The standard model of cosmology is flat ΛCDM:

$$K = 0, \quad a(t) = a_0 \left(\frac{\cosh[\sqrt{3\Lambda}\, t] - 1}{\cosh[\sqrt{3\Lambda}\, t_0] - 1} \right)^{1/3}$$

Our task: Integrate the velocity equation:

$$\frac{d\mathbf{x}}{dt} = \left\{ F \left[-\frac{a'}{\mathcal{E}} j(\mathcal{P})^2 + \frac{a'^2}{\mathcal{E}^2} (\mathcal{L} \cdot \mathcal{P})\, j(\mathcal{P}) \right] + \frac{1}{a\, s} (1+q)\, j(\mathcal{P}) \right\} \mathbf{x}$$
$$+ F \left[\mathcal{P} + \frac{a'}{\mathcal{E}} \mathcal{L} \times \mathcal{P} + \frac{a'^2}{\mathcal{E}^2} (\mathcal{L} \cdot \mathcal{P})\, \mathcal{L} \right] + \frac{1}{a\, s} (1+q)\, \mathcal{L},$$

with

$$F(t) := \frac{q(t)/a(t)}{\mathcal{E} \left[1 + \frac{s^2}{\mathcal{E}^2} a'(t)^2 + \frac{\mathcal{S}^2}{\mathcal{E}^2} \right]}.$$

Let us choose initial conditions at emission time t_e:

$$\mathbf{x}_e = 0, \qquad \mathbf{p}_e = \begin{pmatrix} \|\mathbf{p}_e\| \\ 0 \\ 0 \end{pmatrix}, \qquad \mathbf{s}_e = \begin{pmatrix} s \\ s_e^{\perp} \\ 0 \end{pmatrix}.$$

• Special solutions: straight lines

$$\mathbf{s}_e^{\perp} = 0 \quad \Rightarrow \quad \tilde{\mathbf{x}}(t) = \frac{\mathbf{p}_e}{\|\mathbf{p}_e\|} \int_{t_e}^{t} \frac{d\tau}{a(\tau)}, \quad \mathbf{p}(t) = \frac{a_e}{a(t)} \mathbf{p}_e, \quad \mathbf{s}(t) = s\, \frac{\mathbf{p}_e}{\|\mathbf{p}_e\|}$$

These are the null geodesics (spin is “enslaved”).

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• “Precessing” solutions:
Initial conditions $s_e^{\perp} = \hbar$ (Quantum Mechanics) and e.g. $\lambda_{\mathrm{Ly}_\alpha} = 8.72 \cdot 10^{-34}\,\mathrm{am}$,[1] $z = 2.4$. Then with $\Lambda = 3 \cdot 0.685/\mathrm{am}^2$ and $t_0 = 0.951\,\mathrm{as}$ the time of emission is $t_e = 0.188\,\mathrm{as}$.

For a more modest $\lambda = 1.2 \cdot 10^{-2}\,\mathrm{am}$, Runge & Kutta readily tell us:

- ⋆ $R(S)(S) > 0$.
- ⋆ The longitudinal offset of the trajectory from its companion null geodesic is

$$|x^1(t) - \tilde{x}^1(t)| = \mathcal{O}(\epsilon^2)\,, \quad \epsilon := s_e^{\perp}/\mathcal{E}\,.$$

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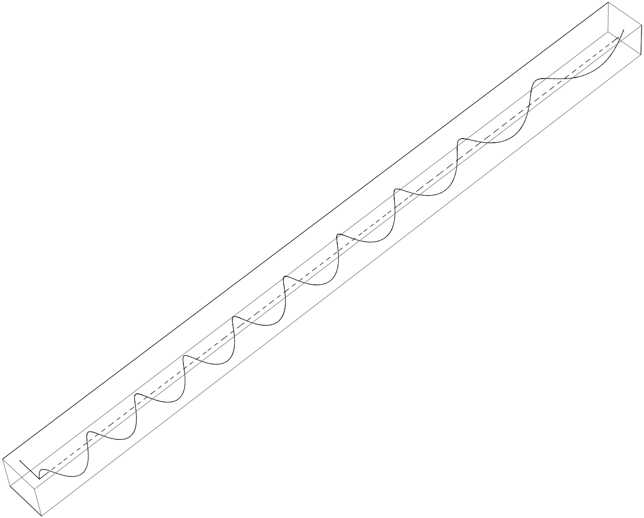

Figure: The trajectory of the photon, $\mathbf{x}(t)$, in comoving coordinates for $s_e^\perp = \hbar$ is the helix. The dashed line is the null geodesic ($s_e^\perp = 0$). The initial transverse spin $\mathbf{s}_e^\perp$ is indicated by the short arrow at the left.

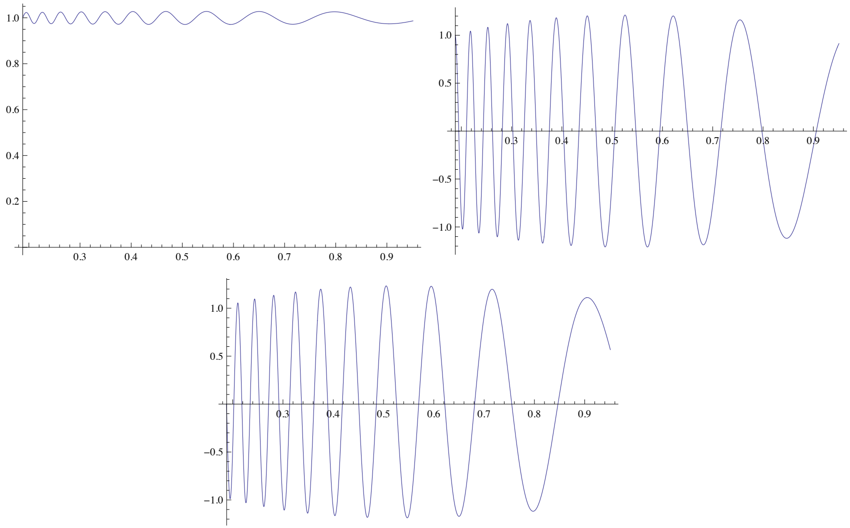

Figure: The three spin components $s^1(t)$, $s^2(t)$, and $s^3(t)$.

Perturbative solutions

We return to *generic*, flat RW spacetimes and *linearize* the equations of motion w.r.t. the small dimensionless parameters

$$\eta := \frac{s}{\mathcal{E}} \qquad \& \qquad \epsilon := \frac{s_e^{\perp}}{\mathcal{E}}.$$

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Put $(x^1, x^2, x^3) = (\tilde{x}^1, \epsilon y^2, \epsilon y^3) + \mathcal{O}(\epsilon^2)$ and linearize $d\mathbf{x}/dt$:

$$\frac{dx^1}{dt} \sim \frac{1}{a} + a_e' \, \frac{1+q}{a} \, y^2 \frac{\epsilon^2}{\eta} \, ,$$

$$\frac{dy^2}{dt} \sim \frac{1+q}{a} \left[1 - a_e' x^1 + y^3\right] \frac{1}{\eta} \, ,$$

$$\frac{dy^3}{dt} \sim - \; \frac{1+q}{a} \, y^2 \frac{1}{\eta}.$$

Birefringence

- Recall that $(x^1(t), 0, 0)$ is (up to second order terms) the null geodesic; with the change of time coordinate

$$t \mapsto \theta(t) \sim \frac{1}{|\eta|}\left[x^1(t) + \frac{1}{a'(t)} - \frac{1}{a'_e}\right]$$

the transverse trajectory is now governed by the equations

$$\frac{dy^2}{d\theta} \sim \mathrm{sign}(\eta)\,(y^3 + 1 - a'_e x^1), \qquad \frac{dy^3}{d\theta} \sim -\mathrm{sign}(\eta)\,y^2.$$

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- With the previous initial conditions and setting $\epsilon = |\eta|$, we obtain:

$$y^2(t) \sim \mathrm{sign}(\eta)\sin\theta(t) \qquad \& \qquad y^3(t) \sim \cos\theta(t) - 1 + a'_e x^1(t).$$

The trajectory is therefore a Left/Right helix depending on the helicity $\mathrm{sign}(\eta) = \mathrm{sign}(s)$ of the photon, i.e. birefringence of light.

Period, center and radius of the helix

- The instantaneous period of the helix in cosmic time is $T_{\rm helix}(t) \sim 2\pi\, dt/d\theta$,

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- Its center at time t is located in comoving coordinates at

$$\mathbf{x}_{\mathrm{center}}(t) \sim \begin{pmatrix} x^1(t) \\ 0 \\ -|\eta| \left(1 - a_e' x^1(t)\right) \end{pmatrix}.$$

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- Its comoving radius is time-independent and equal to $|\eta|$. Its true radius is

$$R_{\mathrm{helix}}(t) \sim \frac{a(t)}{a_e}\,\frac{c\,T_e}{2\pi} = \frac{z+1}{2\pi}\,\lambda_e,$$

λ_e being the wavelength at emission.

Conclusions and open questions

• The gravitational field of an expanding universe produces birefringence of light.

• This birefringence carries information on the acceleration of the universe.

• Can this birefringence of photons be measured?

• Does the gravitational field of a gravitational wave also produce birefringence of light?

• If yes, what information is carried by this birefringence?

• If yes, can this birefringence of photons be measured?

to the memory of Pierre Binétruy