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# HEAVY SPIN-2 DARK MATTER


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IESC Cargèse, May 4 2017

NORDITA

# Heavy spin-2 Dark Matter

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# Based on the work

1604.08564 E. Babichev et al, Phys. Rev. D **94** (2016) no.8

1607.03497 E. Babichev et al, JCAP **1609** (2016) no.09

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Kurt Hinterbichler
Edvard Mörtsell

# Outline of Talk

- Introduction & Motivation
- Basic details & more Motivation
- BM as theory of gravitating massive spin-2
- DM phenomenology
- Summary & Outlook

# Why modify GR?
# And in what way?

# Why modify GR?
# And in what way?

Do you want to ask these questions?
What kind of answers do you want?

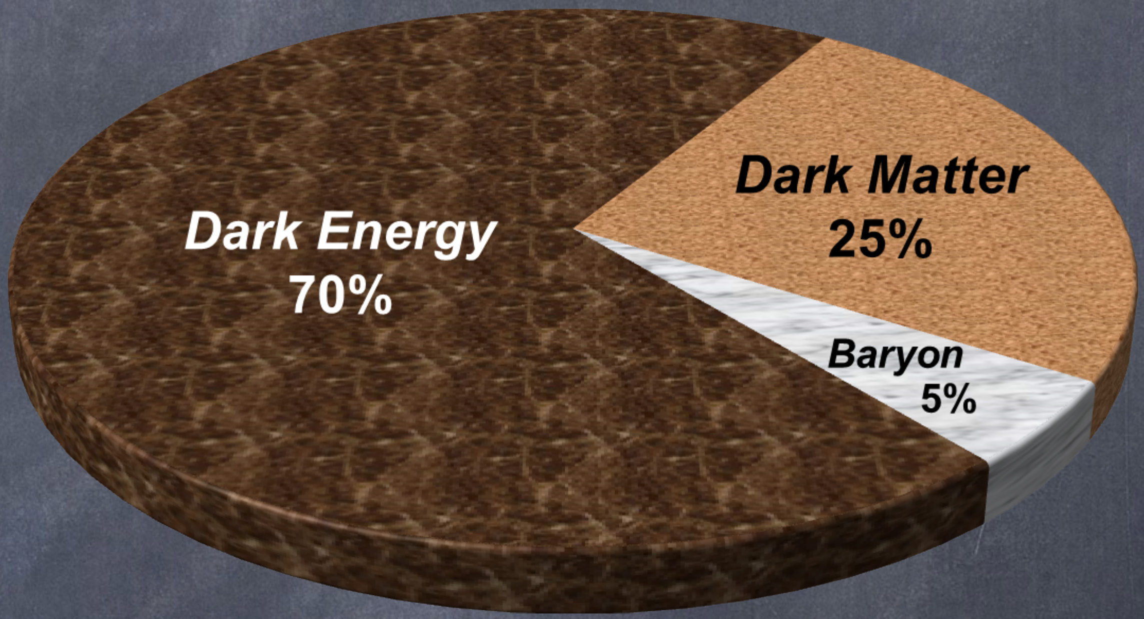
# Introduction & Motivation

I) - Understanding gravity & particularly cosmology and the Dark sectors – plethora of alleys

II) - Natural & direct generalisations of standard field theory (including GR), not ad hoc modifications but based on fundamental principles – very few alleys. Therefore promising for understanding gravity at a deeper level. Two logical alternatives:
    - Either not realised in nature –> why not?
    - Or are realised in nature –> almost certainly will increase our understanding of the Dark sectors –> pointing back to motivation I

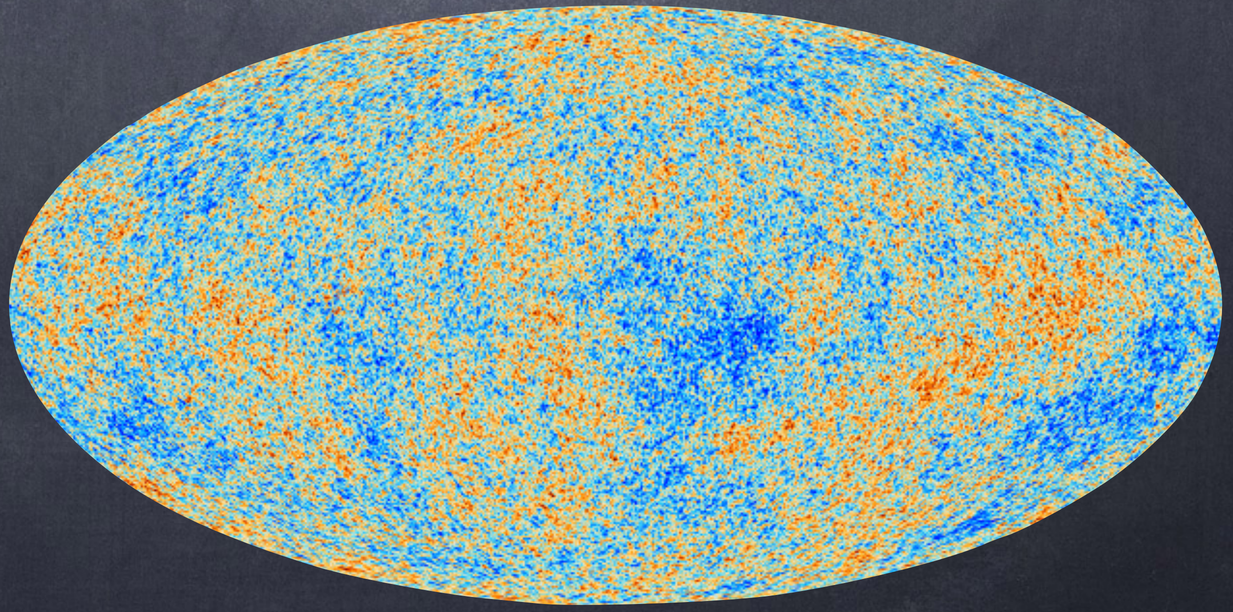
# Motivation I: Cosmology & Dark sectors

GR & the SM are quite adequate to explain observations thus far




Provided ...

- ... we accept the inclusion of, only indirectly inferred, Dark sources which totally dominate the energy budget
- And dont think too seriously about the cosmological constant problem(s) (CCP(s))
- Resolution of the CCP(s) seem to require new understanding of GR, QFT or both
- QFT very robust framework so modification of gravity away from GR appears to be more promising - less radical
- But GR is also a quite robust theory/model so modifications must make sense theoretically



# Motivation II: Field theory

- Lower spin fields well understood and do exist in nature. For the bosonic sector
  - Spin-0:

$$(\nabla^2 - m^2)\phi = 0$$

  - Spin-1:

$$\left(\nabla^2 - m^2 - \Lambda\right) A_\mu = 0\,, \qquad \nabla^\mu A_\mu = 0$$

  - Spin-2:

$$\left(\nabla^2 - m^2 - \tfrac{2\Lambda}{3}\right) h_{\mu\nu} = 0\,, \qquad \nabla^\mu h_{\mu\nu} = 0\,, \qquad h = 0$$

# Motivation II: Field theory

- Lower spin fields well understood and do exist in nature. For the bosonic sector
  - Spin-0:

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  - Spin-1:

    $$\left(\nabla^2 - m^2 - \Lambda\right) A_\mu = 0\,, \qquad \nabla^\mu A_\mu = 0$$

  - Spin-2:

    $$\left(\nabla^2 - m^2 - \tfrac{2\Lambda}{3}\right)\, h_{\mu\nu} = 0\,,$$

Massless and massive fields exist in nature

Massless field exist in nature, BUT ...

# Motivation II: Field theory

- Spin-2 particles carry the charge they mediate! Spin-2 theory beg for non-linear completion; just as massless spin-2 theory beg for GR completion.
- Massless spin-2 is uniquely defined by GR; no interacting massless spin-2 theory

S. Deser, Class. Quant. Grav. **4** (1987)

N. Boulanger et al, Nucl. Phys. B **597** (2001)

- Looking for massive spin-2 theory we must therefore consider a non-linear theory; the corresponding spin-2 particles either exist in nature or they do not
- Possibly, indeed preferably (!), massive spin-2 in conjunction with massless spin-2 --> bimetric/ multimetric theories

# Introduction & Motivation

- History started 1939 with

M. Fierz and W. Pauli, Proc. Roy. Soc. Lond. A 173, 211 (1939)

- Progress halted 1972 by no-go

D. G. Boulware and S. Deser, Phys. Rev. D 6, 3368 (1972)

- Resparked interest in 2000s; A-HGS, DGP, CNPT ...

N. Arkani-Hamed, H. Georgi and M. D. Schwartz and Annals Phys. **305** (2003)

G. R. Dvali, G. Gabadadze and M. Porrati, Phys. Lett. B 485, 208 (2000)

P. Creminelli et al, JHEP **0509** (2005)

- Conjectured resolution in 2010 dRGT

C. de Rham, G. Gabadadze and A. J. Tolley, Phys. Rev. Lett. 106, 231101 (2011)

- Proved & extended by HR in 2011

S. F. Hassan and R. A. Rosen, JHEP 1107, 009 (2011)

M. Fierz & W. Pauli

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DGP,

For more details see review by
A. Schmidt-May & M. von Strauss

A. Schmidt-May and M. von Strauss, J. Phys. A **49**, no. 18, 183001 (2016)

- Conjectured resolution in 2010 dRGT

C. de Rham, G. Gabadadze and A. J. Tolley, Phys. Rev. Lett. 106, 231101 (2011)

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# Basic details

$$\alpha \equiv \frac{m_f}{m_g}$$

Theory is defined by the covariant action

$$S[g,f] = m_g^2 \int \mathrm{d}^4x \left[\sqrt{|g|}\,R(g) + \alpha^2\sqrt{|f|}\,R(f) - 2m^2\sqrt{|g|}\,V(S;\beta_n)\right]$$

with the interactions governed by

$$\sqrt{|g|}\,V(S;\beta_n) = \sqrt{|g|}\sum_{n=0}^{4}\beta_n e_n(S) = \sqrt{|g|}\,\beta_0 + \sqrt{|g|}\sum_{n=1}^{3}\beta_n e_n(S) + \sqrt{|f|}\,\beta_4 = \sqrt{|f|}\,V(S^{-1};\beta_{4-n})$$

in terms of the square-root matrix

$$S = \sqrt{g^{-1}f}, \qquad S^\rho{}_\sigma S^\sigma{}_\nu = g^{\rho\mu}f_{\mu\nu}$$

along with the elementary symmetric polynomials

$$e_n(S) = S^{\mu_1}{}_{[\mu_1}\cdots S^{\mu_n}{}_{\mu_n]} = \tfrac{1}{n!}\delta^{\mu_1\cdots\mu_n}_{\nu_1\cdots\nu_n}S^{\nu_1}{}_{\mu_1}\cdots S^{\nu_n}{}_{\mu_n} = \tfrac{1}{n!(4-n)!}\epsilon^{\mu_1\cdots\mu_n\lambda_{n+1}\cdots\lambda_4}\epsilon_{\nu_1\cdots\nu_n\lambda_{n+1}\cdots\lambda_4}S^{\nu_1}{}_{\mu_1}\cdots S^{\nu_n}{}_{\mu_n}$$

or even more explicitly

$$e_0(S) = 1\,, \quad e_1(S) = \mathrm{Tr}(S)\,, \quad e_2(S) = \tfrac{1}{2}(\mathrm{Tr}(S)^2 - \mathrm{Tr}(S^2))\,, \quad \ldots \quad e_4(S) = \det(S)$$

# Basic details

$$\alpha \equiv \frac{m_f}{m_g}$$

Theory is defined by the covariant action

$$S[g,f] = m_g^2 \int d^4x \left[ \sqrt{|g|}\, R(g) + \dots \right]$$

with

$$\sqrt{|g|} \dots \beta_{4-n})$$

in te…

along…

$$e_n(S) = \dots S^{\nu_n}_{\ \mu_n} = \frac{1}{n!(4-n)!}\epsilon^{\mu_1\cdots\mu_n\lambda_{n+1}\cdots\lambda_4}\epsilon_{\nu_1\cdots\nu_n\lambda_{n+1}\cdots\lambda_4} S^{\nu_1}_{\ \mu_1}\cdots S^{\nu_n}_{\ \mu_n}$$

- Two tensor fields with non-polynomial non-derivative coupling
- Completely symmetric in the tensor fields
- Neither g nor f corresponds to mass eigenstates

or even more explicitly

$$e_0(S) = 1\,, \quad e_1(S) = \mathrm{Tr}(S)\,, \quad e_2(S) = \tfrac{1}{2}(\mathrm{Tr}(S)^2 - \mathrm{Tr}(S^2))\,, \quad \dots \quad e_4(S) = \det(S)$$

# Basic details

This lead to the equations of motion

$$E_{\mu\nu} \equiv \mathcal{G}_{\mu\nu} + m^2 V_{\mu\nu} = 0\,, \qquad V_{\mu\nu} \equiv -\frac{2}{\sqrt{|g|}}\frac{\partial(\sqrt{|g|}V)}{\partial g^{\mu\nu}}$$

$$\tilde{E}_{\mu\nu} \equiv \tilde{\mathcal{G}}_{\mu\nu} + \frac{m^2}{\alpha^2}\tilde{V}_{\mu\nu} = 0\,, \qquad \tilde{V}_{\mu\nu} \equiv -\frac{2}{\sqrt{|f|}}\frac{\partial(\sqrt{|g|}V)}{\partial f^{\mu\nu}}$$

with e.g.

$$V_{\mu\nu} = g_{\mu\rho}\Big[\beta_0\delta^\rho_\nu - \beta_1\left(S^\rho{}_\nu - e_1\delta^\rho_\nu\right) + \beta_2\left([S^2]^\rho{}_\nu - e_1 S^\rho{}_\nu + e_2\delta^\rho_\nu\right)$$
$$- \beta_3\left([S^3]^\rho{}_\nu - e_1[S^2]^\rho{}_\nu + e_2 S^\rho{}_\nu - e_3\delta^\rho_\nu\right)\Big]$$

along with Bianchi constraints

$$\sqrt{|g|}\,g^{\mu\rho}\nabla_\rho V_{\mu\nu} = -\sqrt{|f|}\,f^{\mu\rho}\tilde{\nabla}_\rho\tilde{V}_{\mu\nu} = 0$$

# Basic details

This lead to the equations of motion

$$E_{\mu\nu} \equiv \mathcal{G}_{\mu\nu} + m^2 V_{\mu\nu} = 0\,,$$

with

— Nonderivative corrections to Einsteins equations

— Standard continuity equations hold for matter

along

$$\sqrt{|g|}\, g^{\mu\rho} \nabla_\rho V_{\mu\nu} = -\sqrt{|f|}\, f^{\mu\rho} \tilde{\nabla}_\rho \tilde{V}_{\mu\nu} = 0$$

# Proportional solutions & Mass spectrum

A conformal ansatz $f_{\mu\nu} = c^2 g_{\mu\nu}$ reduce the equations to

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \begin{pmatrix} \Lambda_g \\ \Lambda_f \end{pmatrix} g_{\mu\nu} = 0$$

Consistency requires $\Lambda_g = \Lambda_f$ :

$$\alpha^2 \beta_3 c^4 + (3\alpha^2 \beta_2 - \beta_4) c^3 + 3(\alpha^2 \beta_1 - \beta_3) c^2 + \left(\alpha^2 \beta_0 - 3\beta_2\right) c - \beta_1 = 0$$

Generically determines $c = c(\alpha, \beta_n)$.

# Proportional solutions & Mass spectrum

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Generically determines $c = c(\alpha, \beta_n)$. Decoupled perturbations

$$\tilde{\mathcal{E}}_{\mu\nu}{}^{\rho\sigma} \delta G_{\mu\nu} + \Lambda \delta G_{\mu\nu} = 0$$

$$\tilde{\mathcal{E}}_{\mu\nu}{}^{\rho\sigma} \delta M_{\mu\nu} + \Lambda \delta M_{\mu\nu} + \frac{\tilde{m}^2}{2} \left(\delta M_{\mu\nu} - g_{\mu\nu} \delta M\right) = 0$$

with $\delta G_{\mu\nu} = \delta g_{\mu\nu} + \alpha^2 \delta f_{\mu\nu}\,, \qquad \delta M_{\mu\nu} = \frac{1}{2c} \left(\delta f_{\mu\nu} - c^2 \delta g_{\mu\nu}\right)$

# Matter couplings

Matter can couple minimally to either one of the metrics, but not both. Gives possibility of "strong gravity" à la

C. J. Isham, A. Salam and J. A. Strathdee, Phys. Rev. D **3** (1971)

or more generally a "hidden" sector of "mirror matter"

$$\mathcal{L}_{\text{matter}} = \sqrt{|g|}\mathcal{L}(g, \Phi) + \sqrt{|f|}\mathcal{L}(f, \Psi)$$

Earlier attempts to get DM out of "mirror matter"

Z. Berezhiani et al, JHEP **0907** (2009)

L. Blanchet and L. Heisenberg, Phys. Rev. D **91** (2015)

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Z. Berezhiani et al, JHEP **0907** (2009)

L. Blanchet and L. Heisenberg, Phys. Rev. D **91** (2015)

We only consider standard matter and get DM from presence of additional spin-2

# Recovering GR

Bimetric solution space cover all of GR solution space.
But generic BM solutions are not close to GR solutions and can therefore be ruled out by observations.

–> Look for generic features of BM solutions which keep them close enough to GR solutions

# Spherically symmetric (static) solutions

## Bidiagonal ansätze

$$g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = -e^\nu \mathrm{d}t^2 + e^\lambda \mathrm{d}r^2 + r^2 \mathrm{d}\Omega^2\,,$$

$$f_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = -e^{\tilde\nu} \mathrm{d}t^2 + e^{\tilde\lambda}(r + r\mu)'^2 \mathrm{d}r^2 + (r + r\mu)^2 \mathrm{d}\Omega^2$$

## Linearised solutions

E. Babichev and M. Crisostomi, Phys. Rev. D **88** (2013)

J. Enander and E. Mörtsell, JHEP **1310** (2013)

$$\mu = -\frac{C_2(1+\alpha^2)\, e^{-m_{\rm FP} r}\,(1 + m_{\rm FP} r + m_{\rm FP}^2 r^2)}{3\, m_{\rm FP}^4\, r^3}\,,$$

$$\lambda = \frac{C_1}{r} + \frac{2\, C_2\, \alpha^2\, e^{-m_{\rm FP} r}\,(1 + m_{\rm FP} r)}{3\, m_{\rm FP}^2\, r}\,, \qquad \nu = -\frac{C_1}{r} - \frac{4\, C_2\, \alpha^2 (1+\alpha^2)\, e^{-m_{\rm FP} r}}{3\, m_{\rm FP}^2\, r}$$

$$\tilde\lambda = \frac{C_1}{r} - \frac{2\, C_2\, e^{-m_{\rm FP} r}\,(1 + m_{\rm FP} r)}{3\, m_{\rm FP}^2\, r}\,, \qquad \tilde\nu = -\frac{C_1}{r} + \frac{4\, C_2\, e^{-m_{\rm FP} r}}{3\, m_{\rm FP}^2\, r}$$

with $C_1 = \frac{r_S}{1+\alpha^2}, \quad C_2 = \frac{m_{\rm FP}^2 r_S}{1+\alpha^2}$ $\qquad r_S = \frac{1+\alpha^2}{m_{\rm Pl}^2}\int_0^{R_\odot} \rho\, r^2 dr$

valid for $\nu, \tilde\nu, \lambda, \tilde\lambda \ll 1$ but arbitrary $\mu$

# Spherically symmetric (static) solutions

- Proven that GR is recovered below the Vainshtein radius

$$r_{\rm V} = \left(\frac{r_S}{m_{\rm FP}^2}\right)^{1/3}$$

E. Babichev, Phys. Rev. D **88** (2013)

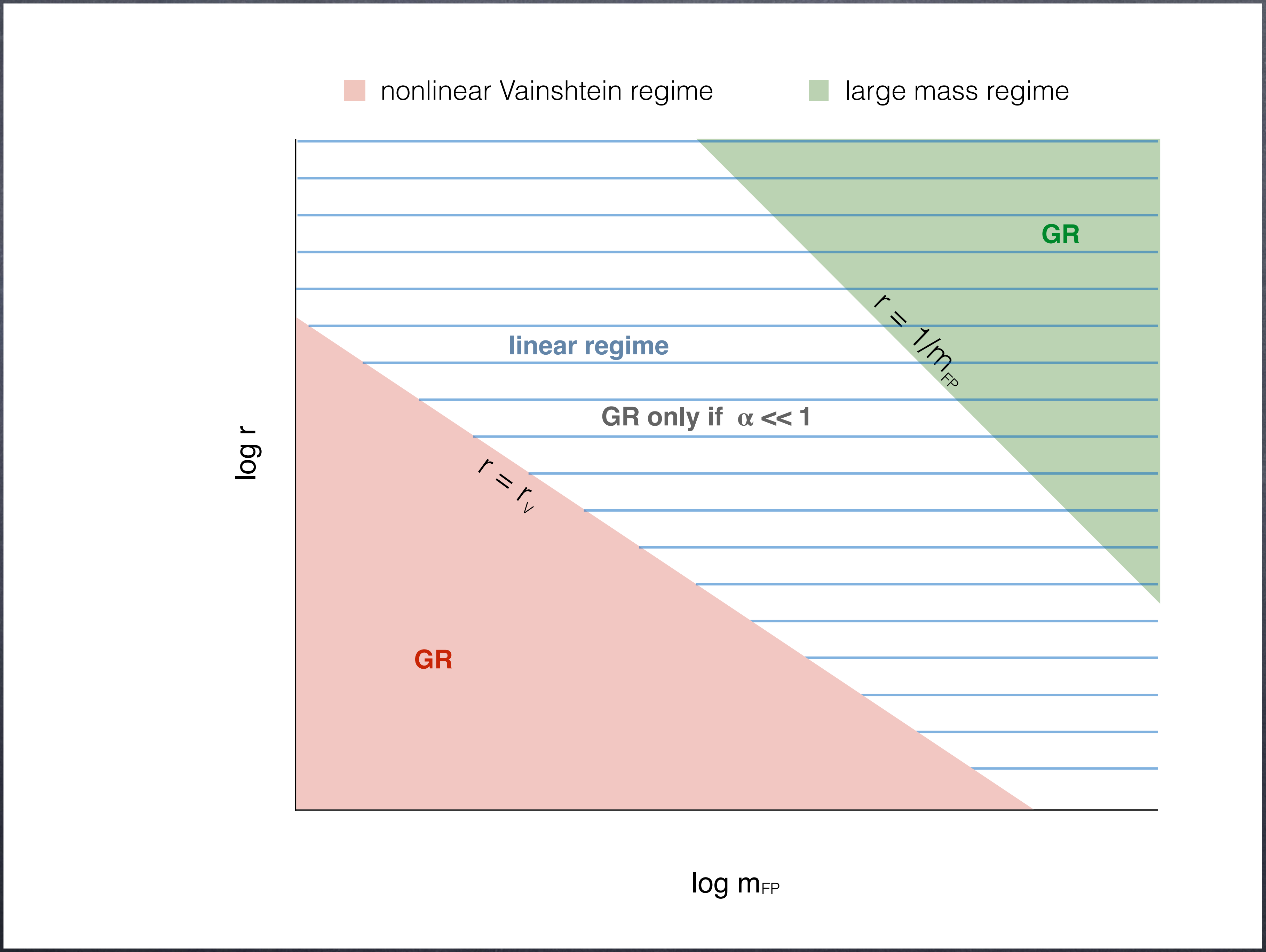
- Easy to see that GR is recovered continuously in the limit of large mass $m_{\rm FP}$ ; or at distances beyond the Compton wavelength

- Can show that GR is recovered also in the limit of vanishing $\alpha$, with Vainshtein mechanism taking over at small enough distances

E. Babichev, Phys. Rev. D **88** (2013)

# Spherically symmetric (static) solutions

## Recovering GR like behaviour



# Cosmological solutions

Bidiagonal ansätze

$$g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = -\mathrm{d}t^2 + a^2(\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2)$$
$$f_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = -X^2\mathrm{d}t^2 + Y^2(\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2)$$

Characterised by the modified Friedmann equation

$$H^2 + \frac{k}{a^2} = \frac{\rho}{3m_g^2} + \frac{m^2}{3}\left(\beta_0 + 3\beta_1\frac{Y}{a} + 3\beta_2\left(\frac{Y}{a}\right)^2 + \beta_3\left(\frac{Y}{a}\right)^3\right)$$

and the polynomial equation

$$\alpha^2\beta_3\left(\frac{Y}{a}\right)^4 + (3\alpha^2\beta_2 - \beta_4)\left(\frac{Y}{a}\right)^3 + 3(\alpha^2\beta_1 - \beta_3)\left(\frac{Y}{a}\right)^2 + \left(\frac{\alpha^2\rho}{m_g m^2} + \alpha^2\beta_0 - 3\beta_2\right)\frac{Y}{a} - \beta_1 = 0$$

Along with standard continuity equation for $\rho$

# Cosmological solutions

## Perturbative corrections to Friedmann equation

$$H^2+\frac{k}{a^2}=\frac{\Lambda}{3}+\frac{\rho}{3m_{\rm Pl}^2}\left[1-\frac{2\alpha^2\left(\Lambda/m_{\rm FP}^2\right)}{3-2\left(\Lambda/m_{\rm FP}^2\right)}\right]+\frac{\rho^2}{m_{\rm Pl}^2m_{\rm FP}^4}\frac{\alpha^2(1+\alpha^2)(\beta_1-\beta_3)\left(\Lambda/m_{\rm FP}^2\right)}{\left(3-2\left(\Lambda/m_{\rm FP}^2\right)\right)^3}$$

$$+\frac{\rho^3}{m_{\rm Pl}^4m_{\rm FP}^6}\frac{\alpha^2(1+\alpha^2)}{3\left(3-2\left(\Lambda/m_{\rm FP}^2\right)\right)^5}\left[9(\beta_1-\beta_3)+3\left((1+3\alpha^2)\beta_1+(1-\alpha^2)\beta_3\right)\frac{\Lambda}{m_{\rm FP}^2}\right.$$

$$\left.-9(1+\alpha^2)(\beta_1-\beta_3)^2\frac{m_{\rm Pl}^2\Lambda}{m_{\rm FP}^4}-2(1+\alpha^2)(3\beta_1-\beta_3)\frac{\Lambda^2}{m_{\rm FP}^4}\right]+\dots$$

# Cosmological solutions

Perturbative corrections to Friedmann equation

$$H^2+\frac{k}{a^2}=\frac{\Lambda}{3}+\frac{\rho}{3m_{\rm Pl}^2}\left[1-\frac{2\alpha^2\left(\Lambda/m_{\rm FP}^2\right)}{3-2\left(\Lambda/m_{\rm FP}^2\right)}\right]+\frac{\rho^2}{m_{\rm Pl}^2m_{\rm FP}^4}\frac{\alpha^2(1+\alpha^2)(\beta_1-\beta_3)\left(\Lambda/m_{\rm FP}^2\right)}{\left(3-2\left(\Lambda/m_{\rm FP}^2\right)\right)^3}$$

$$+\frac{\rho^3}{m_{\rm Pl}^4m_{\rm FP}^6}\frac{\alpha^2(1+\alpha^2)}{3\left(3-2\left(\Lambda/m_{\rm FP}^2\right)\right)^5}\left[9(\beta_1-\beta_3)+3\left((1+3\alpha^2)\beta_1+(1-\alpha^2)\beta_3\right)\frac{\Lambda}{m_{\rm FP}^2}\right.$$

$$\left.-9(1+\alpha^2)(\beta_1-\beta_3)^2\frac{m_{\rm Pl}^2\Lambda}{m_{\rm FP}^4}-2(1+\alpha^2)(3\beta_1-\beta_3)\frac{\Lambda^2}{m_{\rm FP}^4}\right]+\ldots$$

Valid expansion as long as

$$\rho\lesssim m_{\rm Pl}^2m_{\rm FP}^2$$

Safe to use at present epoch if $m_{\rm FP}^2\gg\Lambda$

At early times $\rho\approx T^4$ so valid for temperatures

$$T\lesssim 10^9\,{\rm GeV}\times\left({\rm m_{FP}}/{\rm GeV}\right)^{1/2}\sim 1.2\times10^{22}\,{\rm K}\ \times\left({\rm m_{FP}}/{\rm GeV}\right)^{1/2}$$

# Cosmological solutions

## Perturbative corrections to Friedmann equation

$$H^2 + \frac{k}{a^2} = \frac{\Lambda}{3} + \frac{\rho}{3m_{\rm Pl}^2}\left[1 - \frac{2\alpha^2\left(\Lambda/m_{\rm FP}^2\right)}{3-2\left(\Lambda/m_{\rm FP}^2\right)}\right] + \frac{\rho^2}{m_{\rm Pl}^2 m_{\rm FP}^4}\frac{\alpha^2(1+\alpha^2)(\beta_1-\beta_3)\left(\Lambda/m_{\rm FP}^2\right)}{\left(3-2\left(\Lambda/m_{\rm FP}^2\right)\right)^3}$$

$$+\frac{\rho^3}{m_{\rm Pl}^4 m_{\rm FP}^6}\frac{\alpha^2(1+\alpha^2)}{3\left(3-2\left(\Lambda/m_{\rm FP}^2\right)\right)^5}\left[9(\beta_1-\beta_3) + 3\left((1+3\alpha^2)\beta_1 + (1-\alpha^2)\beta_3\right)\frac{\Lambda}{m_{\rm FP}^2}\right.$$

$$\left.-9(1+\alpha^2)(\beta_1-\beta_3)^2\frac{m_{\rm Pl}^2\Lambda}{m_{\rm FP}^4} - 2(1+\alpha^2)(3\beta_1-\beta_3)\frac{\Lambda^2}{m_{\rm FP}^4}\right] + \dots$$

- Note that both $\Lambda$ and $m_{\rm FP}$ appear in a fundamental way here, despite strictly being defined only on dS

- In the limit of small $\alpha$ the corrections to GR vanish continuously

–In the limit of large mass, $m_{\rm FP}^2 \gg \Lambda$, counterintuitive to massive gravity intuition, most corrections are automatically small. Perturbations also behave well.

A. De Felice et al, JCAP **1406** (2014)

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# BM as theory of gravitating massive spin-2

Expand around vacuum solutions

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu} \qquad f_{\mu\nu} = \bar{g}_{\mu\nu} + l_{\mu\nu}$$

The mass eigenstates

$$\delta G_{\mu\nu} = \frac{m_{\mathrm{Pl}}}{1+\alpha^2}\left(h_{\mu\nu} + \alpha^2 l_{\mu\nu}\right) \qquad h_{\mu\nu} = \frac{1}{m_{\mathrm{Pl}}}\left(\delta G_{\mu\nu} - \alpha\delta M_{\mu\nu}\right)$$

$$\delta M_{\mu\nu} = \frac{\alpha m_{\mathrm{Pl}}}{1+\alpha^2}\left(l_{\mu\nu} - h_{\mu\nu}\right) \qquad l_{\mu\nu} = \frac{1}{m_{\mathrm{Pl}}}\left(\delta G_{\mu\nu} + \alpha^{-1}\delta M_{\mu\nu}\right)$$

diagonalize quadratic action

$$S^{(2)} = \int \mathrm{d}^4x\sqrt{|\bar{g}|}\Big[\mathcal{L}^{(2)}_{\mathrm{GR}}(\delta G) + \mathcal{L}^{(2)}_{\mathrm{GR}}(\delta M) - \frac{m_{\mathrm{FP}}^2}{4}\left(\delta M^{\mu\nu}\delta M_{\mu\nu} - \delta M^2\right) - \frac{1}{m_{\mathrm{Pl}}}\left(\delta G_{\mu\nu} - \alpha\delta M_{\mu\nu}\right)T^{\mu\nu}\Big]$$

- For small $\alpha$ one sees that $\delta G_{\mu\nu} \sim h_{\mu\nu}$ and that $\delta M_{\mu\nu}$ couple weakly to matter

-Going to cubic order one can show that Noether and gravitational stress-energy coincide. Also, massive field source massless field just like DM fluid component

K. Aoki and S. Mukohyama, Phys. Rev. D **94** (2016)

$$\delta G_{\mu\nu} = \frac{m_{\rm Pl}}{1+\alpha^2}\left(h_{\mu\nu} + \alpha^2 l_{\mu\nu}\right) \qquad h_{\mu\nu} = \frac{1}{m_{\rm Pl}}\left(\delta G_{\mu\nu} - \alpha\delta M_{\mu\nu}\right)$$

$$\delta M_{\mu\nu} = \frac{\alpha m_{\rm Pl}}{1+\alpha^2}\left(l_{\mu\nu} - h_{\mu\nu}\right) \qquad l_{\mu\nu} = \frac{1}{m_{\rm Pl}}\left(\delta G_{\mu\nu} + \alpha^{-1}\delta M_{\mu\nu}\right)$$

diagonalize quadratic action

$$S^{(2)} = \int \mathrm{d}^4x\sqrt{|\bar{g}|}\Big[\mathcal{L}^{(2)}_{\rm GR}(\delta G) + \mathcal{L}^{(2)}_{\rm GR}(\delta M) - \frac{m^2_{\rm FP}}{4}\left(\delta M^{\mu\nu}\delta M_{\mu\nu} - \delta M^2\right) - \frac{1}{m_{\rm Pl}}\left(\delta G_{\mu\nu} - \alpha\delta M_{\mu\nu}\right)T^{\mu\nu}\Big]$$

# BM as theory of gravitating massive spin-2

Feedback to massless equation

$$\mathcal{E}_{\mu\nu}{}^{\rho\sigma}\delta G_{\rho\sigma} = \frac{1}{m_{\rm Pl}}\left(T^{(\rm mat)}_{\mu\nu} + T^{(\rm G)}_{\mu\nu} + T^{(\rm M)}_{\mu\nu}\right) + \dots$$

Massive mode contribution

K. Aoki and S. Mukohyama, Phys. Rev. D **94** (2016)

$$T^{(\rm M)}_{\mu\nu} = \frac{1}{4}\langle \partial_\mu \delta M^{\rho\sigma}\partial_\nu \delta M_{\rho\sigma}\rangle$$

# BM as theory of gravitating massive spin-2

Feedback to massless equation

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Massive mode contribution

K. Aoki and S. Mukohyama, Phys. Rev. D **94** (2016)

$$T^{(\rm M)}_{\mu\nu} = \frac{1}{4}\left\langle \partial_\mu \delta M^{\rho\sigma}\partial_\nu \delta M_{\rho\sigma}\right\rangle$$

In non-relativistic restframe of $\delta M_{\mu\nu}$

$$k_\mu = (m_{\rm FP}, 0, 0, 0)$$

It acts just like dust

$$T^{(\rm M)}_{\mu\nu} = \frac{m^2_{\rm FP}}{4}\,\mathrm{diag}\left[\left\langle \delta \mathrm{M}^{\rho\sigma}\delta \mathrm{M}_{\rho\sigma}\right\rangle, 0, 0, 0\right]$$

# BM as theory of gravitating massive spin-2

Expanding to all orders

$$\mathcal{L} \sim \sqrt{G}\left[m_{\rm Pl}^2 R(G) + K(G, \nabla\nabla\delta M) + V(G, \delta M) + \mathcal{L}_{\rm matter}(G, \Phi, \delta M)\right]$$

Generic vertex with n orders of fields in coupling

$$V_n \sim \sum_{k=0}^{n} h^k l^{n-k} \sim \alpha^{qn}\left(1 + \alpha + \alpha^2 + \ldots + \alpha^{2n}\right) \qquad E = \alpha^{1+q} m_{\rm Pl}$$

Double expansion and higher order vertices can contaminate lower orders. Insisting on perturbativity requires care. In general, perturbativity requires

$$E < \alpha m_{\rm Pl} \qquad m_{\rm FP} < \alpha m_{\rm Pl}$$

but demanding no mixing between different orders gives stronger bound

# Higher order vertices & Perturbativity

Structure of quadratic vertices (standard FP)

| $\delta G^2$ | $\delta G \delta M$ | $\delta M^2$ |
|---|---|---|
| $1\,,\Lambda$ | $0$ | $1\,,\Lambda\,,m_{\rm FP}^2$ |

$$\mathcal{L} \sim \delta G \mathcal{E} \delta G + \Lambda \delta G^2$$
$$\delta M \mathcal{E} \delta M + \Lambda \delta M^2 + m_{\rm FP}^2 \delta M^2$$

Structure of cubic vertices (+ overall $1/m_{\rm Pl}$ suppression)

| $\delta G^3$ | $\delta G^2 \delta M$ | $\delta G \delta M^2$ | $\delta M^3$ |
|---|---|---|---|
| $1\,,\Lambda$ | $0$ | $1\,,\Lambda\,,m_{\rm FP}^2$ | $\alpha\,,\alpha\Lambda\,,\alpha m_{\rm FP}^2$<br>$\frac{1}{\alpha}\,,\frac{1}{\alpha}\Lambda\,,\frac{1}{\alpha}m_{\rm FP}^2$ |

Similar structure persists to all orders

$$S(g,f)|_{\delta M=0} = m_{\rm Pl}^2 \int {\rm d}^4x\,\sqrt{|G|}\,\left(R(G) - 2\Lambda\right)$$

# General features

- The massive field couples weakly to matter for small $\alpha$
- The field that couples to matter is mostly massless
- Nonlinear self-interactions of massless field sum up to GR with a CC. Consistent to interpret $\delta G_{\mu\nu}$ as massless
- No linear terms of the massive field present. Implies no direct decay into massless gravitons

# Outline of Talk

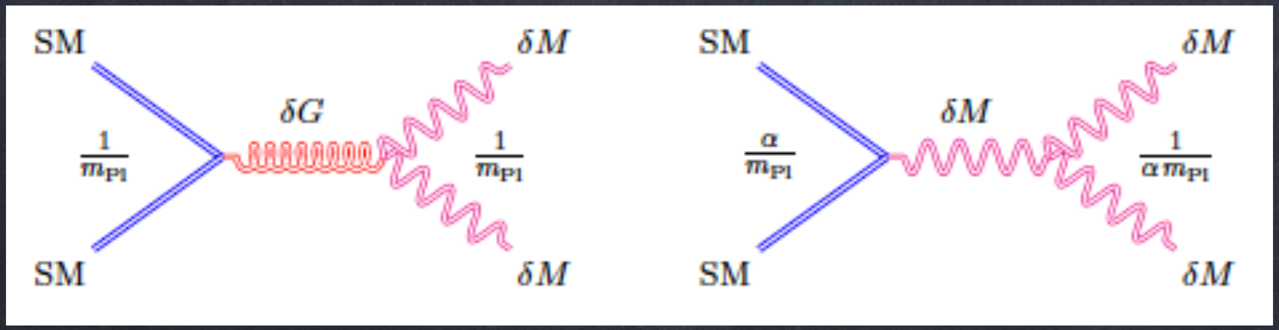
- ~~Introduction & Motivation~~
- ~~Basic details & more Motivation~~
- ~~BM as theory of gravitating massive spin-2~~
- DM phenomenology
- Summary & Outlook

# Spin-2 DM production

- Standard Freeze-out mechanism does not work. Spin-2 DM never in thermal equilibrium in early Universe; expansion rate always dominate over interaction rate.

- Gravitational production (non-adiabatic transition at end of inflation) "does not work". Requires a mass $m_{\rm FP} \geq 10^{10}\,\text{GeV}$ and violates our perturbativity constraint.

- Freeze-in mechanism does work and gives a lower bound on the mass $m_{\rm FP}$ without constraining $\alpha$.



# Spin-2 DM decay

Decay rate of massive spin-2 to two SM particles

$$\Gamma(\delta M \to XX) \sim \frac{\alpha^2 m_{\rm FP}^3}{m_{\rm Pl}^2}$$

Requiring DM to be cosmologically stable

$$1 \text{ TeV} \lesssim m_{\rm FP} \lesssim 6.6 \times 10^3 \, \text{TeV}$$

With observational constraints; PAMELA, AMS-02, EGRB

$$1 \text{ TeV} \lesssim m_{\rm FP} \lesssim 66 \, \text{TeV}$$

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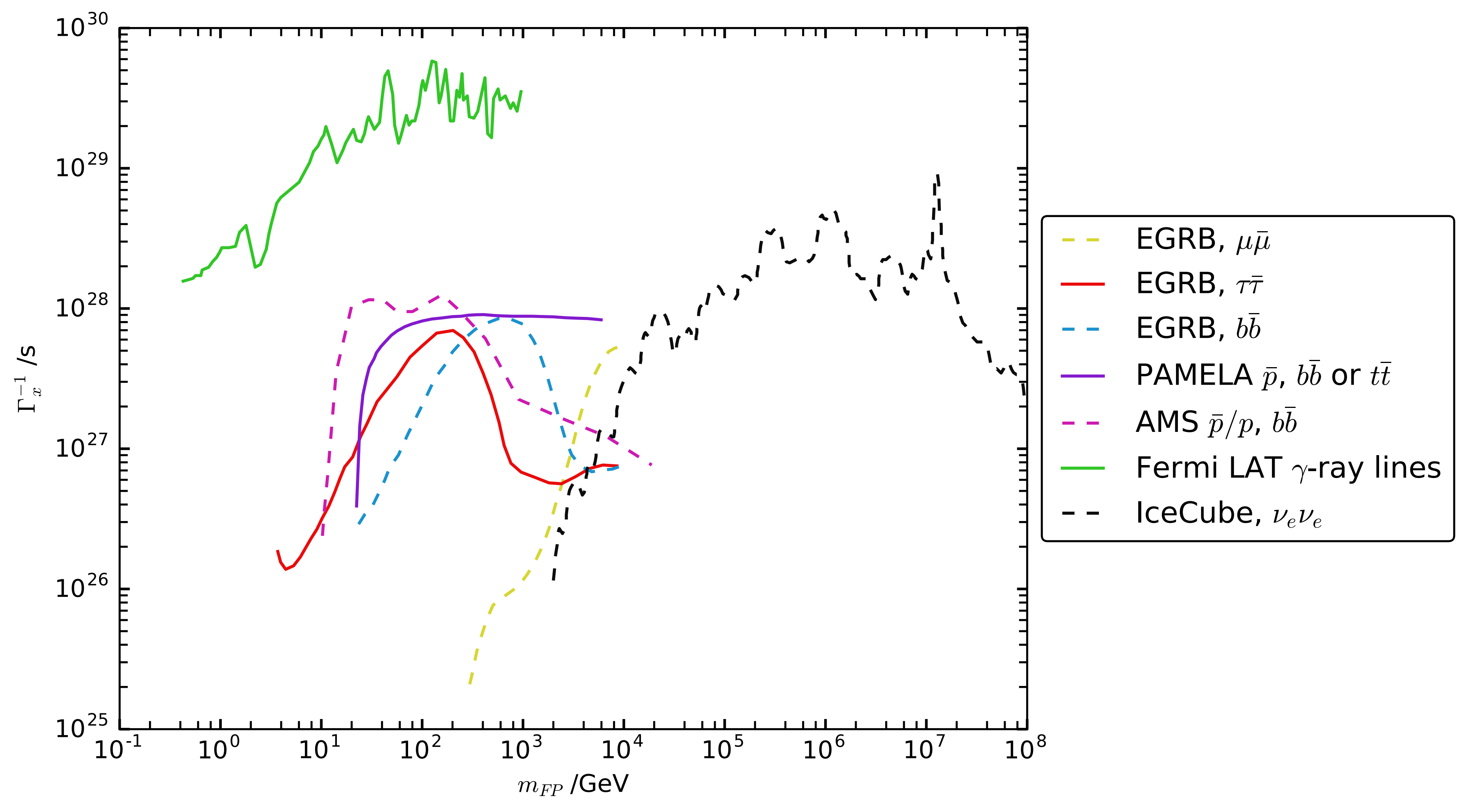
Upper bounds rely on requiring perturbativity!

# Decay constraints

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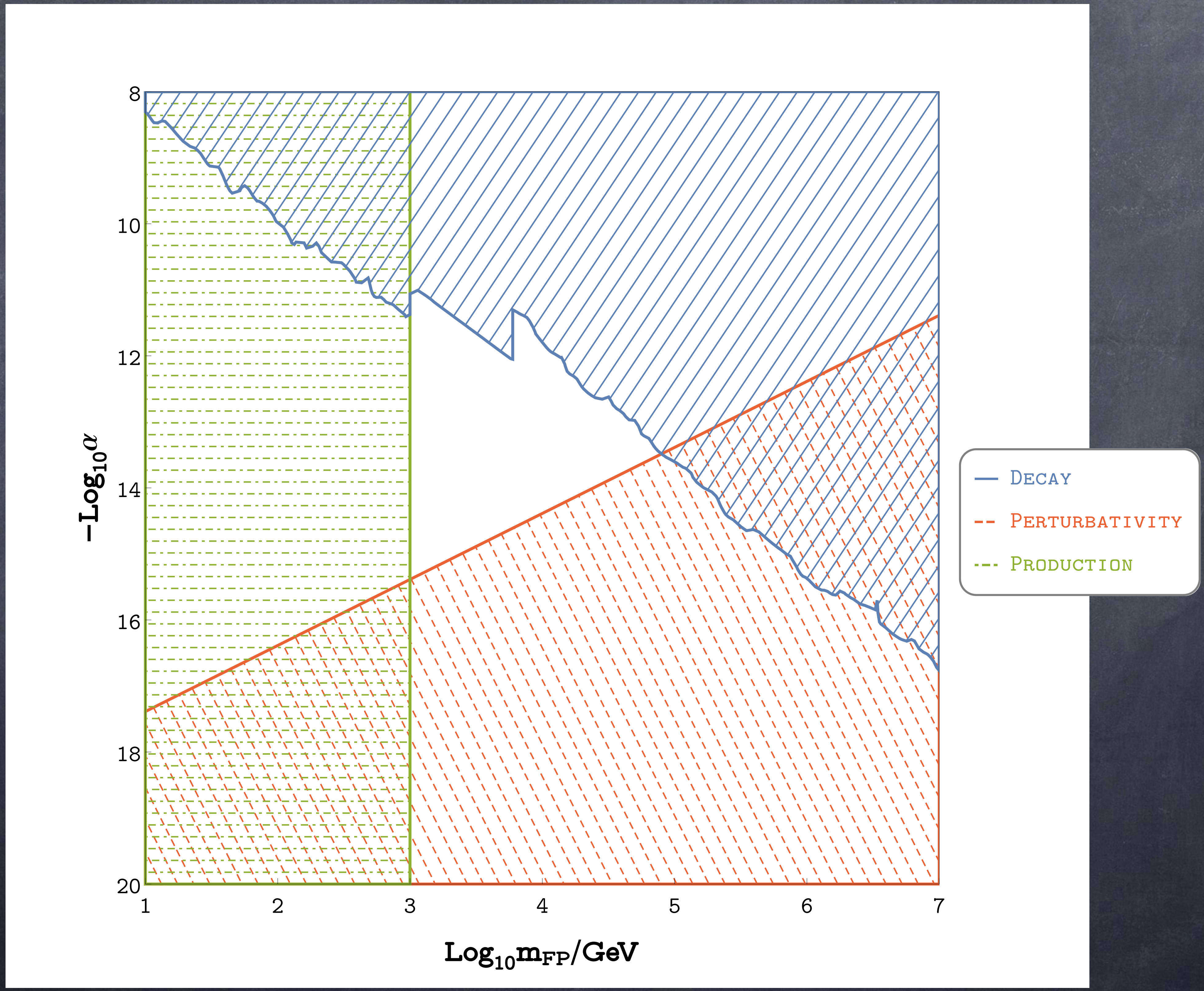
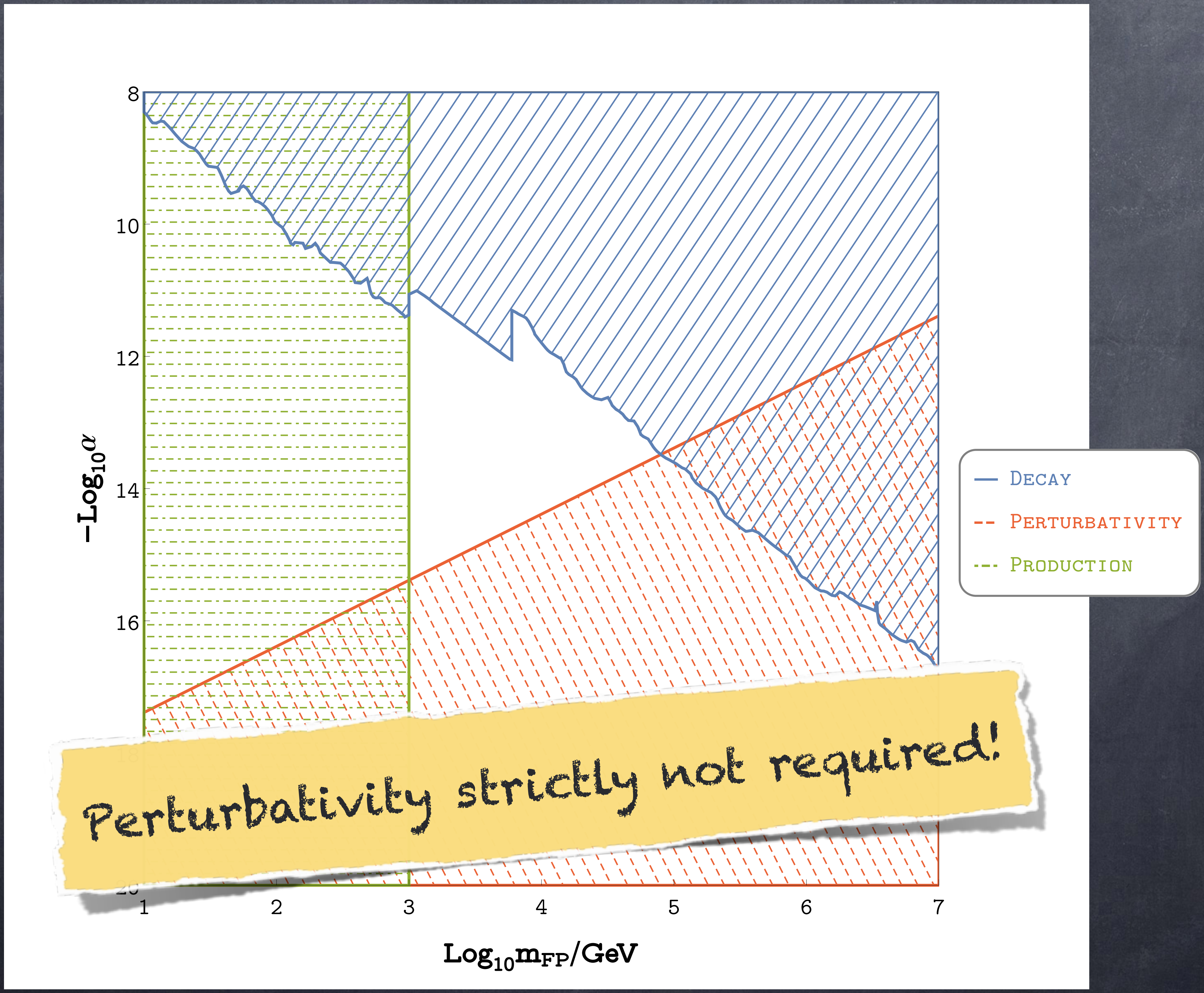
# Combined constraints on mass



# Combined constraints on mass



# Some sanity checks

Our spin-2 DM candidate thus …

- Gravitates with same strength as SM particles
- Has interactions with SM particles which are suppressed by the Planck scale - feebly interacting
- Does not decay into gravitons and has very suppressed decay into SM particles
- Can be produced in the correct abundance; even when restricting theory to perturbative regime

# Outline of Talk

- ~~Introduction & Motivation~~
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# Summary & Outlook

- BM is a theory of modified gravity motivated from first principles
- Can be treated as a theory of a gravitating massive spin-2
- Massive spin-2 behaves like DM in these regimes. Gives purely gravitational origin of DM
- Also gives expansion; but finetuning problem persists
- Production -> a mass in excess of 1 TeV
- Perturbativity and observational constraints combine -> a mass lower than 66 TeV
- Very hard to detect! New ideas may be required …

# Summary & Outlook

- Possible to exclude the non-perturbative regime somehow?
- Not detectable in any current direct (or indirect) detection experiments — boring null prediction
- Any unique and detectable signatures when theory is so close to GR?
- Massive spin-2 may gravitate differently in strong field backgrounds
- Strong self-interactions may give rise to detectable signals in CMB or clustering
- Possible correlations with gravitational wave production - sensitive to inflationary scenario

K. Aoki and S. Mukohyama, Phys. Rev. D **94** (2016)

# Thank you for your kind attention!

Mikael von Strauss
Nordita

IESC Cargèse, May 4 2017

NORDITA

# Linear FP

## Constraints in linear FP theory: The FP equations

$$\delta E_{\mu\nu} = \mathcal{E}_{\mu\nu}{}^{\rho\sigma} h_{\rho\sigma} - \Lambda \left( h_{\mu\nu} - \frac{1}{2} g_{\mu\nu} h \right) + \frac{m^2}{2} \left( h_{\mu\nu} - g_{\mu\nu} h \right) \approx 0$$

$$\left( \mathcal{E}_{\mu\nu}{}^{\rho\sigma} h_{\rho\sigma} = -\frac{1}{2} \left[ \delta^\rho_\mu \delta^\sigma_\nu \nabla^2 + g^{\rho\sigma} \nabla_\mu \nabla_\nu - \delta^\rho_\mu \nabla^\sigma \nabla_\nu - \delta^\rho_\nu \nabla^\sigma \nabla_\mu - g_{\mu\nu} g^{\rho\sigma} \nabla^2 + g_{\mu\nu} \nabla^\rho \nabla^\sigma \right] h_{\rho\sigma} \right)$$

Trace:

$$g^{\mu\nu} \delta E_{\mu\nu} = \nabla^2 h - \nabla^\mu \nabla^\nu h_{\mu\nu} + \left( \Lambda - \frac{3m^2}{2} \right) h \approx 0$$

Divergence:

$$\nabla^\mu \delta E_{\mu\nu} = \frac{m^2}{2} \left( \nabla^\mu h_{\mu\nu} - \nabla_\nu h \right) \approx 0$$

Double divergence:

$$\nabla^\mu \nabla^\nu \delta E_{\mu\nu} = \frac{m^2}{2} \left( \nabla^\mu \nabla^\nu h_{\mu\nu} - \nabla^2 h \right) \approx 0$$

# Linear FP

The linear combination

$$2\nabla^\mu\nabla^\nu\delta E_{\mu\nu} + m^2 g^{\mu\nu}\delta E_{\mu\nu} = \frac{m^2}{2}\left(2\Lambda - 3m^2\right)\, h \approx 0$$

constitutes a scalar constraint. Together with divergence constraints the theory can be written

$$\left(\nabla^2 - m^2 - \tfrac{2\Lambda}{3}\right) h_{\mu\nu} = 0\,, \qquad \nabla^\mu h_{\mu\nu} = 0\,, \qquad h = 0$$

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What about when $2\Lambda=3m^2$ ?

$$\longrightarrow\qquad \left[\nabla^{\mu}\nabla^{\nu}+\frac{m^2}{2}g^{\mu\nu}\right]\delta E_{\mu\nu}=0$$

# Linear FP

Implies the linear gauge symmetry

$$\Delta h_{\mu\nu} = \left[\nabla_\mu \nabla_\nu + \frac{m^2}{2} g_{\mu\nu}\right] \xi(x)$$

Action is trivially invariant since it can be written

$$S[h] \sim \int \mathrm{d}^4x \sqrt{g}\, h^{\mu\nu} \delta E_{\mu\nu}$$

From group theory: coincides with existence of "short" UIRs in de Sitter

# Further motivation

We now have an example of a theory where

$$\Lambda \sim m^2$$

is protected by a symmetry. Similarly

$$m^2 \sim 0$$

may be thought of as "technically natural" due to enhancement of diffeomorphism symmetry. Furthermore dS favoured by unitarity

- Small positive $\Lambda$ may be regarded as technically natural!
- But spin-2 theories require nonlinear completion!