Surprises of quantization in de Sitter space

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Setup

- Based on arXiv:1701.07226
- We consider:

$$S = \int d^D x \sqrt{|g|}\, \left[\frac{1}{2}\, g^{\alpha\beta}\, \partial_\alpha \phi\, \partial_\beta \phi \;\; + \frac{1}{2}\, m^2\, \phi^2 + \frac{\lambda}{4}\, \phi^4 \right],$$

where $[sign(g) = (-, +, +, +)]$.

- The background is the expanding Poincaré patch (EPP) $ds^2 = \frac{1}{\eta^2}\, \left[-d\eta^2 + d\vec{x}^2 \right]$, where $\eta = e^{-t}$. Half of global dS space.
- We set the radius of the dS spacetime to one. Our goal is to check whether the assumption of negligible backreaction is self-consistent or not for various sorts of initial conditions.

- Mode functions in the EPP are factorized as

$$\varphi_p\,(\eta,\vec{x}) = \eta^{(D-1)/2}\, h_\nu(p\eta)\, e^{-i\,\vec{p}\,\vec{x}},$$

where $h_\nu(p\eta)$ is a solution of the Bessel equation of order $\nu = \sqrt{\left(\frac{D-1}{2}\right)^2 - m^2}$.

- The field is expanded as:

$$\phi(\eta,\vec{x}) = \int d^{(D-1)}\vec{p}\,\left[a_{\vec{p}}\,\varphi_p\,(\eta,\vec{x}) + a_{\vec{p}}^{+}\,\varphi_p^{*}\,(\eta,\vec{x})\right].$$

$a_{\vec{p}}$ and $a_{\vec{p}}^{+}$ obey proper commutation relations.

Setup

- Any Bessel function of the order ν behaves as follows:

$$h_\nu(p\eta) = \begin{cases} B_+ \frac{e^{i\,p\eta}}{\sqrt{p\eta}} + B_- \frac{e^{-i\,p\eta}}{\sqrt{p\eta}}, & p\eta \gg \nu \\ A_+ \left(p\eta\right)^{\nu} + A_- \left(p\eta\right)^{-\nu}, & p\eta \ll \nu. \end{cases}$$

  Here $B_\pm$ and $A_\pm$ are normalization complex constants.
- We consider complementary series, $m < (D-1)/2$. Hence, $\nu = \sqrt{\left(\frac{D-1}{2}\right)^2 - m^2}$ is real.
- E.g. the Bunch–Davies (BD) modes are as follows: $h_\nu(p\eta) \propto H_\nu^{(1)}(p\eta)$ — Hankel function.

Tree–level propagators

- In non–stationary situations any field is characterized by three propagators.
- Two of them are retarded and advanced propagators:

$$D_0^{\frac{R}{A}}\left(p\,|\eta_1,\eta_2\right) = \pm\theta\left(\mp\Delta\eta_{12}\right)\,2\,\left(\eta_1\,\eta_2\right)^{\frac{D-1}{2}}\;\mathrm{Im}\left[h_\nu(p\eta_1)\;\;h_\nu^*(p\eta_2)\right].$$

They do not depend on the state, at least at tree–level.

- Another propagator is the Keldysh one:

$$D_0^K\left(p|\eta_1,\eta_2\right) = \left(\eta_1\,\eta_2\right)^{\frac{D-1}{2}}\left[\left(\frac{1}{2} + \left\langle a_{\vec{p}}^+\,a_{\vec{p}}\right\rangle_\Psi\right)\,h_\nu(p\eta_1)\,h_\nu^*(p\eta_2) + \right.$$
$$\left. + \left\langle a_{\vec{p}}\;a_{-\vec{p}}\right\rangle_\Psi h_\nu(p\eta_1)\,h_\nu(p\eta_2) + h.c.\right].$$

If the initial state $|\Psi\rangle$ respects the spatial translational invariance. It does depend on the state Ψ.

Secular growth in one–loop corrections

- The two–loop IR correction to the Keldysh propagator coming from the sunset diagram is as follows:

$$D^K_{0+2}\left(p|\eta_1,\eta_2\right) \approx \frac{A_-^2\,\eta^{D-1}}{(p\,\eta)^{2\nu}}\,\left[1 + A\,\lambda^2 \log\left(\frac{p\eta}{\nu}\right)\right], \quad \eta = \sqrt{\eta_1\eta_2}.$$

- This is the leading contribution in the IR limit $p\eta \to 0$ and $\frac{\eta_1}{\eta_2} = const$.
- When $\lambda^2\ \log(p\eta) \sim 1$ we have a breakdown of the perturbation theory and of the semi–classical approximation.

For the initial state $\left\langle a^{+}_{\vec{p}}\, a_{\vec{p}} \right\rangle_g = \left\langle a_{\vec{p}}\, a_{-\vec{p}} \right\rangle_g = 0$, we have that

$$A \approx \frac{8\, A_- A_+}{3(2\pi)^{2(D-1)}} \left\{ \int_1^{\infty} dv\, v^{-D}\, G(v) \left(-\frac{1}{2\nu} v^{2\nu} + \frac{1}{v^{2\nu}} \right) - \right.$$

$$\left. - \int_0^1 dv\, v^{-D}\, G(v) \left(\frac{1}{2\nu} v^{-2\nu} + v^{2\nu} \right) \right\},$$

where

$$G(v) = \int \int \frac{d^{D-1} q_1}{(2\pi)^{D-1}} \frac{d^{D-1} q_2}{(2\pi)^{D-1}} \times$$

$$h_\nu(q_1 v^2) h^*_\nu(q_1) h_\nu(q_2 v^2) h^*_\nu(q_2) h(|\vec{q}_1 + \vec{q}_2|\, v^2) h^*_\nu(|\vec{q}_1 + \vec{q}_2|).$$

On the physical origin of the secular growth

- In the Gaussian approximation — $\left\langle a^+_{\vec{p}}\, a_{\vec{p}} \right\rangle_\Psi = const$, and $\left\langle a_{\vec{p}}\, a_{-\vec{p}} \right\rangle_\Psi = const$. All time dependence is gone into the modes — $h_\nu(p\eta)$.
- All the quasi–classical results are obtained with the use of the tree–level propagator:

$$D_0^K\left(\eta_1, \eta_2 | p\right) = \left(\eta_1\, \eta_2\right)^{\frac{D-1}{2}} \mathrm{Re}\left[h_\nu(p\eta_1)\, h^*_\nu(p\eta_2)\right]$$

E.g. Bunch–Davies's $\langle T_{\mu\nu}\rangle_0$ in de Sitter space.

- However, in non–stationary situation $\left\langle a^+_{\vec{p}}\, a_{\vec{p}} \right\rangle$ and $\left\langle a_{\vec{p}}\, a_{-\vec{p}} \right\rangle$ start to depend on time. That may strongly modify quasi–classical flux.
- Note that that $\left\langle a^+_{\vec{p}}\, a_{\vec{p}} \right\rangle$ and $\left\langle a_{\vec{p}}\, a_{-\vec{p}} \right\rangle$ are attributed to the comoving volume.

Side remarks about contracting patch and global dS

- Contracting Poincaré patch is just time–reversal of the expanding one.
- For the case of ideal spatial homogeneity, we obtain that

$$\Delta_2 D^K \propto \lambda^2 \log\left(\frac{\nu}{p\eta_0}\right), \quad \text{as} \quad p\eta \to \infty.$$

Here $\eta_0 = e^{t_0}$ is the time after which interactions are adiabatically turned on. Breaking of the dS isometry.

- In this case — IR divergence and, hence, adiabatic catastrophe for any initial state.
- In global de Sitter there is also adiabatic catastrophe for any initial state.

Resummation (general discussion)

To resum the leading corrections from all loops one has to

- Check that there are no leading corrections to the retarded and advanced propagators and that the leading secular growth is present only in the Keldysh propagator.
- Check that there are no leading corrections to the vertexes. This is not true for low enough mass.
- Put in the system of the Dyson–Schwinger equations retarded and advanced propagators (and vertexes, if possible) to their tree–level values. Then this system reduces to the single equation for the Keldysh propagator. What remains to be checked: what type of diagrams contribute leading corrections.

Resummation (dS invariant case)

- If one takes exactly BD state at exactly past infinity of the expanding Poincaré patch, then one can show that dS isometry is respected at every loop order, if $m > 0$. (Polyakov, Higuchi, Marolf, Morrison and Tanaka)
- Moreover, one can show that in this case leading contributions come from the summation of the bubble diagrams. If one puts the above one loop logarithmic correction to the Keldysh propagator into the internal legs of the bubble diagram, the correction is suppressed as $\lambda^4 \log(p\eta)$. The situation is very similar to the standard UV renormalization: if one puts loop corrected expressions again inside the loops they lead to subleading corrections, while the leading corrections come from the multiplication of the bubbles.

Resummation (dS invariant case)

- That is the reason why in the Dyson–Schwinger equation one can put the exact Keldysh propagator only into one of the external legs. As a result in this case the Dyson–Schwinger equation reduces to a system of linear integro–differential equations.
- The result of the resummation for the complementary series at the leading order reduces to a mass renormalization
- At the leading order this result agrees with the Starobinsky–Yokoyama approach. (Serreau et al.)

Resummation (subtle issues)

- For the complementary series, $m < (D-1)/2$, there is a subtle issue because for very low masses, $m < \frac{\sqrt{3}}{4}(D-1)$, there there are secularly growing contributions to the vertexes.
- The last type of secular growth appears due to the presence of bound states or higher power correlations.
- If we would like to perturb the initial BD state by a non–symmetric perturbations, we cannot just put initial $\left\langle a^+_{\vec{p}}\, a_{\vec{p}} \right\rangle$ at the past infinity of the EPP. One has to cut the EPP at the physical momentum $(p\eta)_0 \sim |\nu|$.
- In the latter case internal legs in the loops also provide leading contributions. Bubbles are not enough. One should put the exact Keldysh propagator also into the loops in the Dyson–Schwinger equation. As the result, one obtains a non–linear integro–differential equation.

Resummation (non–invariant perturbations)

- Ansatz for the exact Keldysh propagator

$$D^K\left(p\,|\eta_1,\eta_2\right) = A_-^2\,\frac{\eta^{D-1}}{(p\eta)^{2\nu}}\,N(p\eta),\quad \eta=\sqrt{\eta_1\,\eta_2}.$$

- The summing equation for $\lambda\phi^4$:

$$N(p\eta) - N\left(\nu\right) \approx -\int\limits_{\nu}^{p\eta}\frac{du}{u}\left[N(u)+N\left(\nu\right)\right]\times$$

$$\times\left[\Gamma_1\left(\int\limits_{p\eta}^{\nu} dl\, l^{D-2-2\nu}\,N(l)\right)^2 - \Gamma_2\right],\quad \Gamma_1,\Gamma_2>0.$$

$N(\nu)$ is the initial value of $N(p\eta)$.

Resummation (non–invariant perturbations)

- Stable solution (agrees with the dS invariant situation):

$$N(p\eta) \approx A \cdot (p\eta)^{\alpha}, \quad \alpha \approx -\Gamma_1 A^2 \left[\frac{\nu^{(D-1-2\,\nu+\alpha)}}{D-1-2\,\nu+\alpha}\right]^2 + \Gamma_2.$$

where A is a constant of integration and $\alpha < 0$ and $D-1-2\nu+\alpha > 0$.

- Explosive solution:

$$N(p\eta) \approx \frac{B}{(p\eta - p\eta_*)^{\frac{3}{2}}}, \quad 1 \approx 2\,\Gamma_1\, B^2\, (p\eta_*)^{D-1-2\nu}.$$

where B is a constant of integration.

Conclusions

For the explosive solutions the Keldysh propagator blows up at a finite proper time. Then, also the expectation value of the stress–energy tensor blows up (which would appear at the RHS of the Einstein equations due to the quantum fluctuations). That means that the backreaction is not negligible. One possibility is that that the cosmological constant is secularly screened because the expectation value of the stress–energy tensor under discussion does not respect the dS isometry. This is a subject of a separate study. Here we do not consider the backreaction issue.