A New Formulation Of Lee-Wick Quantum Gravity

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SW11:
Hot Topics in Modern Cosmology
Cargèse
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Outline

The Problem of Renormalizability in QG

The Hilbert-Einstein action

$$S_{HE} = -\frac{1}{2\kappa^2}\int \mathrm{d}^4x\sqrt{-g}R, \qquad \kappa^2 = 8\pi G$$

is perturbatively nonrenormalizable.

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is perturbatively nonrenormalizable.

Renormalization generates an infinite number of counterterms

$$S_Q = -\frac{1}{2\kappa^2}\int \mathrm{d}^4\sqrt{-g}\big[R + c_1R^2 + c_2R_{\mu\nu}R^{\mu\nu} + c_3R^{\mu\nu}_{\rho\sigma}R^{\rho\sigma}_{\alpha\beta}R^{\alpha\beta}_{\mu\nu} + \underbrace{\ldots}_{\infty}\big].$$

A high-energy modification of the theory is necessary.

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Stelle theory provides a renormalizable model of quantum gravity

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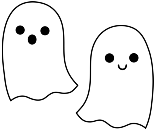

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Optical theorem

$$i(\mathrm{T} + \mathrm{T}^\dagger) = \mathrm{T}^\dagger \mathrm{T}, \qquad \mathrm{S} = \mathrm{I} + i\mathrm{T};$$

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- Complex conjugated poles $\Rightarrow$ trivial imaginary part.

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II)Nonlocal, non-Hermitian divergences (also in 4-dim. QG).

$$\Sigma(p) = -\frac{M^4}{2(4\pi)^3}\left[\frac{M^2}{(p^2)^2} - \frac{i}{p^2}\right] \ln\left(\frac{\Lambda_{UV}}{M^2}\right) + \ldots, \qquad D = 6.$$

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The theory cannot be defined directly on Minkwoski spacetime.

Lee-Wick quantum field theories

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Our formulation inlcudes the answer to all these questions.

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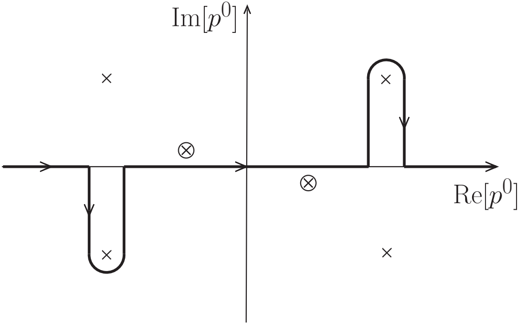

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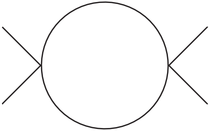

$$\mathcal{J}(p) = \int \frac{\mathrm{d}^D k}{(2\pi)^D} D(k^2, m_1^2, \epsilon_1) D((k-p)^2, m_2^2, \epsilon_2) \tag{1}$$

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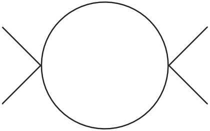

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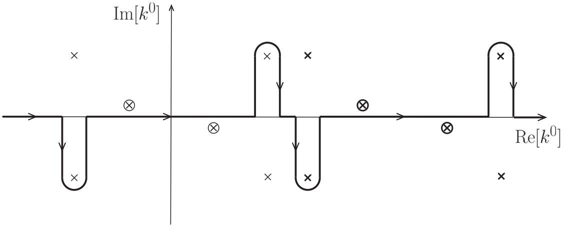

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CLOP prescription (Cutkosky at al.): two different scales M and M' s.t.

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Solution: define LW models as
Nonanalytically Wick rotated Euclidean theories.

D. Anselmi and M. Piva, *A new formulation of Lee-Wick quantum field theory*, 17A1 Renormalization.com and arXiv:1703.04584 [hep-th].

New formulation

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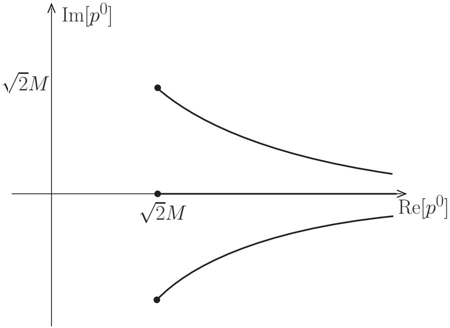

The amplitude is ill-defined on the real axis above the threshold $p^2 = 2M^2$.

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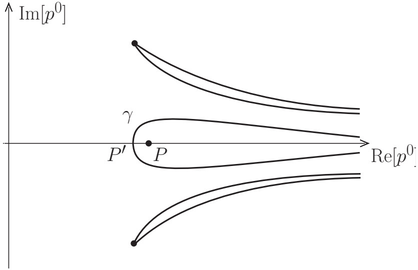

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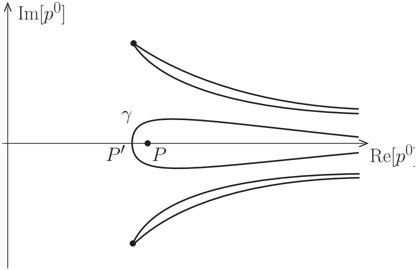
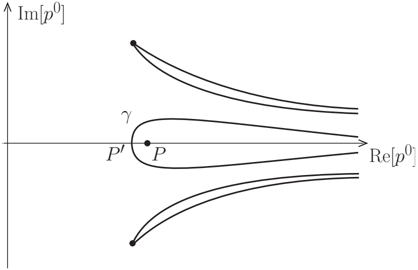


$$E_P \equiv \sqrt{2M^2 + \mathbf{p}^2}, \quad E_{P'} \equiv \sqrt{\frac{\mathbf{p}^2}{2} + \sqrt{\frac{(\mathbf{p}^2)^2}{4} + 4M^4}}.$$

Lorentz invariance seems violated (already noticed by Nakanishi)

We deform the branch cuts



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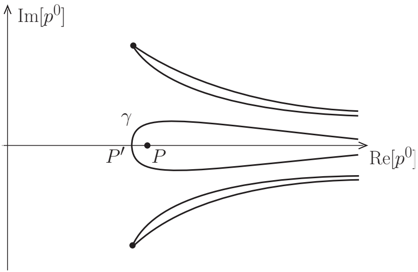

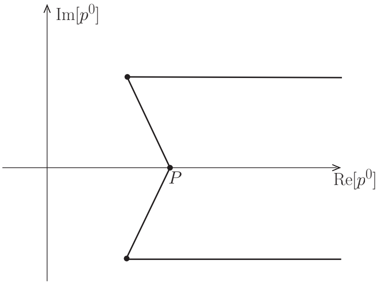

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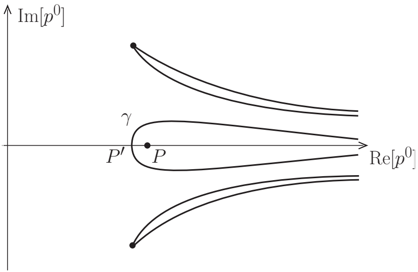

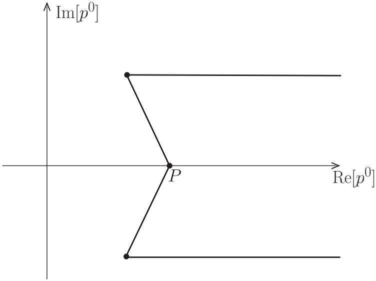

Constraints on the deformation:

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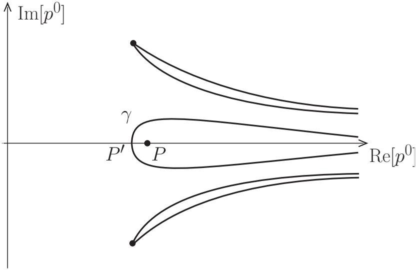

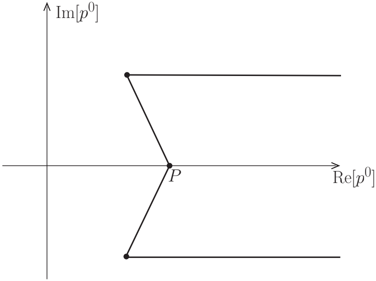

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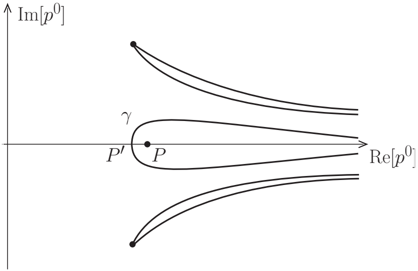

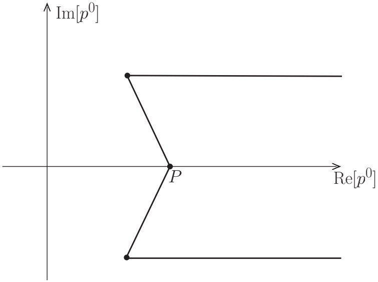

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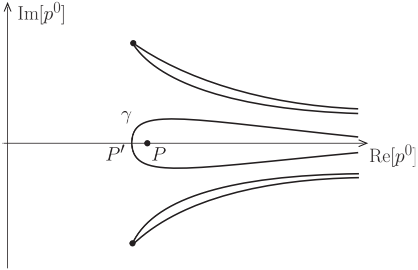

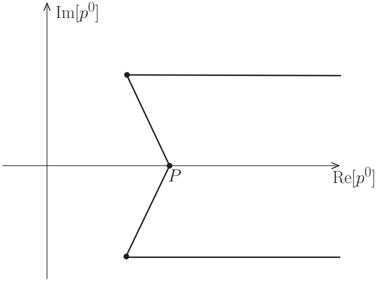

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The amplitude are well defined but nonanalytic.

Expansion around the pinching:

$$k_s = \frac{\sigma_-}{2p^0} + \tau \frac{\sigma_+^2}{2\sigma_-(p^0)^2} + \eta \frac{p_s \sigma_+^2}{4\sigma_- M^2}, \qquad u = \frac{p_s}{2k_s} + \eta \frac{\sigma_+^2}{2\sigma_- M^2},$$

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$$D_\varphi = p^0 \mathrm{e}^{i\varphi} - \omega_M(\mathbf{k}) - \omega_M^*(\mathbf{k} - \mathbf{p}),$$

$$\frac{\mathrm{d}^{D-1}\mathbf{k}}{D_\varphi} = -\frac{2\pi^{(D-2)/2}}{\Gamma\left(\frac{D}{2} - 1\right)} \frac{\sigma_+^4 \left[\sigma_-^2 - (p^0)^2 p_s^2\right]^{(D-4)/2}}{(2p^0)^D M^2} \frac{\mathrm{d}\tau \mathrm{d}\eta}{\tau - i(p_0 \varphi + p_s \eta)}.$$

- First $\varphi \to 0$, then $p_s \to 0$

$$\frac{\mathrm{d}^{D-1}\mathbf{k}}{D_0} \propto \frac{\mathrm{d}\tau \mathrm{d}\eta}{\tau - i p_s \eta} \to \mathrm{d}\tau \mathrm{d}\eta \left[\mathcal{P}\left(\frac{1}{\tau}\right) + i\pi \mathrm{sgn}(\eta)\delta(\tau) \right].$$

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The first result is the correct prescription.

Comparison with other prescriptions

A more general prescription

$$\frac{i}{(8\pi)^2}\frac{\sigma_-}{(p^0)^2}\frac{M^4}{(M^2+im_1^2)(M^2-im_2^2)}\left[\mathcal{P}\left(\frac{1}{\tau}\right)+ia\delta(\tau)\right]\mathrm{d}\tau.$$

Contribution to the amplitude includes complex conjugated

$$\frac{2i\sigma_- M^4}{(8\pi)^2(p^0)^2(M^4+m_1^4)(M^4+m_2^4)}\left[(M^4+m_1^2m_2^2)\mathcal{P}\left(\frac{1}{\tau}\right)+aM^2(m_1^2-m_2^2)\delta(\tau)\right]\mathrm{d}\tau.$$

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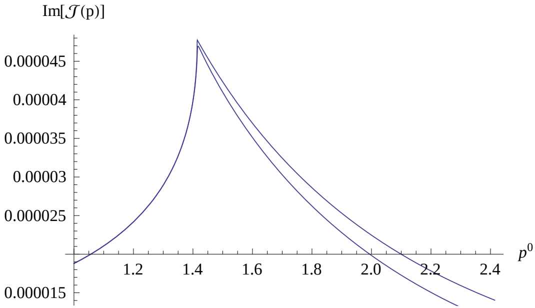
No prescription with nonvanishing a is consistent with this formulation.

CLOP gives $a=\pi\mathrm{sgn}(M'-M)$

Discrepancy above the threshold

$$M = 1,\ \delta = 10^{-3},\ m_1 = 3,\ m_2 = 5.$$



Our formulation gives physical predictions which differ from the previous ones.

Simplest model for Lee-Wick quantum gravity

$$S_{QG} = -\frac{1}{2\kappa^2}\int \mathrm{d}^4x\sqrt{-g}\Big[2\lambda_C + \zeta R + \alpha R_{\mu\nu}R^{\mu\nu} + \beta R^2 - \frac{1}{M^4}(D_\rho R_{\mu\nu})(D^\rho R^{\mu\nu}) + \frac{1}{2M^4}(D_\rho R)(D^\rho R)\Big].$$

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Expansion

$$g_{\mu\nu} = \eta_{\mu\nu} + 2\kappa h_{\mu\nu}, \qquad \eta_{\mu\nu} = \mathrm{diag}(1,-1,-1,-1).$$

De Donder gauge fermion

$$\Psi = -\frac{1}{2}\int \mathrm{d}^4x \bar{C}^\mu \left(1+\frac{\Box^2}{M^4}\right)\left(\frac{1}{2}B_\mu - \frac{1}{\kappa}\mathcal{G}_\mu\right), \qquad \mathcal{G}_\mu = \kappa(2\partial_\nu h^\nu_\mu - \partial_\mu h^\alpha_\alpha).$$

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Propagator

$$\langle h_{\mu\nu}(p)h_{\rho\sigma}(-p)\rangle^{\text{free}}_{\lambda_C=\alpha=\beta=0} = \frac{iM^4}{2(p^2+i\epsilon)}\frac{\eta_{\mu\rho}\eta_{\nu\sigma}+\eta_{\mu\sigma}\eta_{\nu\rho}-\eta_{\mu\nu}\eta_{\rho\sigma}}{(p^2)^2+M^4}.$$

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- Well defined procedure to define the amplitudes and cutting equations.
- Numerators do not spoil unitarity $\to$ suitable for quantum gravity.
- Prediction of nonanalitycity of the amplitudes.
- Physically different from the previous formulations.