# MOND Cosmology

arXiv:1106.4984, 1405.0393, 1608.07858

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# Dark Matter versus Modified Gravity

- GR great for solar system
- But not for galaxies

  Theory: $v^2 = \frac{GM}{r}$

  Observation: $v^2 \rightarrow \sqrt{a_0 GM}$

  $a_0 \sim 10^{-10} \frac{m}{s^2}$

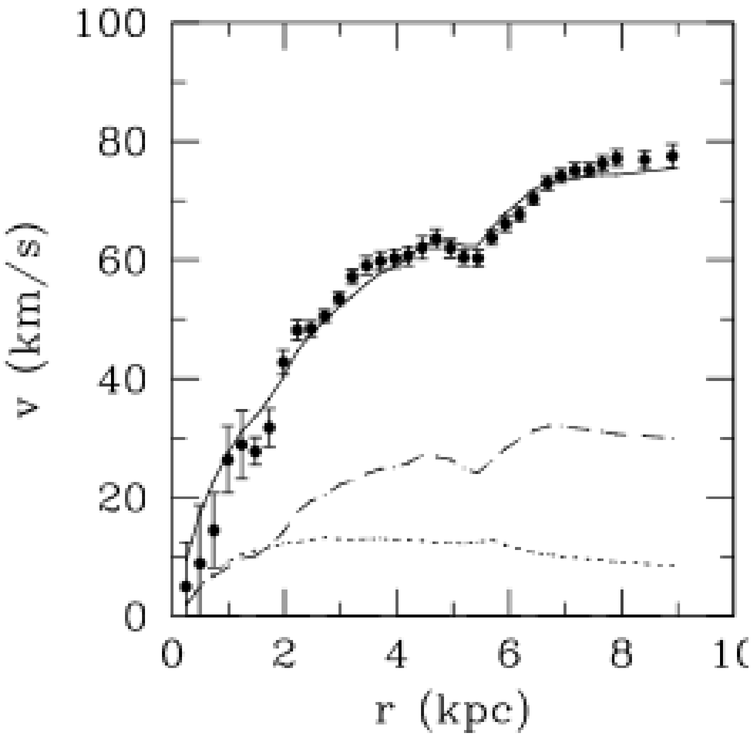
- Maybe missing mass

  But still no direct detection!

  LUX (arXiv:1608.07648)

- Or modified gravity

  MOND (Milgrom 1983)



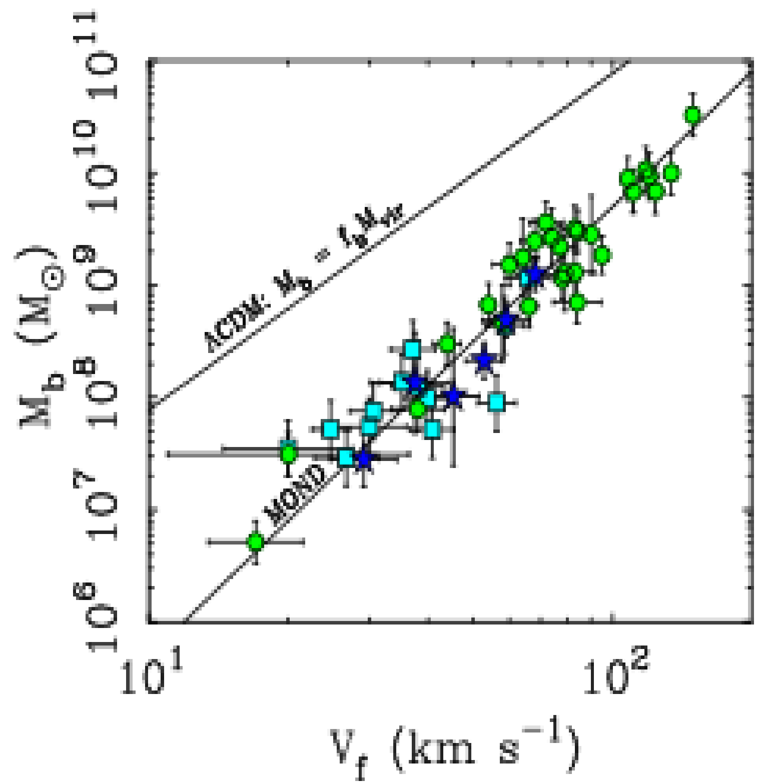
# What is MOND?

- Applies to static, localized mass distributions
  - Can predict Newtonian acceleration $g_N$
  - E.g., a spherical distribution $\rho(r)$

    $M(r) = \frac{4\pi}{c^2}\int_0^r ds\, s^2 \rho(s) \Rightarrow g_N(r) = \frac{GM(r)}{r^2}$
- MOND rule for the actual acceleration $g$
  - $g = \frac{g_N}{1 - \exp\left[-\sqrt{\frac{g_N}{a_0}}\right]}$
    - $g_N > a_0 \Rightarrow g \to g_N$ (Newtonian=GR regime)
    - $g_N < a_0 \Rightarrow g \to \sqrt{a_0 g_N}$ (MOND regime)
  - arXiv:1609.05917 (McGaugh, Lelli & Schombert)
    - Fit 2693 points in 153 late-type galaxies, range of $10^4$ in size
    - $a_0 = (1.20 \pm 0.02) \times 10^{-10}\ m/s^2$

# Observational Evidence for MOND in rotationally supported systems

- Baryonic Tully-Fisher Relation:
  - Asymptotic $v^4 = a_0 GM$
- Milgrom's Law:
  - Start needing DM for $g(r) < a_0$
- Freeman's Law:
  - Surface density $\Sigma < \frac{a_0}{G}$
- Sancisi's Law:
  - Bumps trace baryons
- BOTTOM LINE:
  - This works for galaxies
  - Equally strong for pressure-supported systems



# Fully replacing Dark Matter requires a relativistic extension

- MOND gives $g$ for static, localized systems
- DM contributes to other metric potentials, to evolving systems & to cosmology
  - Gravitational lensing
    - Need same as for GR + DM
  - Recently disturbed systems
    - Bullet Cluster & cluster cores
  - Expansion history in cosmology
    - Only needed for (about) $z < 3500$
  - CMB acoustic oscillations
    - Need 2nd & 3rd Doppler peaks equal
  - Structure formation
    - $E_G = 0.243 \pm 0.060$ (at $z = 0.57$) vs $0.402 \pm 0.012$ for GR + DM

# Our Strategy

- No extra fields or new matter couplings
  - Cf. TeVeS (Bekenstein 2004)
- Retain general coordinate invariance
  - $\mathcal{L} = \frac{c^4\sqrt{-g}}{16\pi G}\left[R - 2\Lambda + \frac{a_0^2}{c^4} f(\text{Ƶ}[g])\right]$
- Find $\text{Ƶ}[g]$ & $f(\text{Ƶ})$ to enforce
  - Tully-Fisher & lensing for small $\text{Ƶ} > 0$
  - GR for large $\text{Ƶ} > 0$
- Find $f(\text{Ƶ})$ for $\text{Ƶ} < 0$ to enforce
  - $\Lambda$CDM expansion history without CDM

# Static, Spherical, Nearly Flat Geometry

- $ds^2 = -[1 + b(r)]c^2dt^2 + [1 + a(r)]dr^2 + r^2 d\Omega^2$
  - Both $|a|$ & $|b|$ less than $10^{-6}$ for our Sun
- $\mathcal{L}_{GR} = \frac{r^2c^4}{16\pi G}\left[-\frac{1}{2}b'^2 + \frac{1}{2}\left(\frac{a}{r} - b'\right)^2\right]$
  - $\frac{\delta S}{\delta b} = \frac{c^4}{16\pi G}(ra)' - \frac{1}{2}r^2\rho$
  - $\frac{\delta S}{\delta a} = \frac{c^4}{16\pi G}(a - rb')$
- Geodesic equation for circular orbits
  - $\mu = t$ component ➔ $\frac{1}{2}b' - \frac{r}{c^2}\dot{\varphi}^2 = 0$
  - $\dot{\varphi} = \frac{v}{r}$ ➔ $v^4 = \frac{1}{4}c^4(rb')^2$

# Inferring equations for $b(r)$ & $a(r)$ from the data (in the MOND regime)

1. Tully-Fisher ➔ $v^4(r) = a_0 G M(r)$
   - Enclosed mass ➔ $M(r) = \frac{4\pi}{c^2}\int_0^r ds\, s^2 \rho(s)$
   - Circular geodesic ➔ $v^4(r) = \frac{1}{4}c^4[r b'(r)]^2$

$$\therefore \frac{c^4}{16\pi G} \times \frac{c^2}{2a_0}\partial_r (rb')^2 = \frac{1}{2}r^2\rho$$

2. Lensing ➔ $a(r) = r b'(r)$ if $b(r)$ obeys Tully-Fisher

GR + MOND Equations

- $\frac{\delta S}{\delta b} = \frac{c^4}{16\pi G}\left[\frac{c^2}{2a_0}\partial_r(r^2 b'^2)\right] - \frac{1}{2}r^2\rho = 0$
- $\frac{\delta S}{\delta a} = \frac{c^4}{16\pi G}[a - rb'] = 0$ unchanged from GR

$$\mathcal{L} = \frac{c^4\sqrt{-g}}{16\pi G}\left[R - 2\Lambda + \frac{a_0^2}{c^4} f(\text{Ƶ}[g])\right]$$

- Recall that Tully-Fisher + Lensing imply
  - $\mathcal{L} = \frac{c^4 r^2}{16\pi G}\left[-\frac{1}{2}b'^2 + \frac{1}{2}\left(\frac{a}{r} - b'\right)^2\right] + \frac{c^4 r^2}{16\pi G}\left[\frac{1}{2}b'^2 - \frac{c^2}{6a_0}b'^3\right]$
- Weak field form of the MOND addition is
  - $f(\text{Ƶ}[g]) = \frac{1}{2}\left(\frac{c^2 b'}{a_0}\right)^2 - \frac{1}{6}\left(\frac{c^2 b'}{a_0}\right)^3$
- Hence we conclude
  - $f(\text{Ƶ}) = \frac{1}{2}\text{Ƶ} - \frac{1}{6}\text{Ƶ}^{\frac{3}{2}} + O(\text{Ƶ}^2)$ for $\text{Ƶ} > 0$
  - $\text{Ƶ}[g] = \left(\frac{c^2 b'}{a_0}\right)^2$ + cubic for static, spherical

# The MOND Invariant $Ƶ[g]$ is NOT Local

- Simple Proof ➔ count the weak fields & $\partial_r$'s
  - MOND equations have TWO $b$'s & THREE $\partial_r$'s
  - Curvatures have ONE $(a \leftrightarrow b)$ & TWO $\left(\partial_r \leftrightarrow \frac{1}{r}\right)$'s
    - E.g. $R = -b'' - \frac{2b'}{r} + \frac{2a'}{r} + \frac{2a}{r^2}$
- Nonlocal reconstruction of $b(r)$ from $R_{00} = \frac{1}{2r}(rb)''$
  - $\square \equiv \frac{1}{\sqrt{-g}} \partial_\mu \left(\sqrt{-g}\, g^{\mu\nu} \partial_\nu\right) \rightarrow \frac{1}{r} \partial_r^2 r$ ➔ $\frac{1}{\square} R_{00} \rightarrow \frac{1}{2} b$
- Achieving an invariant form
  - $X[g] \equiv -\frac{1}{\square} 1$ grows with time ➔ $u^\mu[g] \equiv \frac{g^{\mu\nu} \partial_\nu X}{\sqrt{-g^{\alpha\beta} \partial_\alpha X \partial_\beta X}}$
  - $Ƶ[g] \equiv \frac{4c^4}{a_0^2} g^{\alpha\beta} \left[\partial_\alpha \frac{1}{\square}\left(R_{\mu\nu} u^\mu u^\nu\right)\right] \left[\partial_\beta \frac{1}{\square}\left(R_{\rho\sigma} u^\rho u^\sigma\right)\right]$

# Who ordered THAT?!

- We don’t believe fundamental physics is nonlocal
- But quantum effective actions are
  - Cf. vacuum polarization in QED
  - $\partial_\nu \left[ F^{\nu\mu}(x) + \int d^4x' \Pi(x;x') F^{\nu\mu}(x') \right] = J^\mu(x)$
  - MOND as GR vacuum polarization from inflation?
- No derivation ➔ $Ƶ[g]$ purely phenomenological
- But this does help explain two things:
  1. There is a “beginning of time” for initializing $\frac{1}{\square}$
  2. GR deviations at large scales, not small ones

$$f(\text{Ƶ}) \text{ in } \mathcal{L}_{MOND} = \frac{a_0^2}{16\pi G} f(\text{Ƶ}[g])\sqrt{-g}$$

- $\text{Ƶ}[g] \equiv \frac{4c^4}{a_0^2} g^{\alpha\beta} \left[\partial_\alpha \frac{1}{\Box}\left(R_{\mu\nu}u^\mu u^\nu\right)\right]\left[\partial_\beta \frac{1}{\Box}\left(R_{\rho\sigma}u^\rho u^\sigma\right)\right]$
  - Static Bound ➔ $\frac{1}{\Box}R_{\mu\nu}u^\mu u^\nu = F(\vec{x})$ ➔ $\text{Ƶ} = \frac{4c^4}{a_0^2} g^{ij}\partial_i F \partial_j F > 0$
  - Cosmology ➔ $\frac{1}{\Box}R_{\mu\nu}u^\mu u^\nu = G(t)$ ➔ $\text{Ƶ} = \frac{4c^4}{a_0^2} g^{tt}(\dot{G})^2 < 0$
- Gravitationally bound systems have $\text{Ƶ} > 0$
  - Small Ƶ ➔ $f(\text{Ƶ}) = \frac{1}{2}\text{Ƶ} - \frac{1}{6}\text{Ƶ}^{3/2} + O(\text{Ƶ}^2)$ gives TF & Lensing
  - Large Ƶ ➔ $f(\text{Ƶ}) \to 0$ preserves solar system tests
  - E.g., $f(\text{Ƶ}) = \frac{1}{2}\text{Ƶ}\,\text{Exp}\left[-\frac{1}{3}\sqrt{\text{Ƶ}}\right]$ works
- Cosmology has $\text{Ƶ} < 0$

➔ Choose $f(\text{Ƶ})$ to get ΛCDM expansion history without CDM

# General Field Equations (Absorb Λ into $T_{\mu\nu}$)

- Localize with auxiliary scalars

$$\mathcal{L} = \frac{c^4}{16\pi G}\left\{R + \frac{a_0^2}{c^4} f\left(\frac{g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi}{c^{-4}a_0^2}\right) - \left[\partial_\mu\xi\partial_\nu\phi g^{\mu\nu} + 2\xi R_{\mu\nu}u^\mu u^\nu\right] - \left[\partial_\mu\psi\partial_\nu\chi g^{\mu\nu} - \psi\right]\right\}\sqrt{-g}$$

- Auxiliary scalar equations
  - $\phi[g] = \frac{2}{\Box} R_{\alpha\beta}u^\alpha u^\beta$ , $\chi[g] = -\frac{1}{\Box}1$
  - $\xi[g] = \frac{2}{\Box} D^\mu \left[\partial_\mu\phi\, f'\left(\frac{g^{\rho\sigma}\partial_\rho\phi\partial_\nu}{c^{-4}a_0^2}\right)\right]$ , $\psi[g] = \frac{4}{\Box} D_\mu \left[\frac{\xi(g^{\mu\rho} + u^\mu u^\rho)u^\sigma R_{\rho\sigma}}{\sqrt{-g^{\alpha\beta}\partial_\alpha\chi\partial_\beta\chi}}\right]$

- Modified Einstein equation

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\left[R + \frac{a_0^2}{c^4}f(\mathrm{Z}) - g^{\rho\sigma}(\partial_\rho\xi\partial_\sigma\phi + \partial_\rho\psi\partial_\sigma\chi) - 2\xi u^\rho u^\sigma R_{\rho\sigma} + \psi\right]$$
$$+ \partial_\mu\phi\partial_\nu\phi f'(\mathrm{Z}) - \partial_{(\mu}\xi\partial_{\nu)}\phi - \partial_{(\mu}\psi\partial_{\nu)}\chi - 2\xi\left[2u_{(\mu}R_{\nu)\alpha}u^\alpha + u_\mu u_\nu u^\alpha u^\beta R_{\alpha\beta}\right]$$
$$- \left[\Box\xi u_\mu u_\nu + g_{\mu\nu}D_\alpha D_\beta\left(\xi u^\alpha u^\beta\right) - 2D_\alpha D_{(\mu}\left(\xi u_{\nu)}u^\alpha\right)\right] = \frac{8\pi G}{c^4} T_{\mu\nu}$$

- NB $R_{\mu\nu} = 0$ still vacuum solution ➔ no change to gravitational radiation

# Specialize to FRW

$$ds^2 = -c^2 dt^2 + a^2(t) d\vec{x} \cdot d\vec{x} \text{ with } H(t) \equiv \frac{\dot{a}}{a}$$

- Auxiliary Scalars
  - $\dot{\phi}(t) = \frac{6}{a^3(t)} \int_{t_i}^{t} ds\, a^2(s) \frac{d}{ds}[a(s) H(s)]$ ➔ $Ƶ(t) = -\frac{\dot{\phi}^2(t)}{c^{-2} a_0^2}$
  - $\dot{\chi}(t) = \frac{1}{a^3(t)} \int_{t_i}^{t} ds\, a^3(s)$ ➔ $u^\mu(t) = \delta_0^\mu$
  - $\xi(t) = 2 \int_{t_i}^{t} ds\, \dot{\phi}(s) f'(Ƶ(s))$ , $\psi(t) = 0$
- Modified Friedmann Equations

$$3H^2 + \frac{a_0^2}{2c^2} f(Ƶ) + 3H\dot{\xi} + 6H^2 \xi = \frac{8\pi G}{c^2} \rho$$

$$-2\dot{H} - 3H^2 - \frac{a_0^2}{2c^2} f(Ƶ) - \ddot{\xi} - \left(\tfrac{1}{2}\dot{\phi} + 4H\right)\dot{\xi} - \left(4\dot{H} + 6H^2\right)\xi = \frac{8\pi G}{c^2} p$$

(2nd equation follows from other + conservation ➔ only need 1st)

# Reconstructing $f(\text{Ƶ})$ from

$$3H^2 + \frac{a_0^2}{2c^2} f(\text{Ƶ}) + 3H\dot{\xi} + 3H^2\xi = \frac{8\pi G}{c^2}\rho$$

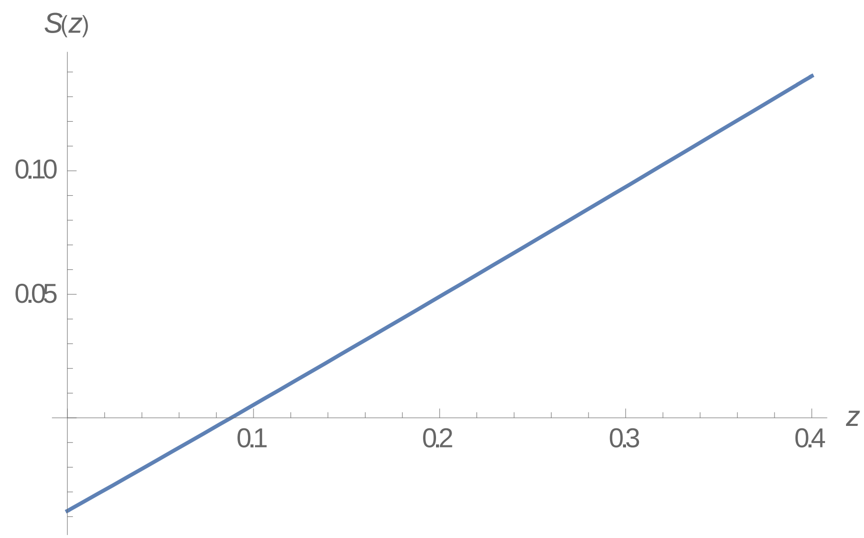
- Switch from time $t$ to redshift $z \equiv \frac{a_0}{a(t)} - 1$
- Factor out $H_0$ ➔ $\widetilde{H}(z) \equiv H(t)/H_0$
  - Dimensionless constant: $\alpha \equiv \frac{6cH_0}{a_0} \approx 33$
- ΛCDM Expansion History
  - $\widetilde{H}^2 = \Omega_r(1+z)^4 + \Omega_m(1+z)^3 + \Omega_\Lambda$
  - $\Omega_r \cong 9.15 \times 10^{-5}$ , $\Omega_m \cong 0.309$ , $\Omega_\Lambda \cong 0.691$
- Energy density without CDM
  - $\frac{8\pi G\rho}{3c^2H_0^2} - \widetilde{H}^2 = -\Omega_c(1+z)^3$ , $\Omega_c \cong 0.260$
- System of equations for $\dot{\phi}/6H_0$ and $f(\text{Ƶ})$
  - $\frac{\dot{\phi}}{6H_0} = (1+z)^3 \int_z^\infty dz' \frac{\widetilde{H}(z') - \widetilde{H}'(z')}{(1+z')^4}$ ➔ $\text{Ƶ} = -\frac{1}{\alpha^2}\left(\frac{\dot{\phi}}{6H_0}\right)^2$
  - $\frac{6}{\alpha^2} f(\text{Ƶ}) + 12\widetilde{H}\frac{\dot{\phi}}{6H_0} f'(\text{Ƶ}) + 12\,\widetilde{H}^2 \int_z^\infty dz' \frac{\dot{\phi}}{6H_0} f'(\text{Ƶ}) = -\Omega_c(1+z)^3$

# Implementing Reconstruction

1. Change dependent variables from $\frac{\dot{\phi}}{6H_0}$ & $f(Ƶ)$ to
   - $S(z) \equiv \frac{\sqrt{-Ƶ}}{\alpha}$ & $\mathcal{F}(z) \equiv -\frac{f(Ƶ)}{\alpha^2 \Omega_c}$
2. Solve equations for $S(z)$ & $\mathcal{F}(z)$
   - $S(z) = (1+z)^3 \int_z^\infty dz' \, \frac{\Omega_r (1+z')^4 + \frac{1}{2}\Omega_m (1+z')^3 - \Omega_\Lambda}{(1+z')^4 \widetilde{H}(z')}$
   - $\frac{1}{2}\mathcal{F}(z) + \frac{\widetilde{H}(z)\mathcal{F}'(z)}{2S'(z)} + \widetilde{H}^2(z) \int_z^\infty dz' \, \frac{\mathcal{F}'(z')}{(1+z')S'(z')\widetilde{H}(z')} = \frac{1}{12}(1+z)^3$
3. Invert $S(z)$ ➔ $z(S)$
   - $f(Ƶ) = -\alpha^2 \Omega_c \mathcal{F}\left(z\left(\frac{\sqrt{-Ƶ}}{\alpha}\right)\right)$

# $\Omega_\Lambda$ makes $S(z)$ change sign at $z_* \cong 0.088$

- For $z \gg 1$
  - $S(z) = \sqrt{\Omega_r}\, z^2 + O(z)$
- $S(z)$ monotonic in $z$
  - $z(S)$ exists but
  - $S = \pm \frac{\sqrt{-z}}{\alpha}$ multivalued
- Choose + root
  - Exactly recovers ΛCDM for $z_* < z < \infty$
  - Small deviations for $0 < z < z_*$
  - These are actually good!

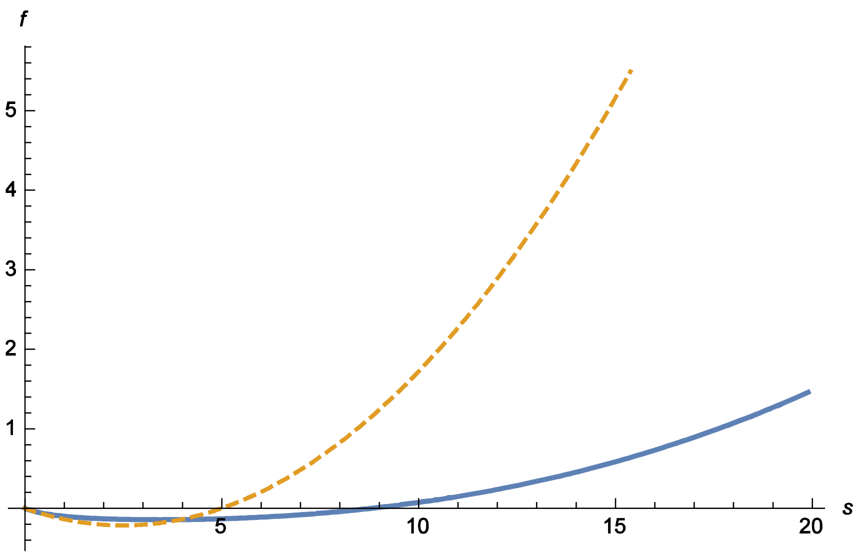


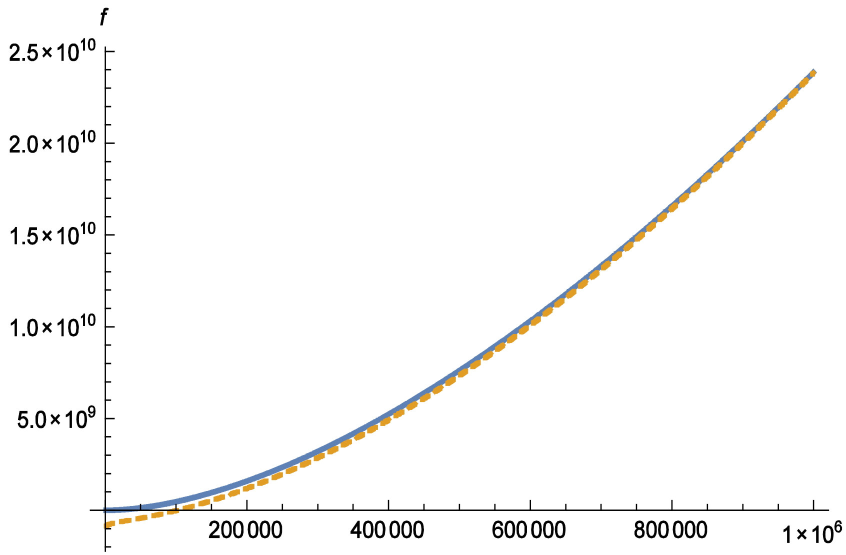
# Asymptotic Expansions

- Large $\zeta \equiv -\text{Ƶ}/\Omega_r$ with $\beta \equiv \sqrt{\alpha}\Omega_m/\Omega_r$
  - $f(\text{Ƶ}) = -\frac{\sqrt{\alpha}\Omega_c}{33}\,\zeta^{\frac{3}{4}}\left[1 - \frac{\beta}{\zeta^{\frac{1}{4}}} + \frac{155}{176}\frac{\beta^2}{\zeta^{\frac{1}{2}}} - \frac{625}{768}\frac{\beta^3}{\zeta^{\frac{3}{4}}} + O\left(\frac{\beta^4}{\zeta}\right)\right]$
- Small $\mathfrak{z} \equiv -\text{Ƶ}/\alpha^2\Omega_\Lambda$
  - $f(\text{Ƶ}) = -\frac{\alpha^2\Omega_\Lambda\Omega_c}{12\Omega_m}\sqrt{\mathfrak{z}}\,[A + B\sqrt{\mathfrak{z}} + O(\mathfrak{z})]$
  - $A \cong -0.764$ and $B \cong +0.127$
  - NB $f(\text{Ƶ}) > 0$ for small $-\text{Ƶ}$
- Numerically solved for all $\text{Ƶ} < 0$

# Comparing to numerical solution

(yellow gives expansions for $-f$ vs $s \equiv \sqrt{-Z}/\alpha$)



# Model Deviates from ΛCDM for $0 < z < z_* \approx 0.088$

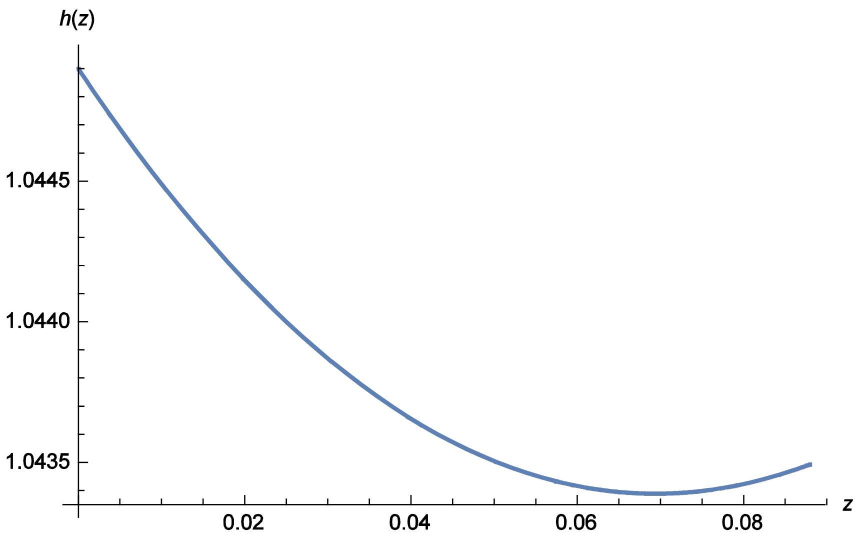
- Rescale

  $\dot{\phi} \equiv H_0 \Phi(z) \quad H \equiv H_0 h(z)$

- Small expansion of $f(\text{Ƶ})$

  1st order ODE's for $\Phi(z), h(z)$

- $h(z_*) \cong 1.043$ GROWS to $h(0) \cong 1.045$
  - ΛCDM FALLS to $h(0) = 1$
- This is actually good!



# Different Measures of $H_0$ in units of km/(s Mpc)

- Inferring it from large $z$ data (CMB,BAO)
  - $H_0 = 67.74 \pm 0.46$
  - arXiv:1502.01589
- Inferring it from small $z$ data (Hubble plots)
  - $H_0 = 73.24 \pm 1.74$ (3.2$\sigma$ discrepancy)
  - arXiv:1604.01424
- This isn't going away as the data improves!
- With large $z$ parameters MOND cosmology predicts the small $z$ measurement should give
  - $H_0 = 1.045 \times (67.74 \pm 0.46) = 70.79 \pm 0.48$
  - Only 1.4$\sigma$ discrepancy

# Conclusions

- Nonlocal, metric-based realization of MOND
  - $\mathcal{L} = \frac{c^4}{16\pi G}(R - 2\Lambda)\sqrt{-g} \; + \; \frac{a_0^2}{16\pi G} f(\text{Ƶ}[g])\sqrt{-g}$
  - View nonlocality as vacuum polarization of inflationary gravitons
- Full causal & conserved field equations derived for any $f(\text{Ƶ})$
  - See arXiv:1405.0393 (Eqn. 17 generally, Eqn. 40 for cosmology)
  - Gravitational radiation unchanged
- Choose function $f(\text{Ƶ})$ to
  - Reproduce Tully-Fisher and lensing (small $\text{Ƶ} > 0$)
  - Preserve solar system tests (large $\text{Ƶ} > 0$)
  - Reproduce $\Lambda$CDM expansion history without DM ($\text{Ƶ} < 0$)
    - Only exact for $z_* < z < \infty$ (gets BBN & recombination time right)
    - $0 < z < z_* \approx 0.088$ resolves tension in different measures of $H_0$!
    - $f(\text{Ƶ})$ is not small for $\text{Ƶ} < 0$ ➔ reasonable chance for good cosmology
- Next step: test model with
  - Evolving systems (cluster cores, Bullet Cluster)
  - CMB & growth of structure