Particle plunging into a black hole in massive gravity : Excitation of quasibound states and quasinormal modes

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Introduction

- In the context of Einstein's general relativity, the construction of the waveform produced by a plunging particle is of fundamental importance because :
  - i The "plunge regime" is the last phase of the evolution of a stellar mass object orbiting near a supermassive BH.
  - ii The waveform generated during this regime encodes the final BH fingerprint.

- With in mind the possibility to test massive gravity in the context of BH physics :
  - i We consider the radiation produced by a particle plunging from slightly below the innermost stable circular orbit (ISCO) into a Schwarzschild BH.
  - ii In order to circumvent the difficulties associated with BH perturbation theory in massive gravity, we use, at first, a toy model where we replace the graviton field by a massive scalar field and consider a linear coupling between the particle and this field.
  - iii We compute the waveform generated by the plunging particle and study its spectral content. We highlight some important effects which are not present for massless fields.
  - iv We compute the waveform generated by the plunging particle for the odd-parity $\ell = 1$ mode in massive gravity.

- Throughout this presentation :
  - i We display our numerical results by using the dimensionless coupling constant

$$\tilde{\alpha} = \frac{2M\mu}{m_p^2} \tag{1}$$

    (here M, μ and m_p denote respectively the mass of BH, the rest mass of the field and the Planck mass).
  - ii We adopt natural units ($\hbar = c = G = 1$).

Our model

- We consider the exterior of the Schwarzschild BH of mass M $(\mathcal{M}, g_{\alpha\beta})$ defined by the metric

$$ds^2 = g_{\mu\nu}(x)dx^\mu dx^\nu = -\left(1-\frac{2M}{r}\right)dt^2 + \frac{1}{\left(1-\frac{2M}{r}\right)}dr^2 + r^2 d\sigma_2^2, \tag{2}$$

where $d\sigma_2^2 = d\theta^2 + \sin^2\theta d\varphi^2$ denotes the metric on the unit $2-$sphere S^2.

- The particle is coupled to a scalar field ϕ with mass μ and the dynamics of the system field-particle is defined by the action :

$$S = S_{\text{field}} + S_{\text{particle}} + S_{\text{interaction}} \tag{3}$$

with

$$S_{\text{field}} = -\frac{1}{2}\int_{\mathcal{M}} d^4x\sqrt{-g(x)} \times \left[g^{\alpha\beta}(x)\nabla_\alpha\Phi(x)\nabla_\beta\Phi(x) + \mu^2\Phi^2(x)\right], \tag{4}$$

$$S_{\text{particle}} = -m_0\int_\gamma d\tau = -m_0\int_\gamma \sqrt{-g_{\alpha\beta}(z(\lambda))\frac{dz^\alpha(\lambda)}{d\lambda}\frac{dz^\beta(\lambda)}{d\lambda}}d\lambda \tag{5}$$

and

$$S_{\text{interaction}} = \int_{\mathcal{M}} d^4x\sqrt{-g(x)}\,\rho(x)\Phi(x) \tag{6}$$

with

$$\rho(x) = q\int_\gamma d\tau\delta^4(x, z(\tau)) \quad \text{charge density.} \tag{7}$$

here, m_0 mass of the particle, q its scalar charge and $z^\alpha = z^\alpha(\tau)$ describe its world line $\gamma(\lambda)$.

Waveform associated with the (ℓ,m) mode

- Due to both the staticity and the spherical symmetry of the Schwarzschild background, the wave equation reduces to the Regge-Wheeler equation with source

$$\left[\frac{d^2}{dr_*^2} + \omega^2 - V_\ell(r)\right]\phi_{\omega\ell m} = -\left(1 - \frac{2M}{r}\right)\rho_{\omega\ell m}. \tag{8}$$

with the effective potential $V_\ell(r)$ given by

$$V_\ell(r) = \left(1 - \frac{2M}{r}\right)\left[\mu^2 + \frac{\ell(\ell+1)}{r^2} + \frac{2M}{r^3}\right]. \tag{9}$$

and the tortoise coordinate $r_*(r)$ is given by

$$r_*(r) = r + 2M\ln[r/2M - 1]. \tag{10}$$

  - For $r \to 2M$ (black hole horizon), $r_*(r) \to -\infty$
  - For $r \to +\infty$ (spatial infinity), $r_*(r) \to +\infty$

- In order to solve the Regge-Wheeler equation, we use the machinery of Green's function. We can show that the solution of Eq 8 is given by

$$\phi_{\ell m}(t,r) = -\frac{1}{\sqrt{2\pi}}\int_{-\infty+ic}^{+\infty+ic} d\omega\, \frac{e^{-i\omega t}}{W_\ell(\omega)}\, \phi^{\text{up}}_{\omega\ell}(r)\int_{2M}^{+\infty} dr'\,\phi^{\text{in}}_{\omega\ell}(r')\rho_{\omega\ell m}(r'). \tag{11}$$

- Here, $\phi^{\text{in}}_{\omega\ell}$ and $\phi^{\text{up}}_{\omega\ell}$ are linearly independent solutions of the homogenous Regge-Wheeler equation with the usual appropriate boundary conditions at the horizon and spatial infinity and $W_\ell(\omega)$ denotes their Wronskian.

Source due to the particle on a plunge trajectory

- We consider a particle plunging into the BH from $r_{\rm ISCO} = 6M$ (*Innermost Stable Circular Orbit*).
- The plunge trajectory lies in the equatorial plane ($\theta = \pi/2$) is given by

$$\frac{t_p(r)}{2M} = \frac{2\sqrt{2}(r-24M)}{2M(6M/r-1)^{1/2}} - 22\sqrt{2}\tan^{-1}\left[(6M/r-1)^{1/2}\right] + 2\tanh^{-1}\left[(3M/r-1/2)^{1/2}\right] + t_0 \quad (12)$$

and

$$\varphi(r) = -\frac{2\sqrt{3r}}{(6M-r)^{1/2}} + \varphi_0 \quad (13)$$

which can be rewritten

$$r(\varphi) = \frac{6M}{\left[1+12/(\varphi-\varphi_0)^2\right]}. \quad (14)$$

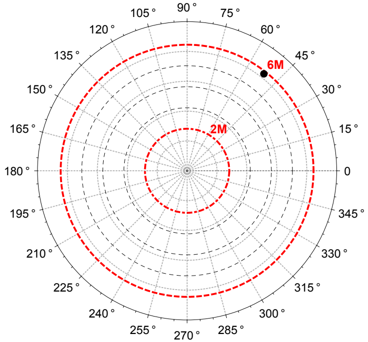


FIGURE 1 – The plunge trajectory. Here, we assume that the particle starts at $r = r_{\rm ISCO}(1-\epsilon)$ with $r_{\rm ISCO} = 6M$ and $\epsilon = 2\times 10^{-2}$.

- We have for the source of the plunging particle

$$\rho_{\omega\ell m}(r) = \frac{3q\sqrt{r}}{\sqrt{2\pi}} \frac{e^{i[\omega t_p(r) - m\varphi_p(r)]}}{(6M-r)^{3/2}} Y^*_{\ell m}\left(\frac{\pi}{2},0\right). \quad (15)$$

Quadrupolar waveforms produced by the plunging particle

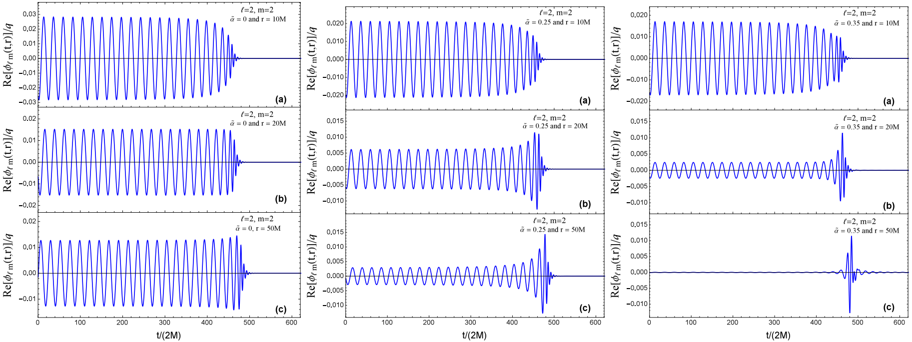


FIGURE 2 – Quadrupolar waveforms produced by the plunging particle.The results are obtained for massless scalar field ($\tilde{\alpha} = 0$) and for massive scalar field ($\tilde{\alpha} = 0.25, 0.35$). The observer is located at (a) $r = 10M$, (b) $r = 20M$ et (c) $r = 50M$.

- For the massless scalar field ($\tilde{\alpha} = 0$), the waveforms can be decomposed in 3 phases :
  - Adiabatic phase
  - A ringdown phase
  - A late-time tail
- For the massive scalar field($\tilde{\alpha} \neq 0$) :
  - Such a decomposition remains roughly valid for low masses.
- For a given distance r, the waveform amplitude decreases as the mass increases.

The adiabatic phase and the circular motion of the particle on the ISCO

- Particle in circular orbit on the ISCO :
  - The trajectory is given by

$$\varphi_p(t) = \Omega_{\text{ISCO}}\, t \quad \text{où} \quad \Omega_{\text{ISCO}} = \frac{1}{6\sqrt{6}M} \approx 0.1361.$$

  - The source is given by

$$\rho_{\omega\ell m}(r) = \frac{\sqrt{\pi}q}{6M} \times \delta(r-6M)\delta(\omega - m\Omega_{\text{ISCO}}) Y^*_{\ell m}\left(\frac{\pi}{2}, 0\right).$$

  - and the waveform in (ℓ, m) mode is given by

$$\phi_{\ell m}(t,r) = -\frac{q}{12\sqrt{2}iM} Y^*_{\ell m}\left(\frac{\pi}{2}, 0\right) \times \frac{\phi^{\text{up}}_{\omega\ell}(r)\phi^{\text{in}}_{\omega\ell}(6M)}{\omega A^{(-)}_{\ell}(\omega)} e^{-i\omega t}\Bigg|_{\omega = m\Omega_{\text{ISCO}}}$$

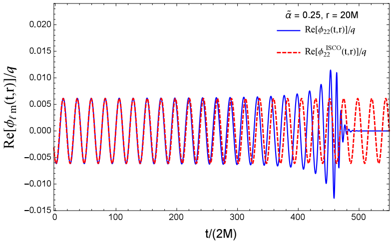


FIGURE 3 – Comparaison of the quadrupolar waveform produced by the plunging particle (blue line) and by a particle orbiting the BH on the ISCO (red dashed line). The results are obtained for a massive scalar field ($\tilde{\alpha} = 0.25$).

- The adiabatic phase is described very accurately by the waveform emitted by a particle living on the ISCO.

- The study of the particle orbiting the BH on the ISCO, permits us to define two regimes :
  - The dispersive regime.
  - The evanescent regime.
- These two regimes are separated by the threshold value $\tilde{\alpha}_c$ corresponding to the mass parameter μ_c

$$\mu_c = 2\Omega_{\text{ISCO}} \approx 0.2722.$$

where Ω_{ISCO} denotes the angular velocity of the particle moving on the ISCO.

The adiabatic phase and the excitation of QBSs

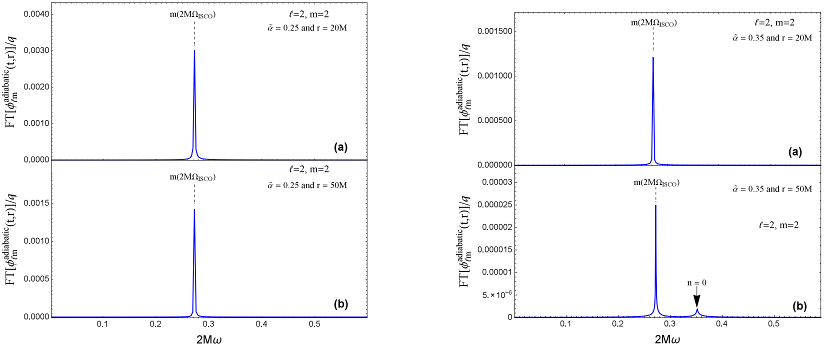


FIGURE 4 – Spectral content of the adiabatic phase of the quadrupolar waveform produced by the plunging particle. The results are obtained for a massive scalar field ($\tilde{\alpha} = 0.25$ et 0.35) and the observer is located at (a) $r = 20M$ et (b) $r = 50M$.

- We observe the signature of the quasi-circular motion of the plunging particle.
- In the evanescent regime :
  - We can observe the signature of the quasi-circular motion.
  - It is important to remark the presence of another peak at the frequency equals to the real part of the complex frequency of the first QBS. In other term we can observe the excitation of the first QBS in adiabatic.

The ringdown phase and the excitation of QNMs

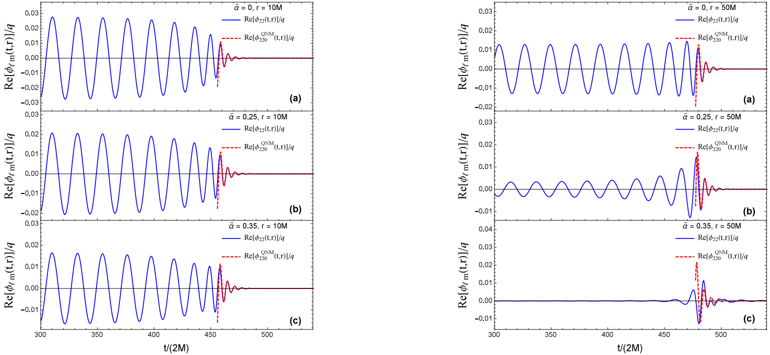


FIGURE 5 – Comparaison of the quadrupolar waveform produced by the plunging particle (blue line) and the quadrupolar quasinormal waveform (red dashed line). The results are obtained for an observer at (left) $10M$ and (right) $50M$.

- When $\tilde{\alpha}$ and the distance r are not too large, the quasinormal waveform describes accurately the ringdown phase.
- However, if $\tilde{\alpha}$ or the distance r increase, the agreement is not so good.

The late-time phase and the excitation of QBSs

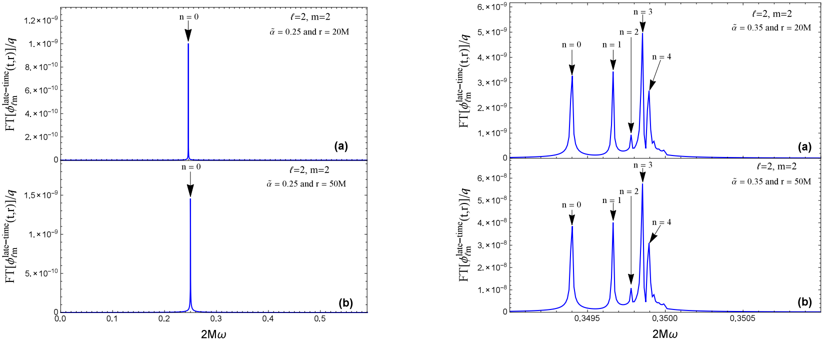


FIGURE 6 – Spectral content of the late-time phase of the quadrupolar waveform produced by the plunging particle. The results are obtained for a massive scalar field ($\tilde{\alpha} = 0.25$ and 0.35) and the observer is located at (a) $r = 20M$ and (b) $r = 50M$.

- We observe the signature of the long-lived QBS.
- We note that, as the reduced mass parameter $\tilde{\alpha}$ increases, the spectrum of the frequencies of the QBSs spreads more and more and it is the possible to separate the different excitation frequencies.

Waveforms in massive gravity

- Our study will be limited to the Fierz-Pauli theory in the Schwarzschild spacetime which can be obtained by linearization of the *"ghost-free bimetric"* theory. The gravitational wave equation (The linearized Fierz-Pauli equation) is given by

$$\Box h_{\mu\nu} + 2R_{\mu\rho\nu\sigma} h^{\rho\sigma} - \mu^2 h_{\mu\nu} = -16\pi \left(\tau_{\mu\nu} - \frac{1}{3} g_{\mu\nu} \tau^{\rho}{}_{\rho} + \frac{1}{3\mu^2} \nabla_\mu \nabla_\nu \tau^{\rho}{}_{\rho} \right), \tag{16}$$

$$\nabla^\mu h_{\mu\nu} = -\nabla_\nu \left(\frac{16\pi}{3\mu^2} \tau^{\rho}{}_{\rho} \right), \tag{17}$$

$$h = -\frac{16\pi}{3\mu^2} \tau. \tag{18}$$

- The field $h_{\mu\nu}(t,r,\theta,\varphi)$ describing the gravitational waves propagating in the Schwarzschild spacetime can be searched

$$h_{\mu\nu}(t,r,\theta,\varphi) = h^{(e)}_{\mu\nu}(t,r,\theta,\varphi) + h^{(o)}_{\mu\nu}(t,r,\theta,\varphi)$$

<u>*Convention* :</u> $h^{\ell m} = 0$ for $\ell = 1$.

$$h^{(o)}_{\mu\nu} = \sum_{\ell=1}^{+\infty} \sum_{m=-\ell}^{m=+\ell} \begin{pmatrix} 0 & 0 & h_t^{\ell m} X_\theta^{\ell m} & h_t^{\ell m} X_\varphi^{\ell m} \\ sym & 0 & h_r^{\ell m} X_\theta^{\ell m} & h_r^{\ell m} X_\varphi^{\ell m} \\ sym & sym & h^{\ell m} X_{\theta\theta}^{\ell m} & h^{\ell m} X_{\theta\varphi}^{\ell m} \\ sym & sym & sym & h^{\ell m} X_{\varphi\varphi}^{\ell m} \end{pmatrix}$$

- The general form of the stress-energy tensor source of the gravitational perturbations of the Schwarzschild black hole is given by

$$\tau_{\mu\nu}(t,r,\theta,\varphi) = \tau^{(e)}_{\mu\nu}(t,r,\theta,\varphi) + \tau^{(o)}_{\mu\nu}(t,r,\theta,\varphi)$$

<u>*Conventions* :</u> $L_t^{\ell m} = L_r^{\ell m} = 0$ for $\ell = 0$ and $L^{\ell m} = 0$ for $\ell = 0, 1$.

$$\tau^{(o)}_{\mu\nu} = \sum_{\ell=0}^{+\infty} \sum_{m=-\ell}^{m=+\ell} \begin{pmatrix} 0 & 0 & L_t^{\ell m} X_\theta^{\ell m} & L_t^{\ell m} X_\varphi^{\ell m} \\ sym & 0 & L_r^{\ell m} X_\theta^{\ell m} & L_r^{\ell m} X_\varphi^{\ell m} \\ sym & sym & L^{\ell m} X_{\theta\theta}^{\ell m} & L^{\ell m} X_{\theta\varphi}^{\ell m} \\ sym & sym & sym & L^{\ell m} X_{\varphi\varphi}^{\ell m} \end{pmatrix}$$

Waveform in massive gravity (odd-parity)

- The system of coupled equations governing the odd-parity modes

$$\begin{cases} \left[\frac{d^2}{dr_*^2}+\omega^2-V_\ell^{(\phi)}(r)\right]\phi_{\omega\ell m}+\frac{(\ell-1)(\ell+2)}{r^2}\left(1-\frac{2M}{r}\right)\left(1-\frac{3M}{r}\right)\psi_{\omega\ell m}=-S_{\omega\ell m}^{(\phi)} \\ \left[\frac{d^2}{dr_*^2}+\omega^2-V_\ell^{(\psi)}(r)\right]\psi_{\omega\ell m}+\frac{4}{r^2}\left(1-\frac{2M}{r}\right)\phi_{\omega\ell m}=-S_{\omega\ell m}^{(\psi)} \end{cases}$$

- With $\phi_{\omega\ell m}$ and $\psi_{\omega\ell m}$

$$\phi_{\omega\ell m}=\left(1-\frac{2M}{r}\right)h_r^{\ell m}$$

$$\psi_{\omega\ell m}=\frac{h^{\ell m}}{r}$$

- The effectives potentials are given by

$$V_\ell^{(\phi)}(r)=\left(1-\frac{2M}{r}\right)\left(\mu^2+\frac{\ell(\ell+1)+4}{r^2}-\frac{16M}{r^3}\right)$$

$$V_\ell^{(\psi)}(r)=\left(1-\frac{2M}{r}\right)\left(\mu^2+\frac{(\ell-1)(\ell+2)}{r^2}+\frac{2M}{r^3}\right)$$

- The source terms are given by

$$S_{\omega\ell m}^{(\phi)}=\frac{16\pi\left(1-\frac{2M}{r}\right)^2}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}dt\,L_r^{\ell m}e^{+i\omega t}$$

$$S_{\omega\ell m}^{(\psi)}=\frac{16\pi\left(1-\frac{2M}{r}\right)}{\sqrt{2\pi}r}\int_{-\infty}^{+\infty}dt\,L^{\ell m}e^{+i\omega t}$$

- **In the case of the odd-parity $\ell=1$ mode :** the system of equation reduces to a single differential equation Regge-Wheeler type

$$\left[\frac{d^2}{dr_*^2}+\omega^2-V_\ell^{(\phi)}(r)\right]\phi_{\omega\ell m}=-S_{\omega\ell m}^{(\phi)}$$

Excitation of the odd-parity $\ell = 1$ mode by the plunging particle

- the source of the plunging particle (odd-parity) is given by

$$S^{(\phi)}_{\omega\ell m} = \frac{16\sqrt{6\pi}}{\ell(\ell+1)} \frac{Mm_0}{r^2} \left(1 - \frac{2M}{r}\right) B(\ell,m)\, e^{i[\omega t_p(r) - m\varphi_p(r)]} \tag{19}$$

with

$$B(\ell,m) = \frac{2^{m+1}}{\sqrt{\pi}} \sqrt{\frac{2\ell+1}{4\pi} \frac{(\ell-m)!}{(\ell+m)!}} \frac{\Gamma\left(\frac{(\ell+m)}{2}+1\right)}{\Gamma\left(\frac{(\ell-m-1)}{2}+1\right)} \sin\left[\frac{\pi}{2}(\ell+m)\right] \tag{20}$$

- We can show that the partial response in the odd-parity $\ell = 1$ mode is given by

$$\phi_{\ell m}(t,r) = -\frac{1}{\sqrt{2\pi}} \int_{-\infty+ic}^{+\infty+ic} d\omega \left(\frac{e^{-i\omega t}}{W_\ell(\omega)}\right) \times \phi^{\mathrm{up}}_{\omega\ell}(r) \int_{2M}^{+\infty} dr' \phi^{\mathrm{in}}_{\omega\ell}(r') S^{(\phi)}_{\omega\ell m}(r'). \tag{21}$$

Excitation of the odd-parity $\ell = 1$ mode by the plunging particle

Waveforms for the odd-parity $\ell = 1$ mode

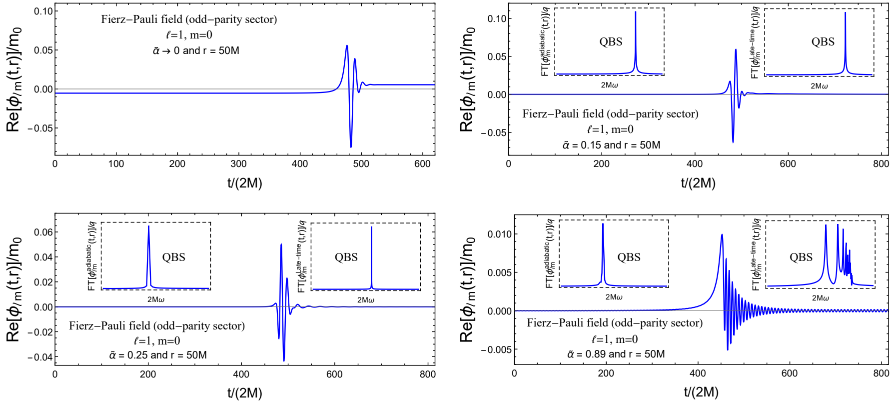


FIGURE 7 – Waveforms ($\ell = 1, m = 0$) produced by the plunging particle.The results are obtained for massless scalar field ($\tilde{\alpha} \to 0$) and for massive scalar field ($\tilde{\alpha} = 0.15, 0.25$ and 0.89). The observer is located at $r = 10M$.

- We can observe for any nonvanishing value of the reduce masse parameter $\tilde{\alpha}$, the QBSs of the Schwarzschild BH are excited, their influence is negligible for $\tilde{\alpha} \to 0$ but increases with $\tilde{\alpha}$.

Conclusion

- In our opinion, the study of the scalar radiation generated by the plunging particle has permitted us to highlight and interpret some important effects occuring in the plunge regime which are not present for massless fields such as :
  - i The decreasing and vanishing, as the mass parameter increases, of the signal amplitude generated when the particle moves on quasicircular orbits near the ISCO.
  - ii In addition to the excitation of the QNMs, the excitation of the QBSs of the BH.
- If the graviton has a mass, the study of the gravitational radiation generated by a particle plunging into a BH and, in particular, the observation of the effects previously discussed, could help us :
  - i To test the various massive gravity theories.
  - ii To impose strong constraints on the graviton mass and to support, in a new way, Einstein's general relativity.

Thank you for your attention.

IN-Mode and UP-Mode

- $\phi^{\rm in}_{\omega\ell}$ and $\phi^{\rm up}_{\omega\ell}$ are linearly independent solutions of the Regge-Wheeler equation

$$\frac{d^2\phi_{\omega\ell}}{dr_*^2} + \left[\omega^2 - V_\ell(r)\right]\phi_{\omega\ell} = 0. \tag{22}$$

  - When $\mathrm{Im}(\omega) > 0$, $\phi^{\rm in}_{\omega\ell}$ is uniquely defined by its ingoing behavior at the event horizon $r = 2M$ (i.e., for $r_* \to -\infty$)

$$\phi^{\rm in}_{\omega\ell}(r) \underset{r_*\to-\infty}{\sim} e^{-i\omega r_*} \tag{23a}$$

    and, at spatial infinity $r \to +\infty$ (i.e., for $r_* \to +\infty$), it has an asymptotic behavior of the form

$$\phi^{\rm in}_{\omega\ell}(r) \underset{r_*\to+\infty}{\sim} \left[\frac{\omega}{p(\omega)}\right]^{1/2} \times \left(A^{(-)}_\ell(\omega) e^{-i[p(\omega)r_* + [M\mu^2/p(\omega)]\ln(r/M)]} + A^{(+)}_\ell(\omega) e^{+i[p(\omega)r_* + [M\mu^2/p(\omega)]\ln(r/M)]}\right) \tag{23b}$$

  - Similarly, $\phi^{\rm up}_{\omega\ell}$ is uniquely defined by its outgoing behavior at spatial infinity

$$\phi^{\rm up}_{\omega\ell}(r) \underset{r_*\to+\infty}{\sim} \left[\frac{\omega}{p(\omega)}\right]^{1/2} e^{+i[p(\omega)r_* + [M\mu^2/p(\omega)]\ln(r/M)]} \tag{24a}$$

    and, at the horizon, it has an asymptotic behavior of the form

$$\phi^{\rm up}_{\omega\ell}(r) \underset{r_*\to-\infty}{\sim} B^{(-)}_\ell(\omega) e^{-i\omega r_*} + B^{(+)}_\ell(\omega) e^{+i\omega r_*}. \tag{24b}$$

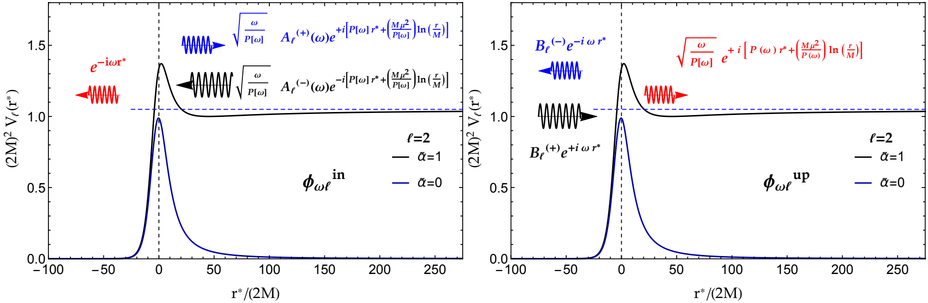
    In Eqs. (23) and (24), $p(\omega) = \left(\omega^2 - \mu^2\right)^{1/2}$ denotes the "wave number" while $A^{(-)}_\ell(\omega)$, $A^{(+)}_\ell(\omega)$, $B^{(-)}_\ell(\omega)$ and $B^{(+)}_\ell(\omega)$ are complex amplitudes.

- By evaluating the Wronskian $W_\ell(\omega)$ at $r_* \to -\infty$ and $r_* \to +\infty$, we obtain

$$W_\ell(\omega) = 2i\omega A^{(-)}_\ell(\omega) = 2i\omega B^{(+)}_\ell(\omega). \tag{25}$$

Quasinormal frequencies

- If the Wronskian $W_\ell(\omega)$ vanishes, the functions $\phi^{\text{in}}_{\omega\ell}$ and $\phi^{\text{up}}_{\omega\ell}$ are linearly dependent and propagate inward at the horizon and outward at spatial infinity, a behavior which defines the QNMs lying in the lower part of the first Riemann sheet associated with the function $p(\omega) = (\omega^2 - \mu^2)^{1/2}$.



- **Quasinormal waveforms** : We deform the contour of integration in Eq. 11 in order to extract a residue séries over the quasinormal frequencies. It is given by

$$\phi^{\text{QNM}}_{\ell m}(t,r) = \sum_{n=0}^{+\infty} \phi^{\text{QNM}}_{\ell mn}(t,r) \tag{26}$$