Cosmological Scenarios in Horndeski Theory

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Based on

- S. Sushkov, Phys.Rev. D80 (2009) 103505
- E. Saridakis, SVS, Phys.Rev. D81 (2010) 083510
- S. Sushkov, Phys.Rev. D85 (2012) 123520
- M. Skugoreva, SVS, A. Toporensky, Phys.Rev. D88 (2013) 083539
- J. Matsumoto, SVS, JCAP 1511, 047 (2015)
- A. Starobinsky, SVS, M. Volkov, JCAP 1606, 007 (2016)
- J. Matsumoto, SVS, arXiv:1703.04966

Collaborators:
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A. Toporensky, M. Volkov, A. Starobinsky

Plan of the talk

- Motivation
- Scalar fields in gravitational physics
- Scalar fields *minimally coupled* to gravity
- Scalar fields *nonminimally coupled* to gravity
- Scalar fields with *nonminimal derivative coupling*
- Horndeski model
- Cosmological models with nonminimal derivative coupling
  - No potential
  - Cosmological constant
  - Power-law potential
  - Higgs-like potential
  - Role of matter
- The screening Horndeski cosmologies
- Summary

Scalar fields in gravitational physics:

- gravitational potential in Newtonian gravity
- variation of "fundamental" constants
- Brans-Dicke theory initially elaborated to solve the Mach problem
- various compactification schemes
- the low-energy limit of the superstring theory
- scalar field as inflaton
- scalar field as dark energy and/or dark matter
- fundamental Higgs bosons, neutrinos, axions, . . .
- etc. . .

Scalar fields minimally coupled to gravity

$$S = \int d^4x\sqrt{-g}\,[L_{GR} + L_S]$$

L_{GR} – *gravitational Lagrangian*

general relativity; $L_{GR} = R$
square gravity; $L_{GR} = R + cR^2$
$f(R)$-theories; $L_{GR} = f(R)$
etc...

L_S – *scalar field Lagrangian*;

ordinary STT; $L_S = -\epsilon(\nabla\phi)^2 - 2V(\phi)$
$\epsilon = +1$ – **canonical** scalar field
$\epsilon = -1$ – **phantom** or **ghost** scalar field with negative kinetic energy
$V(\phi)$ – potential of self-action
K-essence; $L_s = K(X)\ [X = (\nabla\phi)^2]$
etc...

Scalar fields nonminimally coupled to gravity

Bergmann-Wagoner-Nordtvedt scalar-tensor theories

$$S = \int d^4x\sqrt{-g}\left[f(\phi)R - h(\phi)(\nabla\phi)^2 - 2V(\phi)\right]$$

$f(\phi)R \Longrightarrow$ nonminimal coupling between ϕ and R

Scalar fields nonminimally coupled to gravity

Bergmann-Wagoner-Nordtvedt scalar-tensor theories

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$f(\phi)R \Longrightarrow$ nonminimal coupling between ϕ and R

Conformal transformation to the Einstein frame *(Wagoner, 1970)*:

$$\tilde{g}_{\mu\nu} = f(\phi) g_{\mu\nu}; \quad \frac{d\phi}{d\psi} = f \left| fh + \frac{3}{2}\left(\frac{df}{d\phi}\right)^2 \right|^{-1/2}$$

$$S = \int d^4x \sqrt{-\tilde{g}} \left[\tilde{R} - \epsilon(\tilde{\nabla}\psi)^2 - 2U(\psi)\right]$$

$\psi \Longrightarrow$ new scalar field

$U(\psi) \Longrightarrow$ new effective potential

$$\epsilon = \operatorname{sign}\left[fh + \tfrac{3}{2}\left(\tfrac{df}{d\phi}\right)^2\right]$$

Scalar fields nonminimally coupled to gravity

General scalar-tensor theories

$$S = \int d^4x\sqrt{-g}\big[F(\phi, R) - (\nabla\phi)^2 - 2V(\phi)\big]$$

$F(\phi, R) \Longrightarrow$ generalized nonminimal coupling between ϕ and R

Scalar fields nonminimally coupled to gravity

General scalar-tensor theories

$$S = \int d^4x\sqrt{-g}\big[F(\phi, R) - (\nabla\phi)^2 - 2V(\phi)\big]$$

$F(\phi, R) \Longrightarrow$ generalized nonminimal coupling between ϕ and R

Conformal transformation to the Einstein frame *(Maeda, 1989)*:

$$\tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}; \qquad \frac{\Omega^2}{16\pi} \equiv \left|\frac{\partial F(\phi, R)}{\partial R}\right|$$

$$S = \int d^4x\sqrt{-\tilde{g}}\left[\frac{\tilde{R}}{16\pi} - h(\phi)\psi^{-1}(\tilde{\nabla}\phi)^2 - \frac{3}{32\pi}\psi^{-2}(\tilde{\nabla}\psi)^2 + U(\phi, \psi)\right]$$

$\psi \equiv \Omega^2 \Longrightarrow$ new (second!) scalar field

$U(\phi, \psi) \Longrightarrow$ new effective potential

Some remarks:

- A nonminimal scalar field is conformally equivalent to the minimal one possessing some effective potential $V(\phi)$
- A behavior of the scalar field is *"encoded"* in the potential $V(\phi)$
- The potential $V(\phi)$ is a very important ingredient of various cosmological models: a slowly varying potential behaves like an effective cosmological constat providing one or more than one inflationary phases.
  An appropriate choice of $V(\phi)$ is known as a problem of fine tuning of the cosmological constant.

Nonminimal derivative coupling generalization

$$S = \int d^4x\sqrt{-g}\big[R - g^{\mu\nu}\phi_{,\mu}\phi_{,\nu} - 2V(\phi)\big]$$

$F(\phi, R, R_{\mu\nu}, \dots)$

nonminimal coupling generalization!

$K(\phi_{,\mu}, \phi_{;\mu\nu}, \dots, R, R_{\mu\nu}, \dots)$

nonminimal derivative coupling generalization!

Nonminimal derivative coupling generalization

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$\swarrow$ $\searrow$

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Nonminimal derivative coupling generalization

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↙ ↘

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nonminimal coupling generalization!

$K(\phi_{,\mu}, \phi_{;\mu\nu}, \dots, R, R_{\mu\nu}, \dots)$

nonminimal derivative coupling generalization!

↘ ↙

Theories with higher derivatives!

Horndeski theory

In 1974, Horndeski derived the action of the most general scalar-tensor theories with second-order equations of motion
[G.Horndeski, *Second-Order Scalar-Tensor Field Equations in a Four-Dimensional Space*, IJTP **10**, 363 (1974)]

Horndeski Lagrangian:

$$L_{\rm H} = \sqrt{-g}\,(\mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_5)$$

$$\begin{aligned}
\mathcal{L}_2 &= G_2(X,\Phi)\,,\\
\mathcal{L}_3 &= G_3(X,\Phi)\,\Box\Phi\,,\\
\mathcal{L}_4 &= G_4(X,\Phi)\,R + \partial_X G_4(X,\Phi)\,\delta^{\mu\nu}_{\alpha\beta}\,\nabla^\alpha_\mu\Phi\nabla^\beta_\nu\Phi\,,\\
\mathcal{L}_5 &= G_5(X,\Phi)\,G_{\mu\nu}\nabla^{\mu\nu}\Phi - \tfrac{1}{6}\,\partial_X G_5(X,\Phi)\,\delta^{\mu\nu\rho}_{\alpha\beta\gamma}\,\nabla^\alpha_\mu\Phi\nabla^\beta_\nu\Phi\nabla^\gamma_\rho\Phi\,,
\end{aligned}$$

where $X = -\frac{1}{2}(\nabla\phi)^2$, and $G_k(X,\Phi)$ are arbitrary functions,
and $\delta^{\lambda\rho}_{\nu\alpha} = 2!\,\delta^\lambda_{[\nu}\delta^\rho_{\alpha]}$, $\delta^{\lambda\rho\sigma}_{\nu\alpha\beta} = 3!\,\delta^\lambda_{[\nu}\delta^\rho_\alpha\delta^\sigma_{\beta]}$

There is a special subclass of the theory, sometimes called Fab Four (F4), for which the coefficients are chosen such that the Lagrangian becomes

$$L_{\mathrm{F4}} = \sqrt{-g}\,(\mathcal{L}_J + \mathcal{L}_P + \mathcal{L}_G + \mathcal{L}_R - 2\Lambda)$$

with

$$\begin{aligned}
\mathcal{L}_J &= V_J(\Phi)\, G_{\mu\nu} \nabla^\mu \Phi \nabla^\nu \Phi\,, \\
\mathcal{L}_P &= V_P(\Phi)\, P_{\mu\nu\rho\sigma} \nabla^\mu \Phi \nabla^\rho \Phi \nabla^{\nu\sigma} \Phi\,, \\
\mathcal{L}_G &= V_G(\Phi)\, R\,, \\
\mathcal{L}_R &= V_R(\Phi)\, (R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta} - 4 R_{\mu\nu} R^{\mu\nu} + R^2).
\end{aligned}$$

Here the double dual of the Riemann tensor is

$$P^{\mu\nu}{}_{\alpha\beta} = -\frac{1}{4}\,\delta^{\mu\nu\gamma\delta}_{\sigma\lambda\alpha\beta}\, R^{\sigma\lambda}{}_{\gamma\delta} = -R^{\mu\nu}{}_{\alpha\beta} + 2R^\mu_{[\alpha}\delta^\nu_{\beta]} - 2R^\nu_{[\alpha}\delta^\mu_{\beta]} - R\delta^\mu_{[\alpha}\delta^\nu_{\beta]}\,,$$

whose contraction is the Einstein tensor, $P^{\mu\alpha}{}_{\nu\alpha} = G^\mu{}_\nu$.

Fab Four subclass of the Horndeski theory

Fab Four Lagrangian:

$$L_{\mathrm{F4}} = \sqrt{-g}\,(\mathcal{L}_J + \mathcal{L}_P + \mathcal{L}_G + \mathcal{L}_R - 2\Lambda)$$

- The Fab Four model is distinguished by the *screening property* – it is the most general subclass of the Horndeski theory in which flat space is a solution, despite the presence of the cosmological term Λ.
- This property suggests that Λ is actually irrelevant and hence there is no need to explain its value.
- Indeed, however large Λ is, Minkowski space is always a solution and so one may hope that a slowly accelerating universe will be a solution as well.

Theory with nonminimal kinetic coupling

Action:

$$S=\frac{1}{2}\int\left(M_{\rm Pl}^2 R-(\eta\, G_{\mu\nu}+\varepsilon\, g_{\mu\nu})\nabla^\mu\Phi\nabla^\nu\Phi-2V(\phi)\right)\sqrt{-g}\,d^4x+S_{\rm m}$$

Field equations:

$$M_{\rm Pl}^2 G_{\mu\nu}=\eta\,\mathcal{T}_{\mu\nu}+\epsilon\, T_{\mu\nu}^{(\Phi)}+T_{\mu\nu}^{(\rm m)}$$

$$[\epsilon g^{\mu\nu}+\eta G^{\mu\nu}]\nabla_\mu\nabla_\nu\phi=V'_\phi$$

$$T_{\mu\nu}^{(\Phi)}=\epsilon\nabla_\mu\phi\nabla_\nu\phi-\tfrac{1}{2}\epsilon g_{\mu\nu}(\nabla\phi)^2-g_{\mu\nu}V(\phi),$$

$$\begin{aligned}\mathcal{T}_{\mu\nu}=&-\tfrac{1}{2}\nabla_\mu\phi\,\nabla_\nu\phi\,R+2\nabla_\alpha\phi\,\nabla_{(\mu}\phi R^\alpha_{\nu)}-\tfrac{1}{2}(\nabla\phi)^2G_{\mu\nu}+\nabla^\alpha\phi\,\nabla^\beta\phi\,R_{\mu\alpha\nu\beta}\\&+\nabla_\mu\nabla^\alpha\phi\,\nabla_\nu\nabla_\alpha\phi-\nabla_\mu\nabla_\nu\phi\,\Box\phi+g_{\mu\nu}\big[-\tfrac{1}{2}\nabla^\alpha\nabla^\beta\phi\,\nabla_\alpha\nabla_\beta\phi+\tfrac{1}{2}(\Box\phi)^2\\&-\nabla_\alpha\phi\,\nabla_\beta\phi\,R^{\alpha\beta}\big]\end{aligned}$$

$$T_{\mu\nu}^{(\rm m)}=(\rho+p)U_\mu U_\mu+pg_{\mu\nu}\,,$$

Notice: *The field equations are of second order!*

Ansatz:

$$ds^2 = -dt^2 + a^2(t)d\mathbf{x}^2,$$
$$\phi = \phi(t)$$

$a(t)$ *cosmological factor*, $H = \dot{a}/a$ *Hubble parameter*

Field equations:

$$3M_{\rm Pl}^2 H^2 = \frac{1}{2}\dot{\phi}^2\left(\epsilon - 9\eta H^2\right) + V(\phi),$$
$$M_{\rm Pl}^2(2\dot{H} + 3H^2) = -\frac{1}{2}\dot{\phi}^2\left[\epsilon + \eta\left(2\dot{H} + 3H^2 + 4H\ddot{\phi}\dot{\phi}^{-1}\right)\right] + V(\phi),$$
$$\frac{d}{dt}\left[(\epsilon - 3\eta H^2)a^3\dot{\phi}\right] = -a^3\frac{dV(\phi)}{d\phi}$$

Cosmological models: General formulas

Ansatz:

$$ds^2 = -dt^2 + a^2(t)d\mathbf{x}^2,$$
$$\phi = \phi(t)$$

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$$\frac{d}{dt}\left[(\epsilon - 3\eta H^2)a^3\dot{\phi}\right] = -a^3\frac{dV(\phi)}{d\phi}$$

$$V(\phi) \equiv const \quad \Longrightarrow \quad \dot{\phi} = \frac{Q}{a^3(\epsilon - 3\eta H^2)}$$ Q is a scalar charge

Trivial model without kinetic coupling, i.e. $\eta = 0$

$$S = \int d^4x\sqrt{-g}\left[M_{\rm Pl}^2 R - (\nabla\phi)^2\right]$$

Trivial model without kinetic coupling, i.e. $\eta = 0$

$$S = \int d^4x\sqrt{-g}\left[M_{\rm Pl}^2 R - (\nabla\phi)^2\right]$$

Solution:

$$a_0(t) = t^{1/3}; \quad \phi_0(t) = \frac{1}{2\sqrt{3\pi}}\ln t$$

$$ds_0^2 = -dt^2 + t^{2/3}d\mathbf{x}^2$$

$t = 0$ *is an initial singularity*

Model without free kinetic term, i.e. $\epsilon = 0$

$$S = \int d^4x\sqrt{-g}\,\left[M_{\rm Pl}^2 R - \eta G^{\mu\nu}\phi_{,\mu}\phi_{,\nu}\right]$$

Model without free kinetic term, i.e. $\epsilon = 0$

$$S=\int d^4x\sqrt{-g}\left[M_{\rm Pl}^2R-\eta G^{\mu\nu}\phi_{,\mu}\phi_{,\nu}\right]$$

Solution:

$$a(t)=t^{2/3};\quad \phi(t)=\frac{t}{2\sqrt{3\pi|\eta|}},\quad \eta<0$$

$$ds_0^2=-dt^2+t^{4/3}d\mathbf{x}^2$$

$t=0$ *is an initial singularity*

Cosmological models: III. No potential $V(\phi) \equiv 0$

Model for an ordinary scalar field ($\epsilon = 1$) with nonminimal kinetic coupling $\eta \neq 0$

$$S = \int d^4x\sqrt{-g}\left\{M_{\rm Pl}^2 R - (g^{\mu\nu} + \eta G^{\mu\nu})\phi_{,\mu}\phi_{,\nu}\right\}$$

Model for an ordinary scalar field ($\epsilon = 1$) with nonminimal kinetic coupling $\eta \neq 0$

$$S = \int d^4x\sqrt{-g}\left\{M_{\rm Pl}^2 R - (g^{\mu\nu} + \eta G^{\mu\nu})\phi_{,\mu}\phi_{,\nu}\right\}$$

Asymptotic for $t \to \infty$:

$$a(t) \sim a_0(t) = t^{1/3}; \quad \phi(t) \sim \phi_0(t) = \frac{1}{2\sqrt{3\pi}} \ln t$$

Notice: *At large times the model with $\eta \neq 0$ has the same behavior like that with $\eta = 0$*

Asymptotics for early times

The case $\eta < 0$:

$$a_{t\to 0} \approx t^{2/3}; \quad \phi_{t\to 0} \approx \frac{t}{2\sqrt{3\pi|\eta|}}$$

$$ds^2_{t\to 0} = -dt^2 + t^{4/3}d\mathbf{x}^2$$

$t = 0$ *is an initial singularity*

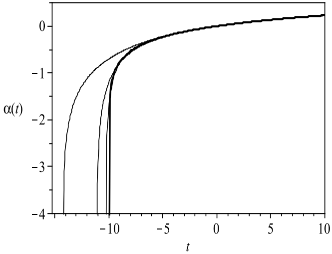
The case $\eta > 0$:

$$a_{t\to -\infty} \approx e^{H_\eta t}; \quad \phi_{t\to -\infty} \approx Ce^{-t/\sqrt{\eta}}$$

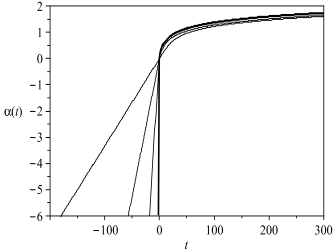
$$ds^2_{t\to -\infty} = -dt^2 + e^{2H_\eta t}d\mathbf{x}^2$$

de Sitter asymptotic with $H_\eta = 1/\sqrt{9\eta}$

Plots of $\alpha = \ln a$ in case $\eta \neq 0$, $\epsilon = 1$, $V = 0$.



(a) $\eta < 0$;
$\eta = 0; -1; -10; -100$



(b) $\eta > 0$;
$\eta = 0; 1; 10; 100$

De Sitter asymptotics: $\alpha(t) = \dfrac{t}{\sqrt{9\eta}} \quad \Rightarrow \quad H = \dfrac{1}{\sqrt{9\eta}}$

Notice: *In the model with nonmnimal kinetic coupling one get de Sitter phase without any potential!*

Models with the constant potential $V(\phi) = M_{\rm Pl}^2\Lambda = const$

$$S = \int d^4x\sqrt{-g}\left[M_{\rm Pl}^2(R - 2\Lambda) - [\epsilon g^{\mu\nu} + \eta G^{\mu\nu}]\phi_{,\mu}\phi_{,\nu}\right]$$

Models with the constant potential $V(\phi) = M_{\rm Pl}^2\Lambda = const$

$$S = \int d^4x\sqrt{-g}\,[M_{\rm Pl}^2(R - 2\Lambda) - [\epsilon g^{\mu\nu} + \eta G^{\mu\nu}]\phi_{,\mu}\phi_{,\nu}]$$

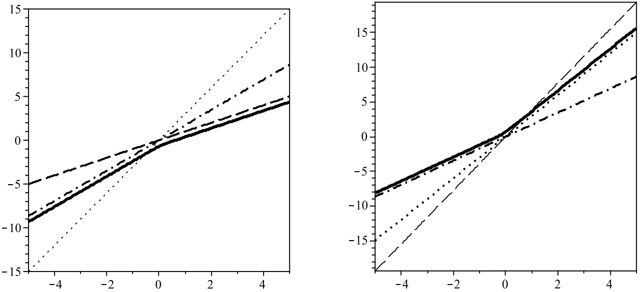
There are two exact de Sitter solutions:

I. $\alpha(t) = H_\Lambda t, \quad \phi(t) = \phi_0 = const,$

II. $\alpha(t) = \dfrac{t}{\sqrt{3|\eta|}}, \quad \phi(t) = M_{\rm Pl}\left|\dfrac{3\eta H_\Lambda^2 - 1}{\eta}\right|^{1/2} t,$

$$H_\Lambda = \sqrt{\Lambda/3}$$

Plots of $\alpha(t)$ in case $\eta > 0$, $\epsilon = 1$, $V = M_{\rm Pl}^2 \Lambda$

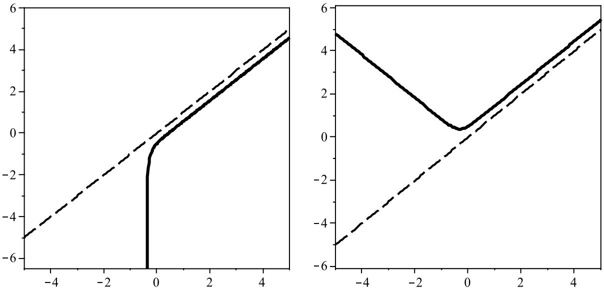


(a) $H_\Lambda^2 < \dot{\alpha}^2 < 1/9\eta$ (b) $1/9\eta < \dot{\alpha}^2 < 1/3\eta < H_\Lambda^2$

De Sitter asymptotics:
$\alpha_1(t) = H_\Lambda t$ *(dashed),*
$\alpha_2(t) = t/\sqrt{9\eta}$ *(dash-dotted),*
$\alpha_3(t) = t/\sqrt{3\eta}$ *(dotted).*

Cosmological models: IV. Cosmological constant

Plots of $\alpha(t)$ in cases $\eta > 0$, $\epsilon = -1$ and $\eta < 0$, $\epsilon = 1$



(a) $\eta < 0$, $\epsilon = 1$ (b) $\eta > 0$, $\epsilon = -1$

De Sitter asymptotic:
$\alpha_1(t) = H_\Lambda t$ *(dashed).*

$$S = \int d^4x\sqrt{-g}\left\{M_{\rm Pl}^2 R - [g^{\mu\nu} + \eta G^{\mu\nu}]\phi_{,\mu}\phi_{,\nu} - 2V(\phi)\right\}$$

↗

What a role does a potential play in cosmological models with the nonminimal kinetic coupling?

Models with the quadratic potential $V(\phi) = \frac{1}{2}m^2\phi^2$

Primary (early-time) "kinetic" inflation:

$$H_{t\to-\infty} \approx \frac{1}{\sqrt{9\eta}}(1 + \tfrac{1}{2}\eta m^2)$$

Late-time cosmological scenarios:

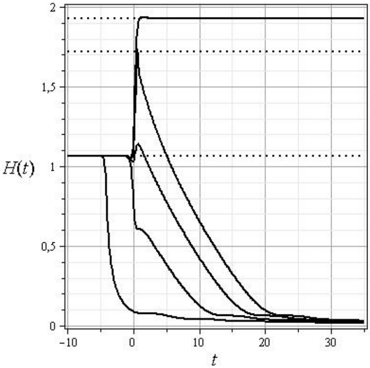
Oscillatory asymptotic or "graceful" exit from inflation

$$H_{t\to\infty} \approx \frac{2}{3t}\left[1 - \frac{\sin 2mt}{2mt}\right]$$

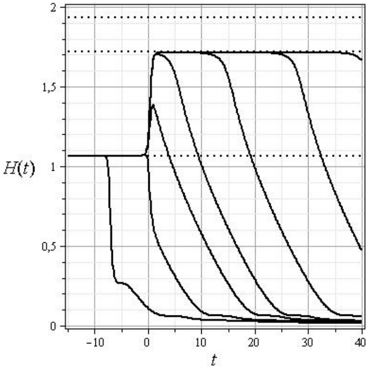
quasi-de Sitter asymptotic or secondary inflation

$$H_{t\to\infty} \approx \frac{1}{\sqrt{3\eta}}\left(1 \pm \sqrt{\tfrac{1}{6}\eta m^2}\right)$$

Cosmological models: Power-law potential



Initial conditions
$\phi_0 = \dot{\phi}_0$



Initial conditions
$\phi_0 = -\dot{\phi}_0$

De Sitter asymptotics: $H_{t\to-\infty} \approx 1/\sqrt{9\eta}(1 + \frac{1}{2}\eta m^2)$,

$$H_{t\to\infty} \approx 1/\sqrt{3\eta}\left(1 \pm \sqrt{\tfrac{1}{6}\eta m^2}\right).$$

Higgs potential $V(\phi) = \frac{\lambda}{4}(\phi^2 - \phi_0^2)^2$

Matsumoto, Sushkov, JCAP2015

Higgs field: $\lambda \simeq 0.14, \quad \phi_0 \simeq 246$ GeV

Field equations

$$H^2 = \frac{\frac{1}{2}\dot{\phi}^2 + V(\phi)}{3(M_{\rm Pl}^2 + \frac{3}{2}\eta\dot{\phi}^2)}$$

$$\begin{aligned}\ddot{\phi} =& [1 + 12\pi\eta\dot{\phi}^2 + 96\pi^2\eta^2\dot{\phi}^4 + 8\pi\eta V(\phi)(12\pi\eta\dot{\phi}^2 - 1)]^{-1} \\ &\times \Big\{ - 2\sqrt{3\pi}\dot{\phi}[1 + 8\pi\eta\dot{\phi}^2 - 8\pi\eta V(\phi)] \\ &\sqrt{[\dot{\phi}^2 + 2V(\phi)](12\pi\eta\dot{\phi}^2 + 1)} - (12\pi\eta\dot{\phi}^2 + 1)(4\pi\eta\dot{\phi}^2 + 1)V_\phi \Big\}\end{aligned}$$

Higgs potential: Dynamical system

Dimensionless variables and parameters:

$$x = \frac{\phi}{\phi_0}, \quad y = \sqrt{8\pi G\eta}\dot{\phi}, \quad \tau = \phi_0 t, \quad V_0 = 2\pi G\eta\lambda\phi_0^4, \quad \gamma \equiv G\phi_0^2$$

Autonomous dynamical system:

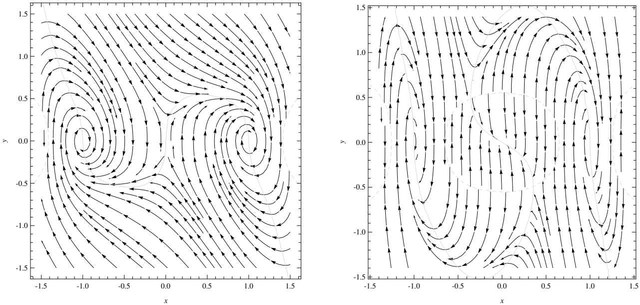
$$\begin{aligned}
\frac{dx}{d\tau} &= \sqrt{\frac{\lambda}{4V_0}}\, y, \\
\frac{dy}{d\tau} &= \frac{1}{\Delta}\Big\{ -\sqrt{3\pi\gamma\lambda V_0^{-1}}\, y \left[1 + y^2 - V_0(x^2-1)^2\right] \\
&\quad \times \sqrt{\left[y^2 + 2V_0(x^2-1)^2\right]\left(\tfrac{3}{2}y^2 + 1\right)} \\
&\quad - 2\sqrt{\lambda V_0}\left(\tfrac{3}{2}y^2 + 1\right)\left(\tfrac{1}{2}y^2 + 1\right) x(x^2-1) \Big\},
\end{aligned}$$

where $\Delta = 1 + \frac{3}{2}y^2 + \frac{3}{2}y^4 + V_0(x^2-1)^2\left(\frac{3}{2}y^2 - 1\right)$.

Stationary points:

$(\pm 1, 0)$	global minima of $V(\phi)$;	$\phi = \pm\phi_0,\ V(\pm\phi_0) = 0$
$(0, 0)$	local maximum of $V(\phi)$;	$\phi = 0,\ V(0) = V_0 = \frac{\lambda}{4}\phi_0^4$
$(\pm\infty, 0)$	"wings" of $V(\phi)$;	$\phi \to \pm\infty$

Phase diagrams:



$$\frac{1}{4}\eta\lambda\phi_0^4 \le M_{\rm Pl}^2$$

$(0, 0)$ – saddle point

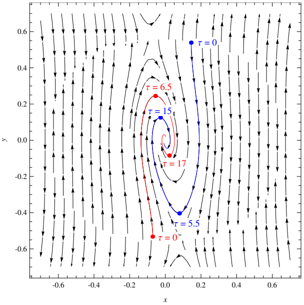
$$\frac{1}{4}\eta\lambda\phi_0^4 > M_{\rm Pl}^2$$

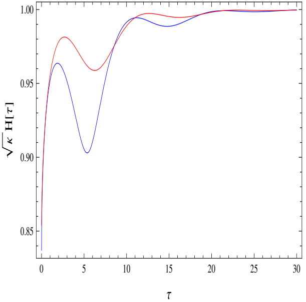
$(0, 0)$ – stable node!

Accelerated cosmological scenarios: Quasi-de Sitter

Quasi-de Sitter scenario

$t \to \infty$ distant future asymptotic





$$\phi \to 0, \quad \dot{\phi} \to 0$$

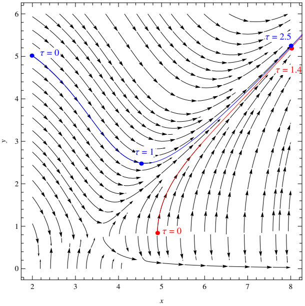
$V(\phi) \to V_0 = \lambda\phi_0^4/4$ (maximum)

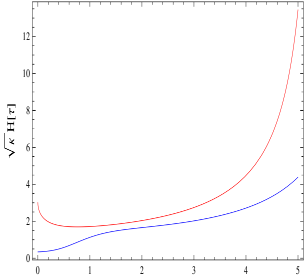
$H(t) \to H_\infty = \sqrt{\frac{2}{3}\pi G\lambda\phi_0^4}$=const

Accelerated cosmological scenarios: Big Rip

Big Rip scenario

$t \to t_*$ finite time asymptotic





$$\phi(t) \simeq \sqrt{\frac{392\eta}{\lambda}} \frac{1}{(t_* - t)^2}$$

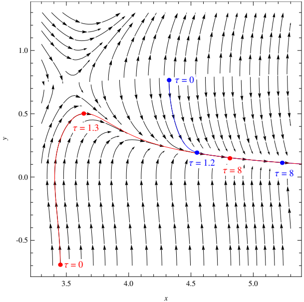
$$\phi \to \infty, \quad \dot{\phi} \to \infty$$

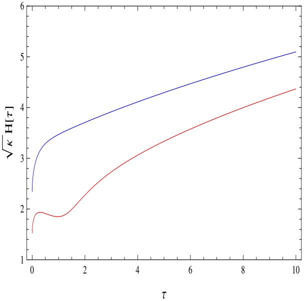
$$V(\phi) \simeq \lambda\phi^4/4 \to \infty$$

$$H^2(t) \simeq \frac{49}{9} \frac{1}{(t_* - t)^2} \to \infty$$

Little Rip scenario

$t \to \infty$ distant future asymptotic





$$\phi(t) \simeq (2/3\pi^3 G^3 \lambda \eta^2)^{1/8}\, t^{1/4}$$

$$\phi \to \infty, \quad \dot{\phi} \to 0$$

$$V(\phi) \simeq \lambda \phi^4/4 \to \infty$$

$$H(t) \simeq (8\lambda/27\pi G \eta^2)^{1/4}\, t^{1/2} \to \infty$$

Cosmological scenarios with nonminimal kinetic coupling and matter Sushkov, PRD 85 (2012) 123520

Role of matter?

$$S=\int d^4x\sqrt{-g}\left\{M_{\rm Pl}^2(R-2\Lambda)-[g^{\mu\nu}+\eta G^{\mu\nu}]\phi_{,\mu}\phi_{,\nu}\right\}+S_{matter}$$

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Stress-energy tensor: $T_{\mu\nu}^{(m)} = \mathrm{diag}(\rho, p, p, p)$
Field equations:

$$3M_{\rm Pl}^2H^2 = \frac{1}{2}\dot\phi^2\left(1-9\eta H^2\right) + M_{\rm Pl}^2\Lambda + \rho,$$

$$M_{\rm Pl}^2(2\dot H + 3H^2) = -\frac{1}{2}\dot\phi^2\left[1+\eta\left(2\dot H + 3H^2 + 4H\ddot\phi\dot\phi^{-1}\right)\right] + M_{\rm Pl}^2\Lambda - p$$

$$\frac{d}{dt}\left[(1-3\eta H^2)a^3\dot\phi\right] = 0$$

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$$\frac{d}{dt}[(1-3\eta H^2)a^3\dot\phi] = 0 \quad \Longrightarrow \quad \dot\phi = \frac{Q}{a^3(1-3\eta H^2)}$$

Cosmological scenarios with nonminimal kinetic coupling and matter

Modified Friedmann equation:

$$H^2 = H_0^2\left[\Omega_{\Lambda 0} + \frac{\Omega_{m0}}{a^3} + \frac{\Omega_{\phi 0}(1-9\eta H^2)}{a^6(1-3\eta H^2)^2}\right]$$

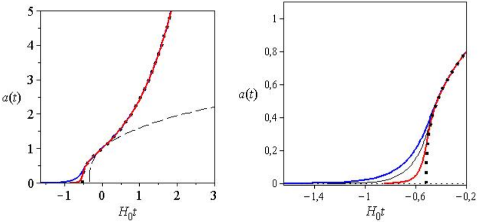
Constraint for parameters:

$$\Omega_{\Lambda 0} + \Omega_{m0} + \frac{\Omega_{\phi 0}(1-9\eta H_0^2)}{(1-3\eta H_0^2)^2} = 1$$

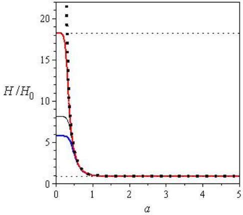
Universal asymptotic:

$$H \to H_\eta = 1/\sqrt{9\eta} \quad \text{at} \quad a \to 0$$

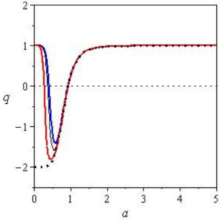
Notice: *The asymptotic $H \approx H_\eta$ at early cosmological times* is only *determined by the coupling parameter η and does* not *depend on other parameters!*



Scale factor $a(t)$



Hubble parameter $H(a)$



Acceleration parameter q

Cosmological scenarios: Estimations

$H_\eta t_f \sim 60$ e-folds
$t_f \simeq 10^{-35}$ sec the end of initial inflationary stage

$\Rightarrow H_\eta = 1/\sqrt{9\eta} \simeq 6 \times 10^{36}$ sec^{-1}

$$\eta \simeq 10^{-74} \text{ sec}^2 \quad \text{or} \quad l_\eta = \eta^{1/2} \simeq 10^{-27} \text{ cm}$$

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$H_0 \sim 70\ (\mathrm{km/sec})/\mathrm{Mpc} \sim 10^{-18}$ sec^{-1} Present Hubble parameter

$\gamma = 3\eta H_0^2 \simeq 10^{-109}$ Extremely small!

$$H^2 = H_0^2 \left[\Omega_{\Lambda 0} + \frac{\Omega_{m0}}{a^3} + \frac{\Omega_{\phi 0}(1 - 9\eta H^2)}{a^6(1 - 3\eta H^2)^2} \right] \Rightarrow \Omega_{\Lambda 0} + \Omega_{m0} + \Omega_{\phi 0} \approx 1$$

$$\Omega_{\Lambda 0} = 0.73, \Omega_{\phi 0} = 0.23, \Omega_{m0} = 0.04 \quad \Rightarrow \quad q_0 = 0.25$$

The FLRW ansatz for the metric:

$$ds^2 = -dt^2 + \mathrm{a}^2(t)\left[\frac{dr^2}{1-Kr^2} + r^2(d\vartheta^2 + \sin^2\vartheta d\varphi^2)\right],$$

$\mathrm{a}(t)$ *cosmological factor*, $H = \dot{\mathrm{a}}/\mathrm{a}$ *Hubble parameter*

Gravitational equations:

$$-3M_{\rm Pl}^2\left(H^2+\frac{K}{\mathrm{a}^2}\right)+\frac{1}{2}\varepsilon\,\psi^2-\frac{3}{2}\eta\,\psi^2\left(3H^2+\frac{K}{\mathrm{a}^2}\right)+\Lambda+\rho=0,$$

$$-M_{\rm Pl}^2\left(2\dot{H}+3H^2+\frac{K}{\mathrm{a}^2}\right)-\frac{1}{2}\varepsilon\,\psi^2-\eta\,\psi^2\left(\dot{H}+\frac{3}{2}H^2-\frac{K}{\mathrm{a}^2}+2H\frac{\dot{\psi}}{\psi}\right)+\Lambda-p=0.$$

The scalar field equation:

$$\frac{1}{\mathrm{a}^3}\frac{d}{dt}\left(\mathrm{a}^3\left(3\eta\left(H^2+\frac{K}{\mathrm{a}^2}\right)-\varepsilon\right)\psi\right)=0,$$

where $\psi = \dot{\phi}$, and $\phi = \phi(t)$ is a homogeneous scalar field

Screening properties of Horndeski model

The first integral of the scalar field equation:

$$\mathrm{a}^3 \left(3\eta \left(H^2 + \frac{K}{\mathrm{a}^2} \right) - \varepsilon \right) \psi = Q,$$

where Q is the Noether charge associated with the shift symmetry $\phi \to \phi + \phi_0$.

Let $Q = 0$. One finds in this case two different solutions:

GR branch: $\psi = 0 \quad \Longrightarrow \quad H^2 + \frac{K}{\mathrm{a}^2} = \frac{\Lambda + \rho}{3M_{\mathrm{Pl}}^2}$

Screening branch: $H^2 + \frac{K}{\mathrm{a}^2} = \frac{\varepsilon}{3\eta} \quad \Longrightarrow \quad \psi^2 = \frac{\eta\,(\Lambda + \rho) - \varepsilon\, M_{\mathrm{Pl}}^2}{\eta\ (\varepsilon - 3\eta\, K/\mathrm{a}^2)}$

NOTICE: The role of the cosmological constant in the screening solution is played by $\varepsilon/3\eta$ while the Λ-term is screened and makes no contribution to the universe acceleration.

Note also that the matter density ρ is screened in the same sense.

Screening properties of Horndeski model

Let $Q \neq 0$, then

$$\psi = \frac{Q}{\mathrm{a}^3 \left[3\eta \left(H^2 + \frac{K}{\mathrm{a}^2}\right) - \varepsilon\right]},$$

and the modified Friedmann equation reads

$$3M_{\mathrm{Pl}}^2 \left(H^2 + \frac{K}{\mathrm{a}^2}\right) = \frac{Q^2 \left[\varepsilon - 3\eta \left(3H^2 + \frac{K}{\mathrm{a}^2}\right)\right]}{2a^6 \left[\varepsilon - 3\eta \left(H^2 + \frac{K}{\mathrm{a}^2}\right)\right]^2} + \Lambda + \rho.$$

Introducing dimensionless values and density parameters

$$H^2 = H_0^2\, y, \quad \mathrm{a} = \mathrm{a}_0\, a\,, \quad \rho_{\mathrm{cr}} = 3M_{\mathrm{Pl}}^2 H_0^2\,, \quad \zeta = \frac{\varepsilon}{3\eta\, H_0^2}\,,$$

$$\Omega_0 = \frac{\Lambda}{\rho_{\mathrm{cr}}}, \; \Omega_2 = -\frac{K}{H_0^2 \mathrm{a}_0^2}, \; \Omega_6 = \frac{Q^2}{6\eta\, \mathrm{a}_0^6\, H_0^2\, \rho_{\mathrm{cr}}}, \; \rho = \rho_{\mathrm{cr}} \left(\frac{\Omega_4}{a^4} + \frac{\Omega_3}{a^3}\right)$$

gives

the master equation:

$$y = \Omega_0 + \frac{\Omega_2}{a^2} + \frac{\Omega_3}{a^3} + \frac{\Omega_4}{a^4} + \frac{\Omega_6 \left[\zeta - 3y + \frac{\Omega_2}{a^2}\right]}{a^6 \left[\zeta - y + \frac{\Omega_2}{a^2}\right]^2}$$

GR branch:

$$y = \Omega_0 + \frac{\Omega_2}{a^2} + \frac{\Omega_3}{a^3} + \frac{\Omega_4}{a^4} + \frac{(\zeta - 3\,\Omega_0)\,\Omega_6}{(\,\Omega_0 - \zeta)^2\,a^6} + \mathcal{O}\left(\frac{1}{a^7}\right) \Longrightarrow \quad H^2 \to \Lambda/3$$

Notice: The GR solution is stable (no ghost) if and only if $\zeta > \Omega_0$.

Screening branches:

$$y_\pm = \zeta + \frac{\Omega_2}{a^2} \pm \frac{\chi}{(\,\Omega_0 - \zeta)\,a^3} \pm \frac{\Omega_2\Omega_6}{\chi\,a^5} - \frac{\Omega_6(\zeta - 3\Omega_0) \pm \Omega_3\chi}{2(\Omega_0 - \zeta)^2\,a^6} + \mathcal{O}\left(\frac{1}{a^7}\right)$$

$$\Longrightarrow \quad H^2 \to \varepsilon/3\alpha$$

Notice: The screening solutions are stable (no ghost) if and only if $0 < \zeta < \Omega_0$.

GR branch:

$$y = \frac{\Omega_4}{a^4} + \frac{\Omega_3}{a^3} + \frac{\Omega_2\Omega_4 - 3\Omega_6}{\Omega_4 a^2} + \frac{3\Omega_3\Omega_6}{\Omega_4 a} + \mathcal{O}(1)$$

Notice: The GR solution is unstable

Screening branch:

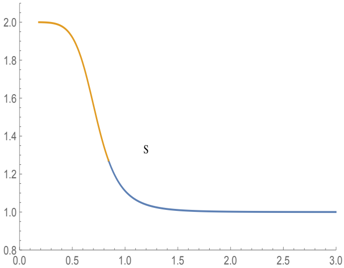
$$y_+ = \frac{3\Omega_6}{\Omega_4\, a^2} - \frac{3\Omega_3\Omega_6}{\Omega_4^2\, a} + \frac{5}{3}\zeta + \frac{3\Omega_6\Omega_3^2 + 9\Omega_6^2}{\Omega_4^3} + \mathcal{O}(a),$$

$$y_- = \frac{\zeta}{3} + \frac{4\,\zeta^2}{27\;\Omega_6}\left(\Omega_4\, a^2 + \Omega_3\, a^3\right) + \mathcal{O}(a^4)$$

Notice: Both screening solutions are stable

Global behavior

$$y = \Omega_0 + \frac{\Omega_2}{a^2} + \frac{\Omega_3}{a^3} + \frac{\Omega_4}{a^4} + \frac{\Omega_6\left[\zeta - 3y + \frac{\Omega_2}{a^2}\right]}{a^6\left[\zeta - y + \frac{\Omega_2}{a^2}\right]^2}$$



Solutions $y(a)$ for $\Omega_0 = \Omega_6 = 1$, $\Omega_2 = 0$, $\Omega_3 = \Omega_4 = 0$ and for $\zeta = 6$

Global behavior

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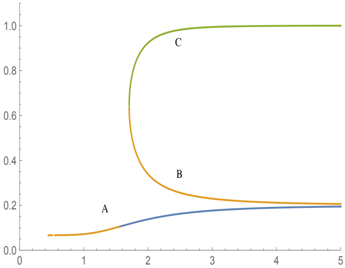
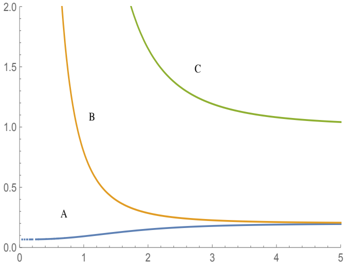


Solutions $y(a)$ for $\Omega_0 = \Omega_6 = 1,\ \Omega_2 = 0,\ \Omega_3 = \Omega_4 = 0,\ \zeta = 0.2$

Global behavior

$$y = \Omega_0 + \frac{\Omega_2}{a^2} + \frac{\Omega_3}{a^3} + \frac{\Omega_4}{a^4} + \frac{\Omega_6\left[\zeta - 3y + \frac{\Omega_2}{a^2}\right]}{a^6\left[\zeta - y + \frac{\Omega_2}{a^2}\right]^2}$$



Solutions $y(a)$ for $\Omega_0 = \Omega_6 = 1$, $\Omega_3 = 5$, $\Omega_4 = 0$, $\zeta = 0.2$. One has $\Omega_2 = 0$.

Summary. I.

- The nonminimal kinetic coupling provides an *essentially new* inflationary mechanism which does not need any fine-tuned potential.
- At early cosmological times the coupling κ-terms in the field equations are dominating and provide the quasi-De Sitter behavior of the scale factor: $a(t) \propto e^{H_\kappa t}$ with $H_\kappa = 1/\sqrt{9\kappa}$ and $\kappa \simeq 10^{-74}$ sec^2 (or $l_\kappa \equiv \kappa^{1/2} \simeq 10^{-27}$ cm)
- The model provides a natural mechanism of epoch change without any fine-tuned potential.
- The nonminimal kinetic coupling crucially changes a role of the scalar potential. Power-law and Higgs-like potentials with kinetic coupling provide accelerated regimes of the Universe evolution.

Summary. II.

- The theory with nonminimal kinetic coupling admits various cosmological solutions.
- Ghost-free solutions exist if $\eta \geq 0$ and $\varepsilon \geq 0$.
- The no-ghost conditions eliminate many solutions, as for example the bounces or the "emerging time" solutions.
- For $\zeta > \Omega_0$ there exists a ghost-free solution. It describes a universe with the standard late time dynamic dominated by the Λ-term, radiation and dust. At early times the matter effects are totally screened and the universe expands with a constant Hubble rate determined by ε/η. Since it contains two independent parameters ζ and $\Omega_0 \sim \Lambda$ in the asymptotics, this solution can have an hierarchy between the Hubble scales at the early and late times. However, at late times it is not screening and dominated by Λ, thus invoking again the cosmological constant problem.

Conclusions

- For $0 < \zeta < \Omega_0$ there exist two ghost-free solutions, A and B. The solution A is sourced by the scalar field, with or without the matter, while the solution B exists only when the matter is present. They both show the screening because their late time behaviour is controlled by $\zeta \sim \varepsilon/\eta$ and not by Λ. Therefore, they could in principle describe the late time acceleration while circumventing the cosmological constant problem, and one might probably find arguments justifying that ε/η should be small. At the same time, these solutions cannot describe the early inflationary phase. Indeed, the near singularity behaviour of the solution B does not correspond to inflation, while the solution A does show an inflationary phase, but with essentially the same Hubble rate as at late times, hence there is no hierarchy between the two Hubble scales.

THANKS FOR YOUR ATTENTION!