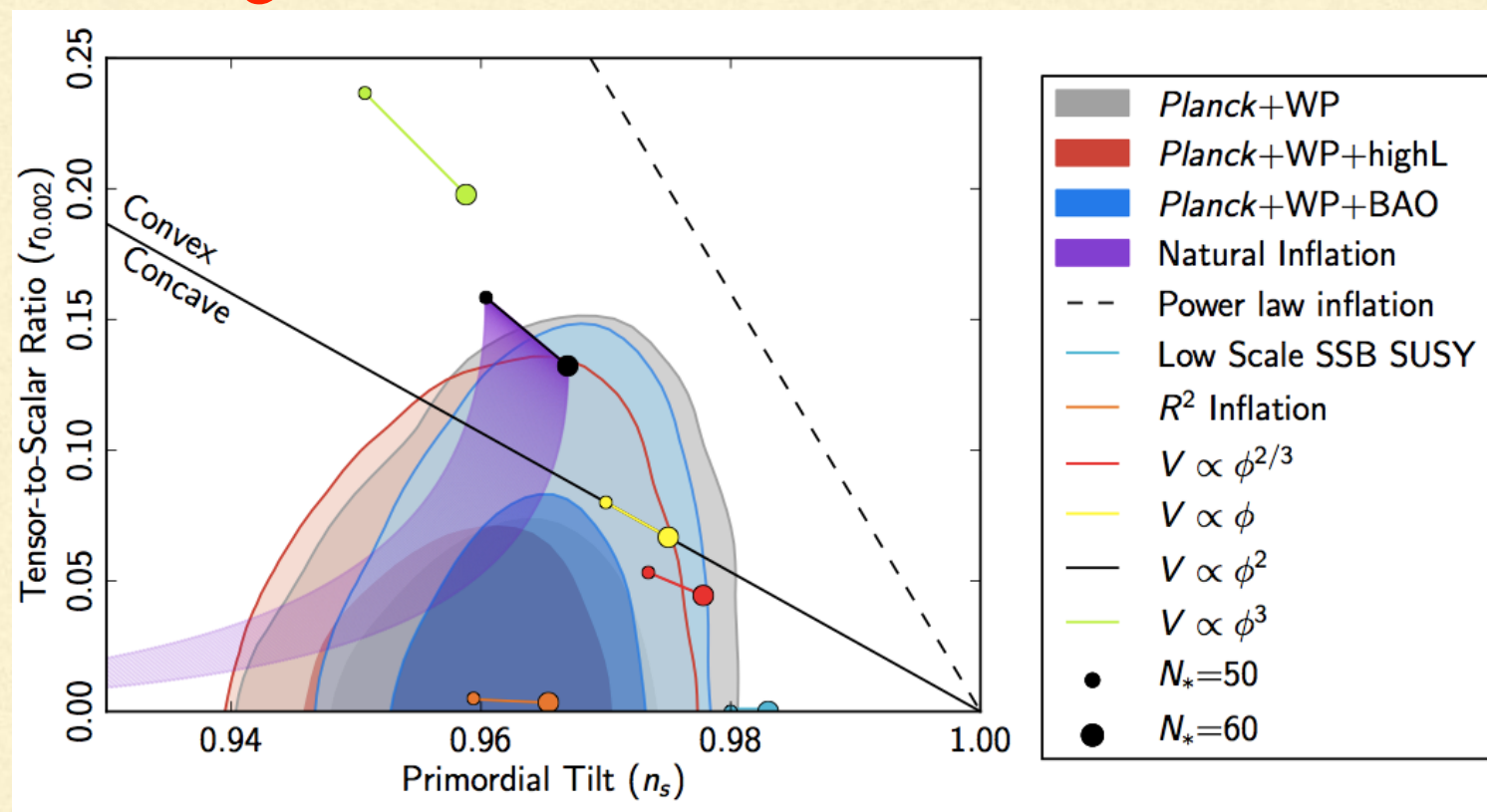
$$(10^{-3} \text{ eV})^4$$

Motivate Quantum Gravity from Observations

$$S \sim M_p^2 R + 10^8 R^2$$

$$S \sim M_p^2 R + \left(\frac{M_p}{M_s}\right)^2 R^2$$

Starobinsky Model



Quadratic Curvature Gravity : No Scale Gravity in UV

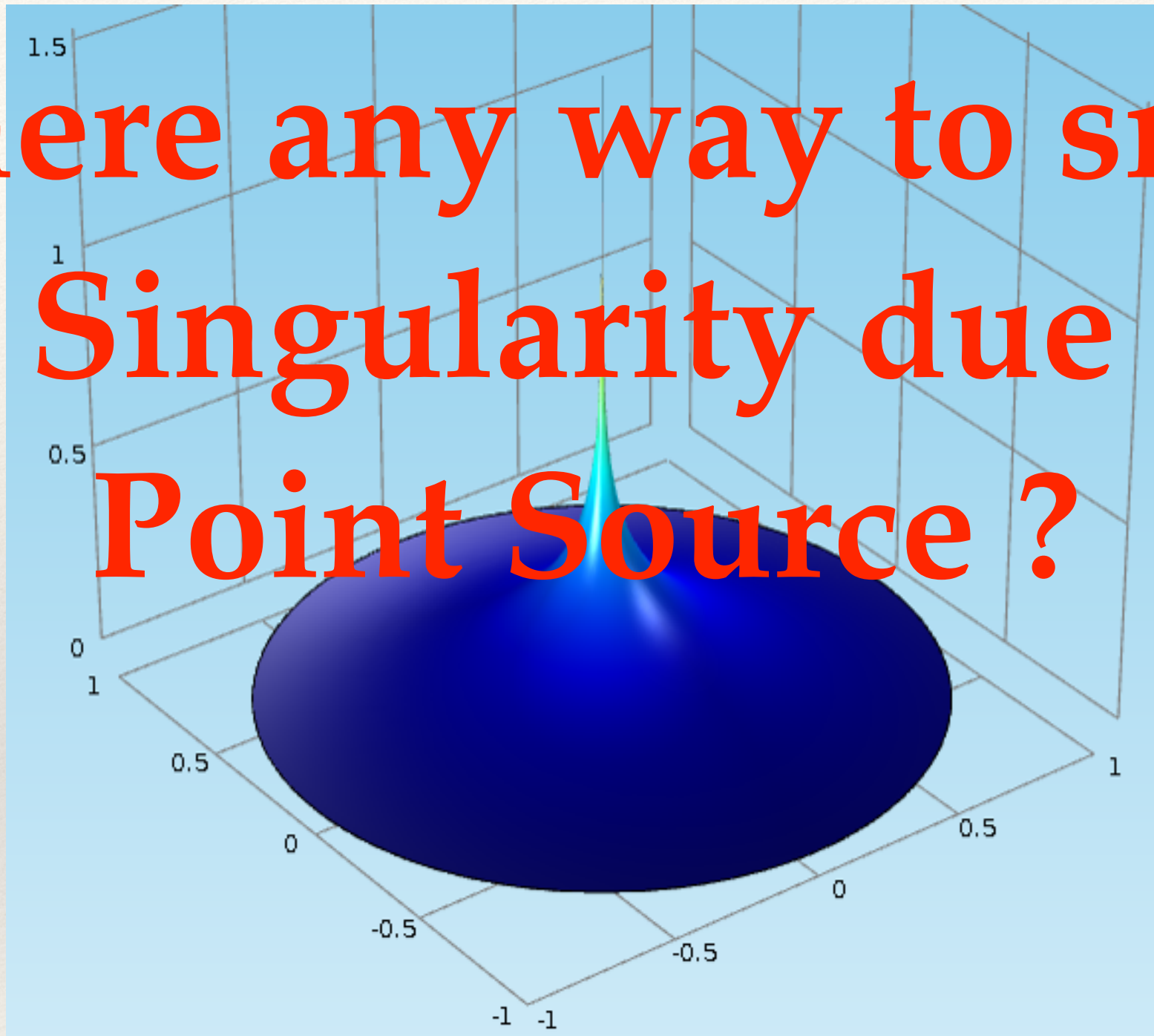
$$S \sim M_p^2 R + \alpha R^2 + \beta R^{mn} R_{mn} + \gamma C^{monp} C_{monp}$$

Numbers

Does a new scale ameliorates Blackholes and Cosmological Singularity Problems ?

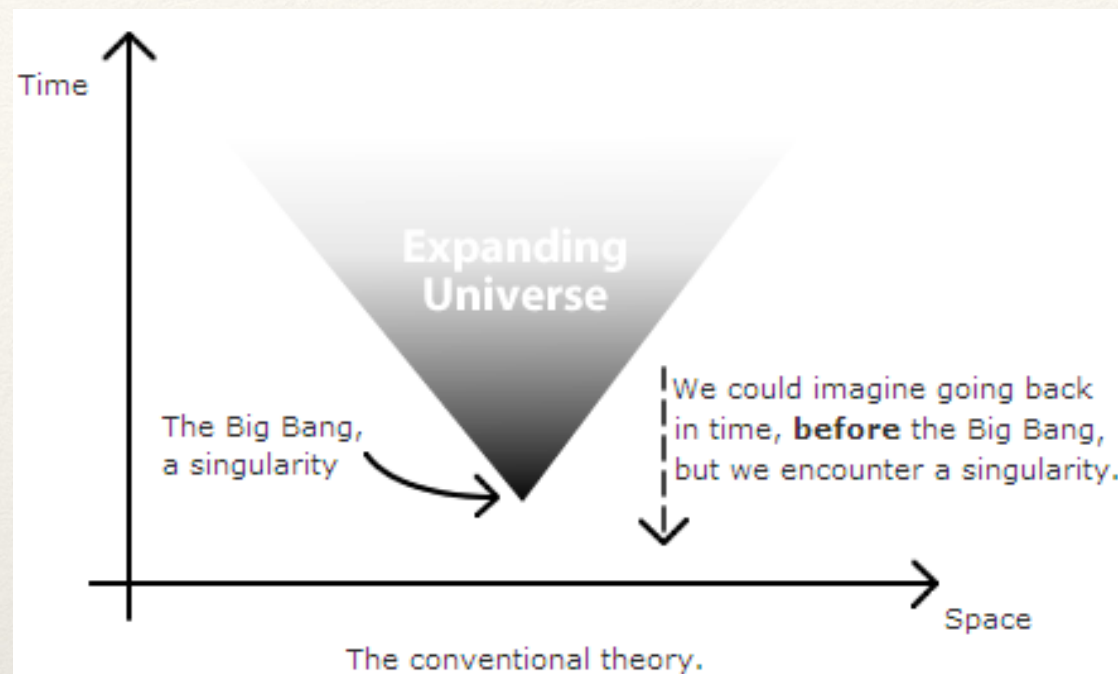
Einstein Gravity

Is there any way to smear
the Singularity due to a
Point Source ?



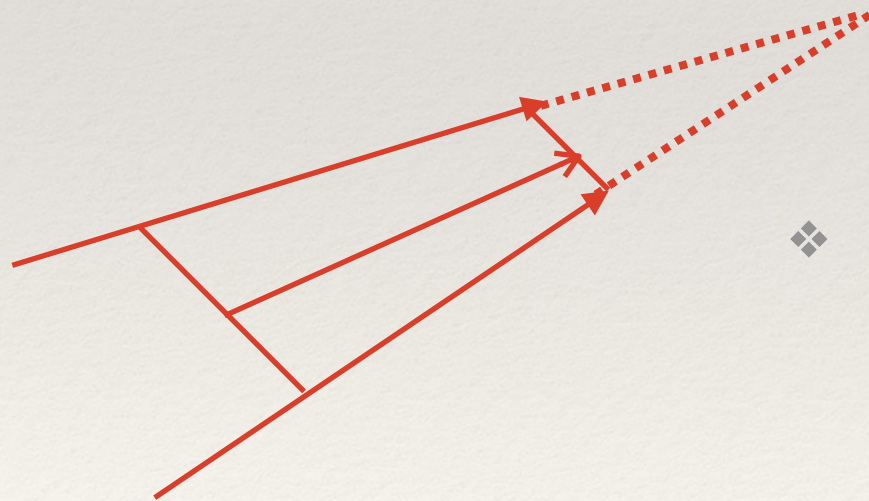
$$ds^2 = \left(1 - \frac{2Gm}{r} \right) dt^2 - \frac{dr^2}{\left(1 - \frac{2Gm}{r} \right)}$$

Cosmological Singularity



Big Bang Singularity,

Space Time have an edge

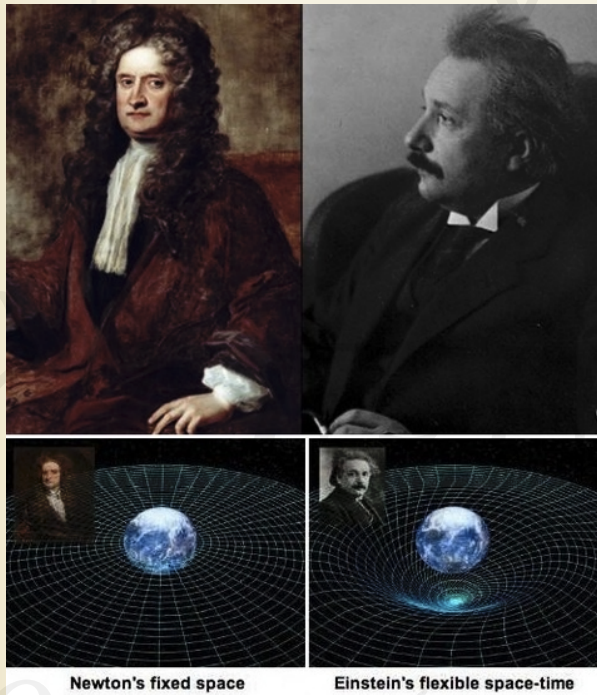


$$\rho + p \geq 0$$

- ❖ A singularity would always imply focusing of geodesics, but **focusing alone cannot imply a singularity**

“Inflation does not solve the singularity problem”

UV Modification of Gravity



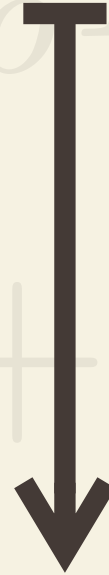
Could we make Gravity sufficiently Weak in UV ?

$$S = \int \sqrt{-g} d^4x \left(\frac{R}{16\pi G} + \dots \right)$$

**Gravity requires modification at small distances
and at early times**

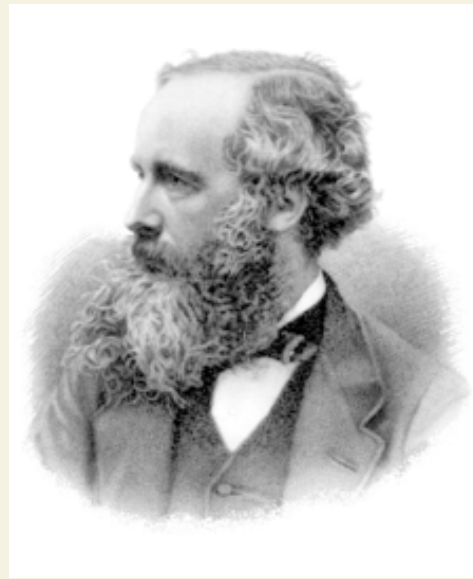
While keeping the General Covariance

**analogous to
Born-Infeld
theory of E & M**

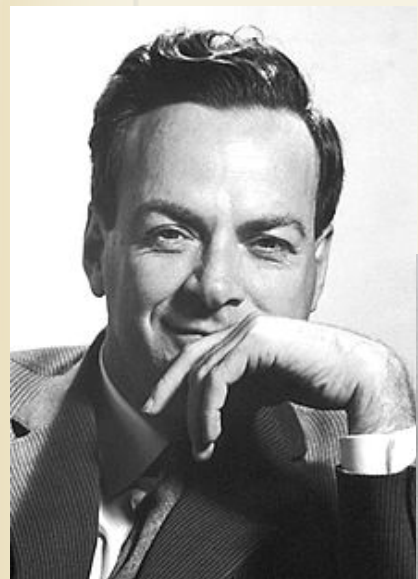

$$S = \int \sqrt{-g} d^4x \left(\frac{R}{16\pi G} \right)$$

Maxwell's Electromagnetism

Self energy of an electron is infinite in Maxwell's theory



$1/r$ -fall of Coulomb's Potential



Quantum
Electrodynamics
(QED)



Classical approach:
Born-Infeld

Born-Infeld resolves $1/r$ singularity in Coulomb Potential

$$\mathcal{L}_{\text{Born-Infeld}} = b^2 \left[1 - \sqrt{1 - (\mathbf{E}^2 - \mathbf{B}^2)/b^2 - (\mathbf{E} \cdot \mathbf{B})^2/b^4} \right]$$

$$b \rightarrow \infty$$

$$\mathcal{L}_{\text{Born-Infeld}} \rightarrow \mathcal{L}_{\text{Maxwell}}$$

Maxwell

$$E_{\text{tot}} = \frac{1}{2} \int (\mathbf{E} \cdot \mathbf{D} + \mathbf{B} \cdot \mathbf{H}) d^3r$$

$$D = e\hat{r}/4\pi r^2, \quad E = e\hat{r}/4\pi\epsilon r^2, \quad B = H = 0$$

$$E_{\text{tot}} = \frac{1}{32\pi^2} \int_0^\infty \frac{e^2}{r^4} 4\pi r^2 dr = \infty$$

Born-Infeld

$$\nabla \cdot \mathbf{D} = e\delta^{(3)}(\mathbf{r}) \quad \mathbf{B} = 0$$

$$\mathbf{D} = \frac{e\mathbf{r}}{4\pi|\mathbf{r}|^3} \quad \mathbf{D}^2/b^2 = \frac{q^2}{r^4}$$

$$\begin{aligned} E_{\text{tot}} &= 4\pi b^2 \int_0^\infty dr r^2 \left(\sqrt{1 + q^2/r^4} - 1 \right) \\ &= \frac{4\Gamma^2(5/4)\sqrt{e^3 b}}{3\pi} = 1.2361\sqrt{e^3 b} \end{aligned}$$

Dealing Pure Gravity similar to QED is extremely hard

$$S = \int \sqrt{-g} d^4x \left(\frac{R}{16\pi G} \right)$$

One loop pure gravitational action is renormalizable

Beyond two loops it is hard to compute, number of Feynman diagrams increases rapidly

Quadratic Curvature Gravity is renormalizable, but contains “Ghosts”: Vacuum is Unstable

Constructing Singularity Free & Ghost Free version for Gravity

Consistent theory of Gravity around Constant Curvature

Backgrounds

Criteria for resolving Cosmological Singularity

Divergence structures in 1 and 2-loops in a scalar Toy model

Without SUSY and SUGRA : SUSY is broken for a generic time dependent scenarios



Consistent General Covariant Quadratic Theories of Gravity with Constant Curvature Backgrounds

“Perturbative Unitarity”

“Ghost Free”

“Tachyon Free”

**“Correct degrees of freedom in
Graviton Propagator”**

Spin-2

&

Spin-0

**components
of a
Graviton
Propagator**

4th Derivative Gravity & Power Counting Renormalizability

$$I = \int d^4x \sqrt{g} \left[\lambda_0 + k R + a R_{\mu\nu} R^{\mu\nu} - \frac{1}{3} (b + a) R^2 \right]$$

$$D \propto \frac{1}{k^4 + Ak^2} = \frac{1}{A} \left(\frac{1}{k^2} - \frac{1}{k^2 + A} \right)$$

Massive Spin-0 & Massive Spin-2 (Ghost) Stelle (1977)

Utiyama, De Witt (1961), Stelle (1977)

Modification of Einstein's GR

Modification
of Graviton
Propagator

Extra propagating
degree of freedom

Challenge: to get rid of the extra dof

Ghosts

Higher Order Derivative Theory Generically Carry Ghosts (-ve Residue) with real “m” (No-Tachyon)

$$S = \int d^4x \phi \square (\square + m^2) \phi \Rightarrow \square (\square + m^2) \phi = 0$$

$$\Delta(p^2) = \frac{1}{p^2(p^2+m^2)} \sim \frac{1}{p^2} - \frac{1}{(p^2+m^2)} \quad \text{Propagator with first order poles}$$

Ghosts cannot be cured order by order, finite terms in perturbative expansion will always lead to Ghosts !!

$$\square e^{-\square} \phi = 0$$

No extra states other than the original dof.

Woodard (1991), Moffat (1991), Tomboulis (1997), Tseytlin (1997), Siegel (2003), Biswas, Grisaru, Siegel (2004), Biswas, Mazumdar, Siegel (2006)

Higher order Construction of Gravity in Any Arbitrary Background

$$S = S_E + S_q$$

$$S_q = \int d^4x \sqrt{-g} R_{\mu_1 \nu_1 \lambda_1 \sigma_1} \mathcal{O}_{\mu_2 \nu_2 \lambda_2 \sigma_2}^{\mu_1 \nu_1 \lambda_1 \sigma_1} R^{\mu_2 \nu_2 \lambda_2 \sigma_2}$$

Covariant derivatives

**Unknown Infinite
Functions of Derivatives**

Well defined Minkowski Limit:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$R \sim \mathcal{O}(h) \quad S_q \sim \int d^4x \sqrt{-g} \mathcal{O}(h^2)$$

Gravitational Form Factors

$$= \int d^4x \sqrt{-g} \left[R + R\mathcal{F}_1(\square)R + R_{\mu\nu}\mathcal{F}_2(\square)R^{\mu\nu} + R_{\mu\nu\alpha\beta}\mathcal{F}_3(\square)R^{\mu\nu\alpha\beta} \right]$$

- (1) GR
- (2) Weyl Gravity
- (3) $F(R)$ Gravity
- (4) Gauss-Bonnet Gravity
- (5) Ghost free Gravity

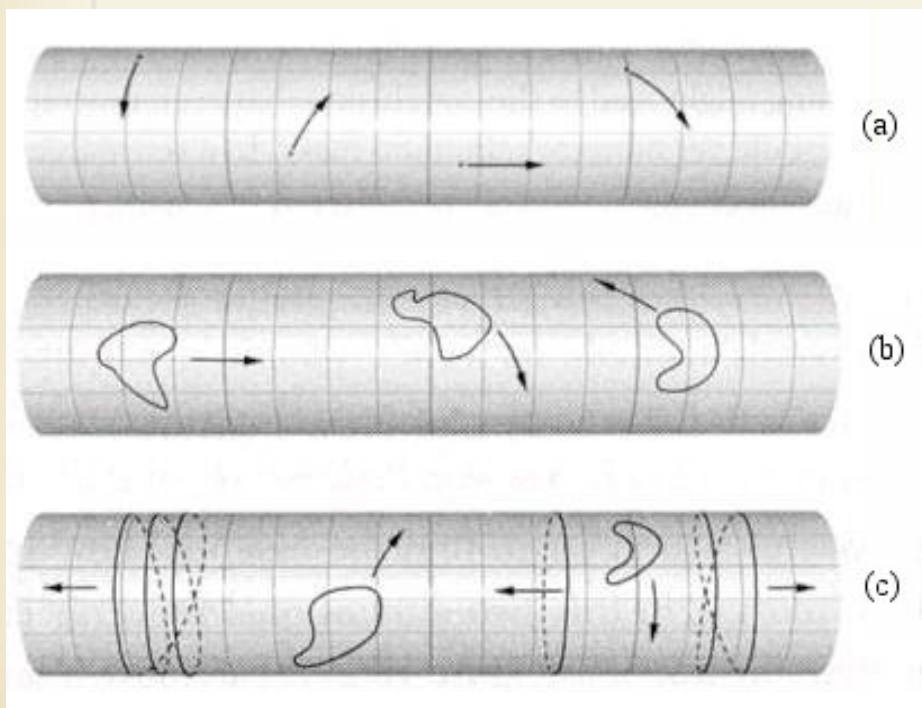
**UV completion of
Starobinsky Inflation up
to quadratic in curvature**

Biswas, Mazumdar, Siegel, 2006,

Chialva, Mazumdar, 2013,

Craps, Koshelev (2014)

Koshelev, Modesto, Rachwal, Starobinsky, (2016)



**Capturing stringy aspects in
Non-local Field theory**

Linearised Equations of Motion around Minkowski

$$= \int d^4x \sqrt{-g} \left[R + R\mathcal{F}_1(\square)R + R_{\mu\nu}\mathcal{F}_2(\square)R^{\mu\nu} + R_{\mu\nu\alpha\beta}\mathcal{F}_3(\square)R^{\mu\nu\alpha\beta} \right]$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$S_q = - \int d^4x \left[\frac{1}{2} h_{\mu\nu} a(\square) \square h^{\mu\nu} + h_{\mu}^{\sigma} b(\square) \partial_{\sigma} \partial_{\nu} h^{\mu\nu} \quad (3) \right. \\ \left. + hc(\square) \partial_{\mu} \partial_{\nu} h^{\mu\nu} + \frac{1}{2} hd(\square) \square h + h^{\lambda\sigma} \frac{f(\square)}{\square} \partial_{\sigma} \partial_{\lambda} \partial_{\mu} \partial_{\nu} h^{\mu\nu} \right]$$

$$a(\square) = 1 - \frac{1}{2} \mathcal{F}_2(\square) \square - 2\mathcal{F}_3(\square) \square$$

$$b(\square) = -1 + \frac{1}{2} \mathcal{F}_2(\square) \square + 2\mathcal{F}_3(\square) \square$$

$$c(\square) = 1 + 2\mathcal{F}_1(\square) \square + \frac{1}{2} \mathcal{F}_2(\square) \square$$

$$d(\square) = -1 - 2\mathcal{F}_1(\square) \square - \frac{1}{2} \mathcal{F}_2(\square) \square$$

$$f(\square) = -2\mathcal{F}_1(\square) \square - \mathcal{F}_2(\square) \square - 2\mathcal{F}_3(\square) \square.$$

$$R_{\mu\nu\lambda\sigma} = \frac{1}{2} (\partial_{[\lambda} \partial_{\nu} h_{\mu\sigma]} - \partial_{[\lambda} \partial_{\mu} h_{\nu\sigma]})$$

$$R_{\mu\nu} = \frac{1}{2} (\partial_{\sigma} \partial_{(\nu} h_{\mu)}^{\sigma} - \partial_{\nu} \partial_{\mu} h - \square h_{\mu\nu})$$

$$R = \partial_{\nu} \partial_{\mu} h^{\mu\nu} - \square h$$

$$a + b = 0$$

$$c + d = 0$$

$$b + c + f = 0$$

Similar analysis has been derived for dS and AdS

Graviton Propagator around Minkowski

$$a(\square)\square h_{\mu\nu} + b(\square)\partial_\sigma\partial_{(\nu}h_{\mu)}^\sigma + c(\square)(\eta_{\mu\nu}\partial_\rho\partial_\sigma h^{\rho\sigma} + \partial_\mu\partial_\nu h) \\ + \eta_{\mu\nu}d(\square)\square h + \frac{1}{4}f(\square)\square^{-1}\partial_\sigma\partial_\lambda\partial_\mu\partial_\nu h^{\lambda\sigma} = -\kappa\tau_{\mu\nu}$$

$$-\kappa\tau\nabla_\mu\tau_\nu^\mu = 0 = (c + d)\square\partial_\nu h + (a + b)\square h_{\nu,\mu}^\mu + (b + c + f)h_{,\alpha\beta\nu}^{\alpha\beta}$$

Bianchi Identity

$$a + b = 0$$

$$c + d = 0$$

$$b + c + f = 0$$

$$\Pi_{\mu\nu}^{-1\lambda\sigma} h_{\lambda\sigma} = \kappa\tau_{\mu\nu} \quad h = h^{TT} + h^L + h^T$$

$$\Pi = \frac{P^2}{ak^2} + \frac{P_s^0}{(a - 3c)k^2}$$

Spin projection operators

Let us introduce

$$\begin{aligned}
 \mathcal{P}^2 &= \frac{1}{2}(\theta_{\mu\rho}\theta_{\nu\sigma} + \theta_{\mu\sigma}\theta_{\nu\rho}) - \frac{1}{3}\theta_{\mu\nu}\theta_{\rho\sigma}, \\
 \mathcal{P}^1 &= \frac{1}{2}(\theta_{\mu\rho}\omega_{\nu\sigma} + \theta_{\mu\sigma}\omega_{\nu\rho} + \theta_{\nu\rho}\omega_{\mu\sigma} + \theta_{\nu\sigma}\omega_{\mu\rho}), \\
 \mathcal{P}_s^0 &= \frac{1}{3}\theta_{\mu\nu}\theta_{\rho\sigma}, \quad \mathcal{P}_w^0 = \omega_{\mu\nu}\omega_{\rho\sigma}, \\
 \mathcal{P}_{sw}^0 &= \frac{1}{\sqrt{3}}\theta_{\mu\nu}\omega_{\rho\sigma}, \quad \mathcal{P}_{ws}^0 = \frac{1}{\sqrt{3}}\omega_{\mu\nu}\theta_{\rho\sigma},
 \end{aligned} \tag{16}$$

where the transversal and longitudinal projectors in the momentum space are respectively

$$\theta_{\mu\nu} = \eta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}, \quad \omega_{\mu\nu} = \frac{k_\mu k_\nu}{k^2}.$$

Note that the operators \mathcal{P}^i are in fact 4-rank tensors, $\mathcal{P}_{\mu\nu\rho\sigma}^i$, but we have suppressed the index notation here.

Out of the six operators four of them, $\{\mathcal{P}^2, \mathcal{P}^1, \mathcal{P}_s^0, \mathcal{P}_w^0\}$, form a complete set of projection operators:

$$\mathcal{P}_a^i \mathcal{P}_b^j = \delta^{ij} \delta_{ab} \mathcal{P}_a^i \quad \text{and} \quad \mathcal{P}^2 + \mathcal{P}^1 + \mathcal{P}_s^0 + \mathcal{P}_w^0 = 1, \tag{17}$$

$$\mathcal{P}_{ij}^0 \mathcal{P}_k^0 = \delta_{jk} \mathcal{P}_{ij}^0, \quad \mathcal{P}_{ij}^0 \mathcal{P}_{kl}^0 = \delta_{il} \delta_{jk} \mathcal{P}_k^0, \quad \mathcal{P}_k^0 \mathcal{P}_{ij}^0 = \delta_{ik} \mathcal{P}_{ij}^0,$$

Ph.D. Thesis : K. J. Barnes, 1963

R. J. Rivers (1963)

P. Van Nieuwenhuizen,

Nucl.Phys. B60 (1973), 478.

For the above action, see:

Biswas, Koivisto, Mazumdar

1302.0532

Tree level Graviton Propagator

$$\Pi = \frac{P^2}{ak^2} + \frac{P_s^0}{(a - 3c)k^2}$$

**No new propagating degree of freedom
other than the massless Graviton**

$$a(\square) = c(\square) \Rightarrow 2\mathcal{F}_1(\square) + \mathcal{F}_2(\square) + 2\mathcal{F}_3(\square) = 0$$

$$S = \int d^4x \sqrt{-g} \left[\frac{R}{2} + R\mathcal{F}_1(\square)R - \frac{1}{2}R^{\mu\nu}\mathcal{F}_2(\square)R_{\mu\nu} \right]$$

Without loss of generality either \mathcal{F}_1 , or \mathcal{F}_2 , or $\mathcal{F}_3 = 0$

Well known Higher Derivative limits

$$a(\square) = 1 - \frac{1}{2}\mathcal{F}_2(\square)\square - 2\mathcal{F}_3(\square)\square$$

$$b(\square) = -1 + \frac{1}{2}\mathcal{F}_2(\square)\square + 2\mathcal{F}_3(\square)\square$$

$$c(\square) = 1 + 2\mathcal{F}_1(\square)\square + \frac{1}{2}\mathcal{F}_2(\square)\square$$

$$d(\square) = -1 - 2\mathcal{F}_1(\square)\square - \frac{1}{2}\mathcal{F}_2(\square)\square$$

$$f(\square) = -2\mathcal{F}_1(\square)\square - \mathcal{F}_2(\square)\square - 2\mathcal{F}_3(\square)\square.$$

(3) GB Gravity:

$$\mathcal{L} = R + \alpha(\square)G,$$

$$a = c = -b = -d = 1$$

$$\Pi = \Pi_{GR}$$

(1) GR:

$$a(0) = c(0) = -b(0) = -d(0) = 1$$

$$\lim_{k^2 \rightarrow 0} \Pi = (\mathcal{P}^2/k^2) - (\mathcal{P}_s^0/2k^2) \equiv \Pi_{GR}$$

(2) F(R) Gravity:

$$\mathcal{L}(R) = \mathcal{L}(0) + \mathcal{L}'(0)R + \frac{1}{2}\mathcal{L}''(0)R^2 + \dots$$

$$a = -b = 1, \quad c = -d = 1 - \mathcal{L}''(0)\square$$

$$\Pi = \frac{\mathcal{P}^2}{k^2} - \frac{\mathcal{P}_s^0}{2k^2(1 + 3\mathcal{L}''(0)k^2)} \quad \Pi = \Pi_{GR} + \frac{1}{2} \frac{\mathcal{P}_s^0}{k^2 + m^2}, \quad m^2 = \frac{1}{3\mathcal{L}''(0)}$$

(4) Weyl Gravity:

$$\mathcal{L} = R - \frac{1}{m^2}C^2 \quad C^2 = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 2R_{\mu\nu}R^{\mu\nu} + \frac{1}{3}R^2$$

$$a = -b = 1 - (k/m)^2$$

$$c = -d = 1 - (k/m)^2/3 \text{ and } f = -2(k/m)^2/3$$

$$\Pi = \frac{\mathcal{P}^2}{k^2(1 - (k/m)^2)} - \frac{\mathcal{P}_s^0}{2k^2} = \Pi_{GR} - \frac{\mathcal{P}^2}{k^2 + m^2}$$

Complete Field Equations

Ghost-free gravity

2.3. The Complete Field Equations

$$S = \int d^4x \sqrt{-g} \left(\frac{R}{2} + R\mathcal{F}_1(\square)R + R^{\mu\nu}\mathcal{F}_2(\square)R_{\mu\nu} + C^{\mu\nu\lambda\sigma}\mathcal{F}_3(\square)C_{\mu\nu\lambda\sigma} \right)$$

Following from this we find the equation of motion for the full action S in (1) to be a combination of S_0 , S_1 , S_2 and S_3 above

$$\begin{aligned} P^{\alpha\beta} &= G^{\alpha\beta} + 4G^{\alpha\beta}\mathcal{F}_1(\square)R + g^{\alpha\beta}R\mathcal{F}_1(\square)R - 4(\nabla^\alpha\nabla^\beta - g^{\alpha\beta}\square)\mathcal{F}_1(\square)R \\ &\quad - 2\Omega_1^{\alpha\beta} + g^{\alpha\beta}(\Omega_{1\sigma}^\sigma + \bar{\Omega}_1) + 4R_\mu^\alpha\mathcal{F}_2(\square)R^{\mu\beta} \\ &\quad - g^{\alpha\beta}R_\nu^\mu\mathcal{F}_2(\square)R_\mu^\nu - 4\nabla_\mu\nabla^\beta(\mathcal{F}_2(\square)R^{\mu\alpha}) + 2\square(\mathcal{F}_2(\square)R^{\alpha\beta}) \\ &\quad + 2g^{\alpha\beta}\nabla_\mu\nabla_\nu(\mathcal{F}_2(\square)R^{\mu\nu}) - 2\Omega_2^{\alpha\beta} + g^{\alpha\beta}(\Omega_{2\sigma}^\sigma + \bar{\Omega}_2) - 4\Delta_2^{\alpha\beta} \\ &\quad - g^{\alpha\beta}C^{\mu\nu\lambda\sigma}\mathcal{F}_3(\square)C_{\mu\nu\lambda\sigma} + 4C_{\mu\nu\sigma}^\alpha\mathcal{F}_3(\square)C^{\beta\mu\nu\sigma} \\ &\quad - 4(R_{\mu\nu} + 2\nabla_\mu\nabla_\nu)(\mathcal{F}_3(\square)C^{\beta\mu\nu\alpha}) - 2\Omega_3^{\alpha\beta} + g^{\alpha\beta}(\Omega_{3\gamma}^\gamma + \bar{\Omega}_3) - 8\Delta_3^{\alpha\beta} \\ &= T^{\alpha\beta}, \end{aligned} \quad (52)$$

where $T^{\alpha\beta}$ is the stress energy tensor for the matter components in the universe and we have defined the following symmetric tensors:

$$\Omega_1^{\alpha\beta} = \sum_{n=1}^{\infty} f_{1n} \sum_{l=0}^{n-1} \nabla^\alpha R^{(l)} \nabla^\beta R^{(n-l-1)}, \quad \bar{\Omega}_1 = \sum_{n=1}^{\infty} f_{1n} \sum_{l=0}^{n-1} R^{(l)} R^{(n-l)}, \quad (53)$$

$$\Omega_2^{\alpha\beta} = \sum_{n=1}^{\infty} f_{2n} \sum_{l=0}^{n-1} R_\nu^{\mu;\alpha(l)} R_\mu^{\nu;\beta(n-l-1)}, \quad \bar{\Omega}_2 = \sum_{n=1}^{\infty} f_{2n} \sum_{l=0}^{n-1} R_\nu^{\mu(l)} R_\mu^{\nu(n-l)}, \quad (54)$$

$$\Delta_2^{\alpha\beta} = \frac{1}{2} \sum_{n=1}^{\infty} f_{2n} \sum_{l=0}^{n-1} [R_\sigma^{\nu(l)} R^{(\beta|\sigma|\alpha)(n-l-1)} - R_\sigma^{\nu;\alpha(l)} R^{\beta\sigma(n-l-1)}]_{;\nu}, \quad (55)$$

$$\Omega_3^{\alpha\beta} = \sum_{n=1}^{\infty} f_{3n} \sum_{l=0}^{n-1} C_{\nu\lambda\sigma}^{\mu;\alpha(l)} C_\mu^{\nu\lambda\sigma;\beta(n-l-1)}, \quad \bar{\Omega}_3 = \sum_{n=1}^{\infty} f_{3n} \sum_{l=0}^{n-1} C_{\nu\lambda\sigma}^{\mu(l)} C_\mu^{\nu\lambda\sigma(n-l)}, \quad (56)$$

$$\Delta_3^{\alpha\beta} = \frac{1}{2} \sum_{n=1}^{\infty} f_{3n} \sum_{l=0}^{n-1} [C_{\sigma\mu}^{\lambda\nu(l)} C_\lambda^{(\beta|\sigma\mu|\alpha)(n-l-1)} - C_{\sigma\mu}^{\lambda\nu;\alpha(l)} C_\lambda^{\beta\sigma\mu(n-l-1)}]_{;\nu}. \quad (57)$$

The trace equation is often particularly useful and below we provide it for the general action (1):

$$\begin{aligned} P &= -R + 12\square\mathcal{F}_1(\square)R + 2\square(\mathcal{F}_2(\square)R) + 4\nabla_\mu\nabla_\nu(\mathcal{F}_2(\square)R^{\mu\nu}) \\ &\quad + 2(\Omega_{1\sigma}^\sigma + 2\bar{\Omega}_1) + 2(\Omega_{2\sigma}^\sigma + 2\bar{\Omega}_2) + 2(\Omega_{3\sigma}^\sigma + 2\bar{\Omega}_3) - 4\Delta_{2\sigma}^\sigma - 8\Delta_{3\sigma}^\sigma \\ &= T \equiv g_{\alpha\beta}T^{\alpha\beta}. \end{aligned} \quad (58)$$

It is worth noting that we have checked special cases of our result against previous work in sixth order gravity given in [24] and found them to be equivalent at least to the cubic order (see Appendix C for details).

$$R^{(m)} \equiv \square^m R$$

Biswas, Conroy, Koshelev, Mazumdar
1308.2319 Class.Quant. Grav. (2014)

Cosmological
Bouncing solution is
known exactly

Biswas, Mazumdar, Siegel, 2006,

Stability of Hamiltonian

Hamiltonian Analysis for Infinite Derivative Field Theories and Gravity

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Abstract

Typically higher-derivative theories are unstable. Instabilities manifest themselves from *extra* propagating degrees of freedom, which are unphysical. In this paper, we will investigate *infinite derivative* field theories and study their *true* dynamical degrees of freedom via Hamiltonian analysis. In particular, we will show that if the infinite derivatives can be captured by a Gaussian kinetic term, i.e. *exponential of entire function*, then it is possible to prove that there are only *finite* number of dynamical degrees of freedom. This conclusion is similar to previous analyses which were performed in the context of Lagrangian analysis. We will further extend our investigation into infinite derivative theory of gravity, and in particular concentrate on *ghost free* and *singularity free* theory of gravity, which has been studied extensively in the Lagrangian approach. Here we will show from the Hamiltonian perspective that there are only finite number of degrees of freedom. For a homogeneous case, we will show that the Hamiltonian density can be bounded from below.

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Consistent theories of Gravity around dS and AdS backgrounds

$$S = \int d^4x \sqrt{-g} \left[\mathcal{P}_0 + \sum_i \mathcal{P}_i \prod_I (\hat{\mathcal{O}}_{iI} \mathcal{Q}_{iI}) \right]$$

Most generic action - “Parity Invariant” and “Torsion Free”

$$R = \bar{R} = \text{const}, \quad R_{\mu\nu} = \frac{\bar{R}}{4} \bar{g}_{\mu\nu}, \quad R^\rho_{\mu\sigma\nu} = \frac{\bar{R}}{12} (\delta^\rho_\sigma \bar{g}_{\mu\nu} - \delta^\rho_\nu \bar{g}_{\mu\sigma})$$

$$S = \int d^4x \sqrt{-g} \left[\frac{M_P^2}{2} R - \Lambda + \frac{\lambda}{2} (R \mathcal{F}_1(\square) R + S_{\mu\nu} \mathcal{F}_2(\square) S^{\mu\nu} + C_{\mu\nu\lambda\sigma} \mathcal{F}_3(\square) C^{\mu\nu\lambda\sigma}) \right]$$

$$h_{\mu\nu} = h^\perp_{\mu\nu} + \bar{\nabla}_\mu A^\perp_\nu + \bar{\nabla}_\nu A^\perp_\mu + (\bar{\nabla}_\mu \bar{\nabla}_\nu - \frac{1}{4} \bar{g}_{\mu\nu} \bar{\square}) B + \frac{1}{4} \bar{g}_{\mu\nu} h$$

For pure EH action, see D’Hoker, Freedman, Mathur, Matusis, Rastelli (hep-th/9902042)

Quadratic order Action for spin-2 and spin-0 components

$$S_2 \equiv \frac{1}{2} \int dx^4 \sqrt{-\bar{g}} \widetilde{h}^{\perp\mu\nu} \left(\bar{\square} - \frac{\bar{R}}{6} \right) \left\{ 1 + \frac{2}{M_p^2} \lambda c_{1,0} \bar{R} + \frac{\lambda}{M_p^2} \left[\left(\bar{\square} - \frac{\bar{R}}{6} \right) \mathcal{F}_2(\bar{\square}) + 2 \left(\bar{\square} - \frac{\bar{R}}{3} \right) \mathcal{F}_3 \left(\bar{\square} + \frac{\bar{R}}{3} \right) \right] \right\} \widetilde{h}^{\perp}_{\mu\nu}$$

$$S_0 \equiv -\frac{1}{2} \int dx^4 \sqrt{-\bar{g}} \widetilde{\phi} \left(\bar{\square} + \frac{\bar{R}}{3} \right) \left\{ 1 + \frac{2}{M_p^2} \lambda c_{1,0} \bar{R} - \frac{\lambda}{M_p^2} \left[2(3\bar{\square} + \bar{R}) \mathcal{F}_1(\bar{\square}) + \frac{1}{2} \bar{\square} \mathcal{F}_2 \left(\bar{\square} + \frac{2}{3} \bar{R} \right) \right] \right\} \widetilde{\phi}$$

**Minkowski limit matches
with our earlier propagator**

$$\Pi_2 = \frac{i}{p^2 \left\{ 1 - \frac{2p^2}{M_p^2} [\mathcal{F}_2(-p^2) + 2\mathcal{F}_3(-p^2)] \right\}},$$

$$\Pi_0 = \frac{-i}{p^2 \left\{ 1 + \frac{2p^2}{M_p^2} [6\mathcal{F}_1(-p^2) + \frac{1}{2}\mathcal{F}_2(-p^2)] \right\}}$$

$$\widetilde{h}^{\perp}_{\mu\nu} = \frac{1}{2} M_p h^{\perp}_{\mu\nu}, \quad \widetilde{\phi} = \sqrt{\frac{3}{32}} M_p \phi$$

Most generic Ghost Free Graviton Propagator in dS/AdS

$$\mathcal{T}(\bar{R}, \bar{\square}) \equiv 1 + \frac{4\bar{R}}{M_p^2} c_{1,0} + \frac{2}{M_p^2} \left[\left(\bar{\square} - \frac{\bar{R}}{6} \right) \mathcal{F}_2(\bar{\square}) + 2 \left(\bar{\square} - \frac{\bar{R}}{3} \right) \mathcal{F}_3 \left(\bar{\square} + \frac{\bar{R}}{3} \right) \right]$$

$$\mathcal{S}(\bar{R}, \bar{\square}) \equiv 1 + \frac{4\bar{R}}{M_p^2} c_{1,0} - \frac{2}{M_p^2} \left[2(3\bar{\square} + \bar{R}) \mathcal{F}_1(\bar{\square}) + \frac{1}{2} \bar{\square} \mathcal{F}_2 \left(\bar{\square} + \frac{2}{3} \bar{R} \right) \right]$$

$$\mathcal{T}(\bar{R}, \bar{\square}) \equiv e^{\tau(\bar{\square})},$$

$$\mathcal{S}(\bar{R}, \bar{\square}) \equiv \left(1 - \frac{\bar{\square}}{m^2} \right)^\epsilon e^{\sigma(\bar{\square})}$$

$\epsilon = 0$, No scalar propagating d.o.f.

Newtonian Limit in Minkowski

$$\Pi = \frac{P^2}{ak^2} + \frac{P_s^0}{(a-3c)k^2} \quad a(\square) = c(\square) = e^{-\square/M^2}$$

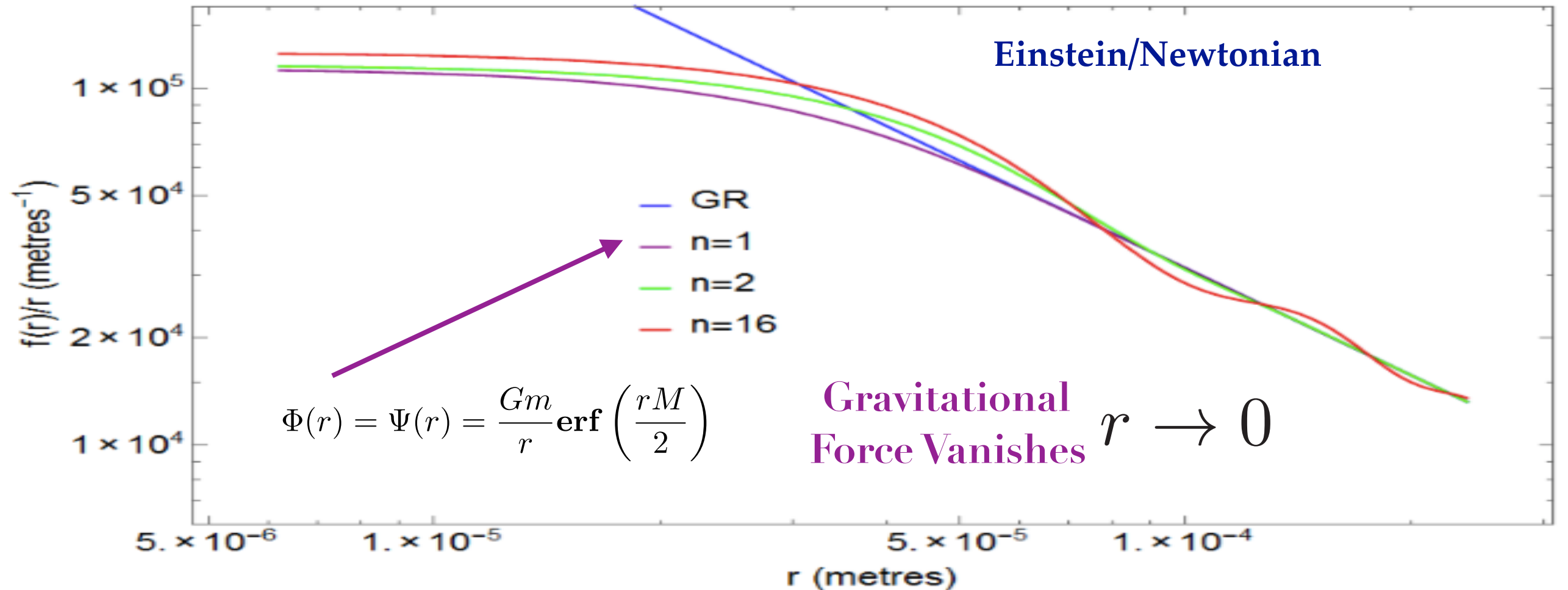
$$S = \int d^4x \sqrt{-g} \left[\frac{R}{2} + R \left[\frac{e^{-\frac{\square}{M^2}} - 1}{\square} \right] R - 2R_{\mu\nu} \left[\frac{e^{-\frac{\square}{M^2}} - 1}{\square} \right] R^{\mu\nu} \right]$$

$$ds^2 = -(1 - 2\Phi)dt^2 + (1 + 2\Psi)dr^2$$

$$\Phi = \Psi = \frac{Gm}{r} \operatorname{erf} \left(\frac{rM}{2} \right)$$

Resolution of Singularity at short distances

$$a(\square) = e^{\gamma(\square)} \quad \text{Any Entire Function: } \gamma(\square) = -\frac{\square}{M^2} - \sum_N a_N \left(\frac{\square}{M^2}\right)^N$$



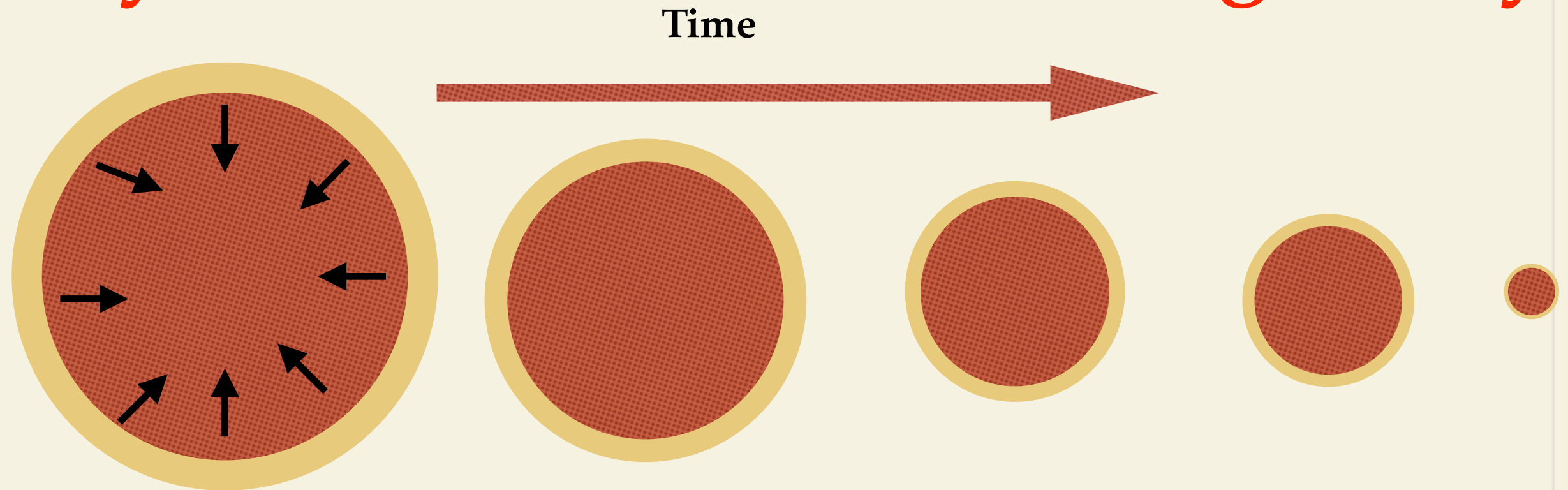
$$mM \ll M_p^2 \implies m \ll M_p$$

Current Bound : $M > 0.01 \text{ eV}$
 $m \leq 10^{25} \text{ grams}$

Edholm, Koshelev, Mazumdar (2016)

Frolov & Zelnikov (2015, 2016)

No dynamical formation of Singularity



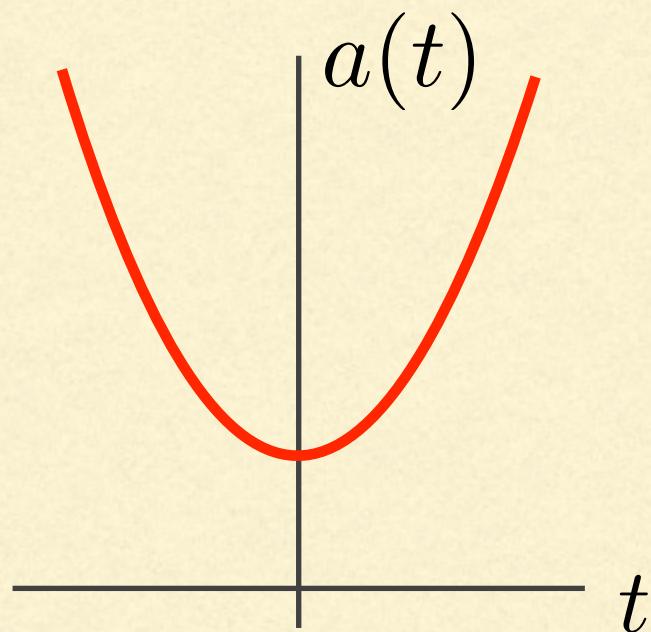
A lump of matter without horizon and without singularity

Valeri Frolov & Andrei Zelnikov (2015, 2016)



Cosmological Singularity can be resolved in a Full Non-linear Regime

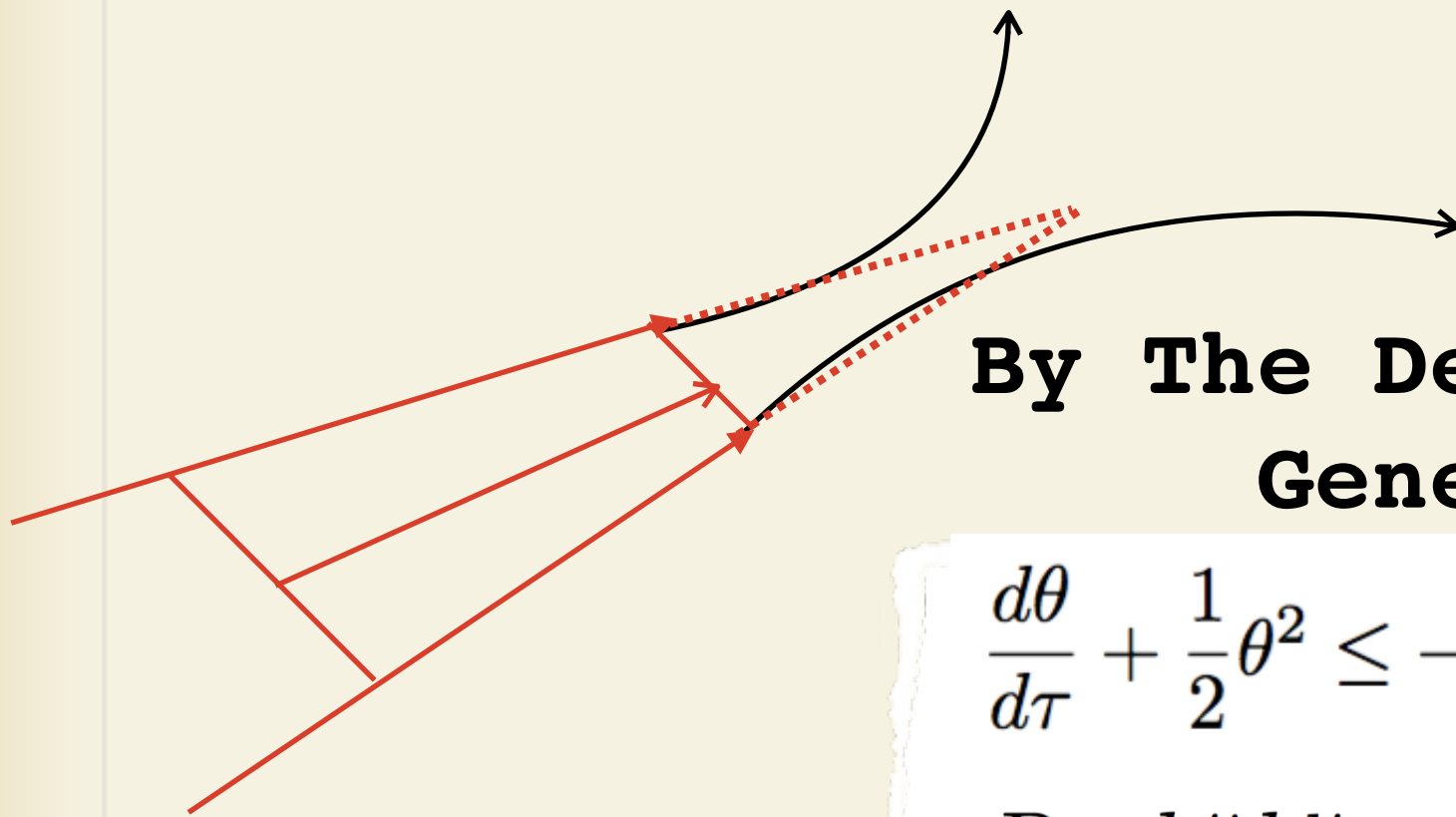
$$S = \int d^4x \sqrt{-g} \left[\frac{R}{2} + R \left[\frac{e^{\frac{-\square}{M^2} - 1}}{\square} \right] R + \Lambda \right]$$



$$a(t) = a_0 \cosh \left(\sqrt{\frac{\Lambda}{6M_{pl}^2}} t \right)$$

$$\square R = c_1 R + c_2$$

Defocusing Null rays



**By The Defocusing Theorem of
General Relativity**

$$\frac{d\theta}{d\tau} + \frac{1}{2}\theta^2 \leq -R_{\mu\nu}k^\mu k^\nu$$

$$R_{\mu\nu}k^\mu k^\nu = \kappa T_{\mu\nu}k^\mu k^\nu$$

$$R_{\mu\nu}k^\mu k^\nu \geq 0, \quad \frac{d\theta}{d\tau} + \frac{1}{2}\theta^2 \leq 0$$

Einstein-Hilbert:

Infinite derivative Gravity:

$$R_{\mu\nu}k^\mu k^\nu \leq 0, \quad \frac{d\theta}{d\tau} + \frac{1}{2}\theta^2 \geq 0$$

3 Criteria for Defocusing Null Congruences without Ghosts & Tachyons

$$\frac{f(\bar{\square})\square}{a(\bar{\square})}R^{(L)} > 0 \Rightarrow \frac{a(\bar{\square}) - c(\bar{\square})}{a(\bar{\square})}R^{(L)} > 0$$

$$c(\bar{\square}) = \frac{a(\bar{\square})}{3} [1 + 2(1 - \alpha M_P^{-2}\square)\tilde{a}(\bar{\square})]$$

$$S = \frac{1}{2} \int d^4x \sqrt{-g} [M_P^2 R + R\mathcal{F}_1(\bar{\square})R]$$

$$\Pi = \frac{P^2}{ak^2} + \frac{P_s^0}{(a-3c)k^2}$$

**Massless Graviton
for : a=c**

(1) Infinite Derivatives

Locality leads to Starobinsky Model, which requires Tachyonic massive Spin-0 states to resolve singularity, but it cannot give Inflation !

(2) Massless Spin-2,

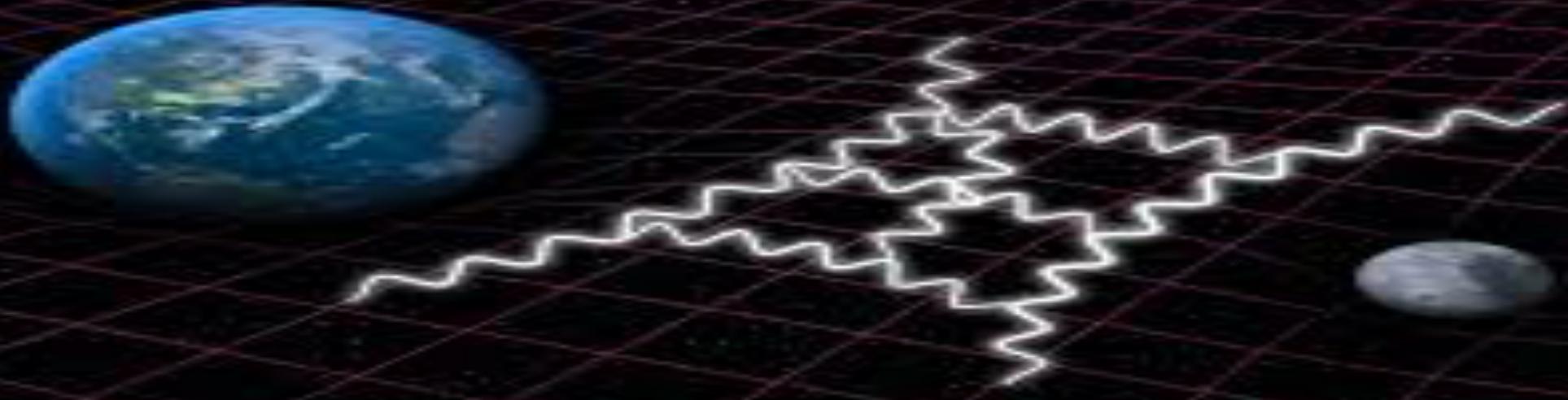
(3) Non-Tachyonic Massive Spin-0

$$\Pi(-k^2) = \frac{1}{a(-k^2)} \left[\frac{\mathcal{P}^2}{k^2} - \frac{1}{2\tilde{a}(-k^2)} \left(\frac{\mathcal{P}_s^0}{k^2} - \frac{\mathcal{P}_s^0}{k^2 + m^2} \right) \right]$$

$$S = \frac{1}{2} \int d^4x \sqrt{-g} [M_p^2 R + cR^2]$$

$$\Pi_{R^2} = \Pi_{GR} + \frac{1}{2} \frac{\mathcal{P}_s^0}{k^2 + m^2}$$

Quantum Aspects



How to make Gravity UV Finite ?

Could we make Gravity weak in UV?

Some interesting progress have been made:

Gravitational entropy, Boundary action, Hamiltonian, Quantum loop corrections, Ultra high energy scatterings, etc.

Quantum aspects

- **Superficial degree of divergence goes as**

$E = V - I$. Use Topological relation : $L = 1 + I - V$

$$E = 1 - L \quad E < 0, \text{ for } L > 1$$

- **At 1-loop, the theory requires counter term, the 1-loop, 2 point function yields Λ^4 divergence**
- **At 2-loops, the theory does not give rise to additional divergences, the UV behaviour becomes finite, at large external momentum, where dressed propagators gives rise to more suppression than the vertex factors**

Toy model based on Symmetries

$$\text{GR e.o.m : } g_{\mu\nu} \rightarrow \Omega g_{\mu\nu}$$

Around Minkowski space the e.o.m are invariant under:

$$h_{\mu\nu} \rightarrow (1 + \epsilon)h_{\mu\nu} + \epsilon\eta_{\mu\nu}$$

Construct a scalar field theory with infinite derivatives whose e.o.m are invariant under

$$\phi \rightarrow (1 + \epsilon)\phi + \epsilon$$

$$S_{free} = \frac{1}{2} \int d^4x (\phi \square a(\square) \phi) \quad a(\square) = e^{-\square/M^2}$$

$$S_{int} = \frac{1}{M_p} \int d^4x \left(\frac{1}{4} \phi \partial_\mu \phi \partial^\mu \phi + \frac{1}{4} \phi \square \phi a(\square) \phi - \frac{1}{4} \phi \partial_\mu \phi a(\square) \partial^\mu \phi \right)$$

$$\Pi(k^2) = -\frac{i}{k^2 e^{\bar{k}^2}}$$

Towards understanding the ultraviolet behavior of quantum loops in infinite-derivative theories of gravity

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Abstract

In this paper we will consider quantum aspects of a non-local, infinite derivative scalar field theory - a *toy model* depiction of a covariant infinite derivative, non-local extension of Einstein's general relativity which has previously been shown to be free from ghosts around the Minkowski background. The graviton propagator in this theory gets an exponential suppression making it *asymptotically free*, thus providing strong prospects of resolving various classical and quantum divergences. In particular, we will find that at 1-loop, the 2-point function is still divergent, but once this amplitude is renormalized by adding appropriate counter terms, the ultraviolet (UV) behavior of all other 1-loop diagrams as well as the 2-loop, 2-point function remains well under control. We will go on to discuss how one may be able to generalize our computations and arguments to arbitrary loops.

Ultra High Energy Scatterings of Scalar Gravitons

High-Energy Scatterings in Infinite-Derivative Field Theory and Ghost-Free Gravity

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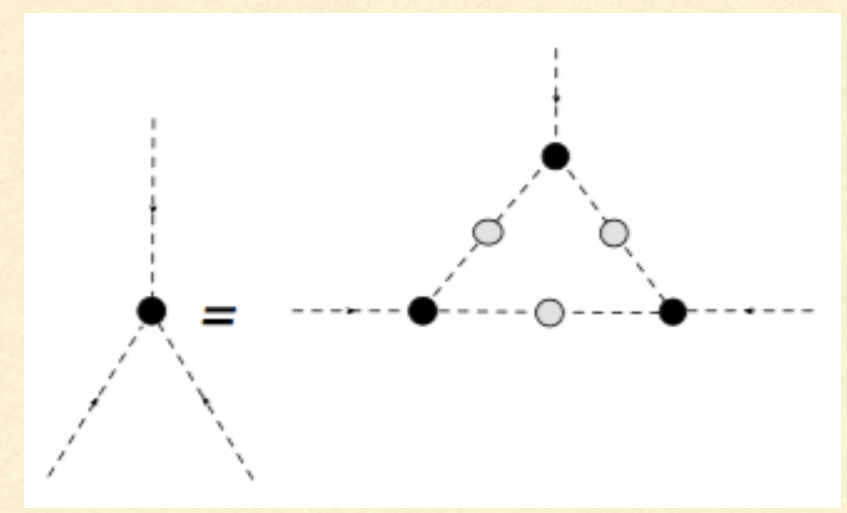
March 14, 2016

Abstract

In this paper, we will consider scattering diagrams in the context of infinite-derivative theories. First, we examine a finite-order higher-derivative scalar field theory and find that we cannot eliminate the external momentum divergences of scattering diagrams in the regime of large external momenta. Then, we employ an infinite-derivative scalar toy model and obtain that the external momentum dependence of scattering diagrams is convergent as the external momenta become very large. In order to eliminate the external momentum divergences, one has to dress the bare vertices of the scattering diagrams by considering renormalised propagator and vertex loop corrections to the bare vertices. Finally, we investigate scattering diagrams in the context of a scalar toy model which is inspired by a *ghost-free* and *singularity-free* infinite-derivative theory of gravity, where we conclude that infinite derivatives can eliminate the external momentum divergences of scattering diagrams and make the scattering diagrams convergent in the ultraviolet.

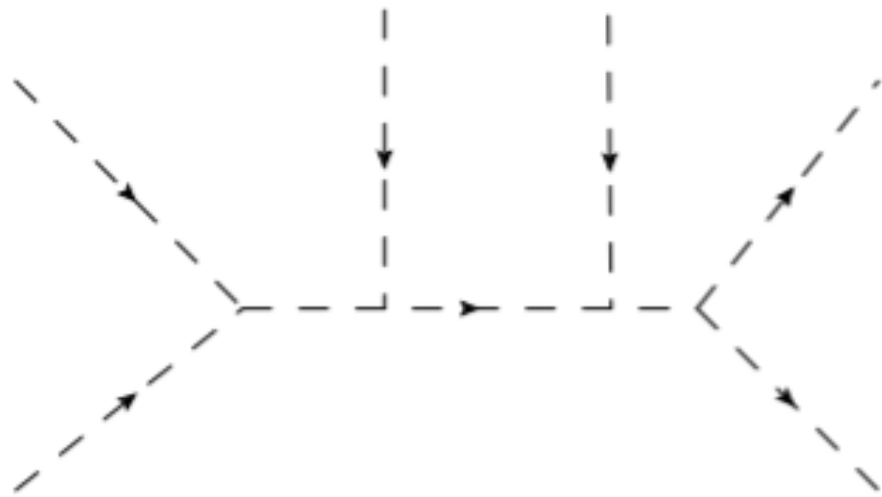


$$\text{grey circle} = \text{dashed line} + \text{black vertex} + \text{black vertex} + \text{black vertex} + \dots$$



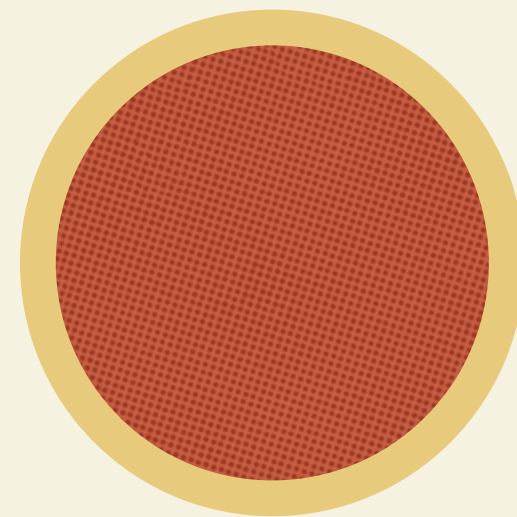
arXiv:1603.03440v1 [hep-th] 10 Mar 2016

Ultra High Energy Scatterings of many Scalar Gravitons :



$$\mathcal{M}' \sim e^{-\left(\frac{p}{M_{eff}}\right)^2},$$
$$M_{eff} = \left(\frac{54}{125n - 206}\right)^{1/2} M$$

Non-Locality can be spread out on Event Horizon scale !

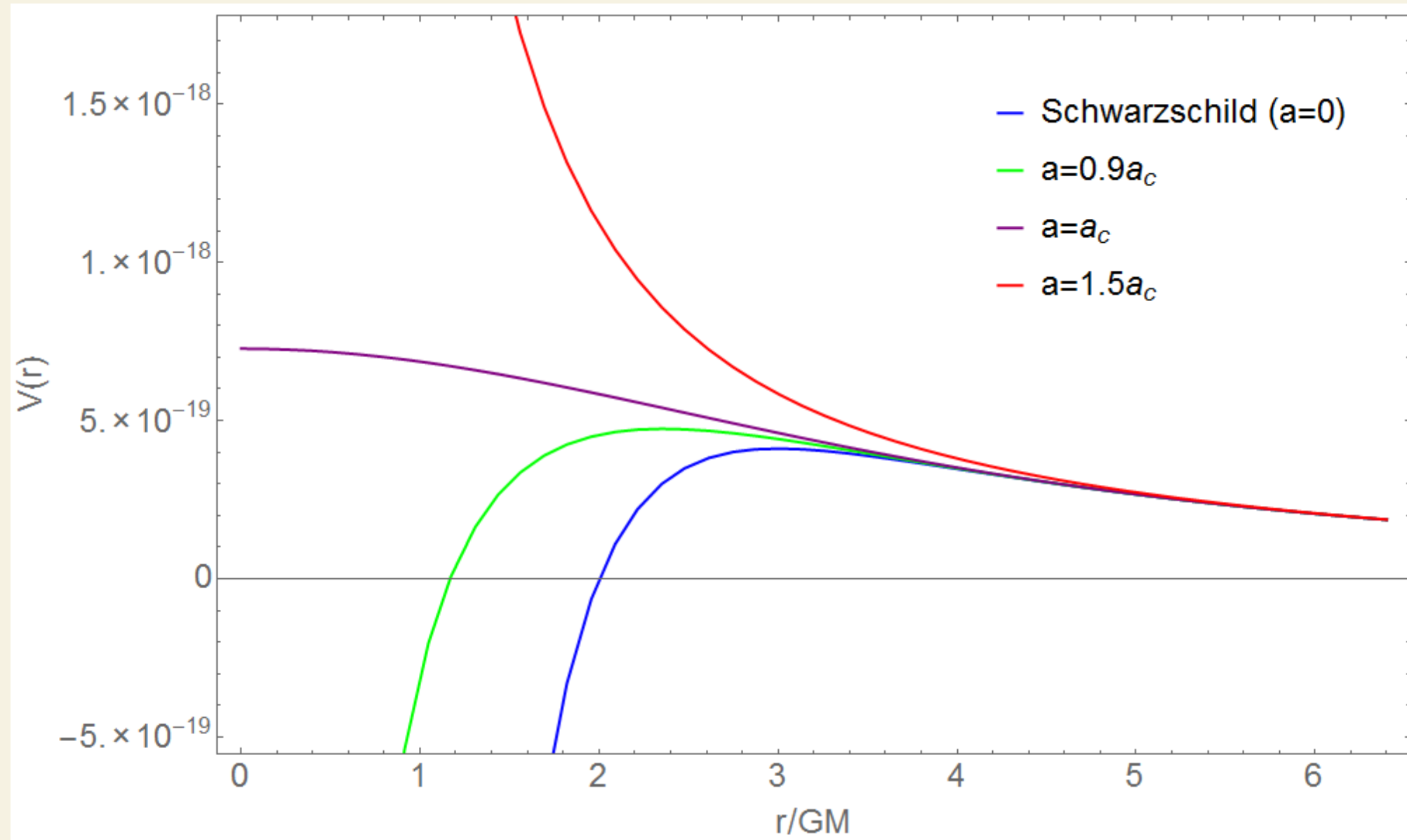


$$r \sim r_* \sim (2GM_{star}) \gg M$$

Conjecture : Gravity can be made weak not to form a trapped surface

Photon Potential

$$ds^2 = - \left(1 - \frac{r_*}{r} \operatorname{Erf} \left(\frac{r}{ar_*} \right) \right) dt^2 + \frac{1}{1 - \frac{r_*}{r} \operatorname{Erf} \left(\frac{r}{ar_*} \right)} dr^2 + r^2 d\theta^2 + r^2 \sin^2(\theta) d\phi^2$$



Conclusions

- We have constructed a Ghost Free & Singularity Free Theory of Gravity.
- Studying singularity theorems, Hawking radiation, Non-Singular Bouncing Cosmology , , many interesting problems has been studied in this framework.
- Quantum computations also show that Infinite Derivative Gravity can ameliorate UV behaviour.
- Ultra-High energy graviton scatterings do not blow up.
- Quantum effects can be seen on Macroscopic scale.

**All these consequences have ramifications for
Blackhole, Inflation & Quantum aspects of Gravity:
Both are Time Dependent Problems**

Extra Slides

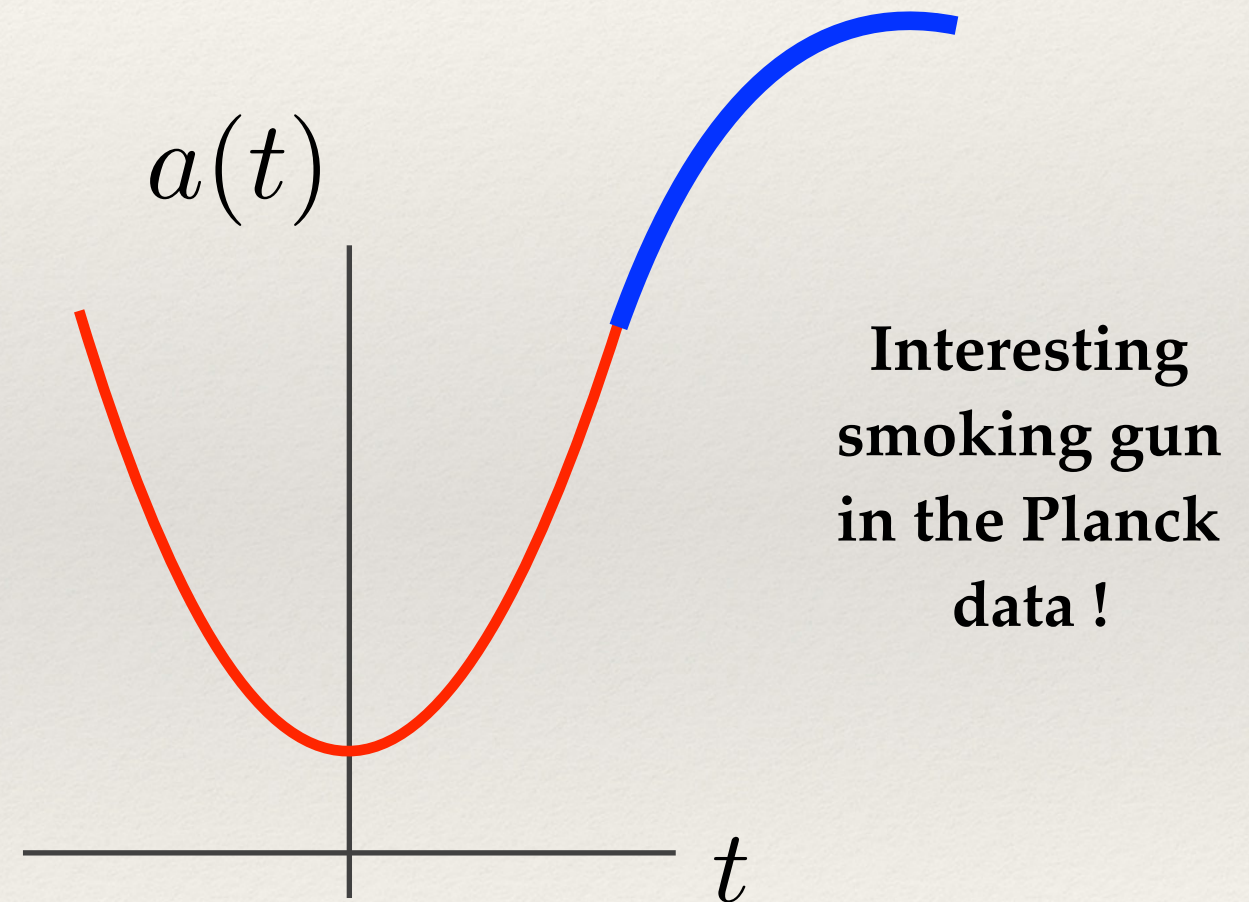
Nonlocal Gravity & Cosmological Singularity

$$S = \int d^4x \sqrt{-g} \left[\frac{R}{2} + R \left[\frac{e^{\frac{-\square}{M^2}} - 1}{\square} \right] R + \Lambda \right]$$

$$a(t) = a_0 \cosh \left(\sqrt{\frac{\Lambda}{6M_{pl}^2}} t \right)$$

**Cosmological
Constant at Bounce**

$$M \sim \Lambda^{1/4}$$



Biswas, AM, PRD (2014)

“Einstein Gravity Does Not Permit Such Solution”

Hawking-Penrose Singularity Theorems & Raychaudhuri Equation

$$\frac{d\theta}{d\tau} + \frac{1}{2}\theta^2 \leq -R_{\mu\nu}k^\mu k^\nu \quad \theta = \nabla_\mu k^\mu$$

$$R_{\mu\nu}k^\mu k^\nu = \kappa T_{\mu\nu}k^\mu k^\nu \quad \text{General Relativity}$$

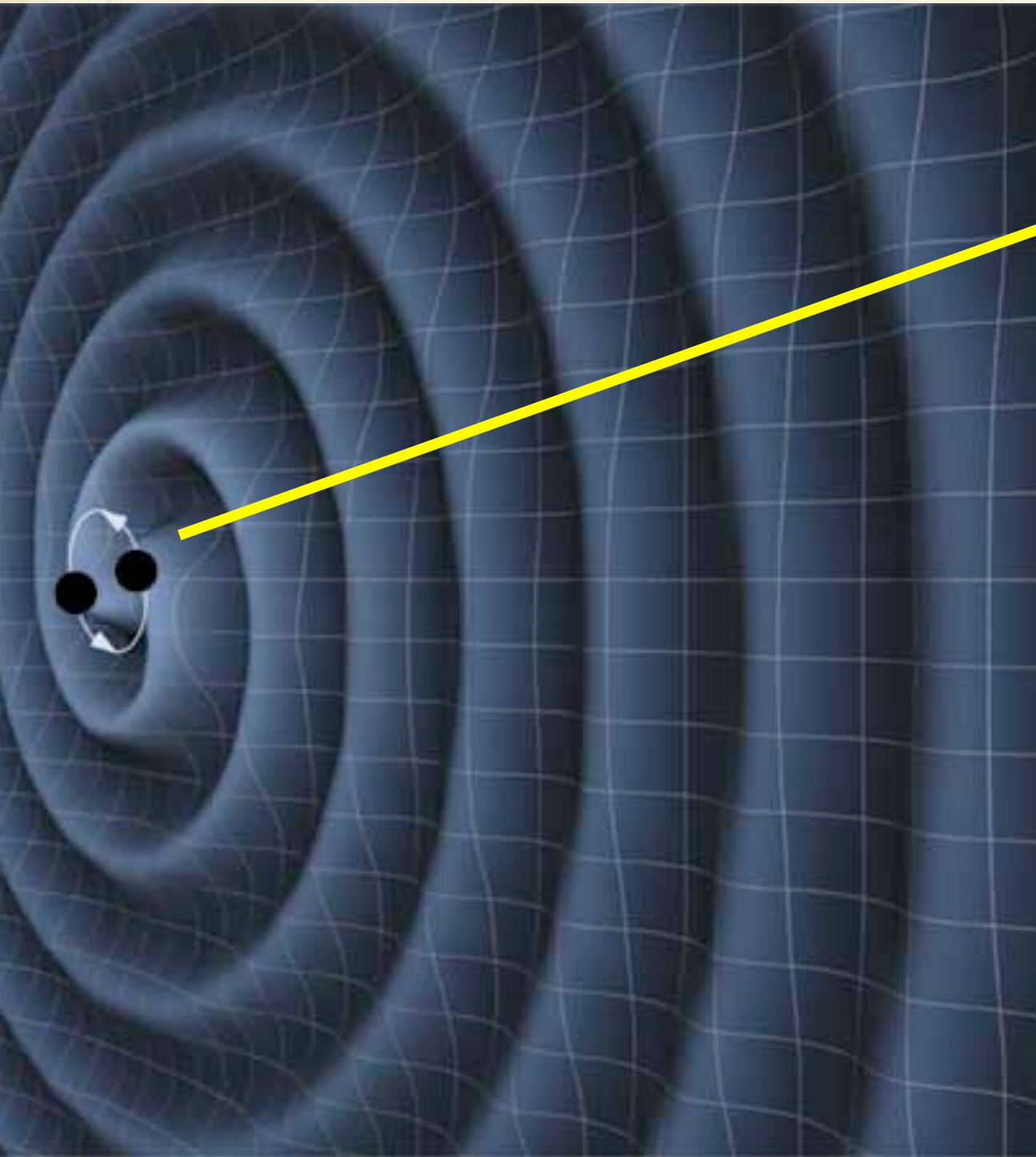
$$R_{\mu\nu}k^\mu k^\nu \geq 0, \quad \frac{d\theta}{d\tau} + \frac{1}{2}\theta^2 \leq 0$$

$$= \int d^4x \sqrt{-g} \left[R + R\mathcal{F}_1(\square)R + R_{\mu\nu}\mathcal{F}_2(\square)R^{\mu\nu} + R_{\mu\nu\alpha\beta}\mathcal{F}_3(\square)R^{\mu\nu\alpha\beta} \right]$$

$$R_{\mu\nu}^{(L)}k^\mu k^\nu = \frac{1}{a(\bar{\square})} \left[\kappa T_{\mu\nu}k^\mu k^\nu - \frac{(k^0)^2}{2} f(\bar{\square})\square R^{(L)} \right]$$

Defocusing: $R_{\mu\nu}^L k^\mu k^\nu \leq 0$

Gravitational Waves



$$\bar{h}_{jk} \approx G \frac{\omega^2 (ML^2)}{r}$$

Large r
limit

$$\bar{h}_{jk} \approx G \frac{\omega^2 (ML^2)}{r} \operatorname{erf} \left(\frac{r M_P}{2} \right)$$

$r \implies 0$, **No Singularity**