Recent results on R^2 and related models of inflation

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Present status of inflation

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Conclusions

Inflation

The inflationary scenario is based on the two cornerstone independent ideas (hypothesis):

1. Existence of inflation (or, quasi-de Sitter stage) – a stage of accelerated, close to exponential expansion of our Universe in the past preceding the hot Big Bang with decelerated, power-law expansion.

2. The origin of all inhomogeneities in the present Universe is the effect of gravitational creation of particles and field fluctuations during inflation from the adiabatic vacuum (no-particle) state for Fourier modes covering all observable range of scales (and possibly somewhat beyond).

NB The latter effect requires breaking of the weak and null energy conditions for matter inhomogeneities.

Outcome of inflation

In the super-Hubble regime ($k \ll aH$) in the coordinate representation:

$$ds^2 = dt^2 - a^2(t)(\delta_{lm} + h_{lm})dx^l dx^m, \quad l, m = 1, 2, 3$$

$$h_{lm} = 2\zeta(\mathbf{r})\delta_{lm} + \sum_{a=1}^{2} g^{(a)}(\mathbf{r})\, e_{lm}^{(a)}$$

$$e_l^{l(a)} = 0, \;\; g^{(a)}_{\;\;,l}\, e_m^{l(a)} = 0, \;\; e_{lm}^{(a)}\, e^{lm(a)} = 1$$

ζ describes primordial scalar perturbations, g – primordial tensor perturbations (primordial gravitational waves (GW)).

The most important quantities:

$$n_s(k) - 1 \equiv \frac{d \ln P_\zeta(k)}{d \ln k}, \quad r(k) \equiv \frac{P_g}{P_\zeta}$$

In fact, metric perturbations h_{lm} are quantum (operators in the Heisenberg representation) and remain quantum up to the present time. But, after omitting of a very small part, decaying with time, they become commuting and, thus, equivalent to classical (c-number) stochastic quantities with the Gaussian statistics (up to small terms quadratic in $\zeta,\ g$).

In particular:

$$\hat{\zeta}_k = \zeta_k\, i(\hat{a}_{\mathbf{k}} - \hat{a}^\dagger_{\mathbf{k}}) + \mathcal{O}\left((\hat{a}_{\mathbf{k}} - \hat{a}^\dagger_{\mathbf{k}})^2\right) + ... + \mathcal{O}(10^{-100})(\hat{a}_{\mathbf{k}} + \hat{a}^\dagger_{\mathbf{k}}) + ,,,$$

The last term is time dependent, it is affected by physical decoherence and may become larger, but not as large as the second term.

Remaining quantum coherence: deterministic correlation between $\mathbf{k}$ and $-\mathbf{k}$ modes - shows itself in the appearance of acoustic oscillations (primordial oscillations in case of GW).

Present status of inflation

Now we have numbers: P. A. R. Ade et al., arXiv:1502.01589

The primordial spectrum of scalar perturbations has been measured and its deviation from the flat spectrum $n_s = 1$ in the first order in $|n_s - 1| \sim N^{-1}$ has been discovered (using the multipole range $\ell > 40$):

$$< \zeta^2(\mathbf{r}) >= \int \frac{P_\zeta(k)}{k}\, dk, \;\; P_\zeta(k) = \left(2.21^{+0.07}_{-0.08}\right) 10^{-9} \left(\frac{k}{k_0}\right)^{n_s - 1}$$

$$k_0 = 0.05 \mathrm{Mpc}^{-1}, \;\; n_s - 1 = -0.035 \pm 0.005$$

Two fundamental observational constants of cosmology in addition to the three known ones (baryon-to-photon ratio, baryon-to-matter density and the cosmological constant). Existing inflationary models can predict (and predicted, in fact) one of them, namely $n_s - 1$, relating it finally to $N_H = \ln \frac{k_B T_\gamma}{\hbar H_0} \approx 67.2$.

From "proving" inflation to using it as a tool

Present status of inflation: transition from "proving" it in general and testing some of its simplest models to applying the inflationary paradigm to investigate particle physics at super-high energies and the actual history of the Universe in the remote past using real observational data on $n_s(k) - 1$ and $r(k)$.

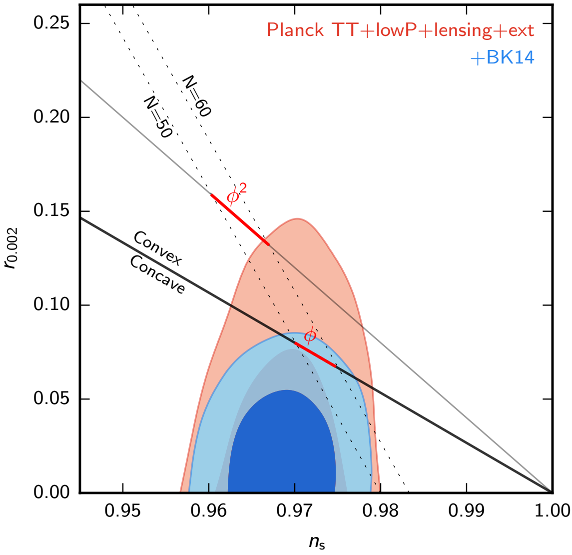
The reconstruction approach – determining curvature and inflaton potential from observational data.

The most important quantities:
1) for classical gravity – $H, \dot{H}$
2) for super-high energy particle physics – m^2_{infl}.

Simple (one-parameter, in particular) models may be good in the first approximation (indeed so), but it is difficult to expect them to be absolutely exact, small corrections due to new physics should exist (indeed so).

Combined results from Planck/BISEP2/Keck Array

P. A. R. Ade et al., Phys. Rev. Lett. 116, 031302 (2016)



Inflation in $f(R)$ gravity

Purely geometrical realization of inflation.

The simplest model of modified gravity (= geometrical dark energy) considered as a phenomenological macroscopic theory in the fully non-linear regime and non-perturbative regime.

$$S = \frac{1}{16\pi G}\int f(R)\sqrt{-g}\, d^4x + S_m$$

$$f(R) = R + F(R), \quad R \equiv R_\mu^\mu$$

Here $f''(R)$ is not identically zero. Usual matter described by the action S_m is minimally coupled to gravity.

Vacuum one-loop corrections depending on R only (not on its derivatives) are assumed to be included into $f(R)$. The normalization point: at laboratory values of R where the scalaron mass (see below) $m_s \approx \mathrm{const}$.

Metric variation is assumed everywhere.

Field equations

$$\frac{1}{8\pi G}\left(R^{\nu}_{\mu} - \frac{1}{2}\,\delta^{\nu}_{\mu}R\right) = -\left(T^{\nu}_{\mu\,(vis)} + T^{\nu}_{\mu\,(DM)} + T^{\nu}_{\mu\,(DE)}\right)\ ,$$

where $G = G_0 = const$ is the Newton gravitational constant measured in laboratory and the effective energy-momentum tensor of DE is

$$8\pi G T^{\nu}_{\mu\,(DE)} = F'(R)\,R^{\nu}_{\mu} - \frac{1}{2}\,F(R)\delta^{\nu}_{\mu} + \left(\nabla_{\mu}\nabla^{\nu} - \delta^{\nu}_{\mu}\nabla_{\gamma}\nabla^{\gamma}\right)F'(R)\,.$$

Because of the need to describe DE, de Sitter solutions in the absence of matter are of special interest. They are given by the roots $R = R_{dS}$ of the algebraic equation

$$Rf'(R) = 2f(R)\ .$$

The special role of $f(R) \propto R^2$ gravity: admits de Sitter solutions with any curvature.

Duality to the GR+scalar field dynamics

In the Einstein frame, free particles of usual matter do not follow geodesics and atomic clocks do not measure proper time.

From the Jordan (physical) frame to the Einstein one:

$$g^E_{\mu\nu} = f' g^J_{\mu\nu}, \quad \kappa\phi = \sqrt{\frac{3}{2}} \ln f', \quad V(\phi) = \frac{f'R - f}{2\kappa^2 f'^2}$$

where $\kappa^2 = 8\pi G$.

Inverse transformation:

$$R = \left(\sqrt{6}\kappa \frac{dV(\phi)}{d\phi} + 4\kappa^2 V(\phi)\right) \exp\left(\sqrt{\frac{2}{3}}\kappa\phi\right)$$

$$f(R) = \left(\sqrt{6}\kappa \frac{dV(\phi)}{d\phi} + 2\kappa^2 V(\phi)\right) \exp\left(2\sqrt{\frac{2}{3}}\kappa\phi\right)$$

$V(\phi)$ should be at least C^1.

Background FRW equations in $f(R)$ gravity

$$ds^2 = dt^2 - a^2(t)\left(dx^2 + dy^2 + dz^2\right)$$

$$H \equiv \frac{\dot{a}}{a}\ , \quad R = 6(\dot{H} + 2H^2)$$

The trace equation (4th order)

$$\frac{3}{a^3}\frac{d}{dt}\left(a^3\frac{df'(R)}{dt}\right) - Rf'(R) + 2f(R) = 8\pi G(\rho_m - 3p_m)$$

The 0-0 equation (3d order)

$$3H\frac{df'(R)}{dt} - 3(\dot{H} + H^2)f'(R) + \frac{f(R)}{2} = 8\pi G\rho_m$$

Reduction to the first order equation

In the absence of spatial curvature and $\rho_m = 0$, it is always possible to reduce these equations to a first order one using either the transformation to the Einstein frame and the Hamilton-Jacobi-like equation for a minimally coupled scalar field in a spatially flat FLRW metric, or by directly transforming the 0-0 equation to the equation for $R(H)$:

$$\frac{dR}{dH} = \frac{(R - 6H^2)f'(R) - f(R)}{H(R - 12H^2)f''(R)}$$

Analogues of large-field (chaotic) inflation: $F(R) \approx R^2 A(R)$ for $R \to \infty$ with $A(R)$ being a slowly varying function of R, namely

$$|A'(R)| \ll \frac{A(R)}{R}\ , \ |A''(R)| \ll \frac{A(R)}{R^2}\ .$$

Analogues of small-field (new) inflation, $R \approx R_1$:

$$F'(R_1) = \frac{2F(R_1)}{R_1}\ , \ F''(R_1) \approx \frac{2F(R_1)}{R_1^2}\ .$$

Thus, all inflationary models in $f(R)$ gravity are close to the simplest one over some range of R.

Perturbation spectra in slow-roll $f(R)$ inflationary models

Let $f(R) = R^2 A(R)$. In the slow-roll approximation $|\ddot{R}| \ll H|\dot{R}|$:

$$P_\zeta(k) = \frac{\kappa^2 A_k}{64\pi^2 A_k'^2 R_k^2}, \quad P_g(k) = \frac{\kappa^2}{12 A_k \pi^2}$$

$$N(k) = -\frac{3}{2}\int_{R_f}^{R_k} dR\, \frac{A}{A' R^2}$$

where the index k means that the quantity is taken at the moment $t = t_k$ of the Hubble radius crossing during inflation for each spatial Fourier mode $k = a(t_k)H(t_k)$.

NB The slow-roll approximation is not specific for inflation only. It was first used in A. A. Starobinsky, Sov. Astron. Lett. 4, 82 (1978) for a bouncing model (a scalar field with $V = \frac{m^2\phi^2}{2}$ in the closed FLRW universe).

The simplest models producing the observed scalar slope

$$f(R) = R + \frac{R^2}{6M^2}$$

$$M = 2.6 \times 10^{-6} \left(\frac{55}{N}\right) M_{Pl} \approx 3.2 \times 10^{13}\,\mathrm{GeV}$$

$$n_s - 1 = -\frac{2}{N} \approx -0.036, \quad r = \frac{12}{N^2} \approx 0.004$$

$$H_{dS}(N = 55) = 1.4 \times 10^{14}\,\mathrm{GeV}$$

The same prediction from a scalar field model with $V(\phi) = \frac{\lambda\phi^4}{4}$ at large ϕ and strong non-minimal coupling to gravity $\xi R\phi^2$ with $\xi < 0,\ |\xi| \gg 1$, including the Brout-Englert-Higgs inflationary model.

The Lagrangian density for the simplest 1-parametric model:

$$\mathcal{L} = \frac{R}{16\pi G} + \frac{N^2}{288\pi^2 P_\zeta(k)} R^2 = \frac{R}{16\pi G} + 5 \times 10^8 R^2$$

1. The specific case of the fourth order gravity in 4D

$$\mathcal{L} = \frac{R}{16\pi G} + AR^2 + BC_{\alpha\beta\gamma\delta}C^{\alpha\beta\gamma\delta} + (\text{small rad. corr.})$$

for which $A \gg 1,\ A \gg |B|$.

2. Another, completely different way: a non-minimally coupled scalar field with a large negative coupling ξ ($\xi_{conf} = \frac{1}{6}$):

$$L = \frac{R}{16\pi G} - \frac{\xi R\phi^2}{2} + \frac{1}{2}\phi_{,\mu}\phi^{,\mu} - V(\phi), \quad \xi < 0, \quad |\xi| \gg 1 .$$

In this limit, the Higgs-like scalar tree level potential $V(\phi) = \frac{\lambda(\phi^2-\phi_0^2)^2}{4}$ just produces $f(R) = \frac{1}{16\pi G}\left(R + \frac{R^2}{6M^2}\right)$ with $M^2 = \lambda/24\pi\xi^2 G$ and $\phi^2 = |\xi|R/\lambda$ (plus small corrections $\propto |\xi|^{-1}$).

Post-inflationary evolution in the $R + R^2$ model

First order equation:

$$x = H^{3/2}, \; y = \frac{1}{2} H^{-1/2} \dot{H}, \; dt = \frac{dx}{3x^{2/3}y}$$

$$\frac{dy}{dx} = -\frac{M^2}{12x^{1/3}y} - 1$$

The y-axis corresponds to inflection points $\dot{a} = \ddot{a} = 0, \dddot{a} \neq 0$. A curve reaching the y-axis at the point $(0, y_0 < 0)$ continues from the point $(0, -y_0)$ to the right.

Late-time asymptotic:

$$a(t) \propto t^{2/3} \left(1 + \frac{2}{3Mt} \sin M(t - t_1)\right), \; R \approx -\frac{8M}{3t} \sin M(t-t_1)$$

$$< R^2 >= \frac{32M^2}{9t^2}, \quad 8\pi G \rho_{s,eff} = \frac{3 < R^2 >}{8M^2} = \frac{4}{3t^2} \propto a^{-3}$$

Scalaron decay and creation of matter

Transition to the FLRWRD stage: occurs through the same mechanism which has been used for generation of perturbations: creation of particle-antiparticle pairs of all quantum matter fields by fast oscillations of R. Technically: one-loop quantum corrections from all matter quantum fields have to be added to the action of the $R + R^2$ gravity. In the particle interpretation: scalaron decays into particles and particles with the energy $E = M/2$.

Thus, the viable $f(R)$ inflationary model is (weakly) non-local already!

The most effective decay channel: into minimally coupled scalars with $m \ll M$. Then the formula obtained in Ya. B. Zeldovich and A. A. Starobinsky, JETP Lett. 26, 252 (1977) can be used:

$$\frac{1}{\sqrt{-g}}\frac{d}{dt}(\sqrt{-g}\,n_s) = \frac{R^2}{576\pi}$$

The corresponding (partial) decay rate is $\Gamma = \frac{GM^3}{24} \sim 10^{24}\ \mathrm{s}^{-1}$, that leads to the maximal temperature $T \approx 3 \times 10^9$ GeV at the beginning of the FLRWRD stage and to $N \approx 53$ for the reference scale in the CMB measurements ($k/a(t_0) = 0.05\ \mathrm{Mpc}^{-1}$), see D. S. Gorbunov, A. G. Panin, Phys. Lett. B 700, 157 (2011) and F. Bezrukov, D. Gorbunov, Phys. Lett. B 713, 365 (2012) for more details.

Slow-roll inflation reconstruction in $f(R)$ gravity

$$A = const - \frac{\kappa^2}{96\pi^2} \int \frac{dN}{P_\zeta(N)}$$

$$\ln R = const + \int dN \sqrt{-\frac{2\, d \ln A}{3\, dN}}$$

The additional "aesthetic" assumptions that $P_\zeta \propto N^\beta$ and that the resulting $f(R)$ can be analytically continued to the region of small R without introducing a new scale, and it has the linear (Einstein) behaviour there, leads to $\beta = 2$ and the $R + R^2$ inflationary model with $r = \frac{12}{N^2} = 3(n_s - 1)^2$ unambiguously.

For $P_\zeta = P_0 N^2$ ("scale-free reconstruction"):

$$A = \frac{1}{6M^2}\left(1 + \frac{N_0}{N}\right), \quad M^2 \equiv \frac{16\pi^2 N_0 P_\zeta}{\kappa^2}$$

Two cases:

1. $N \gg N_0$ always.

$$A = \frac{1}{6M^2}\left(1 + \left(\frac{R_0}{R}\right)^{\sqrt{3/(2N_0)}}\right)$$

For $N_0 = 3/2, \; R_0 = 6M^2$ we return to the simplest $R + R^2$ inflationary model.

2. $N_0 \gg 1$.

$$A = \frac{1}{6M^2}\left(\frac{1 + \left(\frac{R_0}{R}\right)^{\sqrt{3/(2N_0)}}}{1 - \left(\frac{R_0}{R}\right)^{\sqrt{3/(2N_0)}}}\right)^2$$

Constant-roll inflation in $f(R)$ gravity

Search for viable inflationary models outside the slow-roll approximation. Can be done in many ways. A simple and elegant generalization in GR:

$$\ddot{\phi} = \beta H \dot{\phi}\,, \;\; \beta = const$$

The required exact form of $V(\phi)$ for this was found in H. Motohashi, A. A. Starobinsky and J. Yokoyama, JCAP **1509**, 018 (2015).

Natural generalization to $f(R)$ gravity (H. Motohashi and A. A. Starobinsky, arXiv:1704.08188):

$$\frac{d^2 f'(R)}{dt^2} = \beta H \frac{df'(R)}{dt}\,, \;\; \beta = const$$

Then it follows from the field equations:

$$f'(R) \propto H^{2/(1-\beta)}$$

The exact solution for the required $f(R)$ in the parametric form ($\kappa = 1$):

$$f(R) = \frac{2}{3}(3-\beta)e^{2(2-\beta)\phi/\sqrt{6}}\left(3\gamma(\beta+1)e^{(\beta-3)\phi/\sqrt{6}} + (\beta+3)(1-\beta)\right)$$

$$R = \frac{2}{3}(3-\beta)e^{2(1-\beta)\phi/\sqrt{6}}\left(3\gamma(\beta-1)e^{(\beta-3)\phi/\sqrt{6}} + (\beta+3)(2-\beta)\right)$$

Viable inflationary models exist for $-0.1 \lesssim \beta < 0$.

Generality of inflation

Some myths regarding the onset of inflation:

1. Inflation begins with $V(\phi) \sim \dot{\phi}^2 \sim M_{Pl}^2$.
2. As a consequence, its formation is strongly suppressed in models with a plateau-type potentials favored by observations.
3. Beginning of inflation in some patch requires causal connection throughout the patch.
4. One of weaknesses of inflation is that it does not solve the singularity problem.

Theorem. In inflationary models in GR and $f(R)$ gravity, there exists an open set of classical solutions with a non-zero measure in the space of initial conditions at curvatures much exceeding those during inflation which have a metastable inflationary stage with a given number of e-folds.

For the GR inflationary model this follows from the generic late-time asymptotic solution for GR with a cosmological constant found in A. A. Starobinsky, JETP Lett. 37, 55 (1983). For the $R + R^2$ model, this was proved in A. A. Starobinsky and H.-J. Schmidt, Class. Quant. Grav. 4, 695 (1987). For the power-law and $f(R) = R^p$ inflation – in V. Müller, H.-J. Schmidt and A. A. Starobinsky, Class. Quant. Grav. 7, 1163 (1990).

Generic late-time asymptote of classical solutions of GR with a cosmological constant Λ both without and with hydrodynamic matter (also called the Fefferman-Graham expansion):

$$ds^2 = dt^2 - \gamma_{ik} dx^i dx^k$$

$$\gamma_{ik} = e^{2H_0 t} a_{ik} + b_{ik} + e^{-H_0 t} c_{ik} + \ldots$$

where $H_0^2 = \Lambda/3$ and the matrices a_{ik}, b_{ik}, c_{ik} are functions of spatial coordinates. a_{ik} contains two independent physical functions (after 3 spatial rotations and 1 shift in time + spatial dilatation) and can be made unimodular, in particular. b_{ik} is unambiguously defined through the 3-D Ricci tensor constructed from a_{ik}. c_{ik} contains a number of arbitrary physical functions (two - in the vacuum case, or with radiation).

The appearance of an inflating patch does not require that all parts of this patch should be causally connected at the beginning of inflation.

What was before inflation?

In classical gravity (GR or modified $f(R)$): generic space-like curvature singularity.

Generic initial conditions near a curvature singularity in modified gravity models (the $R + R^2$ and Higgs ones).: anisotropic and inhomogeneous (though quasi-homogeneous locally).

Two types singularities with the same structure at $t \to 0$:

$$ds^2 = dt^2 - \sum_{i=1}^{3} |t|^{2p_i}\, a_l^{(i)} a_m^{(i)}\, dx^l dx^m,\ 0 < s \le 3/2,\ u = s(2-s)$$

where $p_i < 1,\ s = \sum_i p_i,\ u = \sum_i p_i^2$ and $a_l^{(i)},\ p_i$ are functions of $\mathbf{r}$. Here $R^2 \ll R_{\alpha\beta}R^{\alpha\beta}$.

Type A. $1 \le s \le 3/2,\ R \propto |t|^{1-s} \to +\infty$

Type B. $0 < s < 1,\ R \to R_0 < 0,\ f'(R_0) = 0$

Spatial gradients may become important for some period before the beginning of inflation.

What is needed for beginning of inflation in classical (modified) gravity, is:
1) the existence of a sufficiently large compact expanding region of space with the Riemann curvature much exceeding that during the end of inflation ($\sim M^2$) – realized near a curvature singularity;
2) the average value $< R >$ over this region positive and much exceeding $\sim M^2$, too, – type A singularity;
3) the average spatial curvature over the region is either negative, or not too positive.

Recent numerical studies confirming this in GR: W. H. East, M. Kleban, A. Linde and L. Senatore, JCAP 1609, 010 (2016); M. Kleban and L. Senatore, JCAP 1610, 022 (2016).

On the other hand, causal connection is certainly needed to have a "graceful exit" from inflation, i.e. to have practically the same amount of the total number of e-folds during inflation N_{tot} in some sub-domain of this inflating patch.

Weakly non-local UV-complete gravity models

$R + R^2$ gravity interacting with quantum matter fields is renormalizable in the scalar sector and can be even asymptotically free. However, in the tensor sector a ghost appears due to the squared Weyl term.

To avoid it, a subclass of weakly non-local (quasi-polynomial) UV-complete quadratic in curvature generalizations of gravity is considered which do not have ghosts and are super-renormalizable (or even finite). Their action is:

$$S = \int d^4x\sqrt{-g}\left[\frac{M_P^2}{2}R + R\mathcal{F}(\Box)R + C_{\mu\nu\rho\sigma}\mathcal{F}_{\rm C}(\Box)C^{\mu\nu\rho\sigma}\right]$$

where $zF(z)$ and $zF_{\rm C}(z)$ are exponentials of entire functions up to constants.

R^2 inflation as a particular solution of non-local gravity

A. S. Koshelev, L. Modesto, L. Rachwal and A. A. Starobinsky, Occurrence of exact R^2 inflation in non-local UV-complete gravity, JHEP **1611**, 067 (2016); arXiv:1604.03127

For the $R + R^2$ model, $\Box R = M^2 R$. Thus, its solutions are also particular solutions of this non-local gravity if, in symbolic notation,

$$\mathcal{F}(M^2) = \frac{M_P^2}{12M^2}, \quad \mathcal{F}'(M^2) = 0$$

Spectrum of scalar perturbations: the same is in the local $R + R^2$ model. For proving it, the fact that these perturbations are conformally flat ($\Phi + \Psi = 0$) at the inflationary stage in the leading slow-roll approximation plays a crucial role.

Tensor perturbations are different. The absence of the tensor ghost requires:

$$1 + \frac{12M^2}{M_P^2}\left(\bar{\Box} - \frac{\bar{R}}{3}\right)\mathcal{F}_{\mathrm{C}}\left(\bar{\Box} + \frac{\bar{R}}{3}\right) = \exp(2\omega(\bar{\Box}))$$

where $\omega(z)$ is some entire function and the bar means a background solution. As a result:

$$r = \frac{12}{N^2}\exp\left(2\omega\left(\frac{\bar{R}}{6}\right)\right)$$

Conclusions

- First quantitative observational evidence for small quantities of the first order in the slow-roll parameters: $n_s(k) - 1$ and $r(k)$.
- The typical inflationary predictions that $|n_s - 1|$ is small and of the order of N_H^{-1}, and that r does not exceed $\sim 8(1 - n_s)$ are confirmed. Typical consequences following without assuming additional small parameters: $H_{55} \sim 10^{14}\,\mathrm{GeV},\ m_{infl} \sim 10^{13}\,\mathrm{GeV}$.
- Though the Einstein gravity plus a minimally coupled inflaton remains sufficient for description of inflation with existing observational data, modified (in particular, scalar-tensor or $f(R)$) gravity can do it as well.
- From the scalar power spectrum $P_\zeta(k)$, it is possible to reconstruct an inflationary model both in the Einstein and $f(R)$ gravity up to one arbitrary physical constant of integration.

- In the $f(R)$ gravity, the simplest $R + R^2$ model is one-parametric and has the preferred values $n_s - 1 = -\frac{2}{N}$ and $r = \frac{12}{N^2} = 3(n_s - 1)^2$. The first value produces the best fit to present observational CMB data.
- Even without using the observed value of $n_s - 1$, the assumptions of the absence of any new physical scale both during inflation and after it and of the model applicability up to the zero values of space-time curvature distinguish the case $P_\zeta(k) \propto \ln^2(k_f/k)$ and $R + R^2$ model unambiguously.
- Thus, it has sense to search for primordial GW from inflation at the level $r > 10^{-3}$!

- Inflation is generic in the $R + R^2$ inflationary model and close ones. Thus, its beginning does not require causal connection of all parts of an inflating patch of space-time (similar to spacelike singularities). However, graceful exit from inflation requires approximately the same number of e-folds during it for a sufficiently large compact set of geodesics. To achieve this, causal connection inside this set is necessary (though still may appear insufficient).
- The fact that inflation does not "solve" the singularity problem, i.e. it does not remove a curvature singularity preceding it, can be an advantage, not its weakness.
- Solutions of the $R + R^2$ inflationary model can also be particular solutions of some non-local UV-complete modifications of gravity without ghosts.