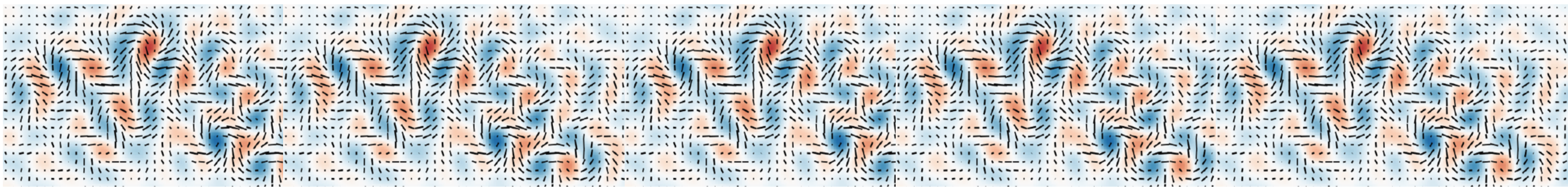

Statical properties of the CMB B-mode polarisation signal

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Santos et al., 2017, JCAP, 01, 043 [arXiv:1612.03564]

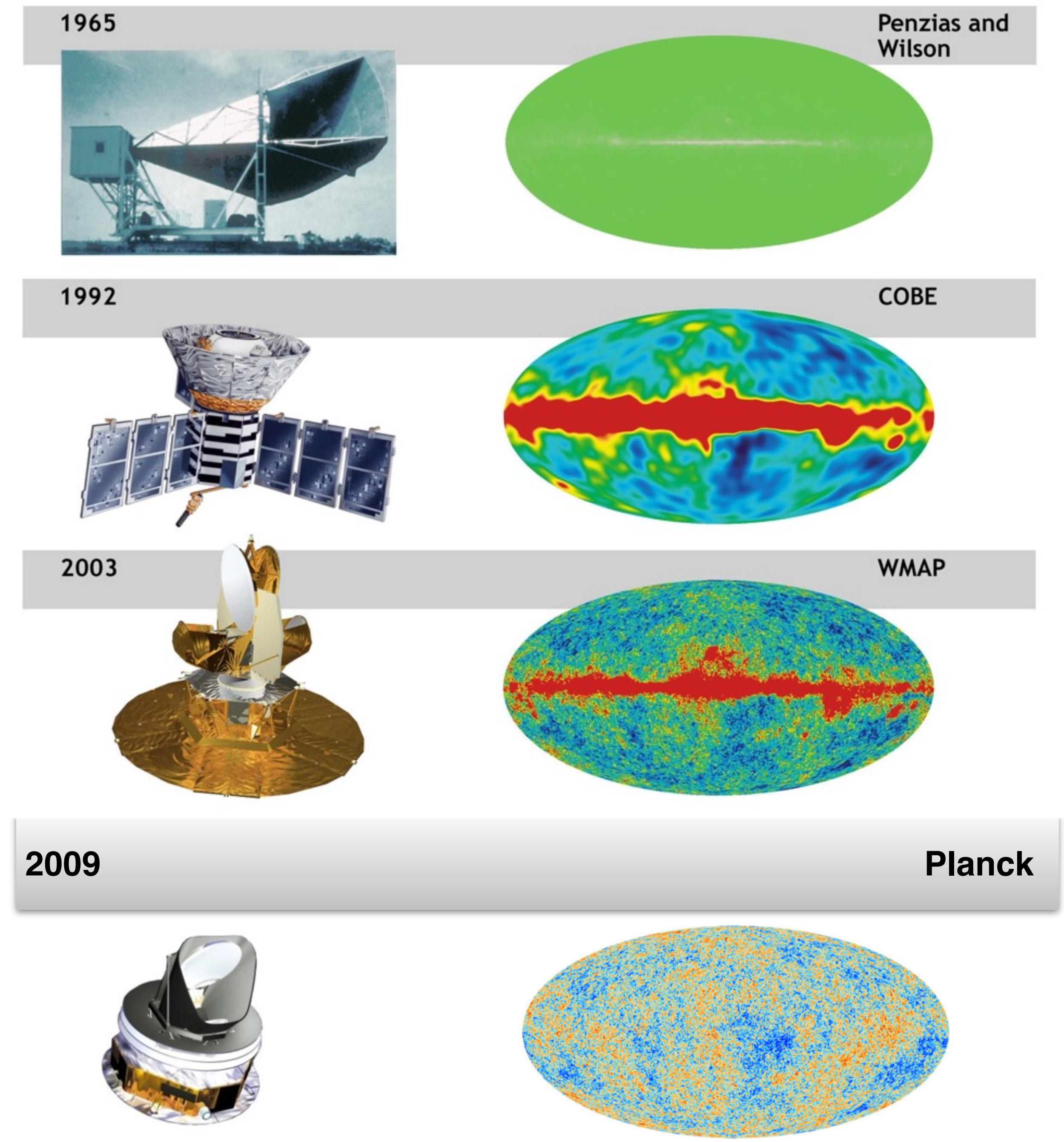
05/04/2017

Outline

- Introduction
    - The cosmic microwave background, CMB
    - The polarised signal
    - The B-map: different contributions
- Motivations
- Statistical method
- The E-to-B leakage contribution

CMB: A brief introduction

- Discovered in1965 by Penzias and Wilson
- Temperature anisotropies measured by COBE in 1992
- Improved measurements by WMAP and Planck



The polarisation signal

-

Q → -Q, U → -U under 90 degree rotation

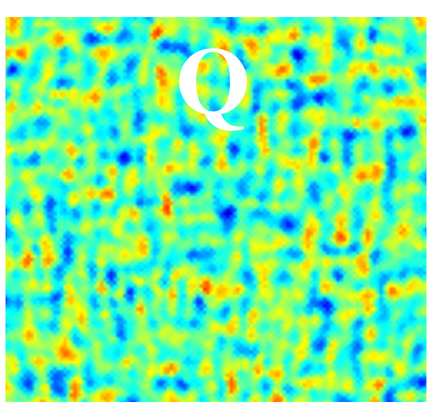
Q

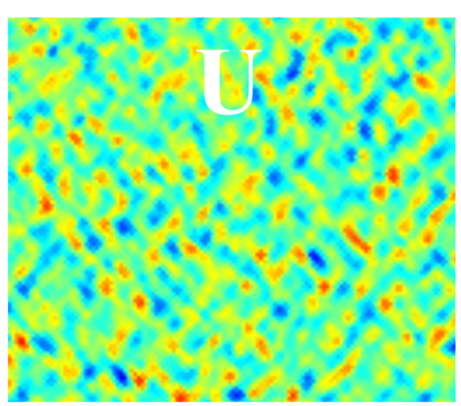
-

U

Q → U, U → -Q under 45 degree rotation

- By rotating the local coordinate system, Q is rotated into U and vice-versa.





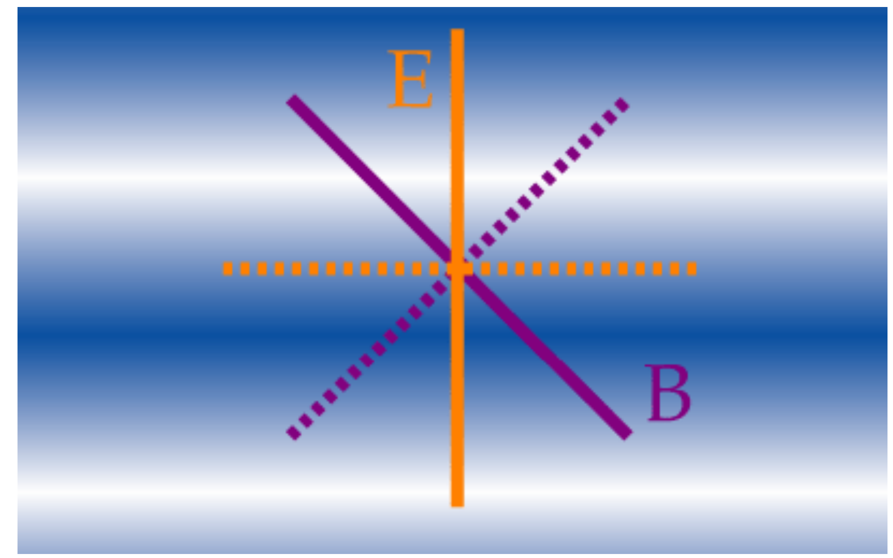
The linearly polarised CMB is completely described by a spin 2 and spin -2 field

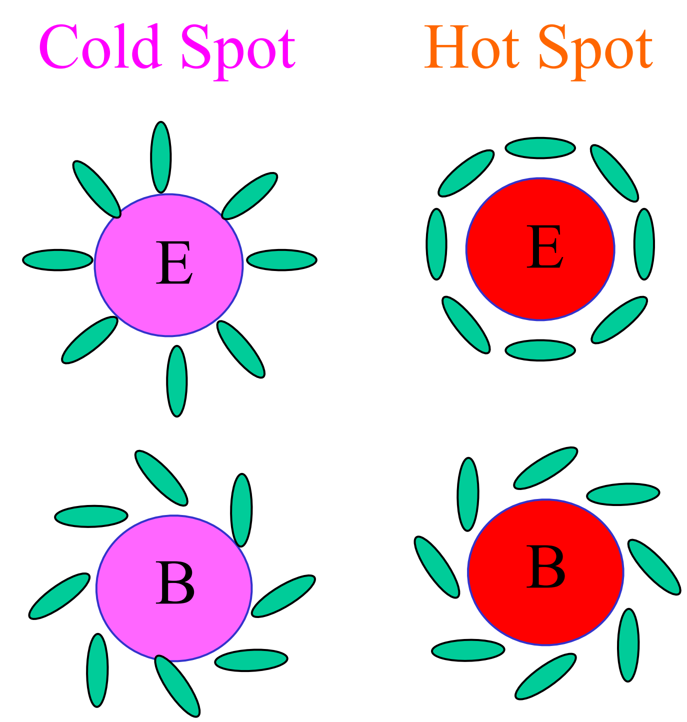
$$P_{\pm 2}(\vec{n}) = Q(\vec{n}) \pm iU(\vec{n})$$

E/B decomposition

- The description North-south and East-West (more formally, the stokes parameters) depends on the arbitrary choices of coordinates

- We then describe the polarisation by its orientation relative to itself: E-mode and B-mode





Spherical harmonics expansion

$$P_{\pm}(\vec{n}) = \sum_{lm} a_{\pm 2,lm}\ {}_{\pm 2}Y_{lm}(\vec{n})$$

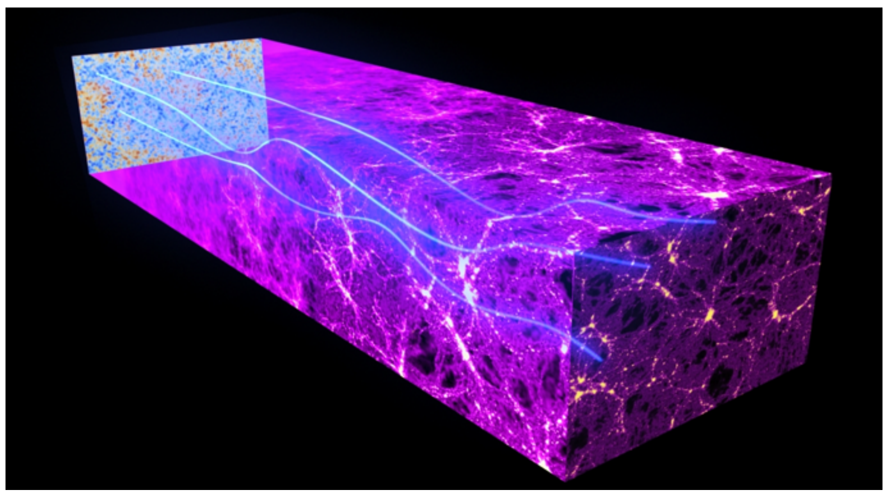
$$E_{lm} \equiv -\frac{1}{2}\left[a_{2,lm} + a_{-2,lm}\right]$$

$$B_{lm} \equiv -\frac{1}{2i}\left[a_{2,lm} - a_{-2,lm}\right]$$

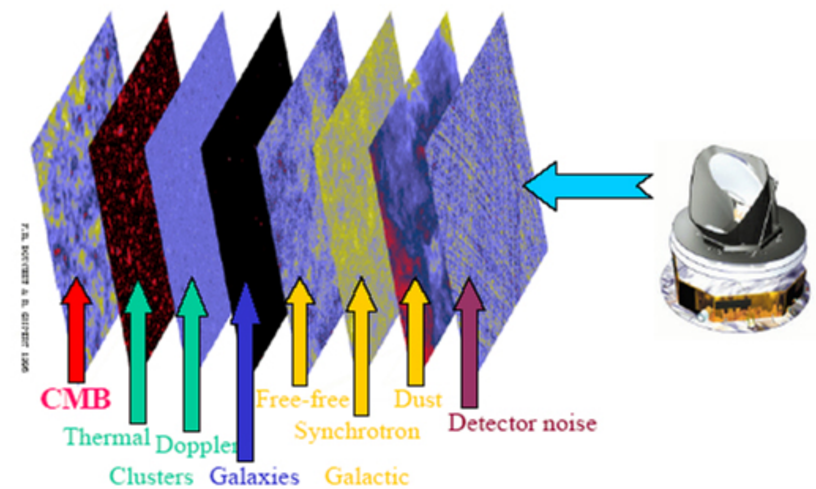
Our target

But, what can generate B-mode?

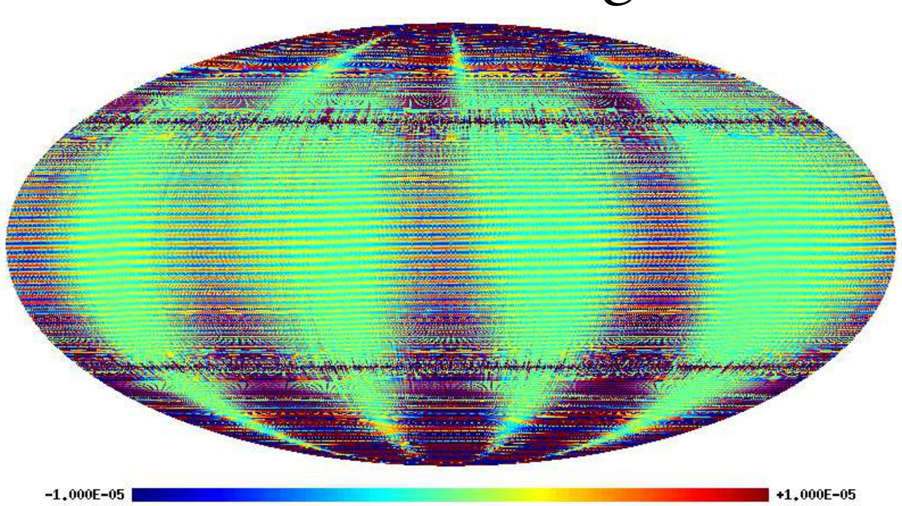
CMB lensing

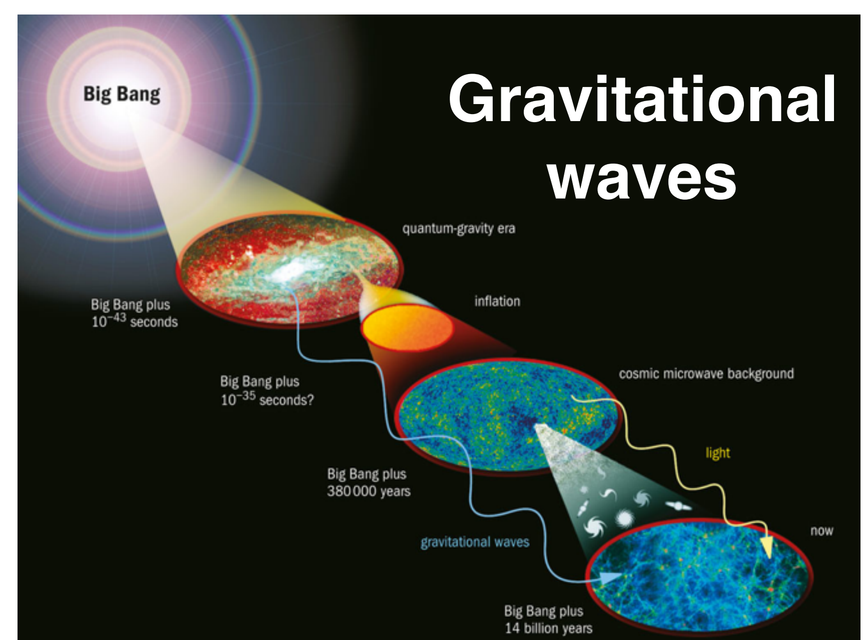


Foreground separation



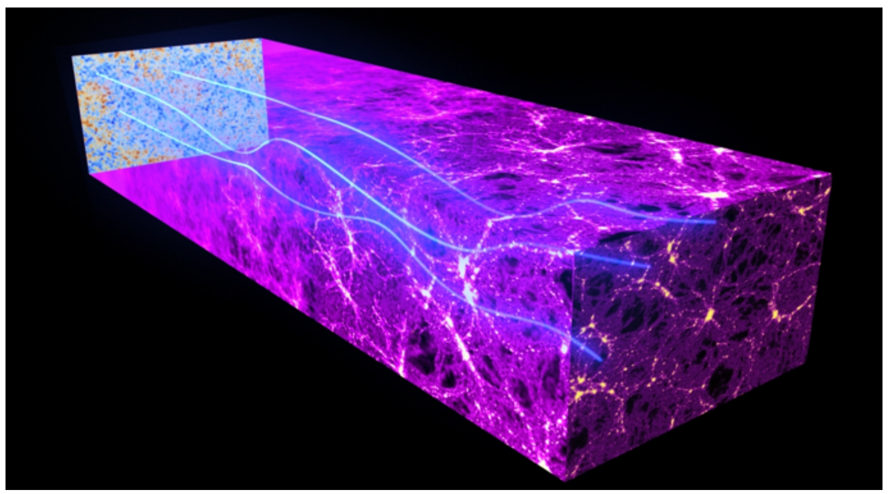
E-to-B leakage



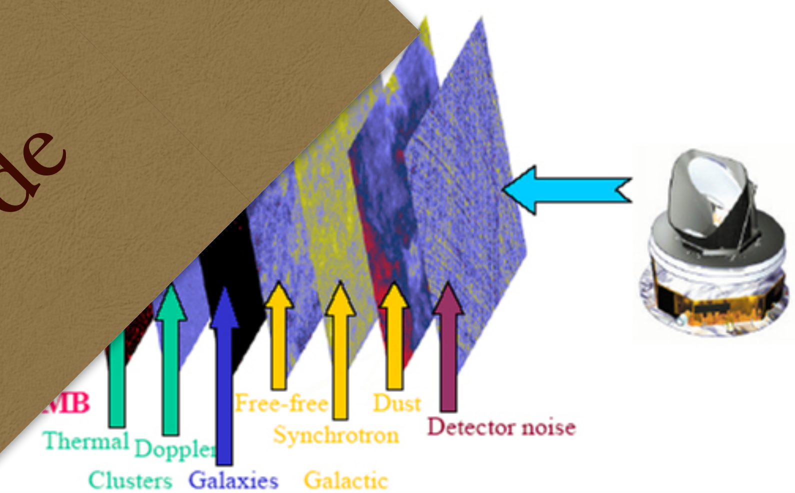


But, what can generate B-mode?

CMB lensing

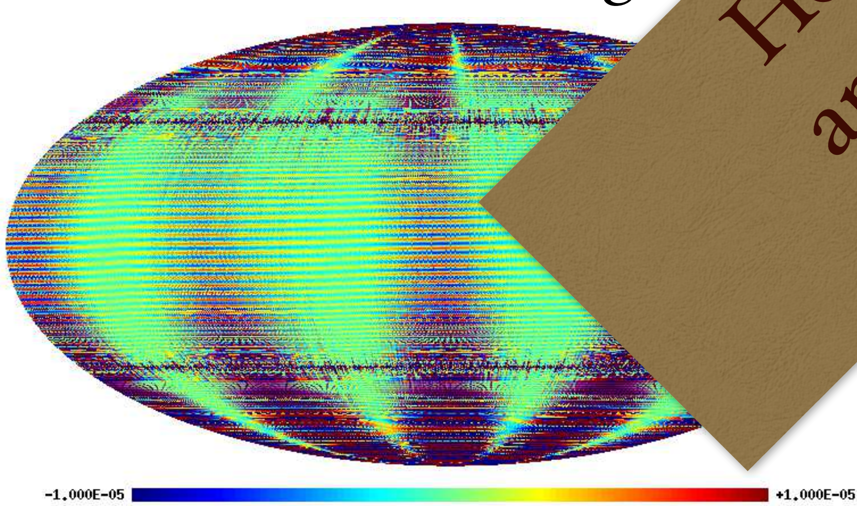


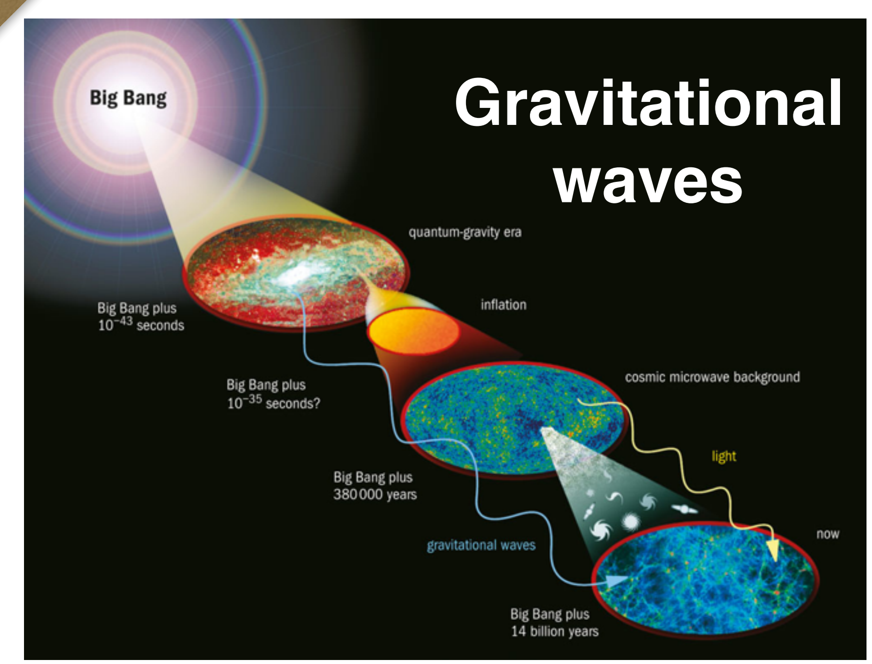
nd separation



How can we distinguish among different B-mode signals?

E-to-B leakage





Motivations

- In the linear theory, the CMB fluctuation satisfy the Gaussian distribution
- For nearly Gaussian random fields (CMB T-map and E-map), the power spectrum can completely describe the statical properties of the field
- The CMB B-mode polarisation field is a non-gaussian random field due to the CMB lensing effect
  - Santos et al., 2016, JCAP, 07, 029 [arXiv:1510.07779]
- To extract all the information from a non-Gaussian field we need extra statical tools besides the power spectrum, as for example, the Minkowski functionals

- Using mainly the Minkowski functionals we aim to look for imprints of different contributions to the B-mode map
  - Today, I will present the results for the E-to-B leakage when considering a partial sky analysis
    - We include the effect of primordial gravitational waves (GW) and extra sky cuts

The Minkowski functionals

- The morphological properties of the excursion sets can be quantified in terms of the MF
- There are 3 MF for the two dimensional CMB excursion sets.

$$v_0(\nu) = \int_{Q_\nu} \frac{da}{4\pi}$$

The total fractional area of the map above a certain threshold

$$v_1(\nu) = \int_{\partial Q_\nu} \frac{dl}{16\pi}$$

The boundary length of the excursion set per unit area

$$v_2(\nu) = \int_{\partial Q_\nu} \frac{k dl}{8\pi^2}$$

The Euler Characteristic per unit area (the difference between the numbers of hot spots and cold spots)

Quantifying the result:

$$\chi^2 = \sum_{aa'} \left[\overline{v}_a^{ideal} - \left\langle v_a^{real} \right\rangle \right] C_{aa'}{}^{-1} \left[\overline{v}_{a'}^{ideal} - \left\langle v_{a'}^{real} \right\rangle \right]$$

where

$$C_{aa'} \equiv \frac{1}{499} \sum_{k=1}^{500} \left[\left(v_a^{k,real} - \overline{v}_a^{real} \right) \left(v_{a'}^{k,real} - \overline{v}_{a'}^{real} \right) \right]$$

A partial sky survey

- In the ideal case (full sky map), we directly derive E and B from Q and U
- However, even for satellite surveys we will not get a full-sky map. Why?
- We must mask out the unavoidable Galactic foreground!

E/B decomposition in partial sky

$$\mathcal{B}(\hat{n}) = -\frac{1}{2i}[\bar{\eth}\bar{\eth}P_{+}(\hat{n}) - \eth\eth P_{-}(\hat{n})]$$

$$\eth f \equiv -\sin^{s}\theta\left(\frac{\partial}{\partial\theta} + \frac{i}{\sin\theta}\frac{\partial}{\partial\phi}\right)(f\sin^{-s}\theta),$$

$$\bar{\eth} f \equiv -\sin^{-s}\theta\left(\frac{\partial}{\partial\theta} - \frac{i}{\sin\theta}\frac{\partial}{\partial\phi}\right)(f\sin^{s}\theta).$$

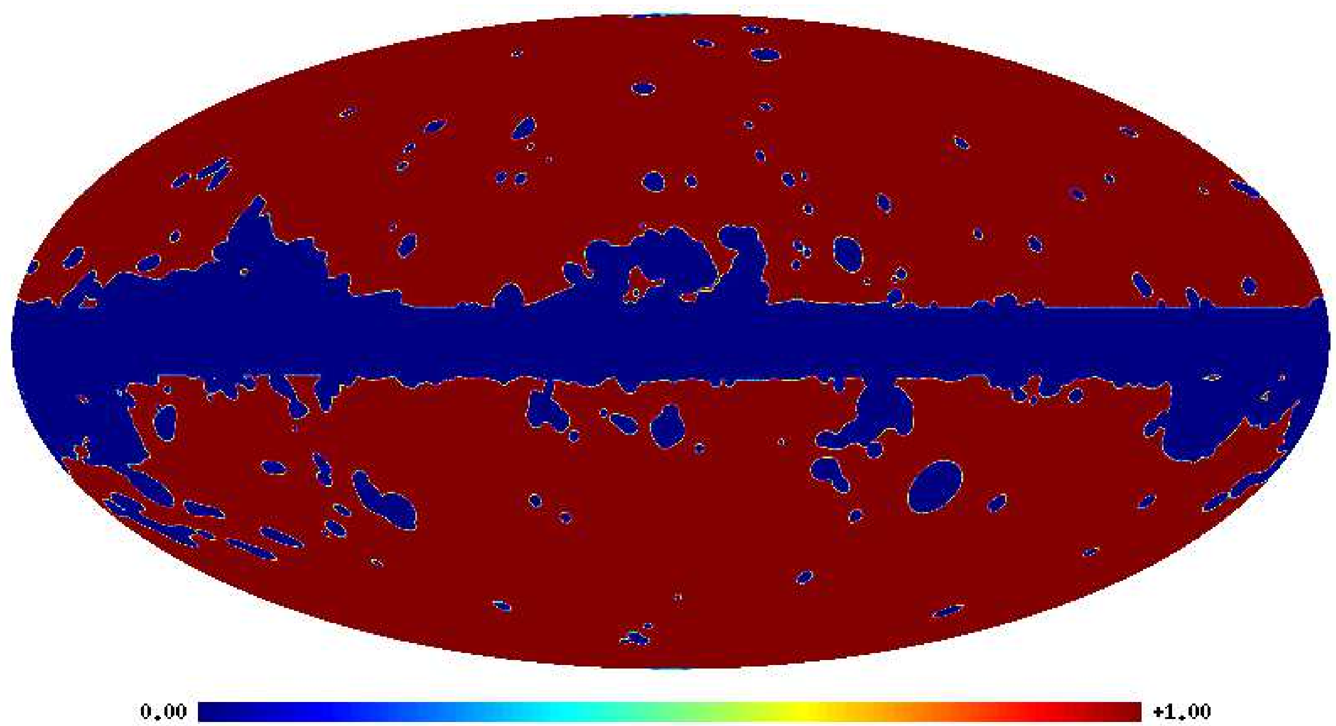
$$\mathcal{B}(\hat{n}) \equiv \sum_{\ell,m}\mathcal{B}_{\ell m}Y_{\ell m}(\hat{n}) \longrightarrow \mathcal{B}_{\ell m} = \int \mathcal{B}(\hat{n})Y^{*}_{\ell m}(\hat{n})d\hat{n}, = N_{\ell,2}B_{\ell m}$$

$$N_{\ell,s} = \sqrt{(\ell+s)!/(\ell-s)!}$$

$$\mathcal{B}^{\text{pure}}_{lm} \equiv -\frac{1}{2i}\int d\hat{n}\left\{P_{+}(\hat{n})\left[\bar{\eth}\bar{\eth}\left(W(\hat{n})Y_{lm}(\hat{n})\right)\right]^{*} - P_{-}(\hat{n})\left[\eth\eth\left(W(\hat{n})Y_{lm}(\hat{n})\right)\right]^{*}\right\}$$

Sky apodization
(Planck polarisation mask)

Yi-Fan Wang, Kai Wang , Wen Zhao, arXiv:1511.01220



$$W_i = \frac{1}{2} + \frac{1}{2} erf\left(\frac{\delta_i - \frac{\delta_c}{2}}{\sqrt{2}\sigma} \right) \quad \delta_i < \delta_c$$

$$W_i = 1 \quad \delta_i > \delta_c$$

E/B decomposition numerical method

- SZ approach [Smith, (2006); Smith and Zaldarriaga (2007)]: the most efficient method for estimating the CMB B-mode power spectrum in partial sky [Fertè et al. (2013)]

- Step 1: To compute the spin-0, spin-1 and spin-2 rendition of the window function

$$W_0 = W \qquad W_1 = \partial W \qquad W_2 = \partial\partial W$$

$$\eth W = -\frac{\partial W}{\partial \theta} - \frac{i}{\sin\theta}\frac{\partial W}{\partial \phi}$$

$$\eth\eth W = -\cot\theta\frac{\partial W}{\partial \theta} + \frac{\partial^2 W}{\partial \theta^2} - \frac{1}{\sin^2\theta}\frac{\partial^2 W}{\partial \phi^2} - \frac{2i\cot\theta}{\sin\theta}\frac{\partial W}{\partial \phi} + \frac{2i}{\sin\theta}\frac{\partial^2 W}{\partial \theta \partial \phi}$$

- Step 2: Construct 3 apodized maps

$$P_{\pm 2} = W_0 P_{\pm 2} \qquad P_{\pm 1} = W_{\mp 1} P_{\pm 2} \qquad P_{\pm 0} = W_{\mp 2} P_{\pm 2}$$

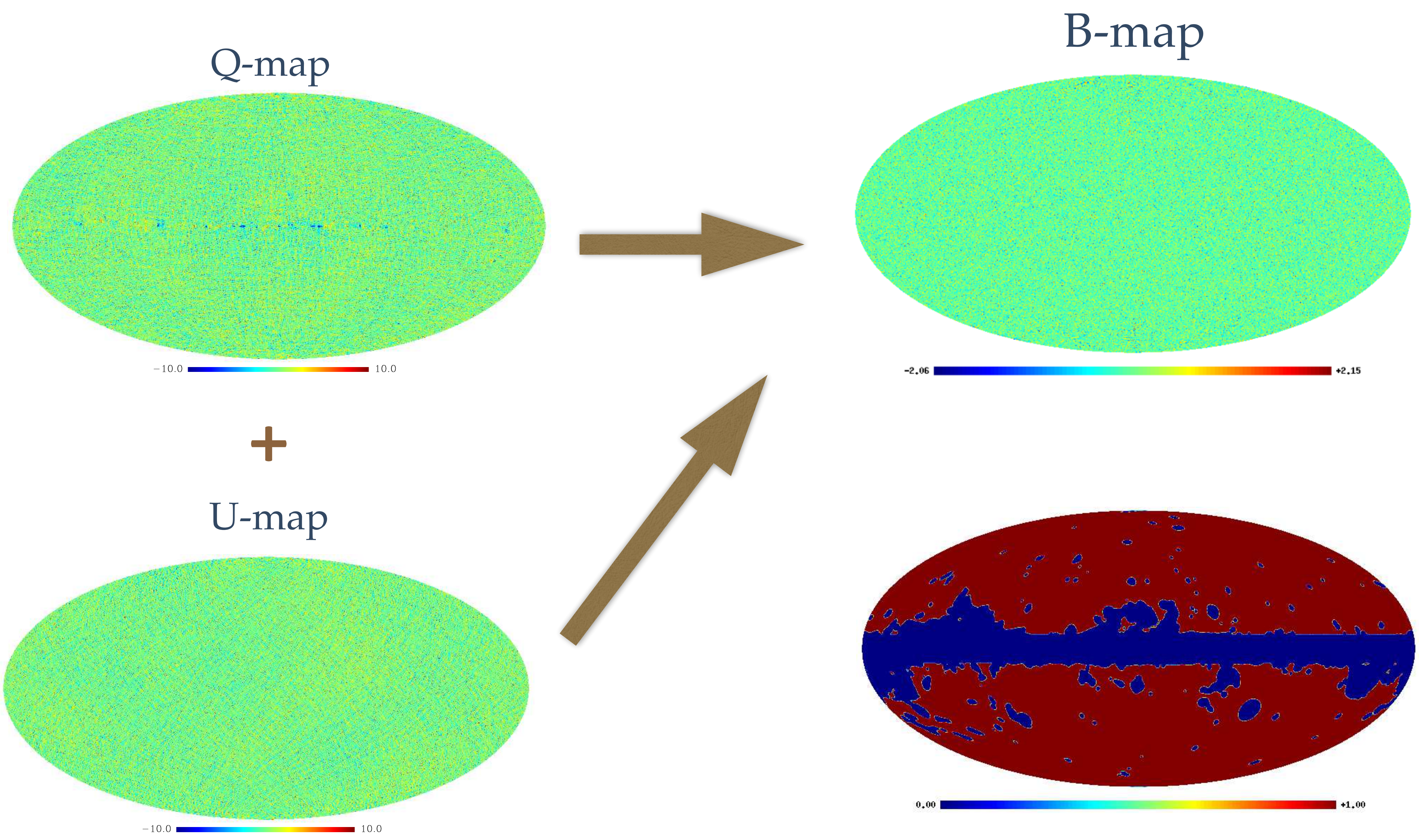
- Step 3: Generating the new B_{lm} and finally the B-map

$$\tilde{B}_{lm} = \left(B_{0,lm} + 2N_{l,1}B_{1,lm} + N_{l,2}B_{2,lm} \right)$$

Simulations

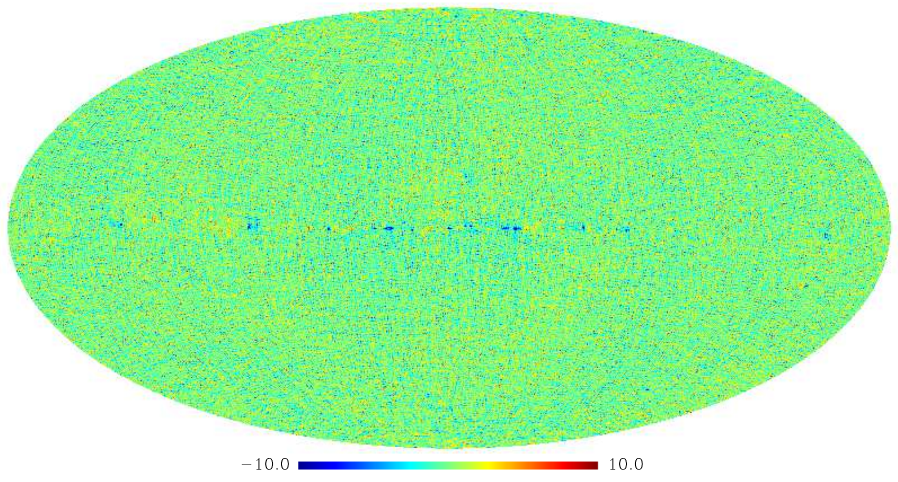
- We used CAMB to generate the lensed CMB power spectra
  - Planck best fit parameters for the LCDM model
- These power spectra are the input of lenspix to produce Q and U maps
  - 500 simulations with nside=1024 (i.e. 1.25829e+07 pixels)

Ideal case



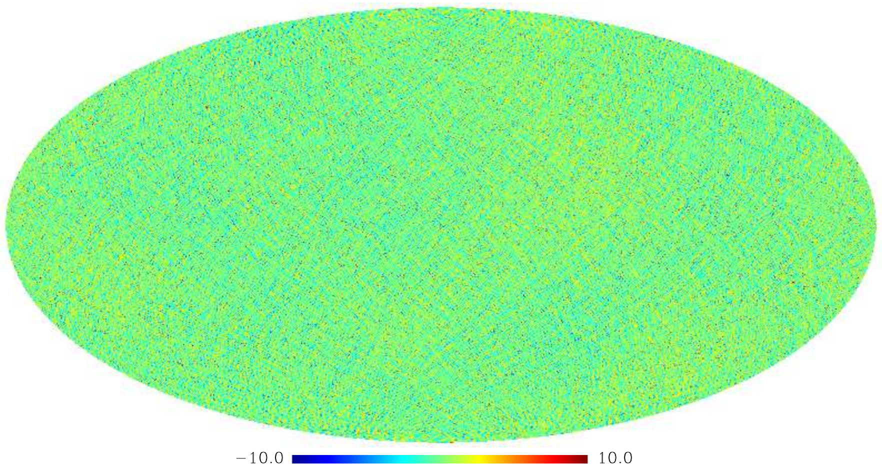
Real case

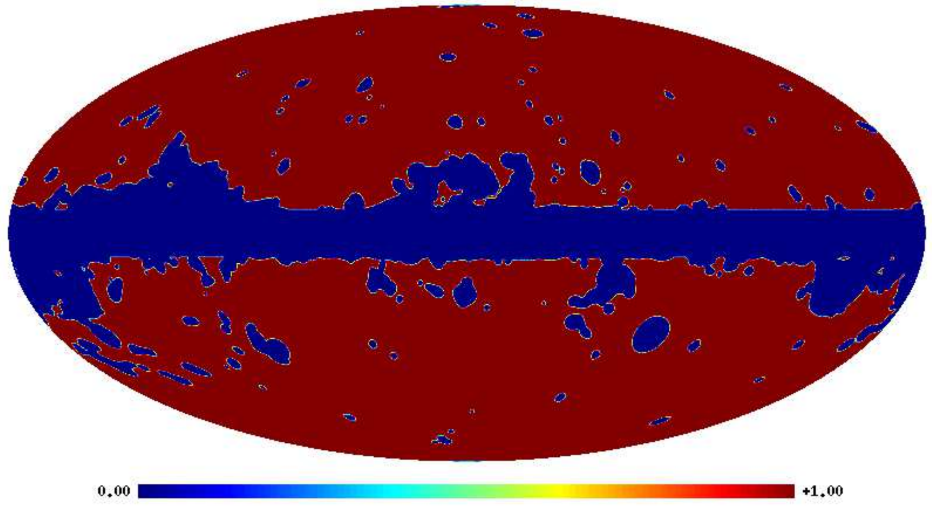
Q-map



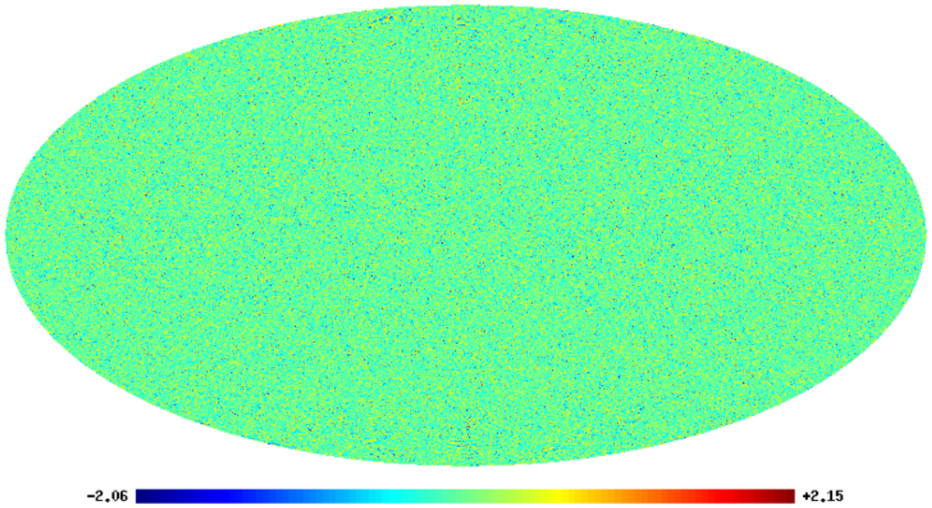
+

U-map

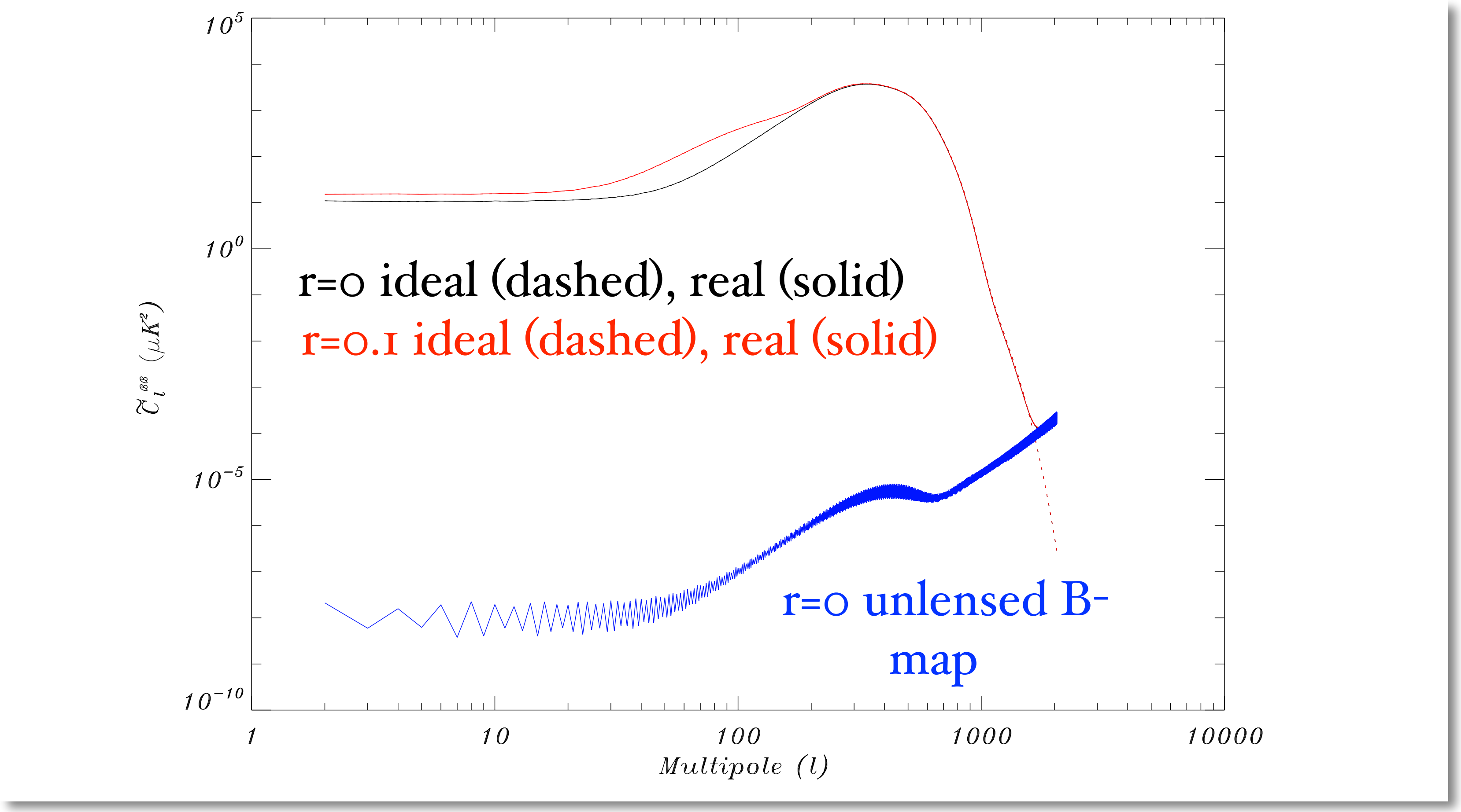


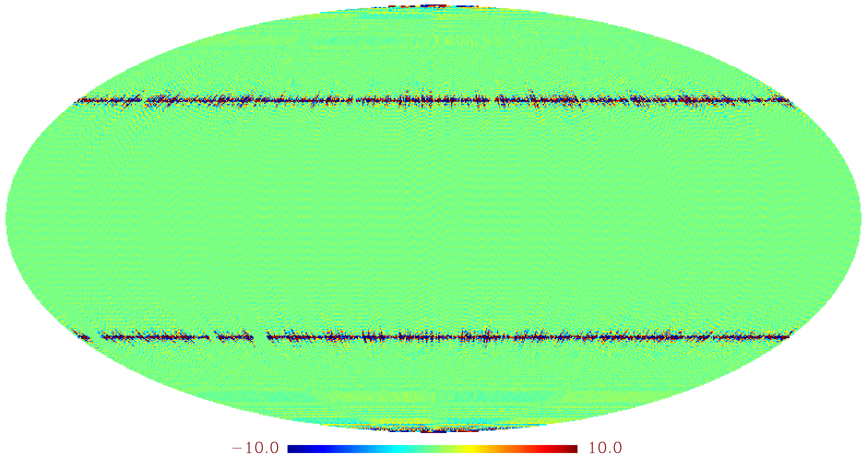


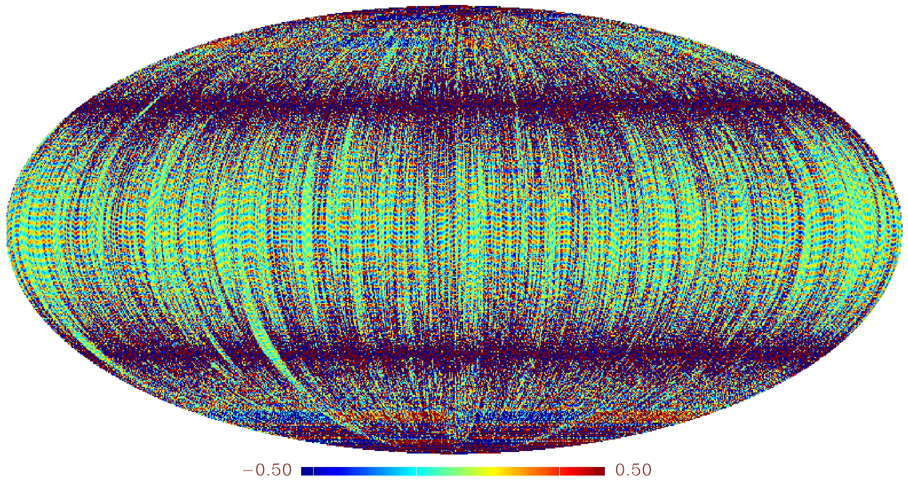
B-map



Real vs ideal







Testing different soothing scales

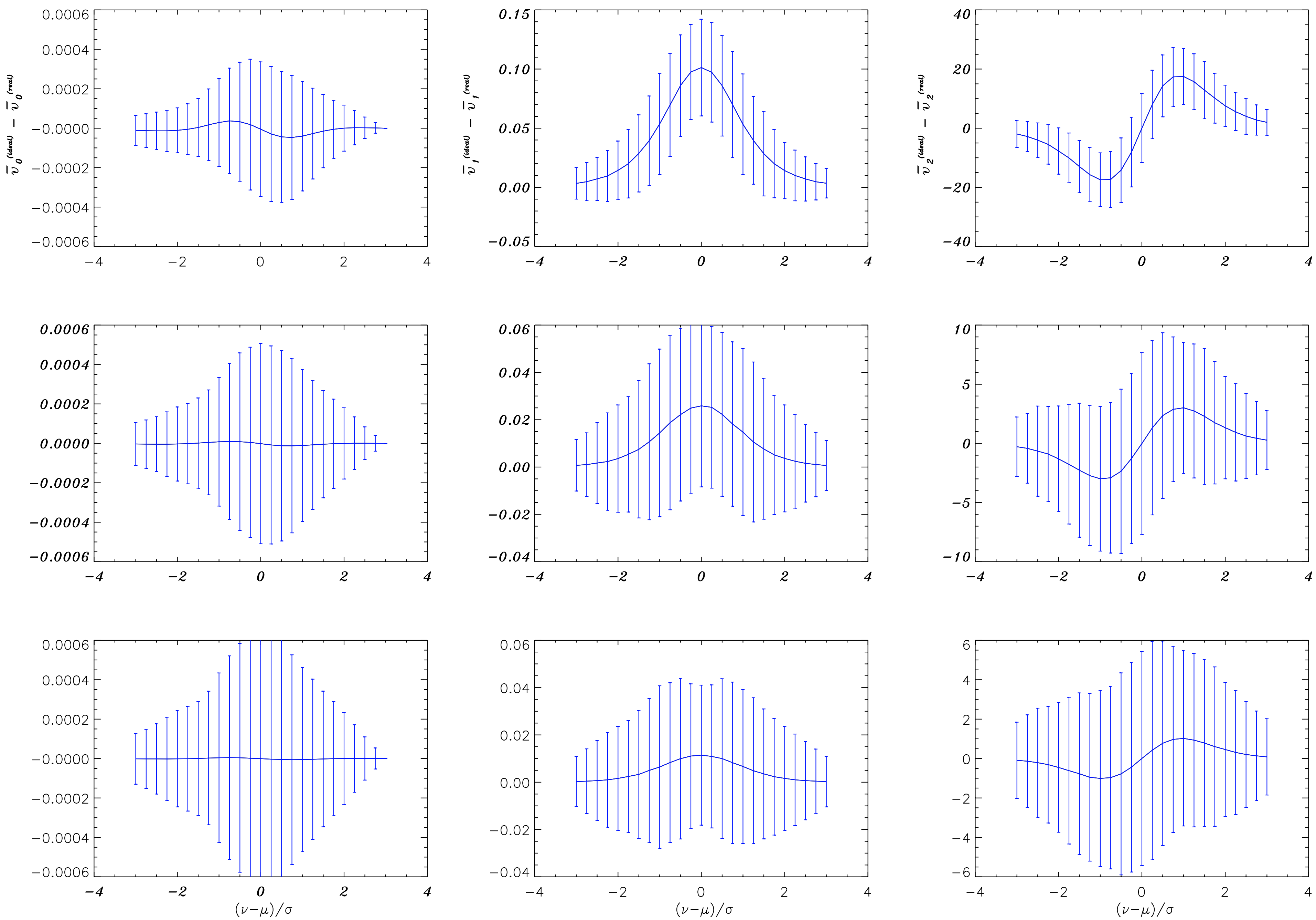
- In order to remove the multipoles dominated by noise, the calculation of the MF requires smoothing the maps to be analysed
- Even though different smoothing scales of the same CMB map have a high correlation, they must be taken into account in order to extract all its available information

$$\theta_s = 10', 20', 30', 40', 50', 60'$$

r=0

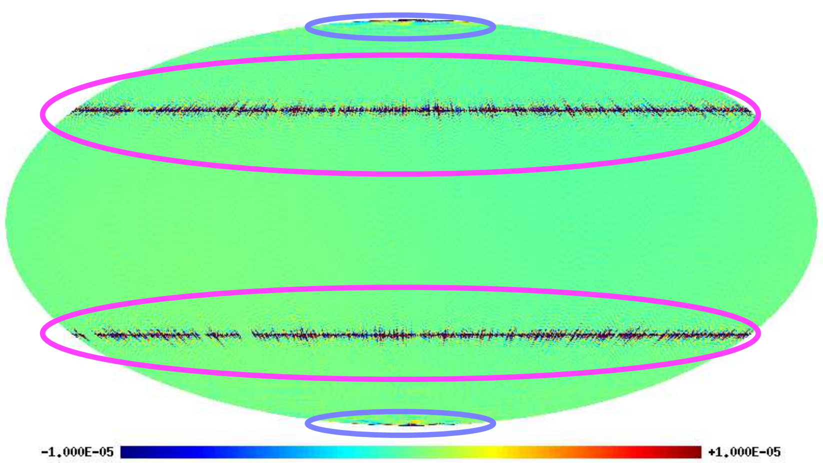
MF: ideal case-real case

$\theta_s = 10'$ $\theta_s = 40'$ $\theta_s = 60'$

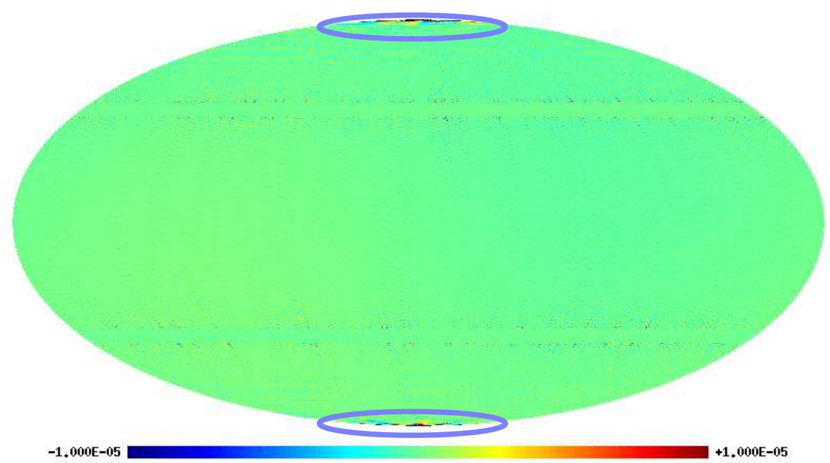


Adding sky cuts and avoiding the leakage signal: χ^2 statistics

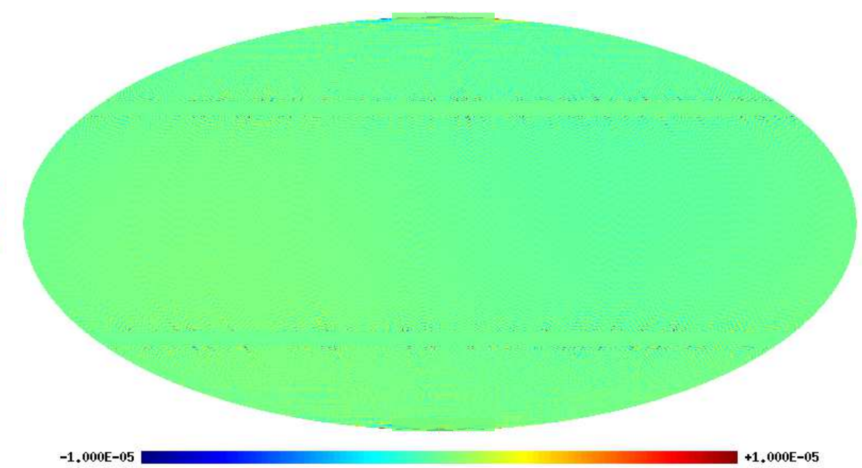
Sky cut 1



Sky cut 2



Sky cut 3



r=0

	10’	20’	30’	40’	50’	60’
cut 1	18.93	9.53	4.27	1.62	0.71	0.34
cut 2	17.27	8.55	3.80	1.42	0.63	0.30
cut 3	15.78	7.97	3.56	1.28	0.57	0.27

Comparing different tensor-to-scalar ratios

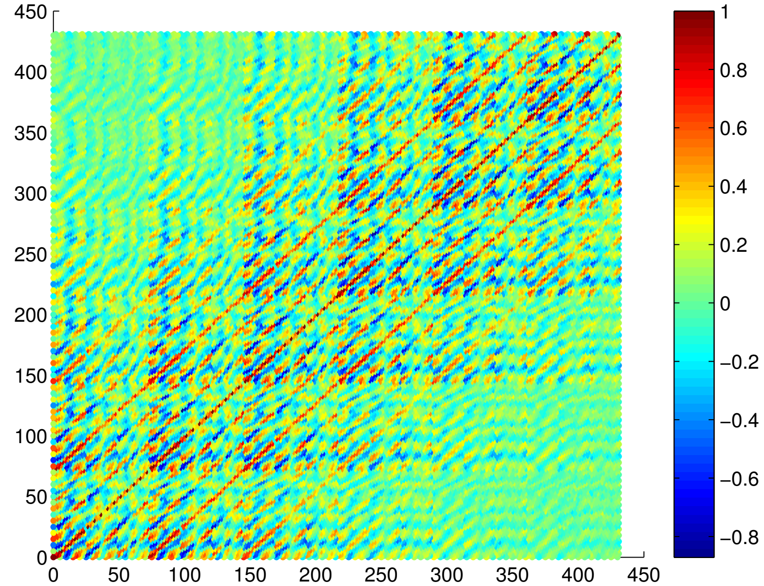
	10’	20’	30’	40’	50’	60’
χ^2 (r=0)	18.93	9.53	4.27	1.62	0.71	0.34
χ^2 (r=0.1)	21.18	11.00	4.41	1.63	0.62	0.29

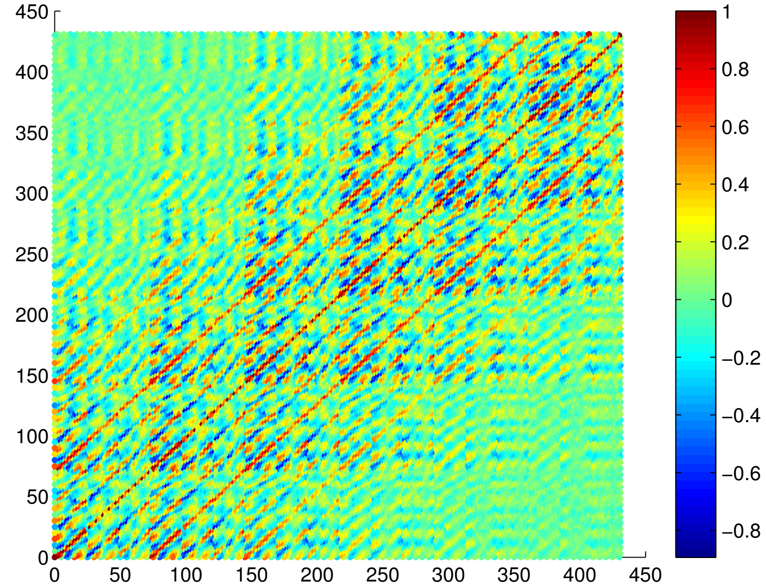
	r=0	r=0.1
Total χ^2	220.23	268.74

The correlation coefficient of the MF

- The large values of the total χ^2 for both r=0 and r=0.1 show that the MF for different smoothing scales are very correlated
  - The E-to-B leakage is not a stochastic noise, and it is always relevant in the same sky regions
  - The total χ^2 depends on the correlation coefficients instead of a direct sum of the χ^2 values for each smoothing scale

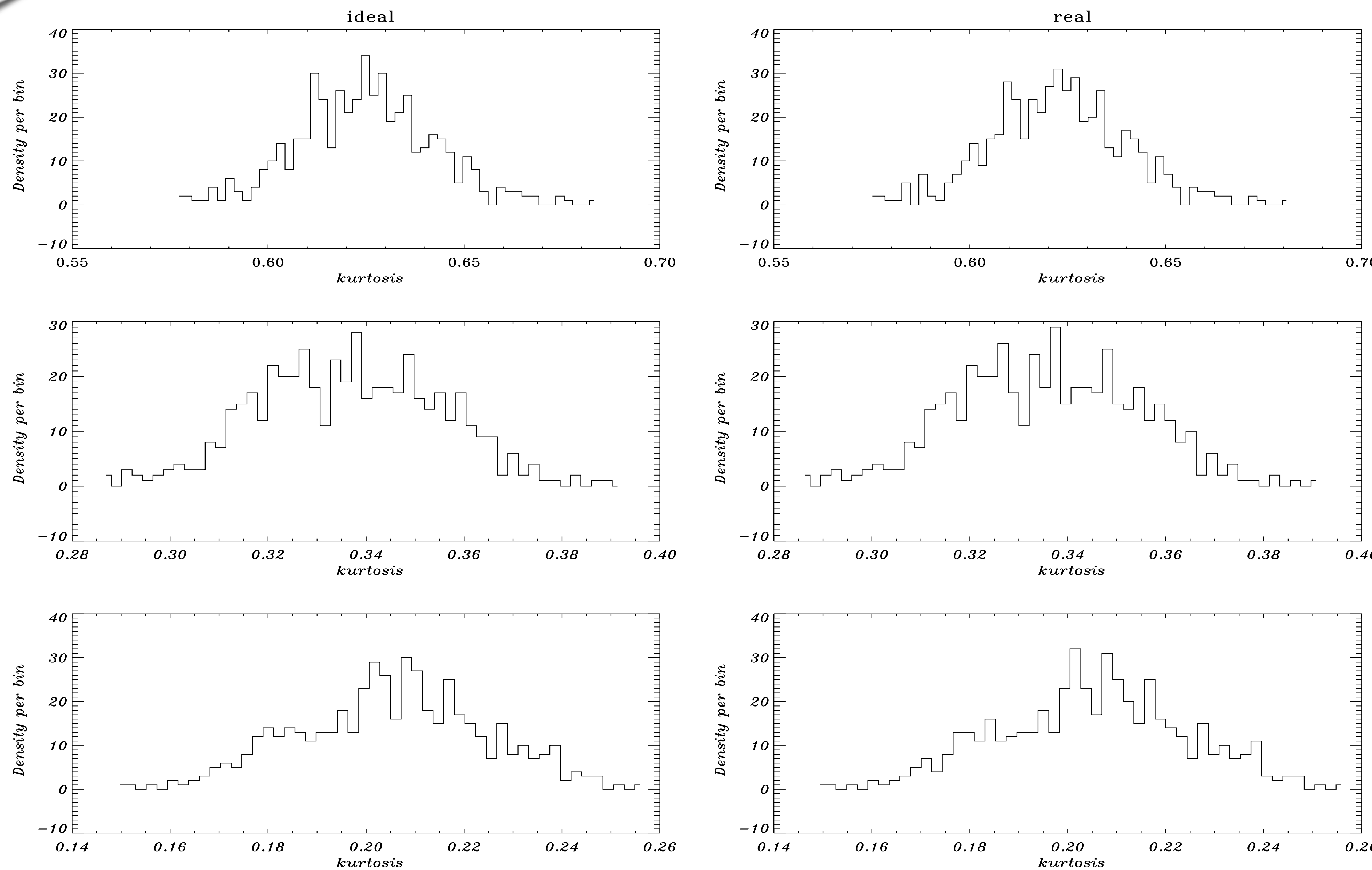
$$\rho_{aa'} = C_{aa'} \Big/ \sqrt{C_{aa} C_{a'a'}}$$





Kurtosis: Ideal case - real case

r=0



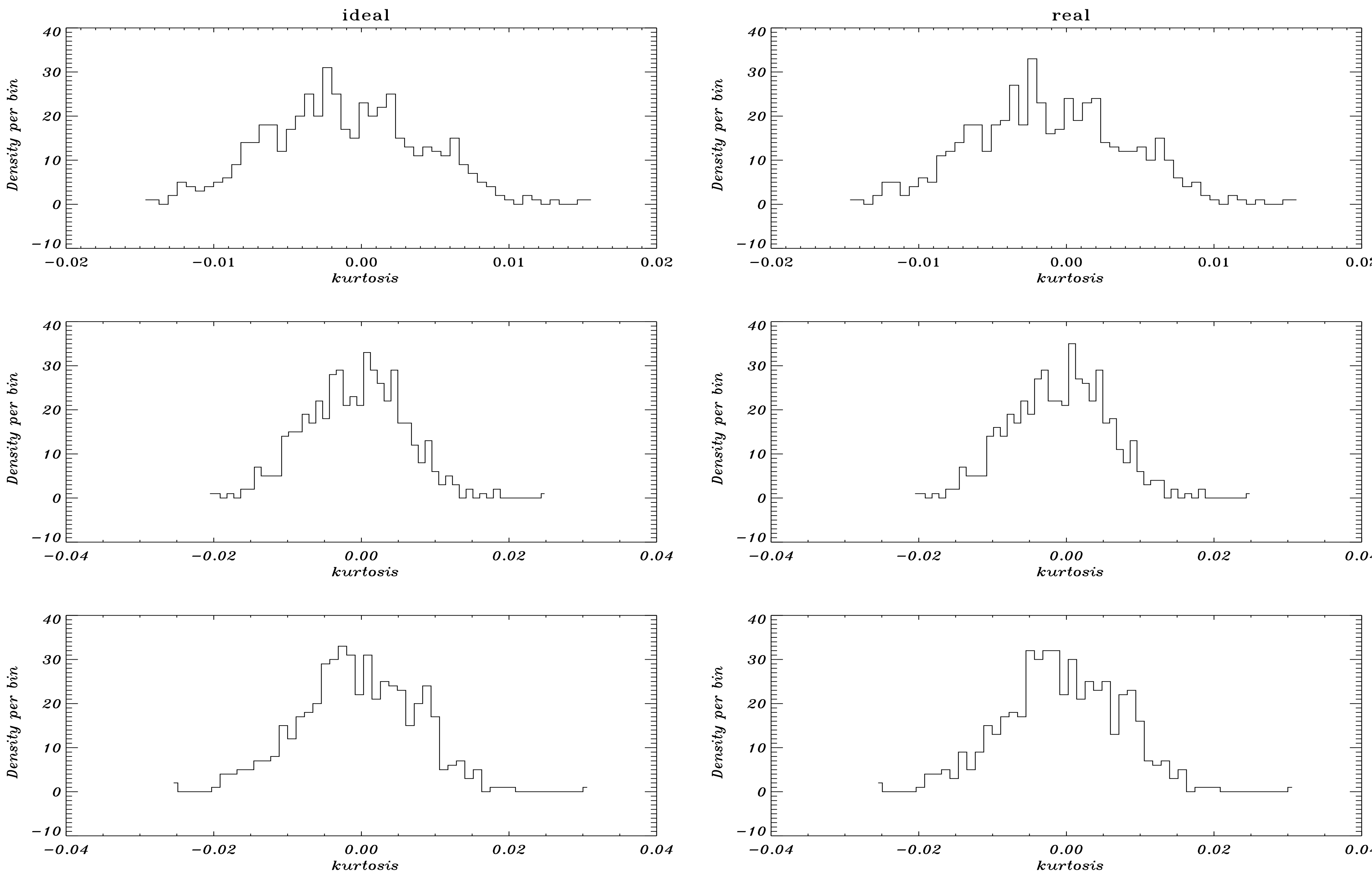
$\theta_s = 10'$

$\theta_s = 40'$

$\theta_s = 60'$

Skewness: ideal case - real case

r=0



$\theta_s = 10'$

$\theta_s = 40'$

$\theta_s = 60'$

Conclusions

- The leakage is dominated by high multipoles: its significance becomes smaller by increasing the smoothing scale
- The imprint of the leakage is still noticeable in the MF analysis even when the contamination bands and the poles are excluded
- These regions do not play an important roll in the overall leakage contribution

- Even though the leakage seems not relevant for individual smoothing scales, it is definitely relevant when they are all combined

- The E-to-B leakage should be carefully considered when analysing partial sky CMB B-mode data in order to avoid misinterpretation of the data

- We did not find any imprint of the E-to-B leakage in the simulated B-maps when considering both skewness and kurtosis statistics for both r=0 and r=0.1