Evolution of thick domain walls. Separated matter and antimatter domains with vanishing domain walls

A.D. Dolgov[1,2,5], S.I. Godunov[2,5], A.S. Rudenko[3,5], I.I. Tkachev[4,5]

[1]Università degli Studi di Ferrara
[2]Institute for Theoretical and Experimental Physics
[3]Budker Institute of Nuclear Physics
[4]Institute for Nuclear Research
[5]Novosibirsk State University

May 4, 2017 / SW11, Cargèse

I. Evolution of thick domain walls in de Sitter universe. Stationary solutions

R. Basu and A. Vilenkin, Phys. Rev. D 50 (1994) 7150

Domain walls (DW) can be formed during inflation by different mechanisms, e.g. by phase transitions.

In their paper Basu, Vilenkin (BV) address the question
what happens to the internal structure of these DW during inflation.

The inflationary Universe is approximated by de Sitter spacetime, which has a constant expansion rate H, and the scale factor evolves as $a(t) = \exp Ht$.

The metric with spatially flat sections is given by

$$ds^2 = dt^2 - e^{2Ht}\left(dx^2 + dy^2 + dz^2\right). \tag{1}$$

BV consider a one component scalar field theory with a simple double-well potential

$$\mathcal{L} = \frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \, \partial_\nu \varphi - \frac{\lambda}{2} \left(\varphi^2 - \eta^2\right)^2 . \qquad (2)$$

The corresponding equation of motion is

$$\frac{1}{\sqrt{-g}} \partial_\mu \left(\sqrt{-g} g^{\mu\nu} \partial_\nu \varphi\right) = -2\lambda\varphi \left(\varphi^2 - \eta^2\right) . \qquad (3)$$

In flat spacetime, $H = 0$, and in one-dimensional static case, $\varphi = \varphi(z)$, the equation takes the form

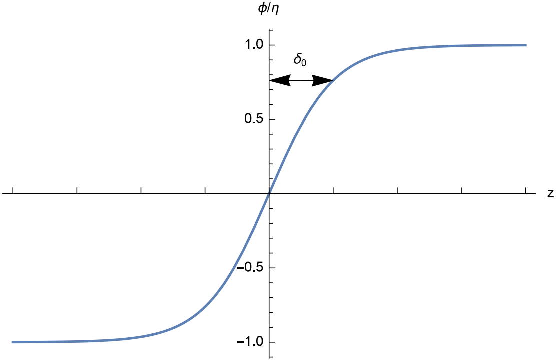
$$\frac{d^2\varphi}{dz^2} = 2\lambda\varphi \left(\varphi^2 - \eta^2\right) . \qquad (4)$$

It has a kink-type solution, which describes a static infinite DW.

One can assume that the wall is situated at $z = 0$ in xy-plane:

$$\varphi(z) = \eta \tanh \frac{z}{\delta_0}, \tag{5}$$

where $\delta_0 = 1/(\sqrt{\lambda}\eta)$ is the flat-spacetime wall thickness (subscript 0 indicates that $H = 0$).



Now consider an expanding universe with constant $H > 0$.
In this case, if one looks for stationary solution, it is reasonable to suggest that the field φ depends only on $za(t) = z \exp Ht$, which is the proper distance from the wall.
So, one can choose the following ansatz for φ:

$$\varphi = \eta \cdot f(u), \quad \text{where} \quad u = Hz \cdot e^{Ht}, \tag{6}$$

here u and f are dimensionless.

Equation of motion takes the form:

$$\left(1 - u^2\right) f'' - 4uf' = -2Cf\left(1 - f^2\right). \tag{7}$$

Here prime means the derivative with respect to u.
It is noteworthy that all parameters of the problem are combined into a single positive constant $C = 1/(H\delta_0)^2 = \lambda\eta^2/H^2 > 0$.

Since one is interested in kink-type solutions, the boundary conditions should be

$$f(0) = 0, \qquad f(\pm\infty) = \pm 1. \tag{8}$$

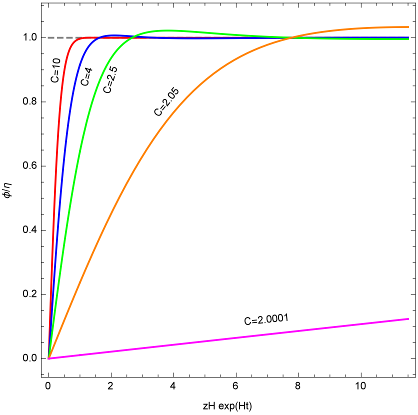


Figure: Stationary field configurations $f(u)$ for different values of C

At large u the field φ approaches its vacuum expectation value (VEV) η.

However, the solutions exhibit an aperiodical damped oscillatory behavior, as opposed to the monotonic approach to the VEV in flat spacetime.

Asymptotic formula for large u:

$$1 - f(u) \sim \frac{\cos\left(\sqrt{4C - 9/4}\,\ln u\right)}{u^{3/2}}. \tag{9}$$

BV noticed that stationary solutions can be found only for $C > 2$, but no explanation of this observation was given.

Naive explanation why the value $C = 2$ is the special one: (A.D. Dolgov, S.I. Godunov, A.S. Rudenko, JCAP 1610 (2016) 10, 026).

from Eq. (7) and condition $f(0) = 0$ it follows that $f''(0) = 0$,

using this condition and expanding $f(u)$ into Taylor series near $u = 0$, one obtains that for sufficiently small positive ϵ and for $f'(0) > 0$:

for $C > 2$, $f''(\epsilon) < 0 \Rightarrow f(u)$ is convex like kink-type solution,

for $C \leq 2$, $f''(\epsilon) > 0 \Rightarrow f(u)$ is concave, it is not the kink-type.

Conclusions for section I

In the case of very thin DW, whose thickness is much smaller than the de Sitter horizon, $\delta << H^{-1}$, i.e. $(C >> 1)$, the solution is well approximated by the flat-spacetime solution.

However, as the flat-spacetime thickness parameter δ_0 increases, a deviation of the solution from the flat-spacetime solution increases too.

Beyond the critical value, $\delta_0 \geq H^{-1}/\sqrt{2}$, i.e. $(C \geq 2)$, there are no stationary solutions at all.

But if we allow for arbitrary dependence of the solution on z and t, it exists for any C, and the case of $C \leq 2$ leads to the expanding kink with rising width, as it is shown in the next section.

II. Evolution of thick domain walls in de Sitter universe. Beyond the stationary solutions

A.D. Dolgov, S.I. Godunov, and A.S. Rudenko, JCAP 1610 (2016) 10, 026

Beyond the stationary approximation we can find solution not only for $C > 2$, but also for $C \leq 2$.
To this end one should solve the original equation of motion (3) in the case when the field φ is a function of at least two independent variables, z and t:

$$\frac{\partial^2 \varphi}{\partial t^2} + 3H\frac{\partial \varphi}{\partial t} - e^{-2Ht}\frac{\partial^2 \varphi}{\partial z^2} = -2\lambda\varphi\left(\varphi^2 - \eta^2\right). \qquad (10)$$

It is convenient to introduce dimensionless variables $\tau = Ht$, $\zeta = Hz$ and dimensionless function $f(\zeta, \tau) = \varphi(z, t)/\eta$.
As a result one obtains the equation

$$\frac{\partial^2 f}{\partial \tau^2} + 3\frac{\partial f}{\partial \tau} - e^{-2\tau}\frac{\partial^2 f}{\partial \zeta^2} = 2Cf\left(1 - f^2\right), \qquad (11)$$

where $C = \lambda\eta^2/H^2 = 1/(H\delta_0)^2 > 0$ as it was above.

The boundary conditions for the kink-type solution are

$$f(0,\tau) = 0, \qquad f(\pm\infty,\tau) = \pm 1, \tag{12}$$

and we choose the initial configuration as DW with "natural" thickness $1/\sqrt{C}$ (with respect to dimensionless coordinate ζ) and zero time derivative:

$$f(\zeta,0) = \tanh\frac{z}{\delta_0} = \tanh\sqrt{C}\zeta, \qquad \left.\frac{\partial f(\zeta,\tau)}{\partial\tau}\right|_{\tau=0} = 0. \tag{13}$$

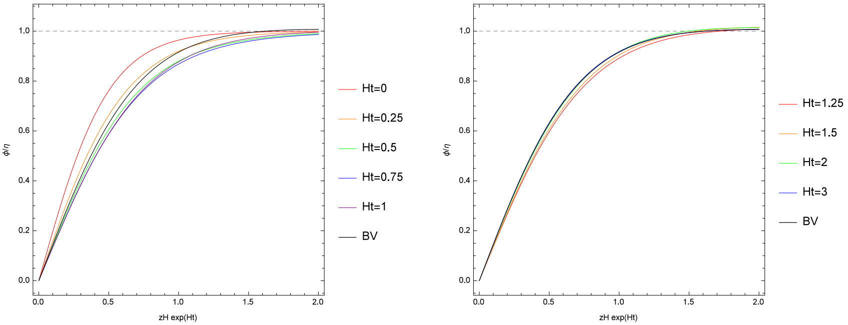


Figure: Evolution of domain wall for $C = 4$.

At the very beginning the wall starts to broaden in terms of the proper distance from the wall, $zH \exp Ht$, and at some moment it becomes wider than the stationary solution (BV).
However, afterwards the wall broadening changes to contraction.
Finally, the wall comes to the stationary configuration after several damped oscillations around it.

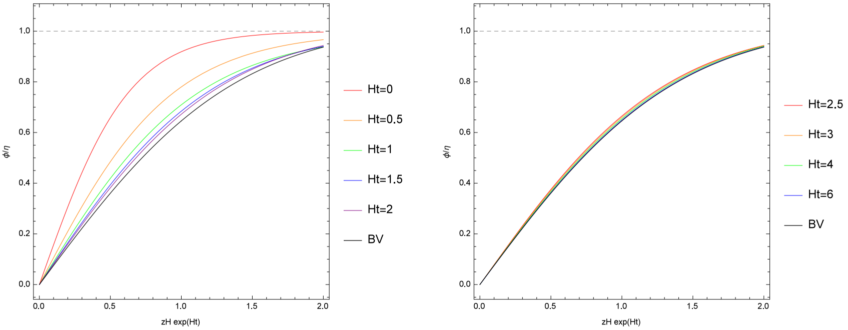


Figure: Evolution of domain wall for $C = 2.5$.

Consider now smaller values of parameter C.
When it is close to its critical value $C = 2$, the stationary DW is quite wide.
Therefore, the non-stationary solution approaches to the stationary one only after quite long time.

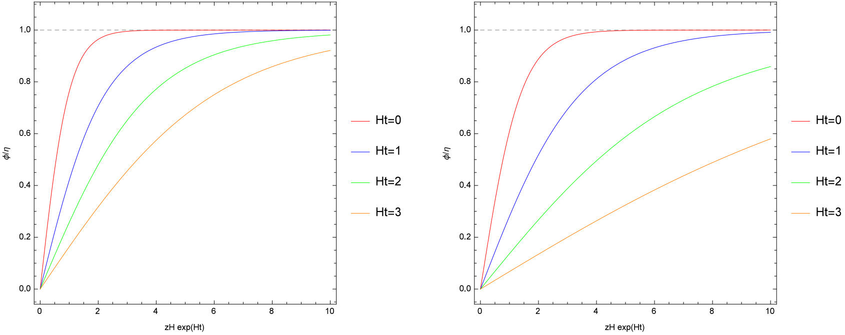


Figure: Evolution of DW for $C = 1$ (left plot) and $C = 0.5$ (right plot).

For $C \leq 2$ there are no stationary solutions at all.
One can see that the DW thickness increases monotonically.

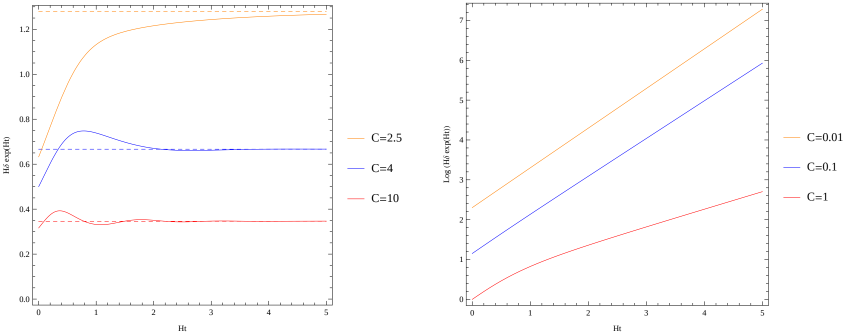


Figure: Time dependence of DW thickness for $C > 2$ and $C < 2$

We use here the definition for the thickness $\delta(t)$ as the value of the coordinate z at the position where the field ϕ reaches the value $\phi/\eta = \tanh 1 \approx 0.76$.

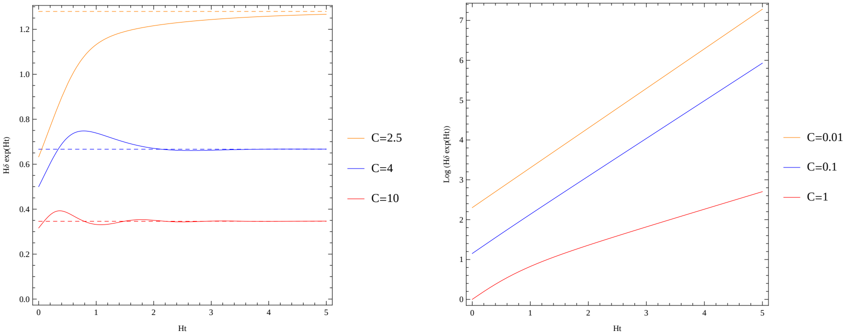


Figure: Time dependence of DW thickness for $C > 2$ and $C < 2$

In the left plot ($C > 2$) one can see that all the curves indeed tend to constant values corresponding to stationary solutions (dashed lines). Oscillatory behavior mentioned above is also apparent.

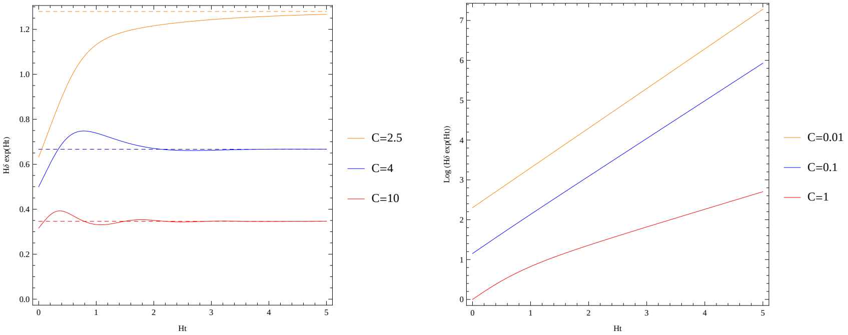


Figure: Time dependence of DW thickness for $C > 2$ and $C < 2$

Right plot contains curves for $C < 2$. Along the vertical axis the logarithm of the DW thickness is shown in order to compare the rate of the DW expansion with the exponential cosmological one.
For $C = 1$, i.e. for not very small values of C, the DW thickness increases slower than exponent.

However, for smaller values of C, e.g. $C \lesssim 0.1$ the rate of the DW expansion is the exponential one with a good accuracy.

Conclusions for section II

We have shown that for large values of parameter $C > 2$ the initial kink configuration in a de Sitter background tends to the stationary solution obtained by Basu and Vilenkin.

We also confirmed the BV result that the width of the stationary wall rises with decreasing value of C.

For $C \leq 2$ the stationary solution does not exist and the width of the wall infinitely grows with time.

For $C \lesssim 0.1$ the rise is close to the exponential one $\Rightarrow$ transition regions between domains might be cosmologically large.

III. Evolution of thick domain walls in universe with $p = w\rho$

A.D. Dolgov, S.I. Godunov, and A.S. Rudenko, in preparation

Let us study how the DW evolves in an expanding universe with the equation of state of matter $p = w\rho$, where constant $w > -1$.
In such universe the scale factor increases as some power of time

$$a(t) = \mathrm{const} \cdot t^{\alpha}, \quad \text{where} \ \ \alpha = \frac{2}{3(1+w)} > 0, \tag{14}$$

and the Hubble parameter decreases as inverse time

$$H(t) = \frac{\dot{a}}{a} = \frac{\alpha}{t}. \tag{15}$$

The value $w = 0$ corresponds to matter-dominated universe, and $w = 1/3$ to radiation-dominated universe.

The equation of motion (3) in the case when the field is a function of two independent variables, z and t, is written as

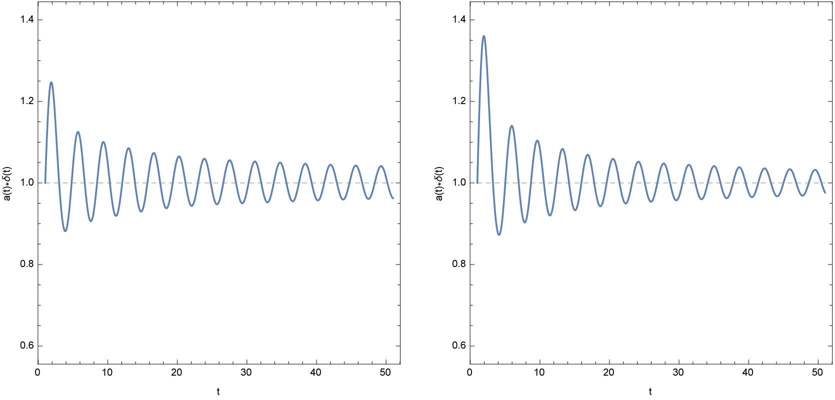
$$\frac{\partial^2 f}{\partial t^2} + 3H(t)\frac{\partial f}{\partial t} - \frac{1}{a^2(t)}\frac{\partial^2 f}{\partial z^2} = \frac{2}{\delta_0^2} f\left(1 - f^2\right), \tag{16}$$

where $f(z,t) = \varphi(z,t)/\eta$.
The boundary conditions for the kink-type solution are

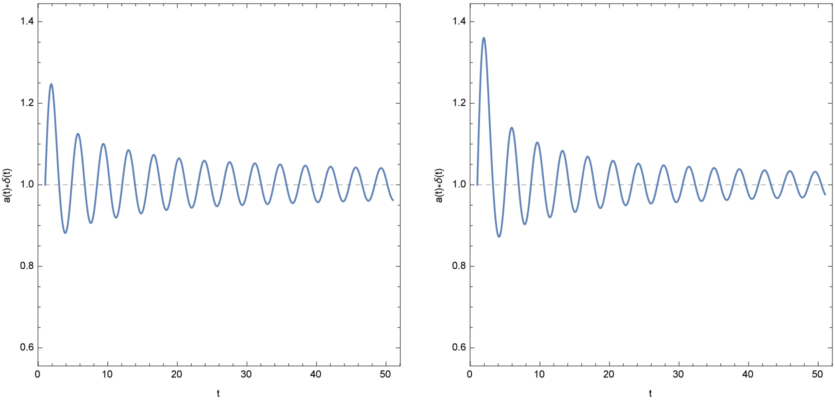
$$f(0,t) = 0, \qquad f(\pm\infty, t) = \pm 1, \tag{17}$$

and we choose the initial configuration as the domain wall with thickness δ_0 and zero time derivative:

$$f(z,t_i) = \tanh\frac{z}{\delta_0}, \qquad \left.\frac{\partial f(z,t)}{\partial t}\right|_{t=t_i} = 0. \tag{18}$$



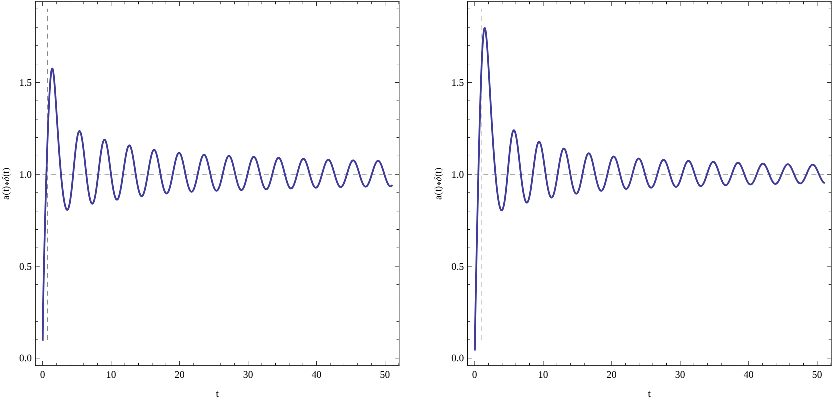
Time dependence of physical width of the wall, $a(t)\delta(t)$.
Left plot: $w = 1/3$ (radiation-dominated universe);
right plot: $w = 0$ (matter-dominated universe).
Dashed lines correspond to δ_0.
$C(t) = 1/(H(t)\delta_0)^2 > 2$ during all the time of the wall evolution.



Here for both plots, $\delta_0 = 1$, $t_{min} = 1$ and $a(t_{min}) = 1$.

When $t \to \infty$ the field configuration tends to

$$f(z,t) = \tanh \frac{z \cdot a(t)}{\delta_0}. \tag{19}$$



Left plot: $w = 1/3$ (radiation-dominated universe);
right plot: $w = 0$ (matter-dominated universe).
Dashed vertical line corresponds to the time when $C(t) = 2$.
Here for both plots, $\delta_0 = 1$, $t_{min} = 0.01$ and $a(t = 1) = 1$.

Conclusions for section III

In the $p = w\rho$ universe, the parameter $C(t)$ increases:
$C(t) \sim 1/H^2(t) \sim t^2$,
and eventually $C(t) > 2$.

Therefore, the DW width $a(t)\delta(t)$ tends to constant value, δ_0, so it is not cosmologically large.

IV. Evolution of thick domain walls in inflationary universe with $U(\phi) = m^2\phi^2/2$

A.D. Dolgov, S.I. Godunov, and A.S. Rudenko, in preparation

Now let us consider simple model of inflation with quadratic inflaton potential $U = m^2\phi^2/2$. Hubble parameter in such model is

$$H = \sqrt{\frac{8\pi\rho}{3m_{pl}^2}} \approx \sqrt{\frac{8\pi}{3m_{pl}^2}\frac{m^2\phi^2}{2}} = \sqrt{\frac{4\pi}{3}}\frac{m}{m_{pl}}\phi, \qquad (20)$$

and equation of motion of inflaton is the following,

$$\dot{\phi} \approx -\frac{m^2\phi}{3H} \approx -\frac{m_{pl}m}{\sqrt{12\pi}}. \qquad (21)$$

Therefore,

$$\phi(t) = \phi_i - \frac{m_{pl} m}{\sqrt{12\pi}} t, \tag{22}$$

$$H(t) = \sqrt{\frac{4\pi}{3}} \frac{m}{m_{pl}} \phi_i - \frac{1}{3} m^2 t, \tag{23}$$

$$a(t) = a_0 \cdot \exp\left(\sqrt{\frac{4\pi}{3}} \frac{m}{m_{pl}} \phi_i t - \frac{1}{6} m^2 t^2 \right). \tag{24}$$

But, in what follows all quantities are dimensionless.
Time t and DW width $\delta(t)$ are measured in units of inverse inflaton mass, m^{-1},
Hubble parameter – in units of m, and
inflaton field ϕ – in units of Planck mass, m_{pl}.

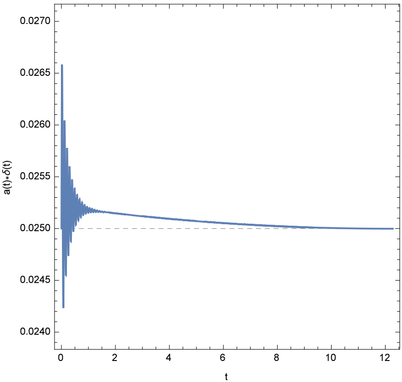
$$H(t) = \sqrt{\frac{4\pi}{3}}\phi_i - \frac{1}{3}t, \tag{25}$$

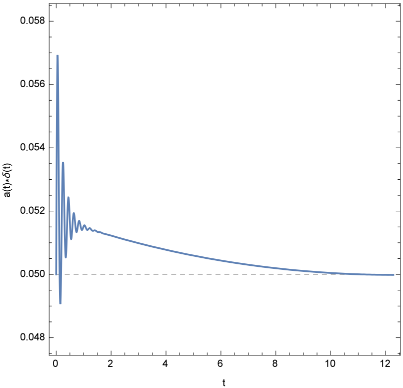
$$a(t) = a_0 \cdot \exp\left(\sqrt{\frac{4\pi}{3}}\phi_i t - \frac{1}{6}t^2\right), \tag{26}$$

$$C(t) = \frac{1}{(H(t)\delta_0)^2}, \tag{27}$$

$\phi_i = 2$, $t_{min} = 0$, and $a_0 = 1$,

Let us introduce t_C which is the time when $C(t) = 2$. Therefore, $t_C = \sqrt{12\pi}\phi_i - 3\sqrt{2}/(2\delta_0)$, here $\delta_0 \geq \sqrt{3}/(2\sqrt{2\pi}\phi_i) \approx 0.173$, since $t_C \geq 0$.





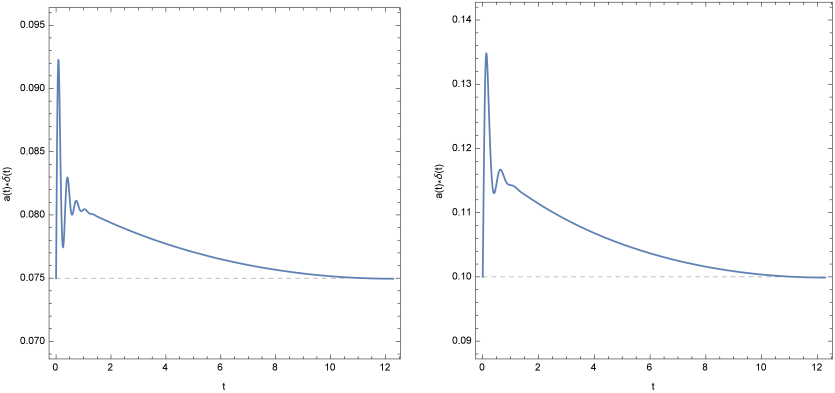
Time dependence of physical width of the wall, $a(t)\delta(t)$, for different values of initial wall width, δ_0.

Left plot: $\delta_0 = 0.025$;

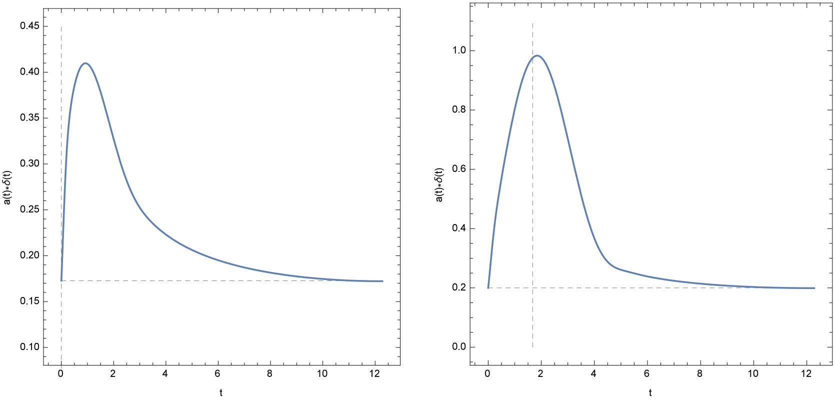
right plot: $\delta_0 = 0.05$.

$C(t) > 2$ during all the time of the wall evolution.

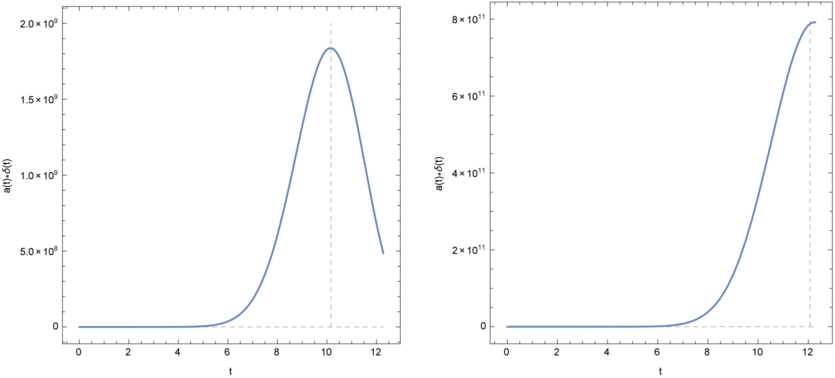
Dashed horizontal line corresponds to δ_0.



Left plot: $\delta_0 = 0.075$;
right plot: $\delta_0 = 0.1$.
$C(t) > 2$ during all the time of the wall evolution.



Left plot: $\delta_0 \approx 0.173$, $t_C = 0$;
right plot: $\delta_0 = 0.2$, $t_C \approx 1.67$.
Dashed vertical line corresponds to t_C.



Left plot: $\delta_0 = 1$, $t_C \approx 10.16$;
right plot: $\delta_0 = 10$, $t_C \approx 12.07$.
Dashed vertical line corresponds to t_C.

Conclusions for section IV

In the inflationary universe with $U(\phi) = m^2\phi^2/2$:

when δ_0 is quite small ($C(t) > 2$ during all the time of inflation), the DW width $a(t)\delta(t)$ tends to constant value, δ_0, so it is not cosmologically large.

when δ_0 is quite large ($C(t) > 2$ only near the end of inflation), the DW width can be cosmologically large at the end of inflation.

V. Separated matter and antimatter domains with vanishing domain walls

A.D. Dolgov, S.I. Godunov, A.S. Rudenko, and I.I. Tkachev,
JCAP 1510 (2015) 10, 027

Introduction

Observations indicate that the universe is 100% baryo-asymmetric. But is it possible that astronomically large pieces of antimatter exist in the universe?

Sakharov conditions for baryogenesis:

1. Non-conservation of baryon number B
2. Breaking of C and CP invariance
3. Deviation from thermal equilibrium

Observed baryon asymmetry cannot be explained by SM $\Rightarrow$ physics beyond Standard Model

There are models where the universe is globally:

1) asymmetric (matter $>$ antimatter)

2) symmetric $\Rightarrow$ domains of matter and antimatter

Domain wall problem – unacceptably high energy density of DW
Solution: domain walls should exist only in the universe past and should disappear by now

Models where a symmetry is broken only in a particular range of temperatures [*V.A. Kuzmin, M.E. Shaposhnikov, I.I. Tkachev, 1981-82*] ⇒ the size of domains is too small from cosmological point of view

We consider another scenario:
domains appeared during inflation ⇒ now they are cosmologically large and separated by cosmologically large distances
[*JCAP 1510 (2015) 10, 027; arXiv:1506.08671*]

Model

Action:

$$\mathcal{S} = \int d^4x \sqrt{-g}\, \left(\mathcal{L}_\Phi + \mathcal{L}_\chi + \mathcal{L}_{\rm int}\right),$$

where

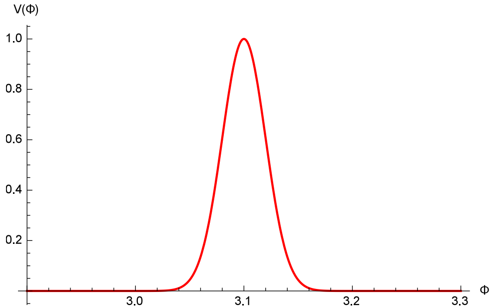
$$\begin{aligned}
\mathcal{L}_\Phi &= \frac{1}{2}\, g^{\mu\nu} \partial_\mu \Phi\, \partial_\nu \Phi - \frac{1}{2} M^2 \Phi^2, \\
\mathcal{L}_\chi &= \frac{1}{2}\, g^{\mu\nu} \partial_\mu \chi\, \partial_\nu \chi - \frac{1}{2} m^2 \chi^2 - \frac{\lambda}{4} \chi^4, \\
\mathcal{L}_{\rm int} &= \mu^2 \chi^2 V(\Phi),
\end{aligned}$$

Φ – inflaton field,
χ – real pseudoscalar field,
M, m, λ, μ – constant parameters,
$V(\Phi)$ – dimensionless function

Model

$V(\Phi)$ is non-zero only in a narrow range of Φ

e.g. in a toy model $\quad V(\Phi) = \exp\left[-\frac{(\Phi-\Phi_0)^2}{2\Phi_1^2}\right]$



$\Phi_0 = 3.1\, m_{Pl}$, (m_{Pl} – Planck mass)

$\Phi_1 = 0.02\, m_{Pl}$

Model

FLRW metric: $ds^2 = dt^2 - a^2(t)\,d\mathbf{x}^2$

Hubble parameter: $H(t) = \dot{a}(t)/a(t) = \sqrt{\frac{8\pi\rho(t)}{3m_{Pl}^2}}$

Energy density:

$$\rho(t) = \frac{\dot{\Phi}^2}{2} + \frac{M^2\Phi^2}{2} + \frac{\dot{\chi}^2}{2} + \frac{m^2\chi^2}{2} + \frac{\lambda\chi^4}{4} - \mu^2\chi^2 V(\Phi)$$

Equations of motion:

$$\begin{aligned}\ddot{\Phi} + 3H\dot{\Phi} + M^2\Phi + \mu^2\chi^2\frac{\Phi - \Phi_0}{\Phi_1^2}V(\Phi) &= 0,\\ \ddot{\chi} + 3H\dot{\chi} + m^2\chi + \lambda\chi^3 - 2\mu^2\chi V(\Phi) &= 0,\end{aligned}$$

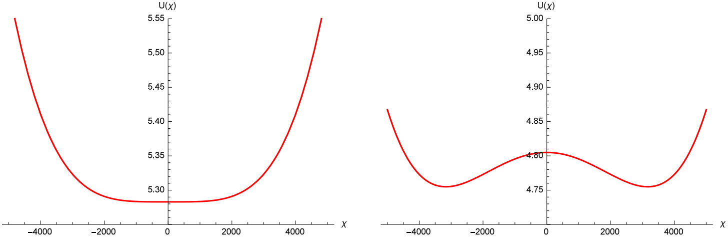
here it is assumed that $\Phi = \Phi(t)$ and $\chi = \chi(t)$

Model

During inflation (usual slow-roll regime) Φ decreases and when it reaches vicinity of Φ_0, two minima appear in the potential

$$U(\chi)\Big|_{\Phi=const} = \frac{\lambda}{4}\chi^4 + \left(\frac{1}{2}m^2 - \mu^2 V(\Phi)\right)\chi^2 + \frac{1}{2}M^2\Phi^2,$$

so the point $\chi = 0$ becomes local maximum



χ – in units of $10^{-6}\, m_{Pl}$, $U(\chi)$ – in units of $10^{-12}\, m_{Pl}^4$

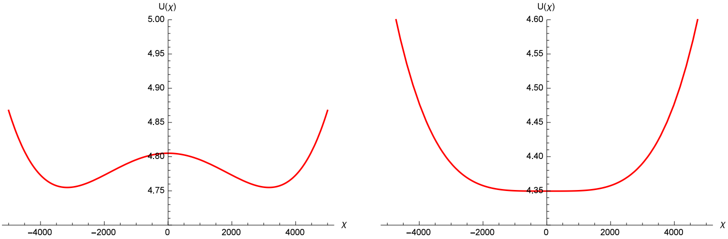
$\lambda = 2\cdot 10^{-3}$; $m = 10^{-10}$, $\mu = 10^{-4}$, $M = 10^{-6}$, $\Phi_{in} = 4$ (in m_{Pl} units)

Model

In spatial regions where χ turns out to be positive/negative (due to fluctuations) it rolls down to the positive/negative minimum $\Rightarrow$ domains with opposite signs of CP violation appear

[*T.D. Lee, 1974; Ya.B. Zeldovich, I.Yu. Kobzarev, L.B. Okun, 1974*]

Such CP violation is operative only when χ sits near the minimum, but in our model this minimum disappeared during inflation $\Rightarrow$ baryon asymmetry (if generated) exponentially inflated away



Successful baryogenesis should take place after the end of inflation

Model

Main features of the model:

1. Inflaton field Φ always gives the main contribution to the energy density $\rho \Rightarrow$ standard slow-roll inflation

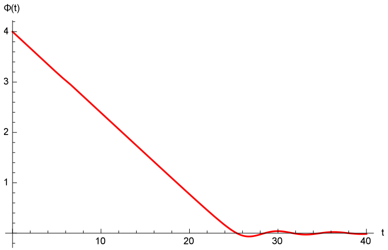
2. Inflation should last quite long after domains were formed (supposing that seed of the domain $\sim 1/H$ and present size of domain ~ 10 Mpc)

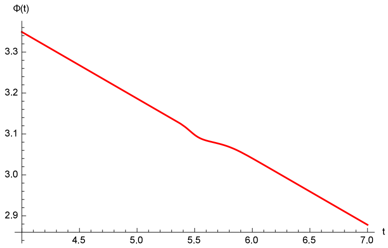
3. The field χ remains non-vanishing for some time after the end of inflation and during baryogenesis
(non-zero χ can induce CP violation)

Evolution of the inflaton field $\Phi(t)$

Standard slow-roll inflation

$\Phi(t)$ only slightly deviates from the straight line around $\Phi_0 = 3.1$
[inflation lasts $\simeq$ 60 e-foldings ($e^{60} \sim 10^{26}$) after $\Phi(t)$ passes Φ_0]

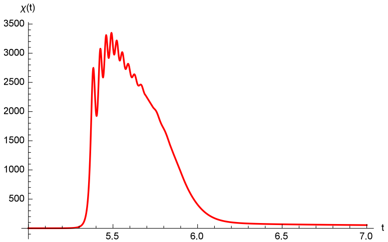


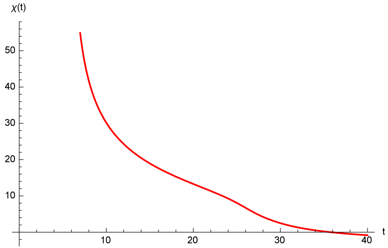


time t – in units of M^{-1}, $\Phi(t)$ – in units of m_{Pl}
$M = 10^{-6}$, $\Phi_{in} = 4$ (in m_{Pl} units)

$\Phi(t) = 0$ at $t \approx 25$ (beginning of reheating)

Evolution of the field $\chi(t)$





time t – in units of M^{-1}, $\chi(t)$ – in units of M

$\chi_{in} = 1\,M$, $M = 10^{-6}\,m_{Pl}$

When reheating starts (at $t \approx 25$) the field χ is quite large:
$\chi \sim 10^{-5}\,m_{Pl} \sim 10^{14}$ GeV

Distances between domains

If matter and antimatter domains exist and they are close enough, the gamma rays produced in annihilation along the boundary would be detectable. Such gamma rays have not been observed.

In our model inflaton Φ always gives the main contribution to $H \Rightarrow$ if $\chi = 0$ the Hubble parameter remains almost the same as in the case of non-zero χ considered above (size of region where $\chi = 0$ is only two times less than the domain size).

Therefore, model predicts the domains of cosmological size with the distances between them of the same order of magnitude

Reheating

The stage of inflation is followed by the stage of reheating, during which the heavy X-particles (e.g. vector bosons) can be produced through decay of inflaton field.

In the case of comparatively large coupling between X-particles and inflaton the decay of the inflaton occurs rapidly, during only one or few oscillations.

Assume that produced X-particles can decay into fermions (not SM in general). If the corresponding couplings are large enough, the X-particles decay very quickly.

Therefore, field χ may remain non-zero yet to the moment when X-particles have been completely decayed:
e.g. $\chi \sim 2\,M \sim 10^{13}$ GeV at $t = 30\,M^{-1}$
(non-zero χ can induce CP violation in the X-boson decays)

Non-zero χ can induce CP violation in X-boson decays

The field χ is real and pseudoscalar,
so it interacts with the produced fermions as

$$\mathcal{L}_{\chi\psi\psi} = g_{kl}\chi\bar{\psi}^k i\gamma_5\psi^l = ig_{kl}\chi(\bar{\psi}^k_L\psi^l_R - \bar{\psi}^k_R\psi^l_L),$$

where k and l denote the fermion flavor.

Free fermion Lagrangian may contain mass terms

$$-m_{\psi kl}\bar{\psi}^k\psi^l = -m_{\psi kl}(\bar{\psi}^k_L\psi^l_R + \bar{\psi}^k_R\psi^l_L)$$

The sum of these terms and $\mathcal{L}_{\chi\psi\psi}$ can be presented in matrix form

$$-(\bar{\psi}_R M_\psi\psi_L + \bar{\psi}_L M_\psi^\dagger\psi_R),$$

here $M_\psi = m_\psi + ig\chi$ is a non-Hermitian matrix,
whereas m_ψ and g are Hermitian ones

Non-zero χ can induce CP violation in X-boson decays

Using simultaneously two unitary transformations
$\psi_R \to \psi'_R = U_R \psi_R$ and $\psi_L \to \psi'_L = U_L \psi_L$
one can always diagonalize the matrix M_ψ.

The elements of transformation matrices U_R and U_L depend on the magnitude of the field χ.

The mass terms take the simple form

$$-m'_{\psi ab} \bar{\psi}^a \psi^b,$$

where ψ^a and ψ^b are the mass eigenstates and
m'_ψ is diagonal matrix with real diagonal elements

Non-zero χ can induce CP violation in X-boson decays

However, the interaction of fermions with vector boson X_μ remains the same under these transformations:

$$g_{Rkl} X_\mu \bar{\psi}^k_R \gamma^\mu \psi^l_R = g'_{Rab} X_\mu \bar{\psi}^a_R \gamma^\mu \psi^b_R,$$

here $g'_R = U_R g_R U^\dagger_R$ is matrix of coupling constants in mass eigenstate basis (analogously $g'_L = U_L g_L U^\dagger_L$).

The constants g'_{ab} are complex in general case, and if there are at least three species of fermions, one cannot rotate away simultaneously all phases in complex matrices $g'_{R,L}$ [*M. Kobayashi, T. Maskawa, 1973*].

The complexity of the coupling constants means that CP is violated in the X-boson decays

Sakharov criteria

The model may satisfy Sakharov criteria for successful baryogenesis without fine tuning of parameters

1. The magnitude of CP violation depends on the value of χ through the matrices $U_{R,L}$ and hence coupling constants $g'_{R,L}$. Since χ is essentially non-zero after the end of inflation and during baryogenesis, CP-odd effects can be large enough.

We assume that interactions with X-boson involve fermions with certain chirality, and thus these interactions break C-invariance

2. State of matter is out of thermal equilibrium

Sakharov criteria

3. Assume also that
the baryon number is not conserved in X-boson decays

Baryon asymmetry generated in the decay of one X-particle:

$$\delta = \frac{1}{\Gamma_{tot}} \sum_f \Gamma(X \to f) B_f$$

The ratio of the baryon number density to the entropy density can be estimated as

$$\Delta_B = \frac{n_B - n_{\bar{B}}}{s} \sim 10^{-3}\delta$$

Thus, to get observed value $\Delta_B \simeq 0.86 \cdot 10^{-10}$ it is sufficient to have only $\delta \sim 10^{-7}$.

Conclusions for section V

The considered model may lead to baryo-symmetric universe with cosmologically large domains of matter and antimatter separated by cosmologically large distances, avoiding the domain wall problem.

Inflation is an essential ingredient of the scenario. A coupling of the pseudoscalar field χ to the inflaton field was introduced on purpose to generate a non-zero value of χ and to keep it during baryogenesis as a source of CP violation.