Anisotropic deformations of spatially open cosmology in massive gravity theory

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Motivations for massive gravity

- Cosmic acceleration $\Rightarrow$ either Λ-term or modification of gravity

$$\text{Newton} \quad \frac{1}{r} \quad \rightarrow \quad \text{Yukawa} \quad \frac{1}{r}\, e^{-mr}$$

$\Rightarrow$ gravity is weaker at large distance = cosmic acceleration, $m \sim 1/(\text{Hubble radius}) \sim 10^{-33}$ eV.

- Small m is more natural than small Λ.
- GW150914 $\Rightarrow$ $m < 1.2 \times 10^{-22}$ eV

Massive gravity – two metrics $g_{\mu\nu}$ and $f_{\mu\nu}$

$$S = M_{\rm Pl}^2 \int \sqrt{-g} \left(\frac{1}{2} R - m^2 \mathcal{U} \right) d^4x$$

where

$$\mathcal{U} = \frac{1}{8} \left(H^\mu_\nu H^\nu_\mu - (H^\alpha_\alpha)^2 \right) + \mathcal{O}(H^3)$$

with

$$H^\mu_\nu = g^{\mu\alpha} f_{\alpha\nu} - \delta^\mu_\nu \qquad\qquad f_{\mu\nu} = \eta_{AB} \partial_\mu \Phi^A \partial_\nu \Phi^B$$

Field equations

$$G_{\mu\nu} = m^2 T_{\mu\nu}$$

where

$$T_{\mu\nu} = 2 \frac{\partial \mathcal{U}}{\partial g^{\mu\nu}} - g_{\mu\nu} \mathcal{U}, \qquad \nabla^\mu T_{\mu\nu} = 0.$$

If $g_{\mu\nu} \approx f_{\mu\nu} \Rightarrow$ Fierz-Pauli with 5 DoF.
In general $2 + 6 = 5 + 1$ DoF. Extra DoF $=$ Boulware-Deser ghost.

dRGT theory

Explicitely

$$S = M_{\rm Pl}^2 \int \left(\frac{1}{2} R - m^2 \mathcal{U} \right) \sqrt{-g}\, d^4x$$

$$\mathcal{U} = b_0 + b_1 \sum_a \lambda_a + b_2 \sum_{a<b} \lambda_a \lambda_b + b_3 \sum_{a<b<c} \lambda_a \lambda_b \lambda_c + b_4 \lambda_0 \lambda_1 \lambda_2 \lambda_3$$

where b_k are parameters and λ_a are eigenvalues of the matrix

$$\boxed{\gamma^\mu_{\ \nu} = \sqrt{g^{\mu\alpha} f_{\alpha\nu}}}$$

Two constraints appear and reduce the number of DoF to 5.

/de Rham, Gabadadze, Tolley 2010/

dRGT cosmologies

Chamseddine, M.S.V, 2011
d'Amico, de Rham, Dubovsky, Gabadadze, Pirskhalava, 2011
Gumrukcuoglu, Lin, Mukhoyama, 2011
de Felice, Gumrukcuoglu, Mukhoyama, 2012
Grata, Hu, Wyman, 2012
M.S.V., 2012
Kobayashi, Siino, Yamaguchi, Yoshida, 2012
M.S.V., 2013
others ...

Compact formulation

Hubble parameter

$$H = \frac{m^2}{3}(b_0 + 2b_1 u_* + b_2 u_*^2), \quad b_1 + 2b_2 u_* + b_3 u_*^2 = 0.$$

g-metric is de Sitter, f-metric is flat

$$\begin{aligned} ds_g^2 &= -(dx^0)^2 + (dx^1)^2 + (dx^2)^2 + (dx^3)^2 + (dx^4)^2 \,, \\ \frac{1}{H^2} &= -(x^0)^2 + (x^1)^2 + (x^2)^2 + (x^3)^2 + (x^4)^2, \\ ds_f^2 &= -(dT)^2 + (dx^1)^2 + (dx^2)^2 + (dx^3)^2 \,, \end{aligned}$$

the Stuckelberg field $T(x^0, x^4)$ fulfills /C.Mazuet, M.S.V '15/

$$\boxed{(\partial_0 T)^2 - (\partial_4 T)^2 = 1}$$

All solutions have the same ds Sitter g-metric but different Stuckelberg scalars. Solution with $\boxed{T = x^0}$ is called Type I. All other solutions are called Type II.

Type I solution in open slicing

$$Ht = \sinh\tau\cosh\rho, \quad Hr = \cosh\tau$$

gives, with $a(\tau) = \cosh\tau$,

$$\begin{aligned} ds_g^2 &= \frac{1}{H^2}\{-d\tau^2 + a^2(\tau)(d\rho^2 + \sinh^2\rho d\Omega^2)\} \\ ds_f^2 &= \frac{u_*^2}{H^2}\{-da^2(\tau) + a^2(\tau)(d\rho^2 + \sinh^2\rho d\Omega^2)\} \end{aligned}$$

Solution is manifestly FLRW – g and f metrics share the same symmetries rotational and translational Killings.

/Gumrukcuoglu, Lin, Mukohyama '11/

Type I solution in flat slicing

$$Ht = \sinh\tau + \frac{\rho^2}{2}\,e^{\tau}, \quad Hr = \cosh\tau - \frac{\rho^2}{2}\,e^{\tau}$$

gives spatially flat FLRW g-metric with $a(\tau) = e^{\tau}$

$$ds_g^2 = \frac{1}{H^2}\{-d\tau^2 + a^2(\tau)(d\rho^2 + \rho^2 d\Omega^2)\}$$

but

$$ds_f^2 = \frac{u_*^2}{H^2}\{-dT^2(\tau,\rho) + dR^2 + R^2 d\Omega^2)\}$$

with inhomogeneous

$$T(\tau,\rho) = -\frac{1}{2}\int\frac{d\tau}{\dot{a}(\tau)} + \frac{1}{2}(1+\rho^2)a(\tau)$$

f-metric does not share the same symmetries with g-metric $\Rightarrow$ no spatially flat FLWR cosmologies /d'Amico, de Rham et al. '11/

Type I solution in static slicing

$$Ht = \sqrt{1-\rho^2}\sinh\tau, \quad Hr = \sqrt{1-\rho^2}\cosh\tau$$

gives, with $a(\tau) = \cosh\tau$

$$\begin{aligned} ds_g^2 &= \frac{1}{H^2}\{-(1-\rho^2)d\tau^2 + \frac{d\rho^2}{1-\rho^2} + \rho^2 d\Omega^2)\} \\ ds_f^2 &= \frac{u_*^2}{H^2}\{-dT^2(\tau,\rho) + d\rho^2 + \rho^2 d\Omega^2)\} \end{aligned}$$

with

$$T(\tau,\rho) = \sqrt{1-\rho^2}\sinh\tau$$

f-metric is not invariant under the action of $\partial/\partial\tau$.

Status of the dRGT cosmology

- Type I solution expressed in open slicing is the best known massive gravity cosmology. Has 6 common Killings, considered to be the only genuinely FLRW solution.
- Type II solutions are less known. The number of common symmetries is less than 6 $\Rightarrow$ perturbation spectrum is expected to be inhomogeneous and/or anisotropic.
- Problem: linear perturbations analysis around Type I reveals only 2 propagating modes because the kinetic term for 3 other modes vanishes in the linearized theory $\Rightarrow$ strong coupling = breakdown of classical description.
- Moreover, Type I solution was claimed to show ghost instability because it admits anisotropic deformations whose perturbations show ghosts at the linear level.
- The latter conclusion has produced overall pessimism

But

- De Felice, Gumrukcuoglu, Mukohyama in "Massive gravity: nonlinear instability of a homogeneous and isotropic universe" /PRL **109**, 171101 (2012)/ actually considered a different massive gravity theory with $f_{\mu\nu}$=de Sitter and not flat. In this theory there is a FLRW solution with spatially flat sections. This solution admits Bianchi I anisotropic deformations which show ghost.

- However, strictly speaking this tells nothing about stability of the original solution in dRGT theory.

- Resumé: following the dFGM, everybody telles the dRGT cosmology is unstable, but nobody has actually checked this.

Anisotropic cosmologies in dRGT

Bianchi types

$$ds_g^2 = -dt^2 + \eta_{ab}(t)\,\omega^a \otimes \omega^b$$

with

$$\omega^a = \omega^a_k(x^m)\,dx^k, \qquad \langle \omega^a, e_b \rangle = \delta^a_b, \qquad e_a = e^k_a(x^m)\,\frac{\partial}{\partial x^k}$$

and

$$[e_a, e_b] = C^c_{ab} e_c$$

Bianchi classification of structure constants C^c_{ab} reveals IX different types. Flat FLRW is contained in Bianchi I,

$$\omega^1 = dx, \quad \omega^2 = dy, \quad \omega^3 = dz.$$

Open FLRW is contained in Bianchi V,

$$\omega^1 = dx, \quad \omega^2 = e^x\,dy, \quad \omega^3 = e^x\,dz.$$

Bianchi V

$$\begin{aligned} ds_g^2 &= -dt^2 + A^2(t)\,dx^2 + e^{2x}\left[B^2(t)\,dy^2 + C^2(t)\,dz^2\right], \\ ds_f^2 &= -(dF)^2 + F^2\left[dX^2 + e^{2X}(dy^2 + dz^2)\right], \end{aligned}$$

with the Stuckelberg fields

$$F = F(t), \qquad X = x + f(t).$$

One can choose

$$B = C$$

Equations for A, B, F, f

$$
\begin{aligned}
-\frac{\ddot B}{B}-\frac{\dot B^2}{B^2}-\frac{\dot A\dot B}{AB}+\frac{2}{A^2} &= -P_0-\frac{1}{2}\,YP_1\,,\\
-\frac{\ddot A}{A}-\frac{2\dot A\dot B}{AB}+\frac{2}{A^2} &= -P_0+\left[u-\frac{1}{2}\,Y-\frac{F}{AY}\left(\dot F+\frac{F}{A}\right)\right]P_1\\
&\quad +\frac{1}{2}\left(Yu-u^2-\frac{F\dot F}{A}\right)\frac{dP_1}{du}\,,\\
\frac{3}{A^2}-\frac{2\dot A\dot B}{AB}-\frac{\dot B^2}{B^2} &= -P_0-\frac{F}{AY}\left(\dot F+\frac{F}{A}\right)P_1,\\
2\,\frac{\dot B}{B}-2\,\frac{\dot A}{A} &= -\frac{F^2\dot f}{Y}\,P_1
\end{aligned}
$$

with

$$
u=\frac{F}{B}\,e^f,\qquad P_m(u)=b_m+2b_{m+1}u+b_{m+2}u^2
$$

Type I isotropic solution

$$A = B = \mathbf{a}, \qquad F = u_*\mathbf{a}, \qquad f = 0, \qquad P_1(u_*) = 0$$

$$\begin{aligned} ds_g^2 &= \frac{1}{H^2}\left(-dt^2 + \mathbf{a}^2\left(dx^2 + e^{2x}\left[dy^2 + dz^2\right]\right)\right), \\ ds_f^2 &= \frac{u_*^2}{H^2}\left\{-(d\mathbf{a})^2 + \mathbf{a}^2\left(dx^2 + e^{2x}[dy^2 + dz^2]\right)\right\}. \end{aligned}$$

with

$$\mathbf{a} = \sinh[H(t - t_0)]$$

The spatial parts of the two metrics are proportional to

$$\begin{aligned} dl^2 &= dx^2 + e^{2x}(dy^2 + dz^2) \\ &= d\rho^2 + \sinh^2(\rho)[d\vartheta^2 + \sin^2\vartheta d\varphi^2] \end{aligned}$$

This is precisely Type I solution.

Type II isotropic solutions

$$F = \frac{u_*}{H}\sqrt{2q^2\,\dot{\mathbf{a}} - 1 - q^4}, \qquad f = \chi + \ln\frac{u_*\mathbf{a}}{F}.$$

with $\mathbf{a} = \sinh[H(t - t_0)]$ while

$$\begin{aligned} ds_g^2 &= -dt^2 + \mathbf{a}^2 dx^2 + e^{2x}\left[\mathbf{a}^2 e^{2\chi}\left[dy^2 + dz^2\right]\right], \\ ds_f^2 &= -dF^2 + F^2\left(dX^2 + e^{2X}[dy^2 + dz^2]\right), \qquad X = x + f(t). \end{aligned}$$

- A family labeled by three continuous parameters q, χ, t_0
- g-metric is de Sitter
- f-metric is flat
- 3 common isometries of the y, z plane.

Generalisation to an infinite family of new Type II

$$
\begin{aligned}
ds_g^2 &= dUdV + (x^1)^2 + (x^2)^2 + (x^4)^2 \,, \\
\frac{1}{u_*^2}\, ds_f^2 &= dUd(V+D) + (x^1)^2 + (x^2)^2 + (x^4)^2 \,,
\end{aligned}
$$

where

$$
UV + (x^1)^2 + (x^2)^2 + (x^4)^2 = \frac{1}{H^2}, \qquad D = \frac{(Hx^4 - q^2)^2}{H^2 U}.
$$

In view of the Gordon relation

$$
f_{\mu\nu} = \omega^2 \left[g_{\mu\nu} + (1-\zeta^2)\mathrm{V}_\mu \mathrm{V}_\nu \right], \qquad g^{\mu\nu}\mathrm{V}_\mu \mathrm{V}_\nu = -1,
$$

it will remain a solution if $D = D(U, x^4)$ such that

$$
\partial_U D + \frac{1}{4}\, (\partial_4 D)^2 = 0.
$$

Small deviations form type I – linear

$$
\begin{aligned}
A &= \mathbf{a}\,(1+\alpha), \qquad B = \mathbf{a}\,(1+\beta), \\
\frac{F}{A} &= u_* + \phi, \qquad f = \psi,
\end{aligned}
$$

Expanding up to the first order

$$
\begin{aligned}
\ddot{\beta} + \frac{\dot{\mathbf{a}}}{\mathbf{a}}\left(5\dot{\beta} + \dot{\alpha}\right) + \frac{4\alpha}{\mathbf{a}} &= \frac{u_*}{2} P_1'(u_*)(\dot{\mathbf{a}} - 1)\,\sigma, \\
\ddot{\alpha} + \frac{\dot{\mathbf{a}}}{\mathbf{a}}\left(2\dot{\beta} + 4\dot{\alpha}\right) + \frac{4\alpha}{\mathbf{a}} &= \frac{u_*}{2} P_1'(u_*)(\dot{\mathbf{a}} - 1)\,\phi, \\
2\frac{\dot{\mathbf{a}}}{\mathbf{a}}\left(\dot{\alpha} + 2\dot{\beta}\right) + \frac{6\alpha}{\mathbf{a}^2} &= 0, \\
2(\dot{\alpha} - \dot{\beta}) &= 0,
\end{aligned}
$$

r.h.s. vanish → strong coupling. The only solution is

$$
\sigma = \phi = 0, \qquad \alpha = \beta = const. \times \dot{\mathbf{a}}/\mathbf{a} \quad \Rightarrow \quad \text{time translations}
$$

Small deviations form type I – quadratic

Expanding up to the second order

$$
\begin{aligned}
\ddot{\beta} + \frac{\dot{\mathbf{a}}}{\mathbf{a}}\left(5\dot{\beta} + \dot{\alpha}\right) + \frac{4\alpha}{\mathbf{a}} &= \frac{u_*}{2} P_1'(u_*)(\dot{\mathbf{a}} - 1)\,\sigma, \\
\ddot{\alpha} + \frac{\dot{\mathbf{a}}}{\mathbf{a}}\left(2\dot{\beta} + 4\dot{\alpha}\right) + \frac{4\alpha}{\mathbf{a}} &= \frac{u_*}{2} P_1'(u_*)(\dot{\mathbf{a}} - 1)\,\phi. \\
2\frac{\dot{\mathbf{a}}}{\mathbf{a}}\left(\dot{\alpha} + 2\dot{\beta}\right) + \frac{6\alpha}{\mathbf{a}^2} &= P_1'(u_*)\,\sigma\,(\phi + \frac{1}{2}\,\sigma), \\
2(\dot{\alpha} - \dot{\beta}) &= P_1'(u_*)\frac{\mathbf{a}^2}{\dot{\mathbf{a}} + 1}\,\sigma\,[\dot{\sigma} - \dot{\phi} + u_*(\dot{\beta} - \dot{\alpha})]
\end{aligned}
$$

Setting

$$
\sigma = \frac{W + Z}{3}, \qquad \phi = \frac{W - 2Z}{3}
$$

Small deviations form type I – solutions

$$\left((W+Z)\dot{Z}\right)^{\cdot} + 4H(W+Z)\dot{Z} + 3u_* H^2 Z = 0,$$
$$W\dot{W} - Z\dot{Z} + 3H(W^2 - Z^2) = 3u_* H^2\, \mathbf{a}\, W.$$

Assuming that $W, Z, \dot{W}, \dot{Z}$ tend to zero simultaneously, the only solution is

$$Z = -\frac{u_* H^2}{2}(t - t_*)^2, \qquad W = \frac{u_* H^2}{2\mathbf{a}}(t - t_*)^3$$

where t_* is an integration constant. Perturbations can be small only for $t \approx t_*$ and diverge for $t \to \infty$ hence they cannot approach zero asymptotically. Therefore when perturbed Type I solution cannot relax back to itself in the long run.

Solution is unstable. What does it decay to ?

Small deviations form type II – linear

$$
\begin{aligned}
A &= \mathbf{a}\,(1+\alpha), \qquad B = \mathbf{a}\,(1+\beta), \\
F &= u_*\mathbf{a}\sqrt{w}\,(1+\phi), \qquad u = u_* + \sigma,
\end{aligned}
$$

$$
\begin{aligned}
\sigma &\to \frac{C_\sigma}{\mathbf{a}^4}\left(1+\mathcal{O}\left(\mathbf{a}^{-1}\right)\right) \\
\alpha &\to \alpha_\infty\left(1+\frac{1}{2H^2\mathbf{a}^2}+\dots\right) - C_\sigma\left(\frac{u_* q^2 P_1'}{9H(q^2+1)\,\mathbf{a}^3}+\dots\right), \\
\beta &\to \beta_\infty + \alpha_\infty\left(\frac{1}{2H^2\mathbf{a}^2}+\dots\right) + C_\sigma\left(\frac{u_* q^2 P_1'}{18H(q^2+1)\,\mathbf{a}^3}+\dots\right) \\
\phi &\to \phi_\infty\,(1+\dots) + \alpha_\infty\left(\frac{q^2}{2H\mathbf{a}}+\dots\right) + C_\sigma\left(\frac{u_* q^2 P_1'}{36(q^2+1)H\,\mathbf{a}^3}+\dots\right)
\end{aligned}
$$

$\alpha_\infty, \beta_\infty, \phi_\infty$ are related to the background moduli parameters.
Perturbed Type II relaxes to itself – late time attractor

Hypothesis

It is plausible that perturbed Type I relaxes to Type II

Numerical analysis shows that

- Weakly perturbed Type I relaxes to Type II indeed.
- Strongly perturbed Type either I collapses or decays into flat space.

Cauchy problem and constraints

Primary constraint

Four field equations contain $\ddot{A}$, $\ddot{B}$, $\dot{F}$, $\dot{f}$.
The first two can be resolved with respect to $\ddot{A}$ and $\ddot{B}$.
Trying to resolve the second two with respect to $\dot{F}$ and $\dot{f}$ gives

$$\dot{F} = a_1(A, B, \dot{A}, \dot{B}, F, f)\,\dot{f} + a_2(A, B, \dot{A}, \dot{B}, F, f)$$

and a constraint which algebraically determines F,

$$\mathcal{C} = A^2\left(\frac{3}{A^2} - \frac{2\dot{A}\dot{B}}{AB} - \frac{\dot{B}^2}{B^2} + P_0\right)^2 - 4\left(\frac{\dot{B}}{B} - \frac{\dot{A}}{A}\right)^2 - (P_1)^2F^2 = 0.$$

The constraint should be stable, hence

$$\dot{\mathcal{C}} = \frac{\partial \mathcal{C}}{\partial A}\,\dot{A} + \frac{\partial \mathcal{C}}{\partial \dot{A}}\,\ddot{A} + \frac{\partial \mathcal{C}}{\partial B}\,\dot{B} + \frac{\partial \mathcal{C}}{\partial \dot{B}}\,\ddot{B} + \frac{\partial \mathcal{C}}{\partial F}\,\dot{F} + \frac{\partial \mathcal{C}}{\partial u}\,\dot{u} = 0,$$

which leads to the secondary constraint $\mathcal{S}(A, B, \dot{A}, \dot{B}, u) = 0$

Secondary constraint

$$
\begin{aligned}
\mathcal{S} \;\equiv\; & \left(P_0\, A^2B^2 - A^2\dot{B}^2 - 2AB\dot{A}\dot{B} - 2AB\dot{B} + 2B^2\dot{A} + 3B^2\right) \times \\
\times\; & \left[\frac{P_0\, A^3B^2\dot{B} - A^3\dot{B}^3 - 2A^2B\dot{A}\dot{B}^2 + AB^2\dot{B} + 2B^3\dot{A}}{A^4B^5}\, P_1' \right. \\
+\; & \left. \frac{u(A\dot{B} + B)}{A^2B^3}\, P_1P_1' - \frac{2A\dot{B} + B\dot{A} - B}{A^2B^3}\, P_1^2\right] \\
+\; & u\,(u\, P_1' - 2P_1)P_1^2 = 0
\end{aligned}
$$

algebraically determines u. The stability condition is

$$\dot{\mathcal{S}} = W(A, B, \dot{A}, \dot{B}, u)\, \dot{f} + V(A, B, \dot{A}, \dot{B}, u)$$

which gives the missing equation for $\dot{f}$,

$$\dot{f} = \mathcal{F}(A, B, \dot{A}, \dot{B}, u).$$

The pair of constraints eliminates the BD ghost

Equations

$$\begin{aligned} \ddot{A} &= \mathcal{D}_A\left(A, B, \dot{A}, \dot{B}, u, F\right) \\ \ddot{B} &= \mathcal{D}_B\left(A, B, \dot{A}, \dot{B}, u, F\right) \end{aligned} \tag{1}$$

with

$$\begin{aligned} \dot{F} &= \mathcal{D}_F\left(A, B, \dot{A}, \dot{B}, u, F\right) \\ \dot{u} &= \mathcal{D}_u\left(A, B, \dot{A}, \dot{B}, u, F\right) \end{aligned} \tag{2}$$

and the constraints

$$\mathcal{C}(A, B, \dot{A}, \dot{B}, u, F) = 0$$
$$\mathcal{S}(A, B, \dot{A}, \dot{B}, u, F) = 0$$

One can impose the constraints only at the initial time moment and then solve (1)+(2). Or one can resolve the constraints at every time moment to find u, F and then solve only (1).

Numerical results

Generic initial values

Choose the parameter values

$$b_0 = 1, \quad b_1 = 1, \quad b_2 = 2, \quad b_3 = -5$$

and the initial values for the metric

$$A_0 = B_0 = 2, \qquad \dot{A}_0 = 0, \qquad \dot{B}_0 = 1$$

The equation $\mathcal{S}(u_0) = 0$ then yields

$$u_0 = 1.4817$$

while the equation $\mathcal{C}(F_0) = 0$ gives

$$F_0 = 4.3649$$

Integrating the equations with these initial values gives

Generic solutions

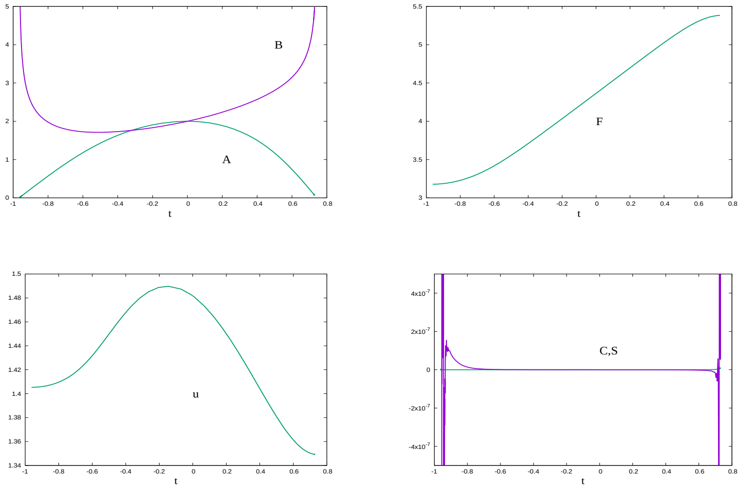


Figure: Generic behaviour of solutions

Weakly perturbed type I

$$b_0 = 1, \quad b_1 = 1, \quad b_2 = 2, \quad b_3 = -5$$

$P_1(u_*) = 0$ has a root $u_* = -0.2$. Let us set

$$A_0 = B_0 = \mathbf{a}, \qquad \dot{A}_0 = \sqrt{1 + H^2(u_*)\mathbf{a}^2}, \qquad \dot{B}_0 = \dot{A} + \delta$$

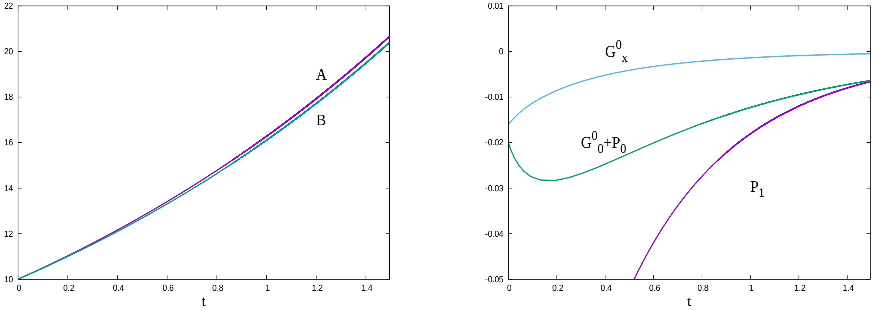
which is exactly Type I for $\delta = 0$. One chooses $\mathbf{a} = 10$, $\delta = 0.1$, the $\mathcal{S} = 0$ constraints yields

$$u_0^{(1)} = -0.231122, \qquad u_0^{(2)} = -0.233943$$

$$u_0^{(3)} = -0.152569, \qquad u_0^{(4)} = -0.645204.$$

One has $u_0^{(1)} \approx u_0^{(2)} \approx u_* = -0.2$, hence this should give a weakly perturbed Type I solution. It approaches Type II (the other two lead to singularity)

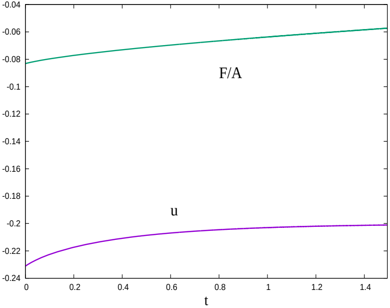
Initial values close to type I

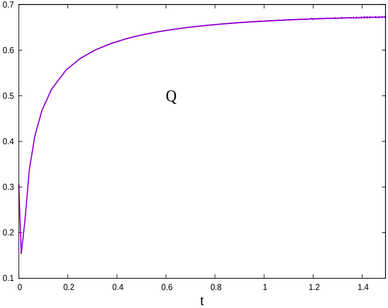


$$A/B \to const., \quad G^0_1 \to 0, \quad G^0_0 + P_0 \to 0, \quad P_1 \to 0.$$

The g-metric relaxes to de Sitter. Stuckelbergs move to Type II.

Initial values close to type I





$$u \to u_*, \qquad Q = \sqrt{\frac{F\dot{F}}{u_*^2 A}} \to q$$

Slightly perturbed Type I evolves to Type II

Strongly perturbed type I

$$b_0 = -19, \quad b_1 = 14, \quad b_2 = -10, \quad b_3 = 7,$$

$P_1(u_*) = 0$ gives $u_* = 1.63$. One sets again

$$A_0 = B_0 = \mathbf{a}, \quad \dot{A}_0 = \sqrt{1 + H^2(u_*)\mathbf{a}^2}, \quad \dot{B}_0 = \dot{A} + \delta$$

with $\mathbf{a} = 10$, $\delta = 0.1$. The $\mathcal{S}$ constraint yields

$$u_0^{(1)} = 1.1222, \quad u_0^{(2)} = 1.5909, \quad u_0^{(3)} = 1.6362, \quad u_0^{(4)} = 1.6680.$$

Here $u_0^{(3)}$ is the closest to u_* and leads to slightly perturbed Type I relaxing towards Type II. $u_0^{(2)}$ and $u_0^{(4)}$ give singular solutions. The root $u_0^{(1)}$ is the farthest from u_* and describes a strong perturbation leading to something new – decay into flat space.

Decay into flat space

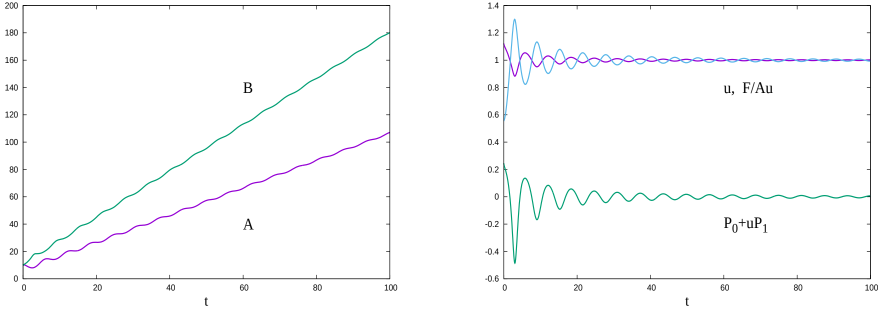


Figure: Strongly perturbed Type I may decay into flat space.

Conclusions

- Anisotropic deformations of the FLRW Type I cosmology in dRGT massive gravity have been studied for the first time.
- When perturbed, Type I cannot relax to itself, hence it is unstable.
- Generic strong perturbations lead to a collapse.
- If perturbed only slightly, the physical geometry relaxes back to de Sitter, hence it is stable.
- However, the Stuckelberg scalars change considerably and the f-metric approaches Type II.
- For some parameter values strong perturbations lead to a decay into flat space.
- A possible presence of ghost has not been studied yet.