Non-local Modifications of Gravity and Cosmic Acceleration

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arXiv:1702.08908

Late time cosmology

- The Universe is expanding with acceleration.
- Possible explanation: vacuum energy

  → Technically unnatural

- Modify General Relativity

  → Interesting not only for cosmology.

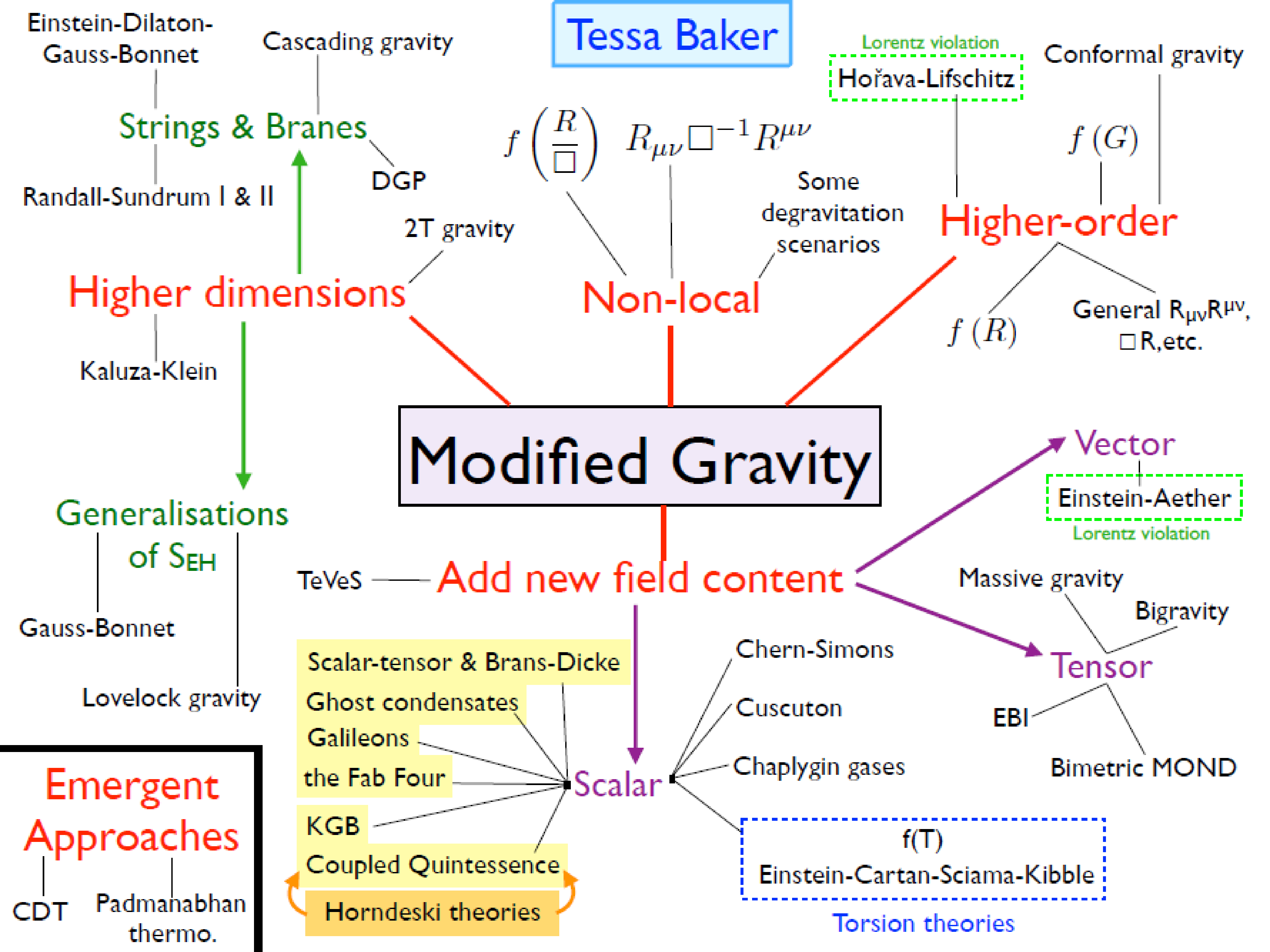
Many consistent ways to modify!

Modify the Gravity!

- Gravity at the cosmological scales is not fully tested!
- Solar System
- Merging Black Holes

- From Solar System to cosmological scales many orders of magnitude.

GR should be tested at cosmological scales!


Tessa Baker
Modified Gravity
Higher dimensions
Strings & Branes
Einstein-Dilaton-Gauss-Bonnet
Cascading gravity
DGP
Randall-Sundrum I & II
2T gravity
Kaluza-Klein
Generalisations of S_{EH}
Gauss-Bonnet
Lovelock gravity
Non-local
$f\left(\frac{R}{\Box}\right)$
$R_{\mu\nu}\Box^{-1}R^{\mu\nu}$
Some degravitation scenarios
Higher-order
Lorentz violation
Hořava-Lifschitz
Conformal gravity
$f(G)$
$f(R)$
General $R_{\mu\nu}R^{\mu\nu}$, $\Box R$, etc.
Add new field content
TeVeS
Vector
Einstein-Aether
Lorentz violation
Massive gravity
Bigravity
Tensor
EBI
Bimetric MOND
Scalar
Scalar-tensor & Brans-Dicke
Ghost condensates
Galileons
the Fab Four
KGB
Coupled Quintessence
Horndeski theories
Chern-Simons
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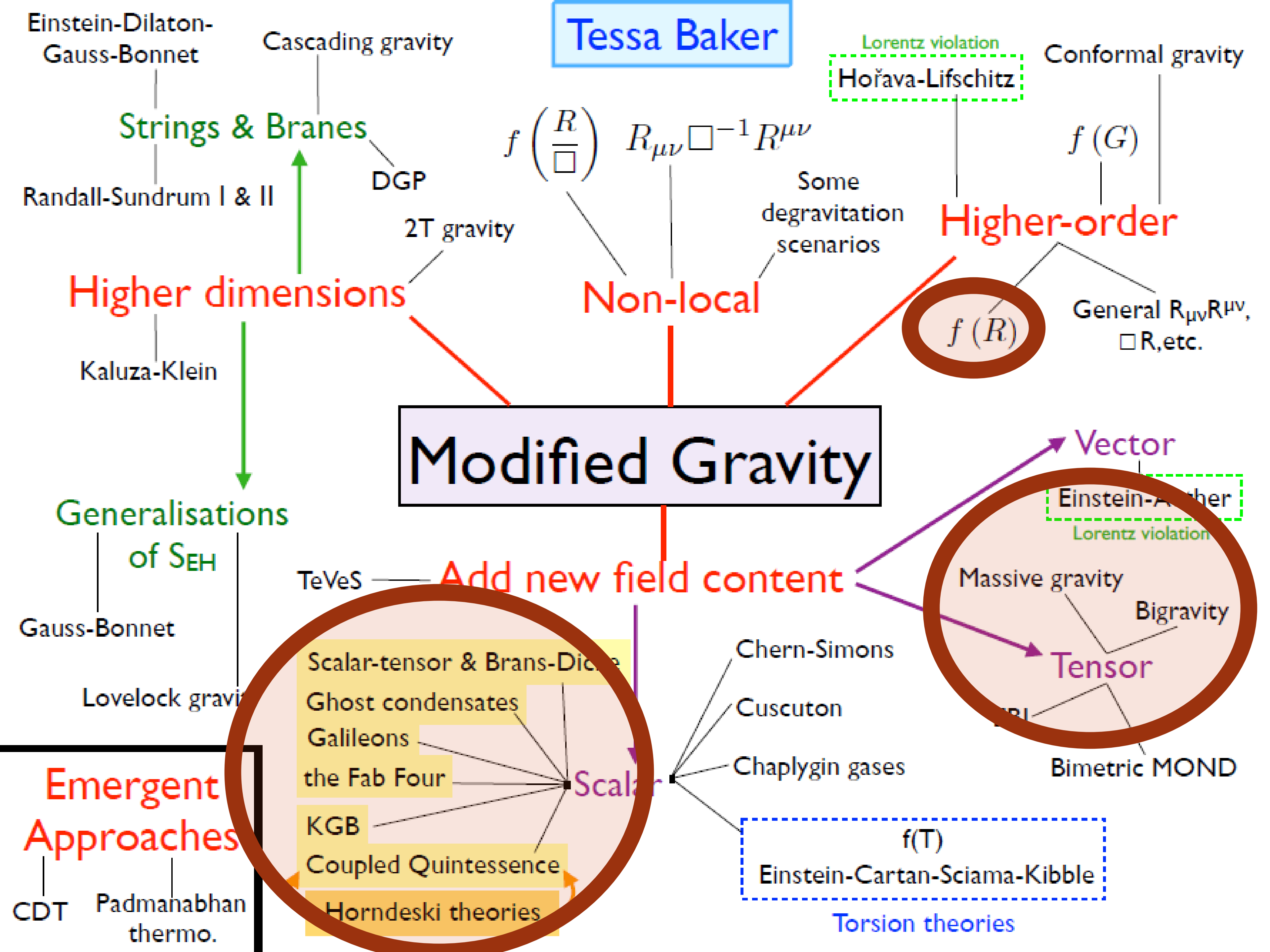
Torsion theories
Emergent Approaches
CDT
Padmanabhan thermo.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Non-Local modifications to GR

- Effective action for Gravity

$$e^{i\Gamma[g_{\mu\nu}]} = e^{iS_{\text{EH}}[g_{\mu\nu}]} \int \mathscr{D}\phi \, e^{iS_m[g_{\mu\nu},\phi]}$$

- The effective action typically contains non-localities.
- IR modifications, hence interesting for late time cosmology
- Modify GR without adding a new d.o.f.
- No need for screening mechanisms
- Help constructing scale-free theories

- Several models interesting for cosmology.
- We are going to suggest another one.

Non-Local modifications to GR

- Degravitation
  - A beautiful idea to make vacuum energy not to gravitate
  - Involves non-local operators

$$\left(1-\frac{m^2}{\Box}\right)G_{\mu\nu}=8\pi G\,T_{\mu\nu}$$

$$G_{\mu\nu}=8\pi G\,[\Box/(\Box-m^2)]\,T_{\mu\nu}$$

N. Arkani Hamed, S. Dimopoulos, G. Dvali, G. Gabadadze, arXiv: 0209227

Non-Local modifications to GR: Degravitation

$$\left(1-\frac{m^2}{\Box}\right)G_{\mu\nu}=8\pi G T_{\mu\nu}$$

$$G_{\mu\nu}=8\pi G\left[\Box/(\Box-m^2)\right]T_{\mu\nu}$$

$$[\nabla_\mu,\Box^{-1}]\neq 0$$

- Energy-Momentum not conserved
  - Not coming from a diff inv action

Deser-Woodard model

- Deser-Woodard $\longrightarrow$ $S = \frac{M_{Pl}^2}{2}\int d^4x\sqrt{-g}[R + \alpha R\frac{1}{\Box}R] + S_{matter}[g,\psi]$

- For FLRW: $\Box \equiv \frac{1}{\sqrt{-g}}\partial_\rho\left(\sqrt{-g}\,g^{\rho\sigma}\partial_\sigma\right) \longrightarrow -\frac{1}{a^3}\frac{d}{dt}\left(a^3\frac{d}{dt}\right)$

$$\left[\frac{1}{\Box}f\right](t) \equiv \mathcal{G}[f](t) = -\int_0^t dt'\frac{1}{a^3(t')}\int_0^{t'} dt'' a^3(t'')f(t'')$$

- Two branches determined by the sign of the free parameter
  - W/ future acceleration
  - W/ future matter domination

S. Deser, R.P. Woodard, arXiv: 0706.2151

Deser-Woodard model

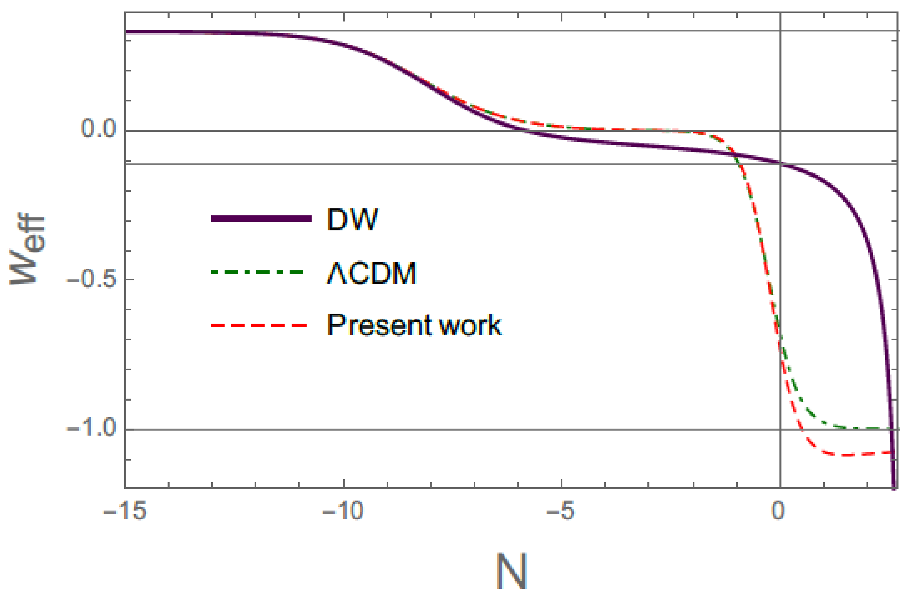
- Deser-Woodard → $S = \frac{M_{Pl}^2}{2}\int d^4x\sqrt{-g}[R + \alpha R\frac{1}{\Box}R] + S_{matter}[g,\psi]$
  - No **new scale** introduced
  - Constraint equations of GR are untouched, meaning no new propagating d.o.f.s
  - Graviton modes are healthy (kinetic term does not flip the sign, at least for FLRW bckgr)
  - No need for an additional screening mechanism!

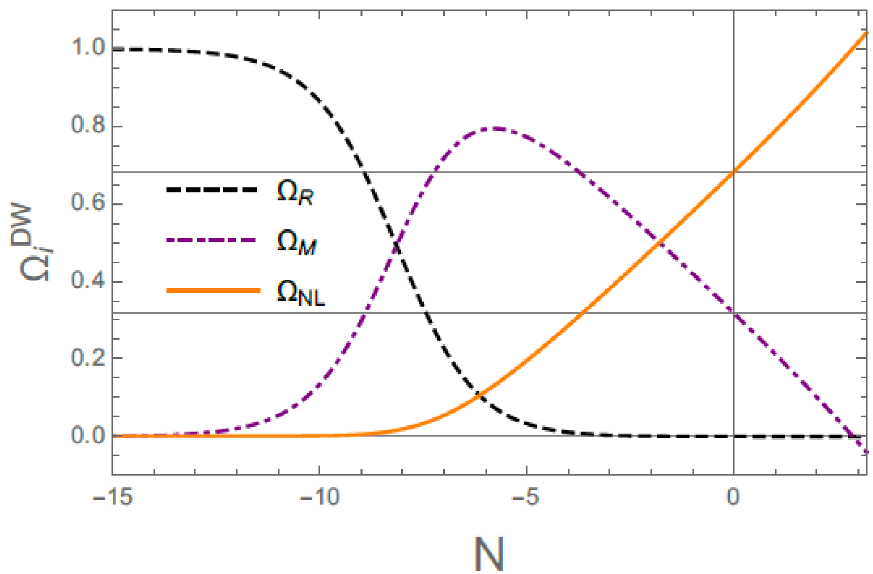
Unfortunately this simple model does not reproduce the correct cosmological background

Deser-Woodard model

- Deser-Woodard $$S = \frac{M_{Pl}^2}{2}\int d^4x\sqrt{-g}[R + \alpha R\frac{1}{\Box}R] + S_{matter}[g,\psi]$$
- Two branches determined by the sign of the free parameter:

  Positive Sign → Accelerating Branch



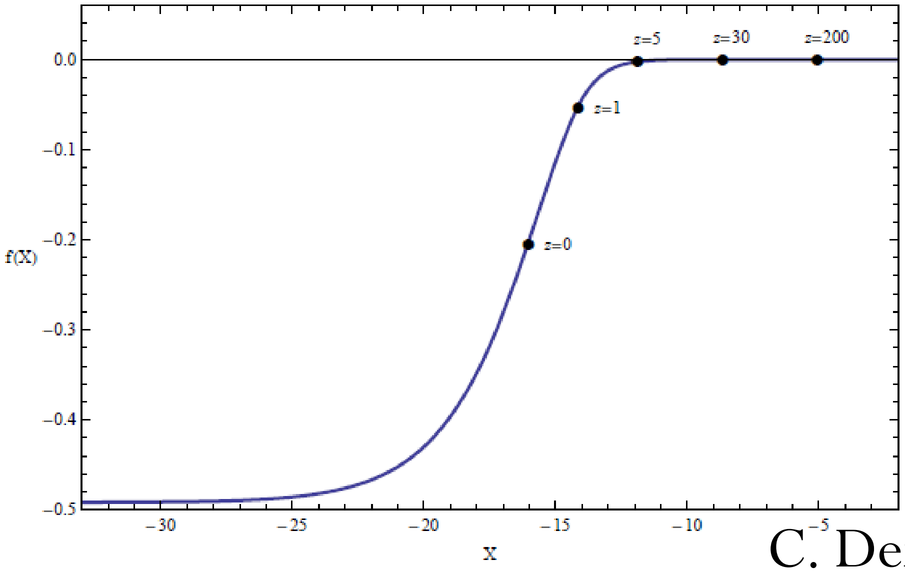


T. Koivisto, arXiv: 0803.3399

More complicated version of DW

- Deser-Woodard → $S = \frac{M_{Pl}^2}{2}\int d^4x\sqrt{-g}[R + \alpha R\frac{1}{\Box}R] + S_{matter}[g,\psi]$
  - Similar to reconstructing scalar potentials or the general function in f(R) theories.

Introduce a general function here



Reconstructed to reproduce LCDM.

T. Koivisto, arXiv: 0803.3399

C. Deffayet, R. Woodard, arXiv: 0904.0961

About bckgr and perturbations

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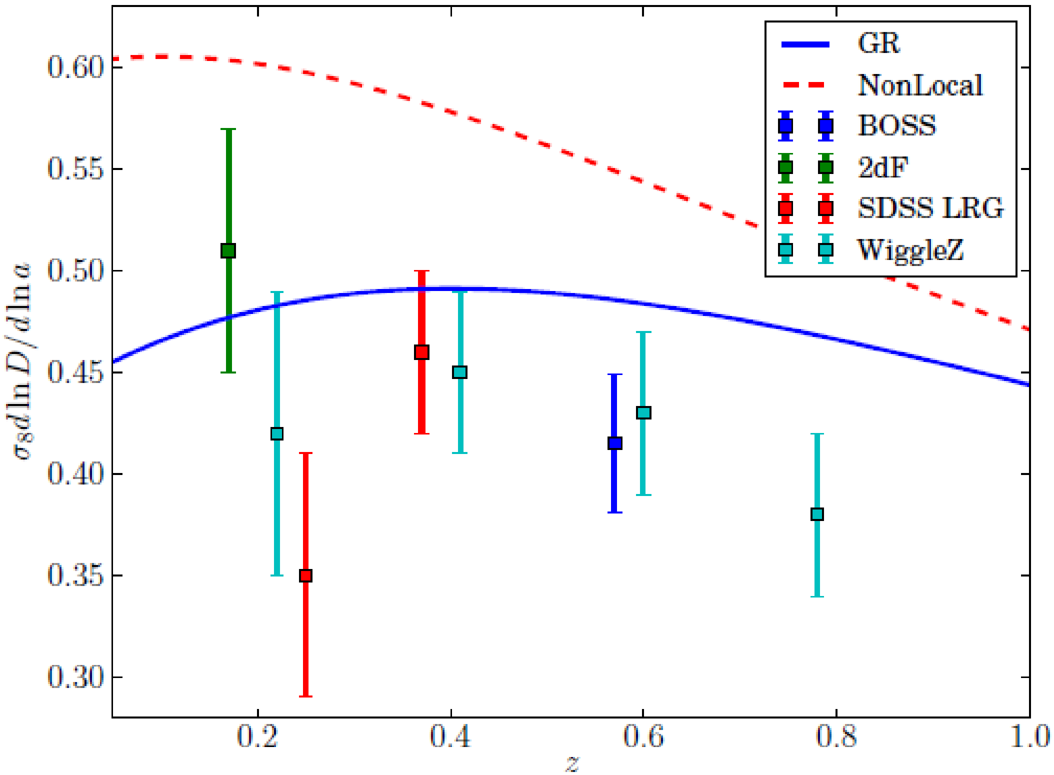
Introduce a general function here



S. Dodelson and S. Park, arXiv: 1310.4329

About bckgr and perturbations

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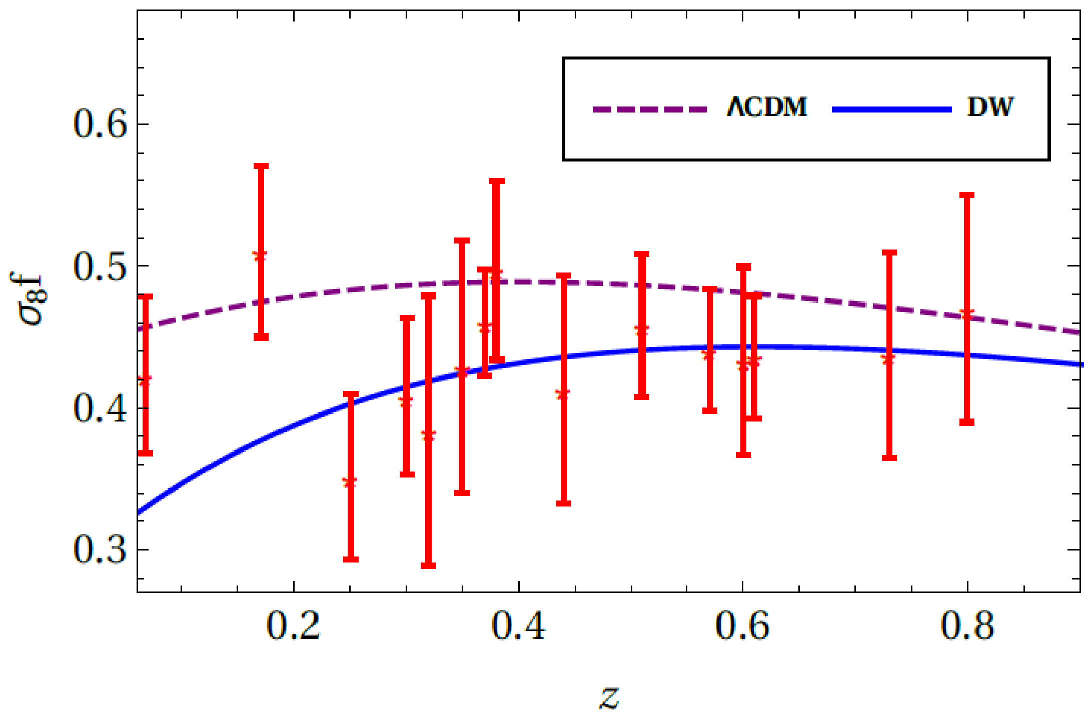

Introduce a general function here

H. Nersisyan, A. Cid, L. Amendola, arXiv: 1701.00434

About bckgr and perturbations

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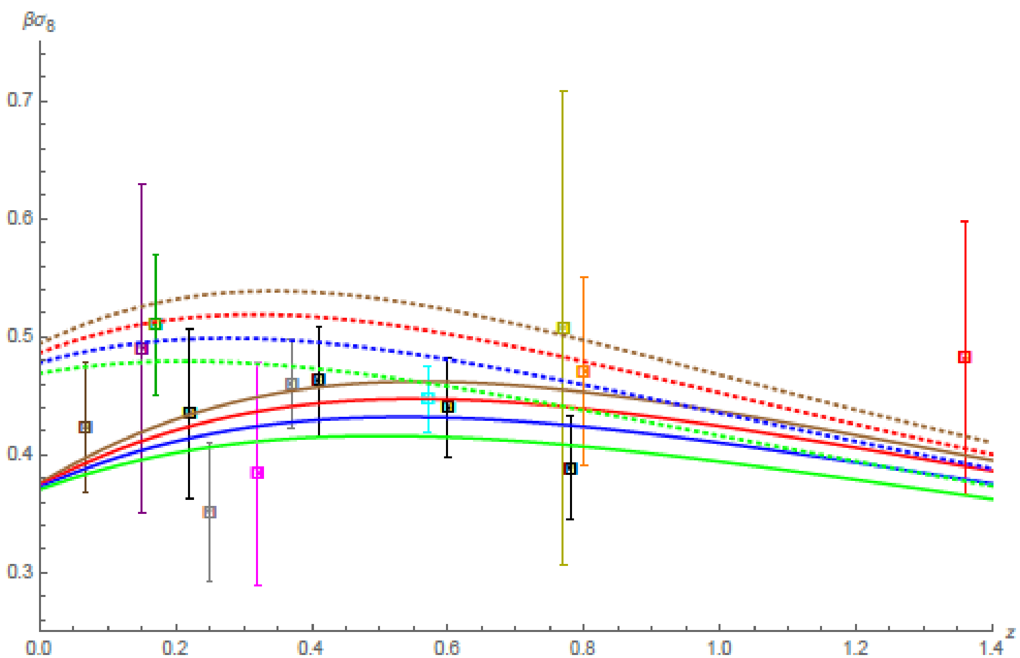


Introduce a general function here

Reconstructed to reproduce Non-LCDM bckgr.

S. Park and A. Shafieloo, arXiv: 1608.02541

Maggiore-Mancarella Model

- So called RR model ⟶ $\Gamma_{\rm RR} = \frac{m_{\rm Pl}^2}{2} \int d^4x \sqrt{-g} \left[R - \frac{m^2}{6} R \frac{1}{\Box^2} R \right]$

- Works well at the background and perturbation levels!
- BUT, theoretically the mass-scale turns out to be too small for cosmology.

M. Maggiore and M. Mancarella, arXiv: 1402.0448

Tensorial non-localities

- There are other possible terms as well:
  Tensorial Non-Localities
- Those are very interesting, i.e. for the degravitation idea.

$$S = \frac{M_{\rm Pl}^2}{2}\int d^4x\sqrt{-g}\left(-R + Rf(\triangle)R + R^{\alpha\beta}g(\triangle)R_{\alpha\beta} + R^{\mu\nu\alpha\beta}h(\triangle)R_{\mu\nu\alpha\beta}\right) + \int d^4x\sqrt{-g}\mathcal{L}_m$$

$$g(\triangle) \equiv \frac{\bar{M}_1^{\;2}}{6\triangle}\,, \qquad h(\triangle) \equiv \frac{\bar{M}_2^{\;2}}{6\triangle}\,,$$

$$\triangle \equiv m^4 + \alpha_1\Box + \alpha_2\Box^2 + \beta_1 R_{\alpha\beta}\nabla^\alpha\nabla^\beta + \beta_2 R\Box + \gamma\left(\nabla^\alpha R_{\alpha\beta}\right)\nabla^\beta$$

There are instabilities.

H. Nersisyan, Y. Akrami, L. Amendola, T. Koivisto, J. Rubio, A. Solomon, arXiv: 1610.01799

Connection to Massive/Multimetric Gravity

- Interesting interactions (Non-Local) might exist
- Bigravity is very restrictive, so this can help to enrich the phenomenology.
- Restore gauge (diffeomorphism) invariance

- Perhaps ghost-free spin-2 fields interact through extra fields.
  Underlying theory unknown. Package the unknown sector into non-localities.

Massive gravity/bi-gravity

- Conceptually interesting modification:
  - give the graviton a small mass
  - This leads naturally to a theory with two metrics

- Also: field theory motivation:
  - how to construct interacting spin-2 fields?

A Brief History of Massive Gravity

- 1939: Fierz and Pauli develop linear theory
- 1970s: Various problems discovered beyond linear order
  - Ghost!! Boulware-Deser
  - Discontinuity in limit m=0 van Dam-Veltman-Zakharov
  - Funny nonlinear effects Vainshtein
- 2010-11: Loophole found!
  - Nonlinear massive gravity finally constructed
    de Rham-Gabadadze-Tolley (dRGT)
  - Proved to be ghost-free Hassan-Rosen

There are ghost and gradient instabilities.

A Proposal of a New Model

- Non-Local interaction of two metrics
- Scale-free at the action level
- Turns out to have very interesting late time cosmology

$$S = \frac{M_{Pl}^2}{2}\int d^4x\sqrt{-g}[R + \alpha R\frac{1}{\Box}R] + S_{matter}[g,\psi]$$

↓

$$S = \frac{M_{\rm Pl}^2}{2}\int \mathrm{d}^4x\sqrt{-g}R + \frac{M_f^2}{2}\int \mathrm{d}^4x\sqrt{-f}R_f - \frac{M_{\rm Pl}^2}{2}\int \mathrm{d}^4x\sqrt{-g}\alpha(R_f\frac{1}{\Box}R + R\frac{1}{\Box}R_f) + S_{\rm matter}[g,\Psi],$$

V.V., Akrami, Amendola, Silvestri, arXiv:1702.08908

Non-Local E.O.M.s

$$S = \frac{M_{\rm Pl}^2}{2}\int \mathrm{d}^4x\sqrt{-g}R + \frac{M_f^2}{2}\int \mathrm{d}^4x\sqrt{-f}R_f - \frac{M_{\rm Pl}^2}{2}\int \mathrm{d}^4x\sqrt{-g}\alpha(R_f\frac{1}{\Box}R + R\frac{1}{\Box}R_f) + S_{\rm matter}[g,\Psi],$$

$$G^f_{\mu\nu} + \Delta G^f_{\mu\nu} = 0, \qquad G_{\mu\nu} + \Delta G_{\mu\nu} = \frac{1}{M_{\rm Pl}^2}T_{\mu\nu}$$

$$\Delta G_{\mu\nu} = -2\alpha[(\frac{1}{\Box}R_f)G_{\mu\nu} + g_{\mu\nu}R_f(1-\frac{1}{2\Box}R) - \nabla_\mu\nabla_\nu(\frac{1}{\Box}R_f) - \frac{1}{2}g_{\mu\nu}\nabla^\rho(\frac{1}{\Box}R)\nabla_\rho(\frac{1}{\Box}R_f) + \nabla_{(\mu}(\frac{1}{\Box}R_f)\nabla_{\nu)}(\frac{1}{\Box}R)]$$

$$\Delta G^f_{\mu\nu} = -2\alpha\frac{M_{\rm Pl}^2}{M_f^2}[\sqrt{f^{-1}g}(\frac{1}{\Box}R)R^f_{\mu\nu} + f_{\mu\nu}\Box_f(\sqrt{f^{-1}g}\frac{1}{\Box}R) - \nabla^f_\mu\nabla^f_\nu(\sqrt{f^{-1}g}\frac{1}{\Box}R)]$$

V.V., Akrami, Amendola, Silvestri, arXiv:1702.08908

D.O.F. counting in the non-local formulation

- Dynamical: $G_{ij} = \frac{1}{2}\ddot{h}_{ij} - \frac{1}{2}h_{ij}\partial_t^2 \log h + \mathcal{O}(\partial_t)\,, \qquad h \equiv \det h_{ij}$
- Constraint: $G_{0\mu}$
- We explicitly check that: $\Delta G_{0\mu}(t_0) = 0$
- Similarly for the reference metric.
- Additionally we check that there are no degrees of freedom which become ghost (See later)

The Localized Version

- It's easier to study such models in terms of auxiliary (nonlocal in terms of scale factor) scalar fields.

$$U \equiv \frac{1}{\Box} R,$$
$$V \equiv \frac{1}{\Box} R_f .$$

$$S = \frac{M_{\rm Pl}^2}{2} \int \mathrm{d}^4x \sqrt{-g} R + \frac{M_f^2}{2} \int \mathrm{d}^4x \sqrt{-f} R_f - \frac{M_{\rm Pl}^2}{2} \int \mathrm{d}^4x \sqrt{-g}\, \alpha (R_f U + RV) + \int \mathrm{d}^4x \sqrt{-g} \lambda_1 (R - \Box U) + \int \mathrm{d}^4x \sqrt{-g} \lambda_2 (R_f - \Box V) + S_{\rm matter}[g, \Psi],$$

$$G^f_{\mu\nu} + \Delta G^f_{\mu\nu} = 0, \qquad G_{\mu\nu} + \Delta G_{\mu\nu} = \frac{1}{M_{\rm Pl}^2} T_{\mu\nu}$$

$$\Delta G_{\mu\nu} = -2\alpha [V G_{\mu\nu} + g_{\mu\nu} R_f (1 - \frac{1}{2} U) - \nabla_\mu \nabla_\nu V - \frac{1}{2} g_{\mu\nu} \nabla^\rho V \nabla_\rho U + \nabla_{(\mu} V \nabla_{\nu)} U],$$
$$\Delta G^f_{\mu\nu} = -2\alpha [\sqrt{f^{-1} g} U R^f_{\mu\nu} + f_{\mu\nu} \Box_f (\sqrt{f^{-1} g} U) - \nabla^f_\mu \nabla^f_\nu (\sqrt{f^{-1} g} U)],$$

Ghosts?

- If we treat the non-local version as a scalar-tensor theory, there is a ghost d.o.f.
- Those are just localization artifacts.

$$S^{(2)}_{\mathrm{EH}} + S_{\mathrm{int}} = \int d^4x \left[\frac{1}{2} h_{\mu\nu} \mathscr{E}^{\mu\nu,\rho\sigma} h_{\rho\sigma} + \frac{\kappa}{2} h_{\mu\nu} T^{\mu\nu}\right]$$

$$h_{\mu\nu} = h^{\mathrm{TT}}_{\mu\nu} + (\partial_\mu \varepsilon_\nu + \partial_\nu \varepsilon_\mu) + \frac{1}{3}\eta_{\mu\nu} s$$

$$\partial^\mu h^{\mathrm{TT}}_{\mu\nu} = 0, \qquad \eta^{\mu\nu} h^{\mathrm{TT}}_{\mu\nu} = 0$$

$$S^{(2)}_{\mathrm{EH}} + S_{\mathrm{int}} = \int d^4x \frac{1}{2}\left[h^{\mathrm{TT}}_{\mu\nu} \Box (h^{\mu\nu})^{\mathrm{TT}} - \frac{2}{3} s \Box s\right] + \frac{\kappa}{2}\left[h^{\mathrm{TT}}_{\mu\nu} (T^{\mu\nu})^{\mathrm{TT}} + \frac{1}{3} s T\right]$$

Bianchi Constraints

- From the metric e.o.m.: $-\frac{1}{2}(\frac{1}{\Box}R)\nabla_\nu R_f = 0.$
- From the reference metric e.o.m.: $\sqrt{f^{-1}g}(\frac{1}{\Box}R)\nabla^\mu_f R^f_{\mu\nu} = 0.$

$$\nabla_\nu R_f = 0$$

- Too strong constraint!
- At least for background we find a solution compatible w/ the constraint
- Perturbations compatible?
- Motivates to add more interesting terms (e.g. tensorial)

The Equivalent Phenomenological model

- For all practical purposes we can use:

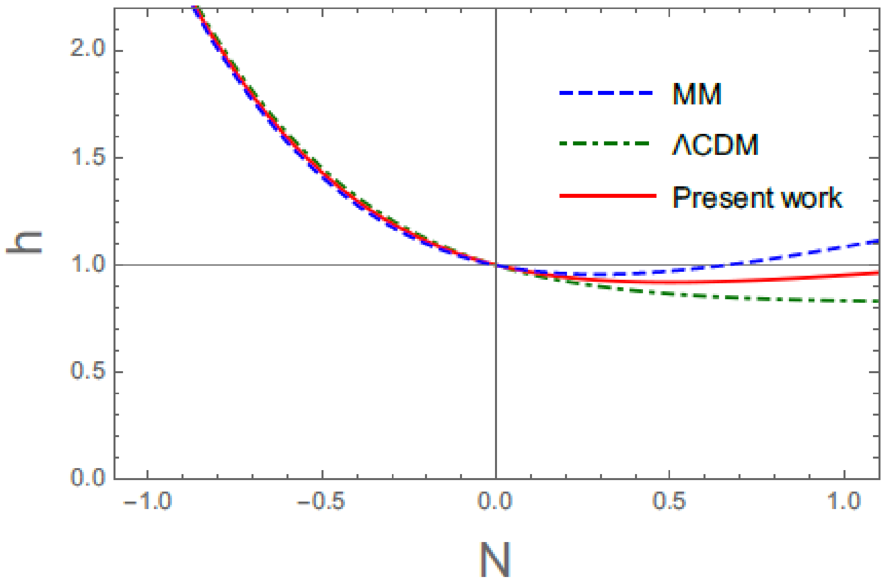
$$S = \frac{M_{\mathrm{Pl}}^2}{2} \int \mathrm{d}^4 x \sqrt{-g} (R + m^2 \frac{1}{\Box} R) + S_{\mathrm{matter}}[g, \Psi]$$

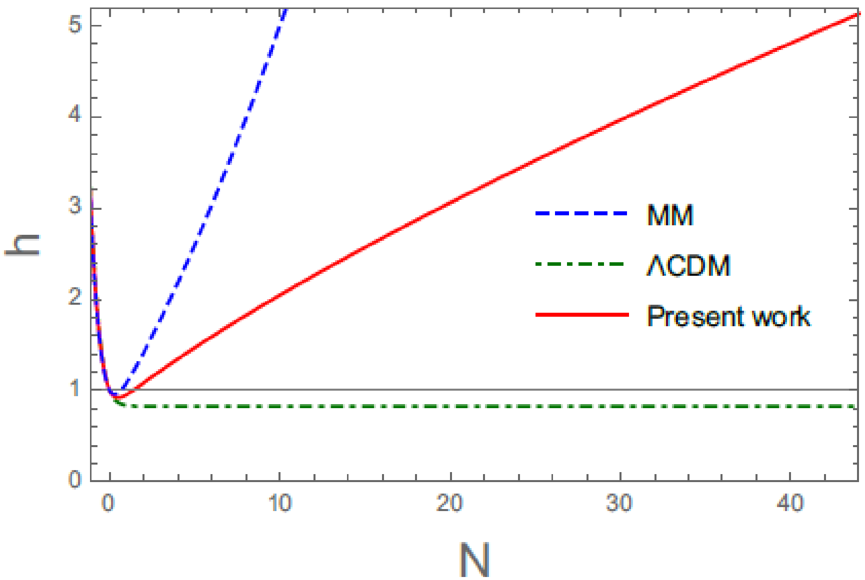
- One-parameter model of cosmic acceleration
- Interesting without the connection to bigravity
- Could also originate from somewhere else

Cosmology

$$g_{\mu\nu}\mathrm{d}x^{\mu}\mathrm{d}x^{\nu} = -\mathrm{d}t^2 + a^2(t)\delta_{ij}\mathrm{d}x^i\mathrm{d}x^j \qquad h \equiv H/H_0$$

$$h^2 = \Omega_{\mathrm{M}}^0 e^{-3N} + \Omega_{\mathrm{R}}^0 e^{-4N} + 2v - \frac{1}{6}\frac{m^2}{H_0^2}u - (2\xi v - v')(2 + \frac{1}{3}u')$$

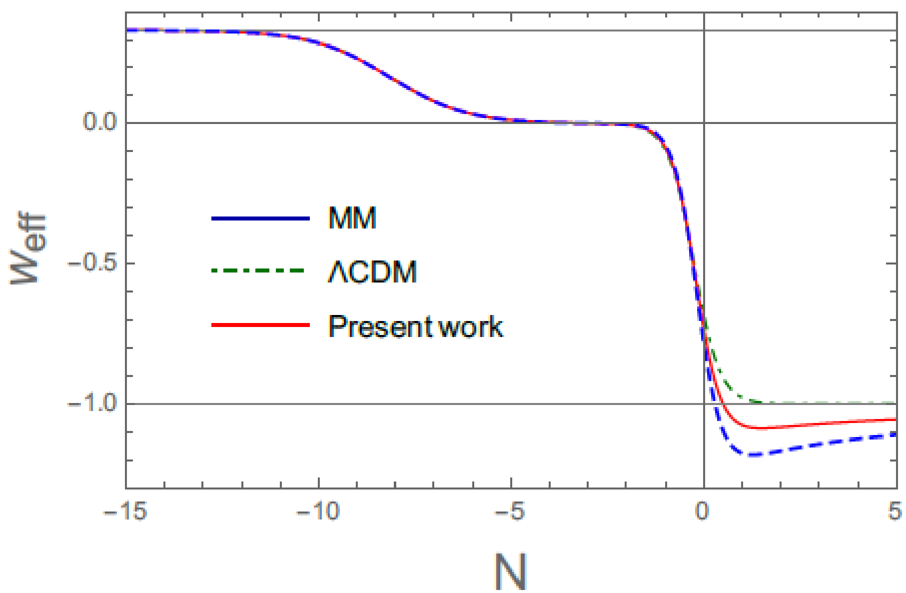


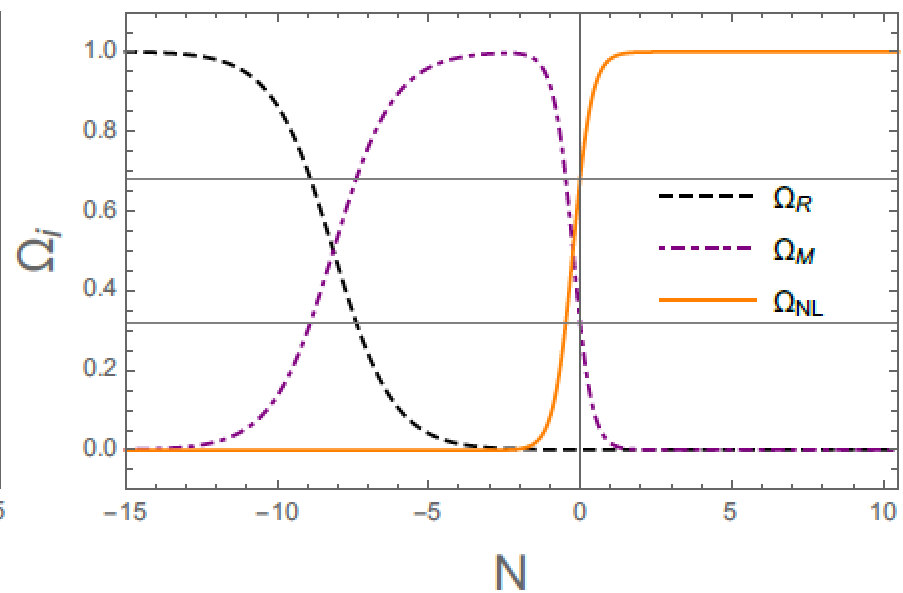


Phenomenology

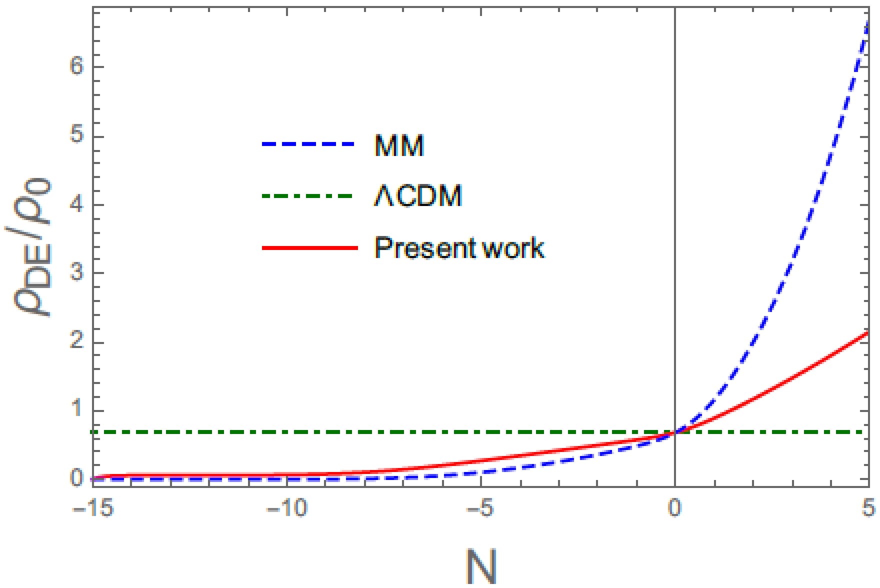
$$\Omega_{\rm NL} \equiv \frac{\rho_{\rm NL}}{\rho_{\rm tot}} = h^{-2}(2v - \frac{1}{6}\frac{m^2}{H_0^2}u - (2\xi v - v')(2 + \frac{1}{3}u'))$$

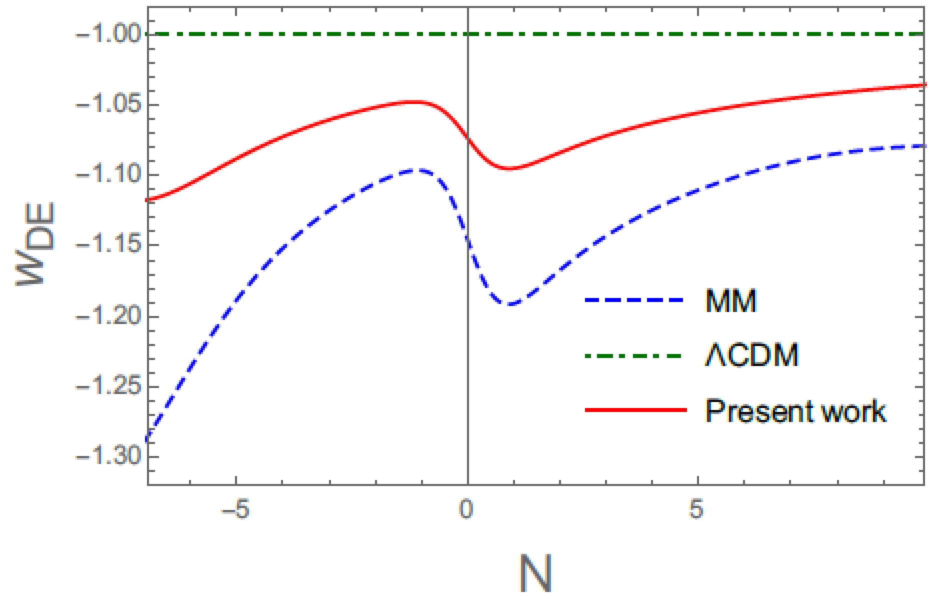
$$w_{\rm eff} = -1 - \frac{2}{3}\frac{h'}{h}$$





Phenomenology: CPL

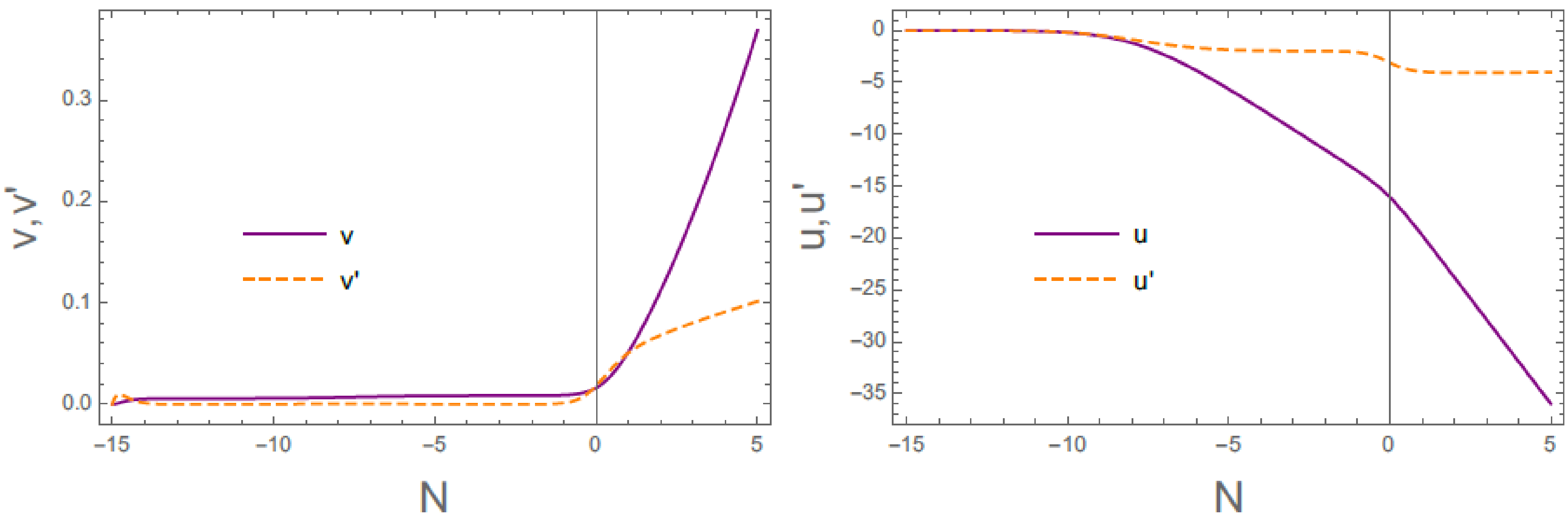




$$\rho'_{\mathrm{NL}} + 3\rho_{\mathrm{NL}}(1 + w_{\mathrm{NL}}) = 0$$

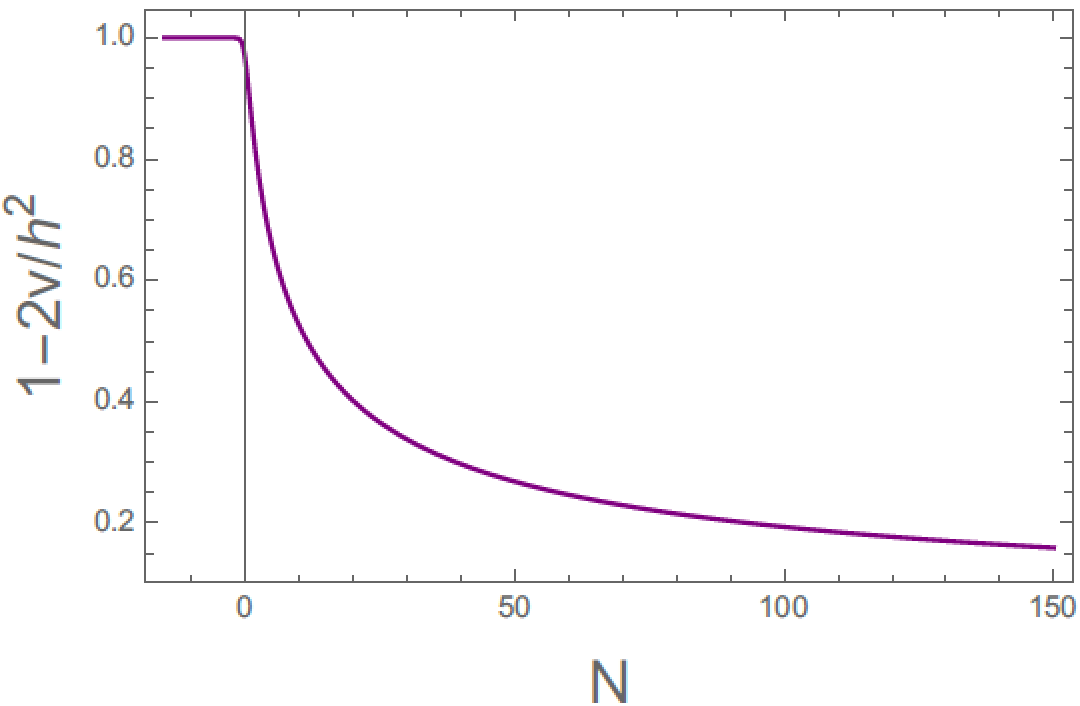
$$w(z) = w_0 + w_a z/(1+z)$$

$$w_0 = -1.075 \text{ and } w_a = 0.045$$



v, v'
v
v'
0.0
0.1
0.2
0.3
−15
−10
−5
0
5
N
u, u'
u
u'
0
−5
−10
−15
−20
−25
−30
−35
−15
−10
−5
0
5
N

No ghosts



On Testing Modified Gravity Theories

- Linear Perturbations:

$$ds^2 = a^2(\eta)\left[-(1+2\Psi)d\eta^2 + (1-2\Phi)dx^2\right]$$

$$k^2\Psi = -4\pi\mu(a,k)Ga^2\left[\rho\Delta + 3(\rho+P)\sigma\right]$$

$$k^2\left[\Phi - \gamma(a,k)\Psi\right] = 12\pi\mu(a,k)Ga^2(\rho+P)\sigma$$

- Phenomenological Parametrization (MGCAMB)
- Map to EFT Language (EFTCAMB)

www.eftcamb.org

What else can be done?

- Non-local modifications are quite appealing for cosmology.

- The model presented here leads to an interesting accelerating background.
- We should still study the perturbations of this model.
- Tensorial structures: might cure some of the instabilities.
- Derive more realistic interaction models.
- It can also potentially solve some issues within Massive/bi-metric theories.