Evolution of domain walls in early universe

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based on

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Observations indicate that the universe is 100% baryo-asymmetric. But is it possible that astronomically large pieces of antimatter exist in the universe?

Observed baryon asymmetry cannot be explained by Standard Model $\Rightarrow$ physics beyond SM

There are models where the universe is globally:
1) asymmetric (matter $>$ antimatter)
2) symmetric $\Rightarrow$ domains of matter and antimatter
Domain wall problem – unacceptably high energy density of DW

Solution: domain walls should exist only in the universe past and should disappear by now

Models where a symmetry is broken only in a particular range of temperatures, i.e. it is restored at the highest as well as at the lowest temperatures [V.A. Kuzmin, M.E. Shaposhnikov, I.I. Tkachev, 1981-82] ⇒ the size of domains is too small from cosmological point of view
Another problem – if matter and antimatter domains exist and they are close enough, the gamma rays produced in annihilation along the boundary would be detectable. 

**Such gamma rays have not been observed**

Solution: matter and antimatter domains should be separated by cosmologically large distances, i.e. 

**domain wall thickness should be cosmologically large**
A few years ago we considered a model which may lead to a baryo-symmetric universe with \textit{cosmologically large domains} of matter and antimatter separated by \textit{cosmologically large distances}, avoiding the domain wall problem [A.D. Dolgov, S.I. Godunov, A.S. Rudenko, and I.I. Tkachev, JCAP 1510 (2015) 10, 027; arXiv:1506.08671].

Inflation is an essential ingredient of this scenario. We considered therein $\phi^2$-inflation with the inflaton potential $U = m^2 \phi^2 / 2$.

A coupling of the pseudoscalar field $\chi$ to the inflaton field $\phi$ was introduced on purpose to generate a non-zero value of $\chi$ and to keep it during baryogenesis as a source of $CP$ violation.
But the questions are

- How does the domain wall thickness depend on time in other models?
- Can domain wall thickness become cosmologically large?
I. Evolution of thick domain walls in de Sitter universe.
Stationary solutions

R. Basu and A. Vilenkin
In their paper Basu, Vilenkin (BV) address the question what happens to the internal structure of the DW during inflation.

The inflationary Universe is approximated by de Sitter spacetime, which has a constant expansion rate $H$, and the scale factor evolves as $a(t) = \exp Ht$.

The metric with spatially flat sections is given by

$$ds^2 = dt^2 - e^{2Ht} (dx^2 + dy^2 + dz^2)$$  \hspace{1cm} (1)
BV consider a one component scalar field theory with a simple double-well potential

\[ \mathcal{L} = \frac{1}{2} g^{\mu \nu} \partial_\mu \varphi \partial_\nu \varphi - \frac{\lambda}{2} \left( \varphi^2 - \eta^2 \right)^2 \]  

(2)

The corresponding equation of motion is

\[ \frac{1}{\sqrt{-g}} \partial_\mu \left( \sqrt{-g} g^{\mu \nu} \partial_\nu \varphi \right) = -2\lambda \varphi \left( \varphi^2 - \eta^2 \right) \]  

(3)

In flat spacetime, \( H = 0 \), and in one-dimensional static case, \( \varphi = \varphi(z) \), the equation takes the form

\[ \frac{d^2 \varphi}{dz^2} = 2\lambda \varphi \left( \varphi^2 - \eta^2 \right) \]  

(4)

It has a kink-type solution, which describes a static infinite DW.
One can assume that the wall is situated at \( z = 0 \) in \( xy \)-plane:

\[
\varphi(z) = \eta \tanh \frac{z}{\delta_0},
\]  

(5)

where \( \delta_0 = 1/\sqrt{\lambda \eta} \) is the flat-spacetime wall thickness (subscript 0 indicates that \( H = 0 \)).
Now consider an expanding universe with constant $H > 0$
In this case, if one looks for stationary solution, it is reasonable to suggest that the field $\varphi$ depends only on $z a(t) = z \exp H t$, which is the proper distance from the wall
So, one can choose the following ansatz for $\varphi$:

$$\varphi = \eta \cdot f(u), \quad \text{where} \quad u = Hz \cdot e^{Ht},$$  

(6)

here $u$ and $f$ are dimensionless.

Equation of motion takes the form:

$$(1 - u^2) f'' - 4uf' = -2 Cf (1 - f^2)$$  

(7)

Here prime means the derivative with respect to $u$.
It is noteworthy that all parameters of the problem are combined into a single positive constant $C = 1/(H\delta_0)^2 = \lambda \eta^2 / H^2 > 0$
Since one is interested in kink-type solutions, the boundary conditions should be

\[ f(0) = 0, \quad f(\pm \infty) = \pm 1 \] (8)

**Figure:** Stationary field configurations \( f(u) \) for different values of \( C \)
At large $u$ the field $\varphi$ approaches its vacuum expectation value (VEV) $\eta$.

However, the solutions exhibit an aperiodical damped oscillatory behavior, as opposed to the monotonic approach to the VEV in flat spacetime.

Asymptotic formula for large $u$:

$$1 - f(u) \sim \frac{\cos \left( \sqrt{4C - \frac{9}{4}} \ln u \right)}{u^{3/2}}$$  \hspace{1cm} (9)
BV noticed that stationary solutions can be found only for $C > 2$, but no explanation of this observation was given.

Naive explanation why the value $C = 2$ is the special one:


eq. of motion (7) and condition $f(0) = 0 \Rightarrow f''(0) = 0$,

then expanding $f(u)$ into Taylor series near $u = 0$, one obtains that for sufficiently small positive $\epsilon$ and for $f'(0) > 0$:

for $C > 2$, $f''(\epsilon) < 0 \Rightarrow f(u)$ is concave downward like kink-type solution,

for $C \leq 2$, $f''(\epsilon) > 0 \Rightarrow f(u)$ is convex downward, it is not the kink-type.
In the case of very thin DW, when thickness is much smaller than the de Sitter horizon, $\delta \ll H^{-1}$, i.e. $(C \gg 1)$, the solution is well approximated by the flat-spacetime solution.

Beyond the critical value, $\delta_0 \geq H^{-1}/\sqrt{2}$, i.e. $(C \leq 2)$, there are no stationary solutions at all.
II. Evolution of thick domain walls in de Sitter universe. Beyond the stationary solutions

A.D. Dolgov, S.I. Godunov, and A.S. Rudenko

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Beyond the stationary approximation we can find solution not only for \( C > 2 \), but also for \( C \leq 2 \).
To this end one should solve the original equation of motion (3) in the case when the field \( \phi \) is a function of at least two independent variables, \( z \) and \( t \):

\[
\frac{\partial^2 \phi}{\partial t^2} + 3H \frac{\partial \phi}{\partial t} - e^{-2Ht} \frac{\partial^2 \phi}{\partial z^2} = -2\lambda \phi (\phi^2 - \eta^2). \tag{10}
\]

It is convenient to introduce dimensionless variables \( \tau = Ht \), \( \zeta = Hz \) and dimensionless function \( f(\zeta, \tau) = \phi(z, t)/\eta \)
As a result one obtains the equation

\[
\frac{\partial^2 f}{\partial \tau^2} + 3 \frac{\partial f}{\partial \tau} - e^{-2\tau} \frac{\partial^2 f}{\partial \zeta^2} = 2Cf (1 - f^2), \tag{11}
\]

where \( C = \lambda \eta^2 / H^2 = 1/(H\delta_0)^2 > 0 \) as it was above.
The boundary conditions for the kink-type solution are

\[ f(0, \tau) = 0, \quad f(\pm \infty, \tau) = \pm 1, \quad (12) \]

and we choose the initial configuration as DW with ”natural”
thickness \(1/\sqrt{C}\) (with respect to dimensionless coordinate \(\zeta\)) and
zero time derivative:

\[ f(\zeta, 0) = \tanh \frac{z}{\delta_0} = \tanh \sqrt{C}\zeta, \quad \frac{\partial f(\zeta, \tau)}{\partial \tau} \bigg|_{\tau=0} = 0. \quad (13) \]
Figure: Time dependence of DW thickness for $C > 2$ and $C < 2$

We use here the definition for the thickness $\delta(t)$ as the value of the coordinate $z$ at the position where the field $\varphi$ reaches the value $\varphi/\eta = \tanh 1 \approx 0.76$
Figure: Time dependence of DW thickness for $C > 2$ and $C < 2$

In the left plot ($C > 2$) one can see that all the curves indeed tend to constant values corresponding to stationary solutions (dashed lines). Oscillatory behavior mentioned above is also apparent.
**Figure:** Time dependence of DW thickness for $C > 2$ and $C < 2$

Right plot contains curves for $C < 2$. Along the vertical axis the logarithm of the DW thickness is shown in order to compare the rate of the DW expansion with the exponential cosmological one.

For $C = 1$, i.e. for not very small values of $C$, the DW thickness increases slower than exponent.

However, for smaller values of $C$, e.g. $C \lesssim 0.1$ the rate of the DW expansion is the exponential one with a good accuracy.
We have shown that for large values of parameter $C > 2$ the initial kink configuration in a de Sitter background tends to the stationary solution obtained by Basu and Vilenkin.

We also confirmed the BV result that the thickness of the stationary wall rises with decreasing value of $C$.

For $C \leq 2$ the stationary solution does not exist and the thickness of the wall infinitely grows with time.

For $C \lesssim 0.1$ the rise is close to the exponential one ⇒ transition regions between domains might be cosmologically large.
III. Evolution of thick domain walls in inflationary universe with $U(\phi) = m^2 \phi^2 / 2$

A.D. Dolgov, S.I. Godunov, and A.S. Rudenko
arXiv:1711.04704 [gr-qc]
Consider simple model of inflation with quadratic inflaton potential $U = m^2 \phi^2 / 2$

In the slow-roll regime Hubble parameter is

$$H(t) = \sqrt{\frac{8\pi \rho(t)}{3m_{pl}^2}} \approx \sqrt{\frac{8\pi}{3m_{pl}^2}} \frac{m^2 \phi^2(t)}{2} = \sqrt{\frac{4\pi}{3}} \frac{m}{m_{pl}} \phi(t) \neq \text{const}$$

and equation of motion of inflaton is

$$\dot{\phi}(t) \approx -\frac{m^2 \phi(t)}{3H} \approx -\frac{m_{pl} m}{\sqrt{12\pi}}$$

(14)
Therefore,

$$\phi(t) = \phi_i - \frac{m_{pl}m}{\sqrt{12\pi}}t,$$  \hspace{1cm} (16)

where $\phi_i$ is the initial value of inflaton field.

$$H(t) = \sqrt{\frac{4\pi}{3} \frac{m}{m_{pl}}} \phi_i - \frac{1}{3}m^2t,$$  \hspace{1cm} (17)

$$a(t) = a_0 \cdot \exp \left( \sqrt{\frac{4\pi}{3} \frac{m}{m_{pl}}} \phi_it - \frac{1}{6}m^2t^2 \right).$$  \hspace{1cm} (18)

These formulas are valid only till the end of slow-roll regime,

$$mt < m_{te} \simeq \sqrt{12\pi} \frac{\phi_i}{m_{pl}} - \sqrt{3}$$  \hspace{1cm} (19)
In numerical calculations we use the following values: 
\(\phi_i = 2\, m_{pl}, \ t_i = 0, \text{ and } a_0 = 1.\)

Parameter \(C(t)\) is still defined as \(C(t) = 1/(H(t)\delta_0)^2.\) Since \(H(t)\) is decreasing, \(C(t)\) increases with time.

Time \(t_C\) at which \(C(t_C) = 2\) is determined as

\[
mt_C = \sqrt{12\pi \frac{\phi_i}{m_{pl}}} - \frac{3\sqrt{2}}{2m\delta_0}. \tag{20}
\]

Obviously \(t_C \geq 0 \Rightarrow C = 2\) if

\[
m\delta_0 \geq \sqrt{\frac{3}{8\pi} \frac{m_{pl}}{\phi_i}} \approx 0.173 \tag{21}
\]
\[ \delta_0 = 0.025 \cdot m^{-1} \]

\[ \delta_0 = 0.05 \cdot m^{-1} \]

\( C(t) > 2 \) during all this time
\[ \delta_0 = 0.2 \cdot m^{-1} \]

Vertical dashed line – moment \( t_C \) at which \( C(t_C) = 2 \).
\( C(t) < 2 \) for \( t < t_C \) and \( C(t) > 2 \) for \( t > t_C \).
\[ \delta_0 = 10 \cdot m^{-1} \]
\[ \delta_0 = 100 \cdot m^{-1} \]

\[ C(t) < 2 \text{ during all this time.} \]

Red dashed line corresponds to \( a(t) \cdot \delta_0 \cdot m \)

For large \( \delta_0 \) this line coincides with \( a(t) \cdot \delta(t) \cdot m \), therefore

\( \delta(t) \approx \delta_0 \) till the end of slow-roll regime with very good accuracy.
IV. Evolution of thick domain walls in a “hilltop” model of inflation
Potential

\[ V(\phi) = \Lambda^4 \left[ 1 - \left( \frac{\phi}{\mu} \right)^4 + \ldots \right] \quad (22) \]

We choose \( \mu = 2.5 \, m_{Pl} \) in consistency with Planck data,
\( \Lambda = 10^{-3} \, m_{Pl} \)
\( \phi_i = \phi(0) = 1.43 \, m_{Pl} \)
\( \phi_e = 2.3 \, m_{Pl} \)
$\delta_0 = 0.03$

$\delta_0 = 0.05$

$C(t) > 2$ during all the time of the wall evolution $t$ and $a(t)\delta(t)$ are shown in $(10^{-6} m_{Pl})^{-1}$ units

A.D. Dolgov, S.I. Godunov, A.S. Rudenko

Evolution of domain walls in early universe
\[ \delta_0 = 0.3 \]

Vertical dashed line – moment \( t_C \) at which \( C(t_C) = 2 \).

\( C(t) < 2 \) during all the time of the wall evolution for \( \delta_0 = 0.5 \)
V. Evolution of thick domain walls in $R^2$-inflation
Modified theory of gravity with the action

\[ S = -\frac{m_{Pl}^2}{16\pi} \int d^4x \sqrt{-g} \left( R - \frac{R^2}{6M^2} \right) \]  

is equivalent to the general relativity with a scalar field \( \phi \) with the potential

\[ V(\phi) = \frac{3M^2m_{Pl}^2}{32\pi} \left( 1 - e^{-4\sqrt{\pi/3}\phi/m_{Pl}} \right)^2 \]  

We choose \( M = 2.6 \cdot 10^{-6} m_{Pl} \)
\( \phi_i = \phi(0) = 0.7 m_{Pl} \)
\( \phi_e = 0.3 m_{Pl} \)
\[ \delta_0 = 0.1 \]

\[ \delta_0 = 0.3 \]

\( C(t) > 2 \) during all the time of the wall evolution

\( t \) and \( a(t)\delta(t) \) are shown in \( (10^{-6} m_{Pl})^{-1} \) units

A.D. Dolgov, S.I. Godunov, A.S. Rudenko
Vertical dashed line – moment $t_C$ at which $C(t_C) = 2$. $C(t) < 2$ during all the time of the wall evolution for $\delta_0 = 10$. 

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Evolution of domain walls in early universe
In the inflationary universe

- If $\delta_0$ is quite small ($C(t) > 2$ during all the time of inflation), then DW thickness, $a(t)\delta(t)$, tends to constant value, $\delta_0$, therefore is not cosmologically large.

- If $\delta_0 \sim H(t)^{-1}$, then DW thickness can grow up to quite large values during inflation, but at some moment it starts to diminish and tends to $\delta_0$ at the end of inflation.

- Domain walls, which were thick initially ($C(t) < 2$ during all the time of inflation), expand until the end of inflation. In this case coordinate thickness almost does not change, $\delta(t) \approx \delta_0$, and domain wall expansion is entirely due to growth of scale factor, $a(t)$. Thickness of such DW can be cosmologically large at the end of inflation.
VI. Evolution of thick domain walls in universe with $p = w\rho$

A.D. Dolgov, S.I. Godunov, and A.S. Rudenko
arXiv:1711.04704 [gr-qc]
Let us study how the DW evolves in an expanding universe with the equation of state of matter $p = w \rho$, where constant $w > -1$. In such universe the scale factor increases as some power of time

$$a(t) = a_0 \cdot \left(\frac{t}{t_i}\right)^\alpha,$$

where $\alpha = \frac{2}{3(1 + w)} > 0$, \hspace{1cm} (25)

and the Hubble parameter decreases as inverse time

$$H(t) = \frac{\dot{a}}{a} = \frac{\alpha}{t}.$$

(26)

The values $w = 0 \ (\alpha = 2/3)$ and $w = 1/3 \ (\alpha = 1/2)$ correspond to the matter-dominated and radiation-dominated universe, respectively.
The equation of motion

\[ \frac{\partial^2 f}{\partial t^2} + 3H(t) \frac{\partial f}{\partial t} - \frac{1}{a^2(t)} \frac{\partial^2 f}{\partial z^2} = \frac{2}{\delta_0^2} f \left( 1 - f^2 \right) \]  

(27)

After the substitution \( \tau = t/\delta_0 \) and \( \zeta = z/\delta_0 \) one gets

\[ \frac{\partial^2 \tilde{f}}{\partial \tau^2} + \frac{3}{\sqrt{C(\tau)}} \frac{\partial \tilde{f}}{\partial \tau} - \frac{1}{\tilde{a}^2(\tau)} \frac{\partial^2 \tilde{f}}{\partial \zeta^2} = 2\tilde{f} \left( 1 - \tilde{f}^2 \right) \]  

(28)

where \( C(\tau) = (H(\tau \cdot \delta_0) \cdot \delta_0)^{-2} = H^{-2}(\tau) \)

No explicit dependence on \( \delta_0 \)!

The equation of motion is the same for different \( \delta_0 \) if \( t \) and \( z \) are measured in units of \( \delta_0 \).
The evolution of the domain wall is basically defined by the parameter \( C(t) \).

In the \( p = w \rho \) universe the parameter \( C(t) \) increases as

\[
C(t) = \frac{1}{(H(t) \delta_0)^2} = \frac{t^2}{(\alpha \delta_0)^2} \propto t^2. \tag{29}
\]

Since \( C = 2 \) is the critical value, let us introduce the time \( t_C \) at which \( C(t_C) = 2 \). In \( p = w \rho \) universe

\[
\frac{t_C}{\delta_0} = \sqrt{2\alpha}. \tag{30}
\]

We obtain that \( t_C > t_i \) for

\[
w < \frac{2\sqrt{2} \delta_0}{3} \frac{1}{t_i} - 1. \tag{31}
\]
Figure: Time dependence of the physical thickness of the wall, \( a(t)\delta(t) \), for \( t_i/\delta_0 = 0.5 \) and different \( w \). Dashed horizontal line corresponds to \( \delta_0 \). Vertical dashed lines correspond to the moment \( t_C \) at which \( C'(t_C) = 2 \).
Figure: Time dependence of the physical thickness of the wall, \( a(t)\delta(t) \), for \( t_i/\delta_0 = 1.0 \) and different \( w \). Dashed horizontal line corresponds to \( \delta_0 \). Vertical dashed lines correspond to the moment \( t_C \) at which \( C (t_C) = 2 \).
As for behaviour at $t \to \infty$, one sees in these Figures that physical thickness oscillates with slowly decreasing amplitude around the value $\delta_0$. Therefore, when $t \to \infty$ the field configuration tends to

$$f(z, t) = \tanh \frac{z \cdot a(t)}{\delta_0} \quad (32)$$
Also one can consider the evolution of the wall thickness from different initial configurations, e.g. from the configurations stretched along $z$ by factor $k$:

$$f(z, t_i) = \tanh \frac{z \cdot a(t_i)}{k \cdot \delta_0}$$  \hspace{1cm} (33)
Figure: Time dependence of the physical thickness of the wall, $a(t)\delta(t)$, for $w = 1/3$, $t_i/\delta_0 = 1.0$ and different $k$. Dashed horizontal line $- \delta_0$. Vertical dashed lines – moment $t_C$ at which $C(t_C) = 2$. 

A.D. Dolgov, S.I. Godunov, A.S. Rudenko
Evolution of domain walls in early universe
Figure: Time dependence of the physical thickness of the wall, $a(t)\delta(t)$, for $w = -2/3$, $t_i/\delta_0 = 1.0$ and different $k$. Dashed horizontal line $-\delta_0$. Vertical dashed lines - moment $t_C$ at which $C(t_C) = 2$. 
Parameter $C(t)$ increases as $t^2$, so at some moment physical thickness of the wall, $a(t)\delta(t)$, starts to diminish and eventually (for $t/\delta_0 \gg \alpha$) goes to the constant value, $\delta_0$, which is microscopically small.

Domain walls with cosmologically large thickness can exist only at the beginning of $p = w\rho$ stage.
Thank you for your attention!
Backup
VII. Separated matter and antimatter domains with vanishing domain walls

Introduction

Observations indicate that the universe is 100% baryo-asymmetric. But is it possible that astronomically large pieces of antimatter exist in the universe?

Sakharov conditions for baryogenesis:
1. Non-conservation of baryon number $B$
2. Breaking of $C$ and $CP$ invariance
3. Deviation from thermal equilibrium

Observed baryon asymmetry cannot be explained by SM $\Rightarrow$ physics beyond Standard Model

There are models where the universe is globally:
1) asymmetric (matter $>\,$ antimatter)
2) symmetric $\Rightarrow$ domains of matter and antimatter
Domain wall problem – unacceptably high energy density of DW
Solution: domain walls should exist only in the universe past and
should disappear by now

Models where a symmetry is broken only in a particular range of
the size of domains is too small from cosmological point of view

We consider another scenario:
domains appeared during inflation ⇒ now they are cosmologically
large and separated by cosmologically large distances

Model

Action:

\[ S = \int d^4 x \sqrt{-g} \left( \mathcal{L}_\Phi + \mathcal{L}_\chi + \mathcal{L}_{\text{int}} \right), \]

where

\[ \mathcal{L}_\Phi = \frac{1}{2} g^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi - \frac{1}{2} M^2 \Phi^2, \]

\[ \mathcal{L}_\chi = \frac{1}{2} g^{\mu\nu} \partial_\mu \chi \partial_\nu \chi - \frac{1}{2} m^2 \chi^2 - \frac{\lambda}{4} \chi^4, \]

\[ \mathcal{L}_{\text{int}} = \mu^2 \chi^2 V(\Phi), \]

\( \Phi \) – inflaton field,
\( \chi \) – real pseudoscalar field,
\( M, m, \lambda, \mu \) – constant parameters,
\( V(\Phi) \) – dimensionless function
$V(\Phi)$ is non-zero only in a narrow range of $\Phi$

e.g. in a toy model \[ V(\Phi) = \exp \left[ - \frac{(\Phi - \Phi_0)^2}{2\Phi_1^2} \right] \]

$\Phi_0 = 3.1 \, m_{Pl}$, \((m_{Pl} - \text{Planck mass})\)

$\Phi_1 = 0.02 \, m_{Pl}$
FLRW metric: \[ ds^2 = dt^2 - a^2(t) \, dx^2 \]

Hubble parameter: \[ H(t) = \frac{\dot{a}(t)}{a(t)} = \sqrt{\frac{8\pi \rho(t)}{3m_{Pl}^2}} \]

Energy density:
\[ \rho(t) = \frac{\dot{\Phi}^2}{2} + \frac{M^2 \Phi^2}{2} + \frac{\dot{\chi}^2}{2} + \frac{m^2 \chi^2}{2} + \frac{\lambda \chi^4}{4} - \mu^2 \chi^2 V(\Phi) \]

Equations of motion:
\[ \ddot{\Phi} + 3H \dot{\Phi} + M^2 \Phi + \mu^2 \chi^2 \frac{\Phi - \Phi_0}{\Phi_1^2} V(\Phi) = 0, \]
\[ \ddot{\chi} + 3H \dot{\chi} + m^2 \chi + \lambda \chi^3 - 2\mu^2 \chi V(\Phi) = 0, \]

here it is assumed that \( \Phi = \Phi(t) \) and \( \chi = \chi(t) \)
During inflation (usual **slow-roll regime**) $\Phi$ decreases and when it reaches vicinity of $\Phi_0$, two minima appear in the potential

$$U(\chi) \bigg|_{\Phi=\text{const}} = \frac{\lambda}{4} \chi^4 + \left( \frac{1}{2} m^2 - \mu^2 V(\Phi) \right) \chi^2 + \frac{1}{2} M^2 \Phi^2,$$

so the point $\chi = 0$ becomes local maximum

$\chi$ – in units of $10^{-6} m_{Pl}$, $U(\chi)$ – in units of $10^{-12} m_{Pl}^4$

$\lambda = 2 \cdot 10^{-3}$; $m = 10^{-10}$, $\mu = 10^{-4}$, $M = 10^{-6}$, $\Phi_{in} = 4$ (in $m_{Pl}$ units)
In spatial regions where $\chi$ turns out to be positive/negative (due to fluctuations) it rolls down to the positive/negative minimum $\Rightarrow$ domains with opposite signs of $CP$ violation appear


Such $CP$ violation is operative only when $\chi$ sits near the minimum, but in our model this minimum disappeared during inflation $\Rightarrow$ baryon asymmetry (if generated) exponentially inflated away

Successful baryogenesis should take place after the end of inflation
Main features of the model:

1. Inflaton field $\Phi$ always gives the main contribution to the energy density $\rho \Rightarrow$ standard slow-roll inflation

2. Inflation should last quite long after domains were formed (supposing that seed of the domain $\sim 1/H$ and present size of domain $\sim 10\text{ Mpc}$)

3. The field $\chi$ remains non-vanishing for some time after the end of inflation and during baryogenesis (non-zero $\chi$ can induce $CP$ violation)
Standard slow-roll inflation

\( \Phi(t) \) only slightly deviates from the straight line around \( \Phi_0 = 3.1 \)
inflation lasts \( \sim 60 \) e-foldings (\( e^{60} \sim 10^{26} \)) after \( \Phi(t) \) passes \( \Phi_0 \)

\[ \Phi(t) = 0 \] at \( t \approx 25 \) (beginning of reheating)
Evolution of the field $\chi(t)$

- Time $t$ — in units of $M^{-1}$, $\chi(t)$ — in units of $M$
- $\chi_{in} = 1 M, \ M = 10^{-6} m_{Pl}$

When reheating starts (at $t \approx 25$) the field $\chi$ is quite large:
- $\chi \sim 10^{-5} m_{Pl} \sim 10^{14}$ GeV
If matter and antimatter domains exist and they are close enough, the gamma rays produced in annihilation along the boundary would be detectable. Such gamma rays have not been observed.

In our model inflaton $\Phi$ always gives the main contribution to $H \Rightarrow$ if $\chi = 0$ the Hubble parameter remains almost the same as in the case of non-zero $\chi$ considered above (size of region where $\chi = 0$ is only two times less than the domain size).

Therefore, model predicts the **domains of cosmological size with the distances between them of the same order of magnitude**
The stage of inflation is followed by the stage of reheating, during which the heavy $X$-particles (e.g. vector bosons) can be produced through decay of inflaton field.

In the case of comparatively large coupling between $X$-particles and inflaton the decay of the inflaton occurs rapidly, during only one or few oscillations.

Assume that produced $X$-particles can decay into fermions (not SM in general). If the corresponding couplings are large enough, the $X$-particles decay very quickly.

Therefore, field $\chi$ may remain non-zero yet to the moment when $X$-particles have been completely decayed:

- e.g. $\chi \sim 2M \sim 10^{13}$ GeV at $t = 30M^{-1}$

(non-zero $\chi$ can induce $CP$ violation in the $X$-boson decays)
Non-zero $\chi$ can induce $CP$ violation in $X$-boson decays

The field $\chi$ is real and pseudoscalar, so it interacts with the produced fermions as

$$\mathcal{L}_{\chi\psi\psi} = g_{kl} \chi \bar{\psi}^k i\gamma_5 \psi^l = ig_{kl} \chi (\bar{\psi}_L^k \psi_R^l - \bar{\psi}_R^k \psi_L^l),$$

where $k$ and $l$ denote the fermion flavor.

Free fermion Lagrangian may contain mass terms

$$-m_{\psi kl} \bar{\psi}^k \psi^l = -m_{\psi kl} (\bar{\psi}_L^k \psi_R^l + \bar{\psi}_R^k \psi_L^l)$$

The sum of these terms and $\mathcal{L}_{\chi\psi\psi}$ can be presented in matrix form

$$-(\bar{\psi}_R M_\psi \psi_L + \bar{\psi}_L M_\psi^\dagger \psi_R),$$

here $M_\psi = m_\psi + ig \chi$ is a non-Hermitian matrix, whereas $m_\psi$ and $g$ are Hermitian ones.
Non-zero $\chi$ can induce $CP$ violation in $X$-boson decays

Using simultaneously two unitary transformations

\[ \psi_R \rightarrow \psi'_R = U_R \psi_R \quad \text{and} \quad \psi_L \rightarrow \psi'_L = U_L \psi_L \]

one can always diagonalize the matrix $M_\psi$.

The elements of transformation matrices $U_R$ and $U_L$ depend on the magnitude of the field $\chi$.

The mass terms take the simple form

\[ -m'_\psi_{ab} \bar{\psi}^a \psi^b , \]

where $\psi^a$ and $\psi^b$ are the mass eigenstates and $m'_\psi$ is diagonal matrix with real diagonal elements.
Non-zero $\chi$ can induce $CP$ violation in $X$-boson decays

However, the interaction of fermions with vector boson $X_\mu$ remains the same under these transformations:

$$g_{Rkl}X_\mu \bar{\psi}_R^k \gamma^\mu \psi_R^l = g'_{Rab}X_\mu \bar{\psi}_R^a \gamma^\mu \psi_R^b,$$

here $g'_R = U_R g_R U_R^\dagger$ is matrix of coupling constants in mass eigenstate basis (analogously $g'_L = U_L g_L U_L^\dagger$).

The constants $g'_{ab}$ are complex in general case, and if there are at least three species of fermions, one cannot rotate away simultaneously all phases in complex matrices $g'_{R,L}$ [M. Kobayashi, T. Maskawa, 1973].

The complexity of the coupling constants means that $CP$ is violated in the $X$-boson decays
The model may satisfy Sakharov criteria for successful baryogenesis without fine tuning of parameters

1. The magnitude of $CP$ violation depends on the value of $\chi$ through the matrices $U_{R,L}$ and hence coupling constants $g'_{R,L}$. Since $\chi$ is essentially non-zero after the end of inflation and during baryogenesis, $CP$-odd effects can be large enough.

We assume that interactions with $X$-boson involve fermions with certain chirality, and thus these interactions break $C$-invariance

2. State of matter is out of thermal equilibrium
3. Assume also that the baryon number is not conserved in $X$-boson decays

Baryon asymmetry generated in the decay of one $X$-particle:

$$\delta = \frac{1}{\Gamma_{tot}} \sum_f \Gamma(X \rightarrow f) B_f$$

The ratio of the baryon number density to the entropy density can be estimated as

$$\Delta_B = \frac{n_B - n_{\bar{B}}}{s} \sim 10^{-3} \delta$$

Thus, to get observed value $\Delta_B \simeq 0.86 \cdot 10^{-10}$ it is sufficient to have only $\delta \sim 10^{-7}$. 
The considered model may lead to baryo-symmetric universe with cosmologically large domains of matter and antimatter separated by cosmologically large distances, avoiding the domain wall problem.

Inflation is an essential ingredient of the scenario. A coupling of the pseudoscalar field $\chi$ to the inflaton field was introduced on purpose to generate a non-zero value of $\chi$ and to keep it during baryogenesis as a source of $CP$ violation.