Optical drift effects in cosmology: covariant approach

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in collaboration with

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Michele Grasso (CFT PAN Warsaw)
Julius Serbenta (CFT PAN Warsaw)

Hot Topics in Modern Cosmology
Spontaneous Workshop XII

IESC Cargèse, 14th-19th May 2018

Papers

Based on:

- M.K., J. Kopiński *“Optical drift effects in general relativity”*, JCAP 03 (2018) 012, e-print: 1711.00584 [gr-qc]
- M.K., M. Grasso, J. Serbenta *“Geometric optics in general relativity using bi-local operators”*, in preparation

NCN project SONATA BIS No 2016/22/E/ST9/00578
“Local relativistic perturbative framework in hydrodynamics and general relativity and its application to cosmology”

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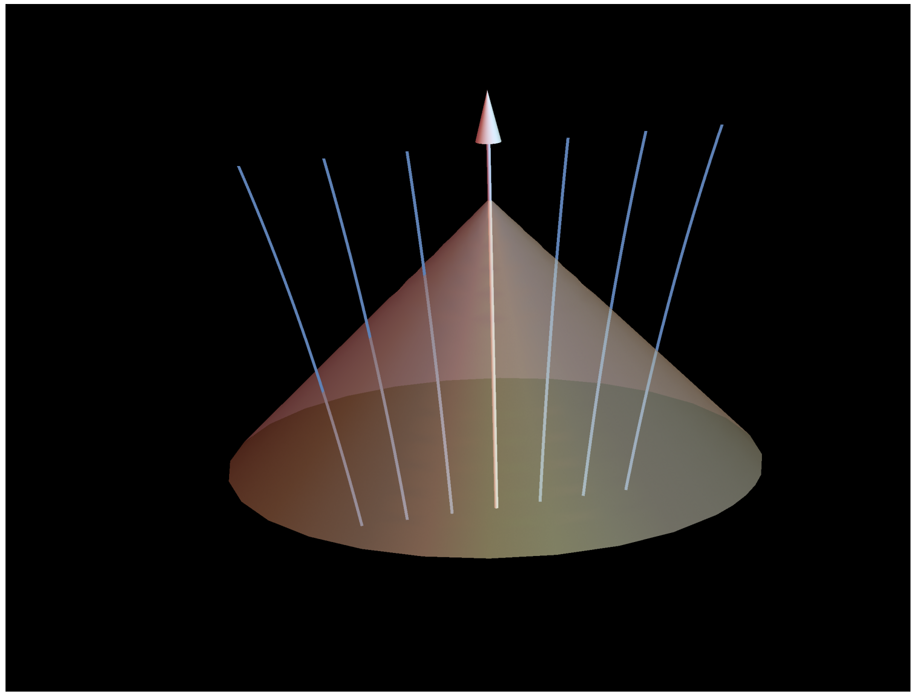

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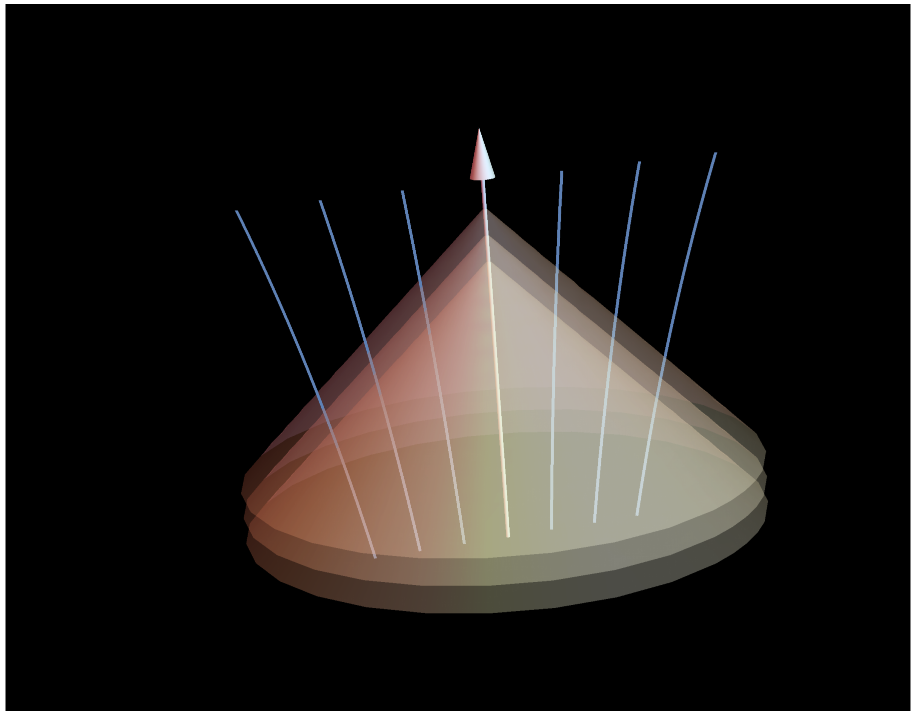

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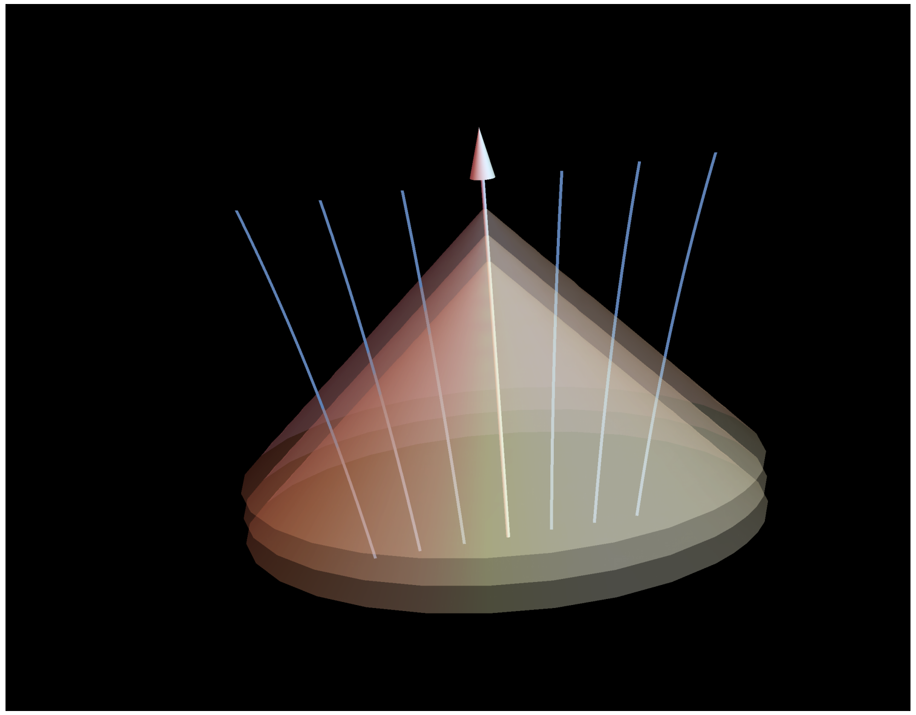

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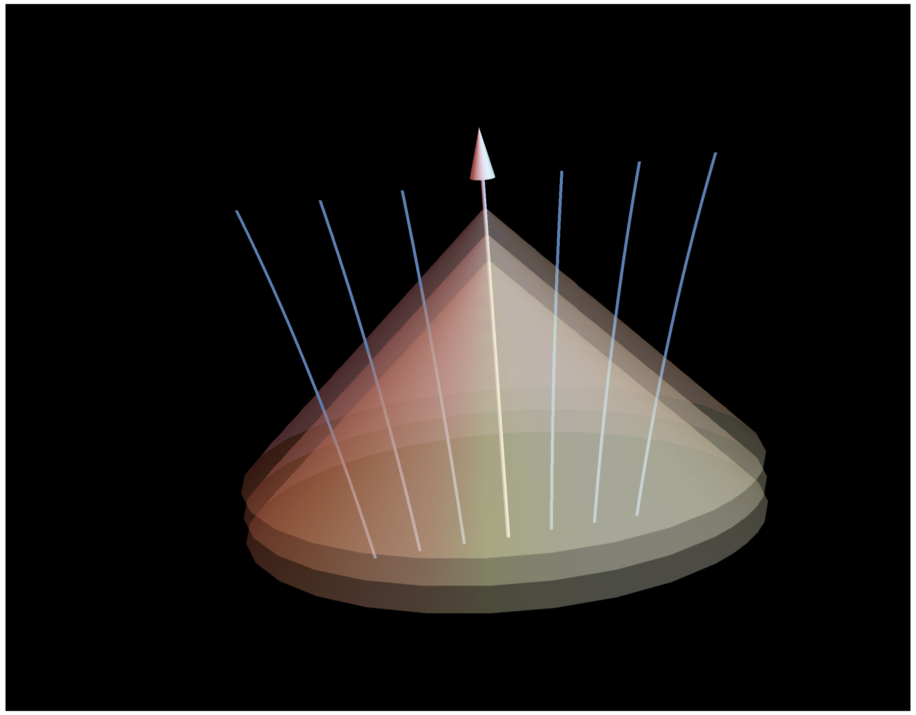

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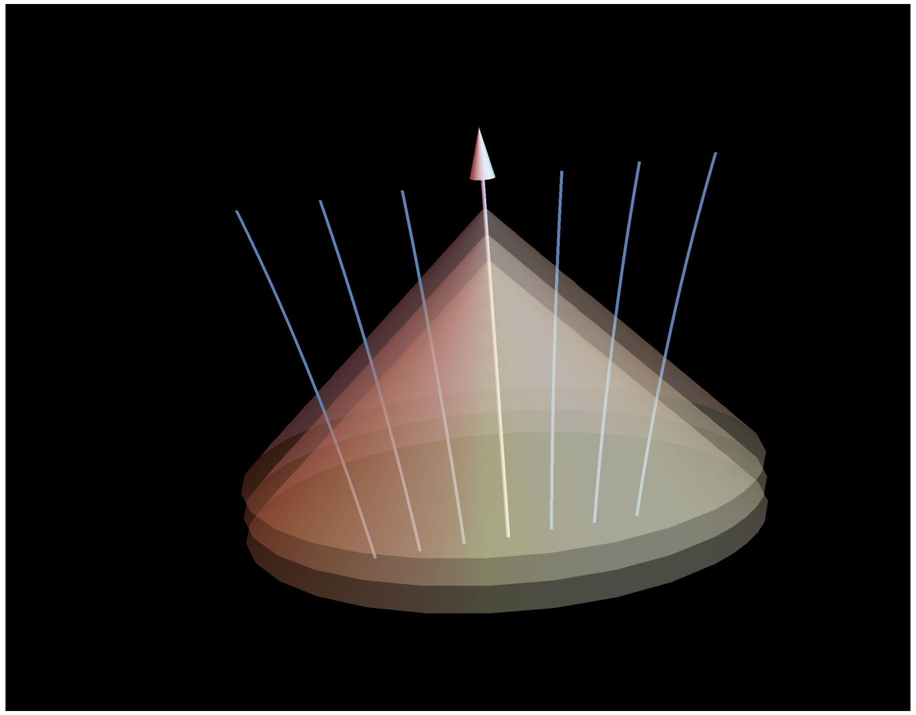

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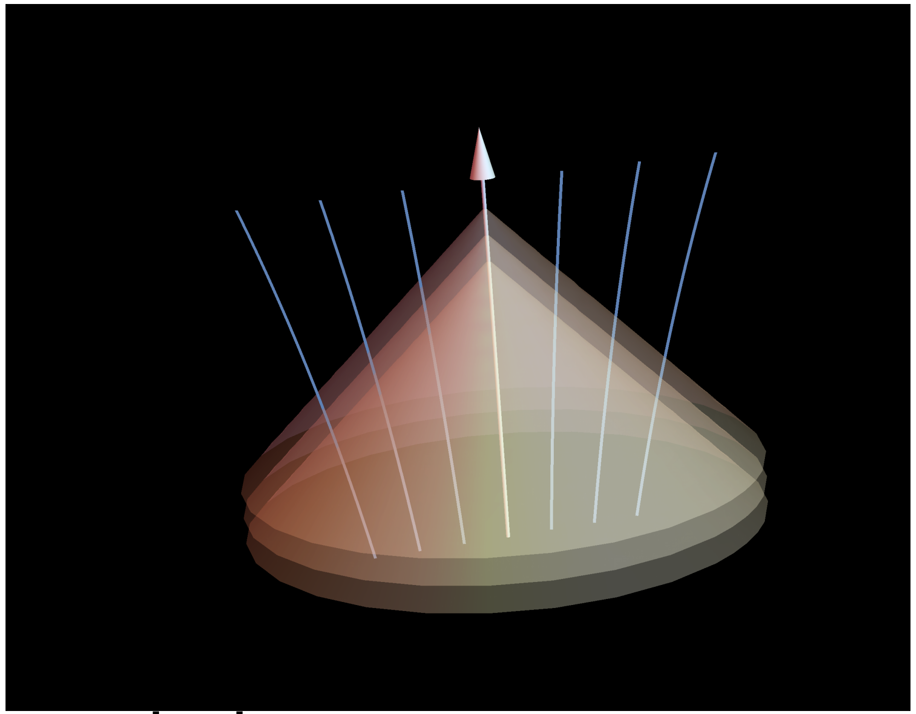

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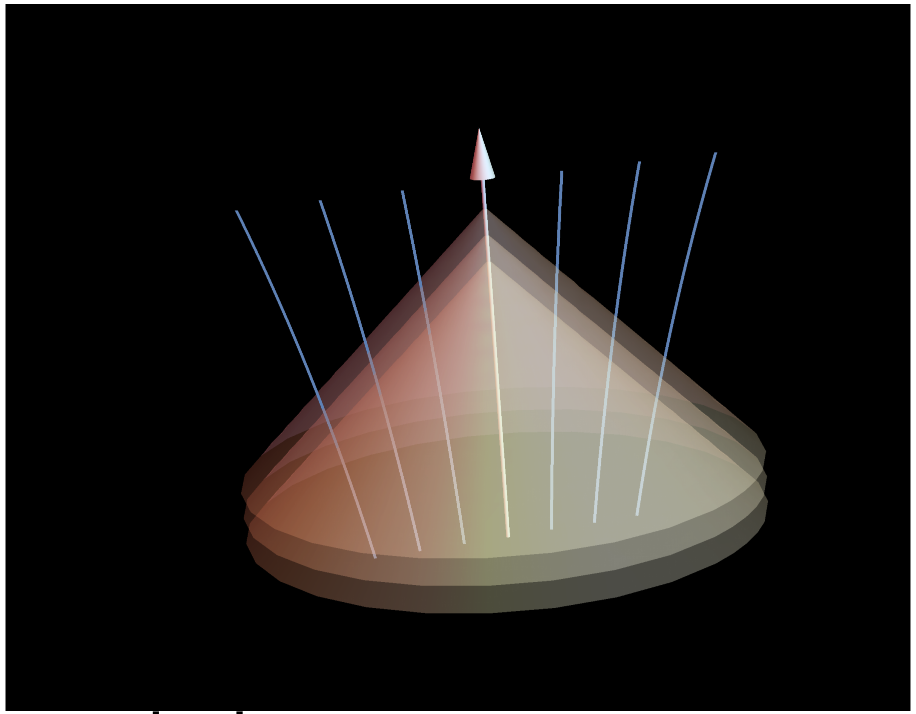

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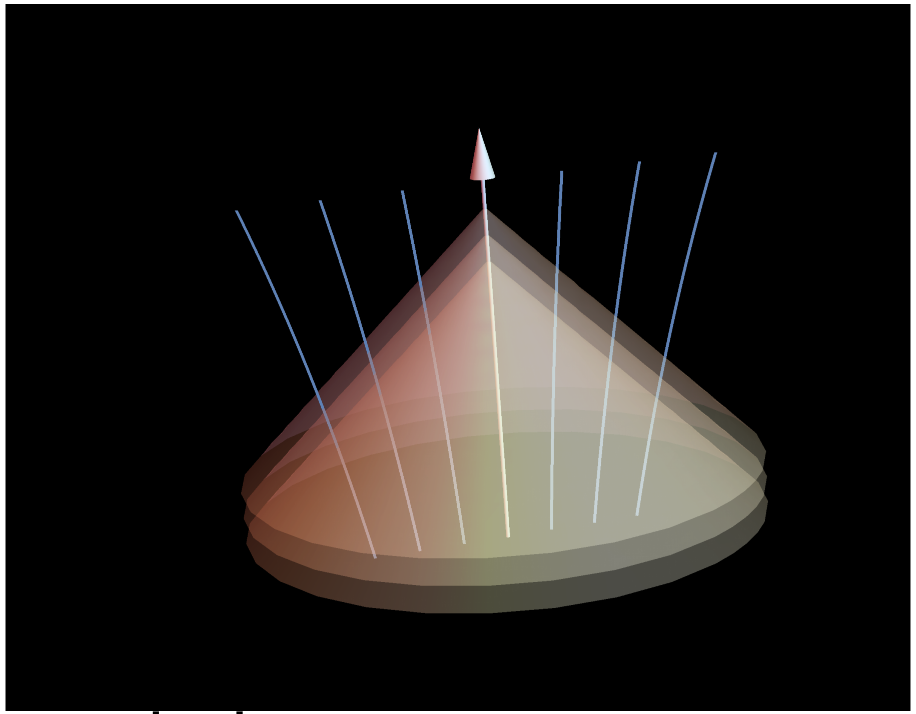

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Quercellini *et al “Real-time cosmology”* Phys. Rept. **521** (2012) 95

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FLRW without perturbation (non-rotating frame, cosmic flow observer and emitter):

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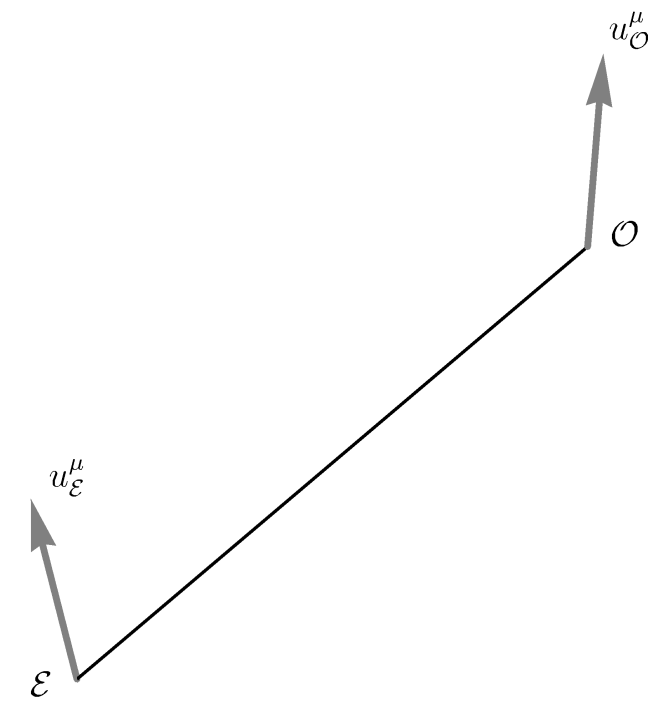
Need for a general formalism for optical drift effects in GR (applicable to *any* emitter and observer in *any* spacetime)



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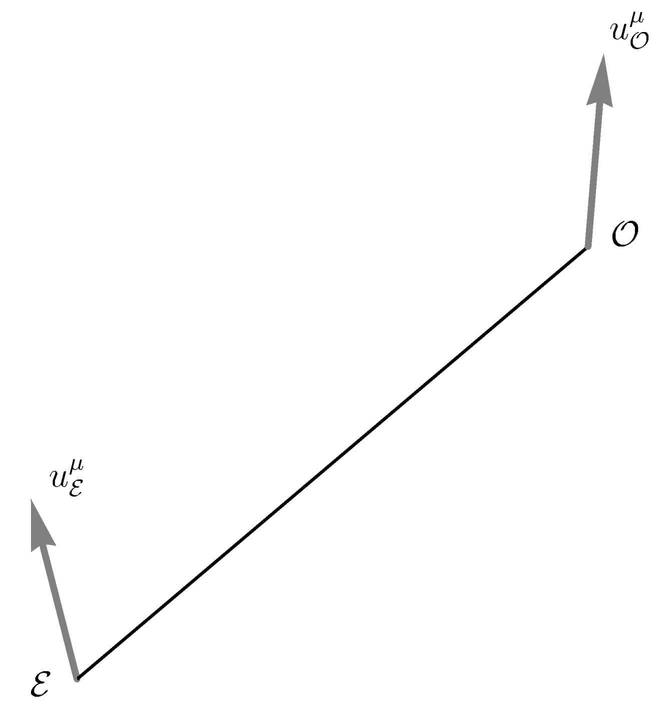

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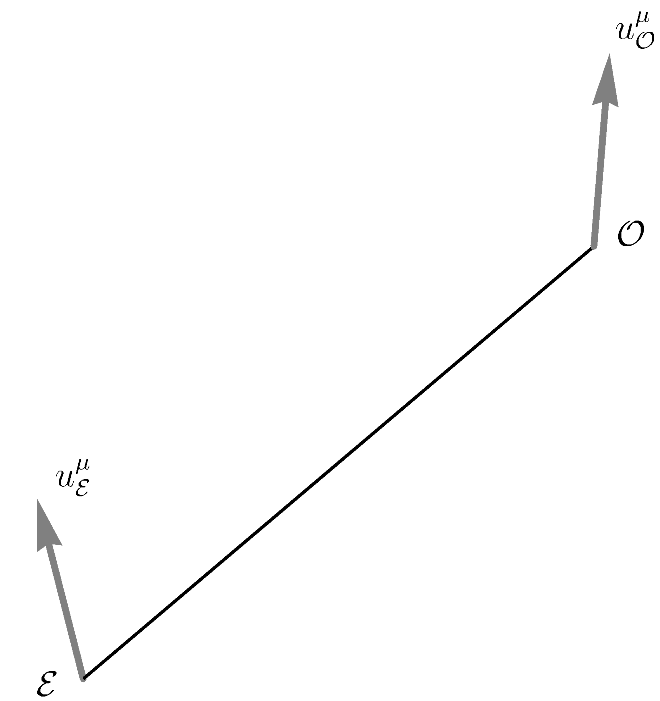

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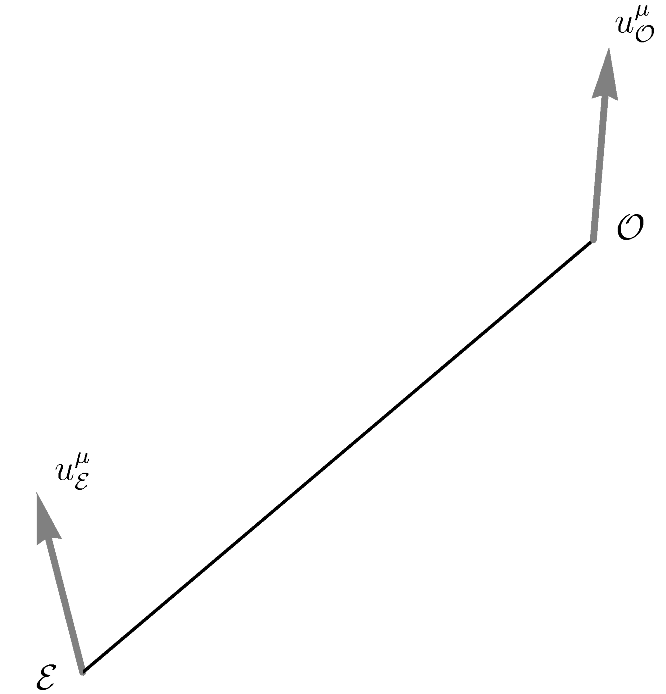

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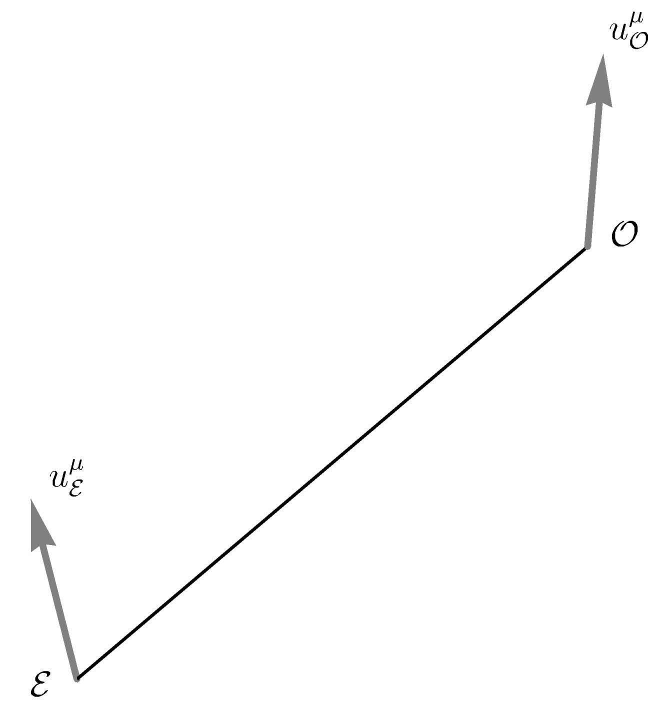

Separate the dependence on the emitter's and observer's peculiar motions
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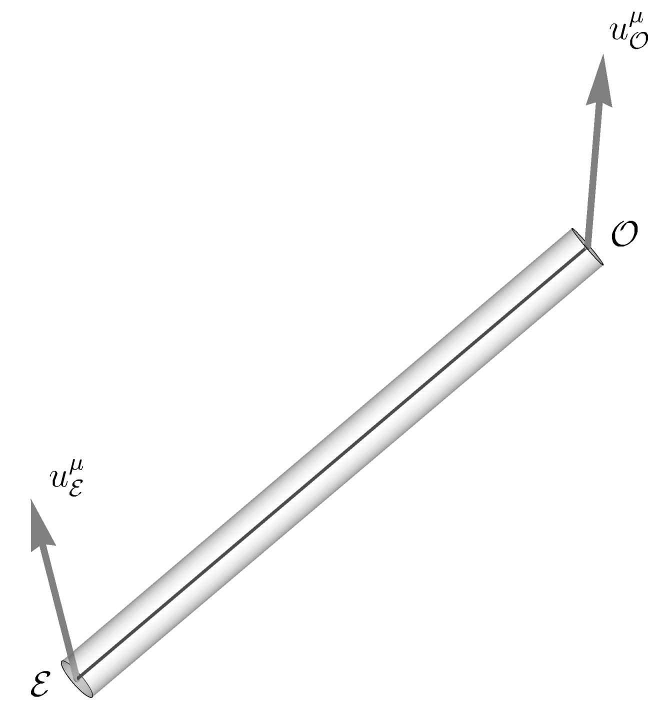

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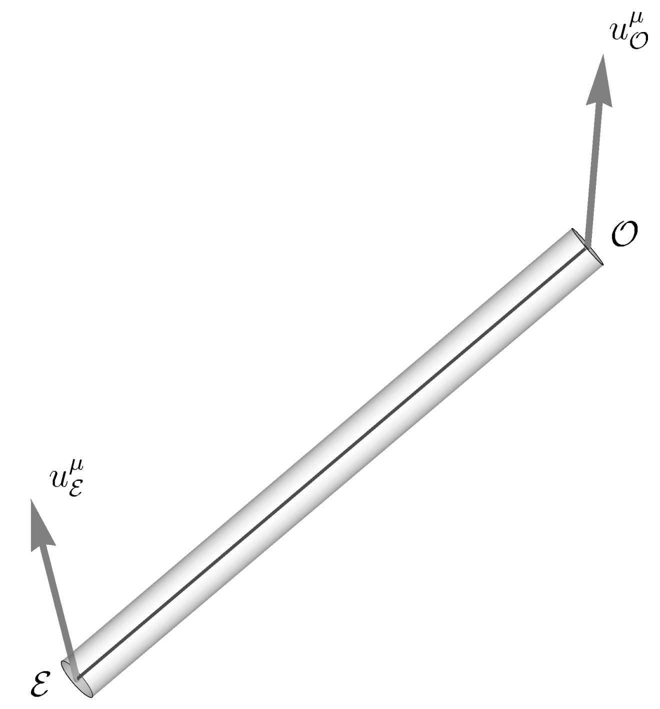

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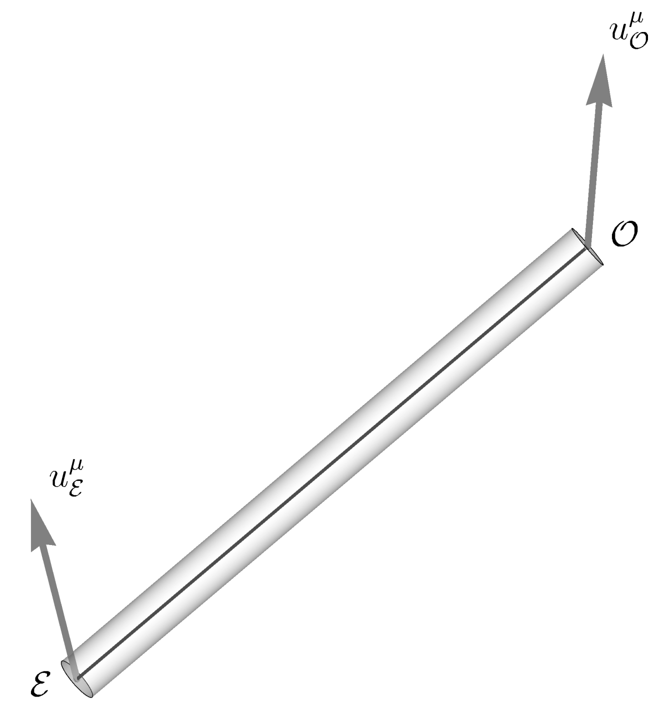

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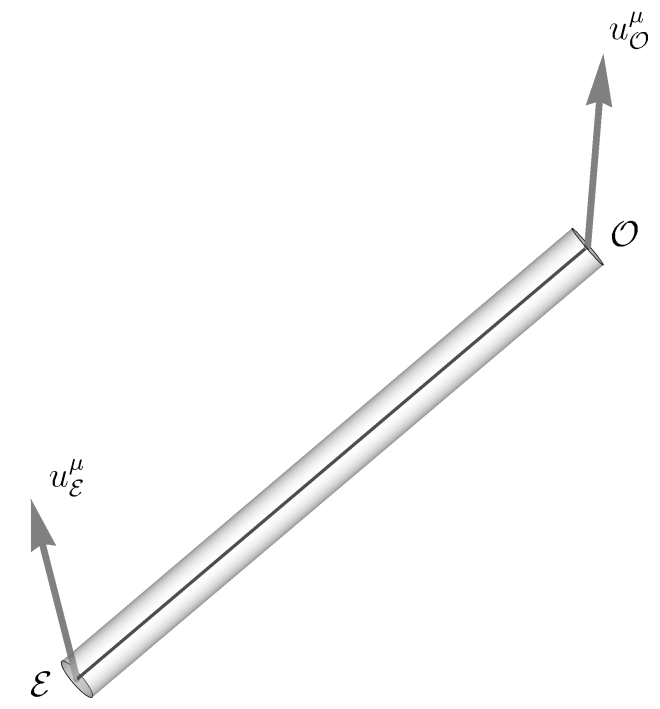

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Extension of the Sachs formalism (Sachs 1961, Etherington 1930's, Ehlers…)

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General expression for drift effects

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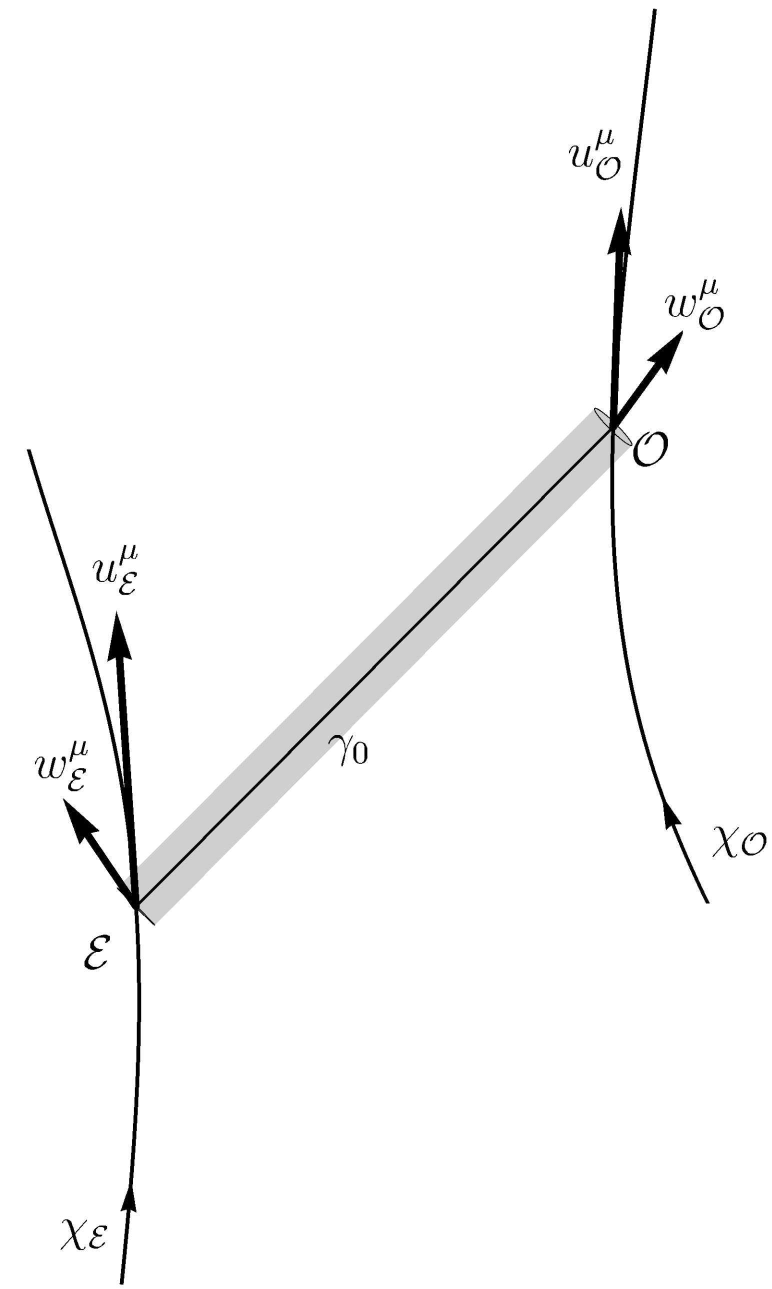

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General expression for drift effects

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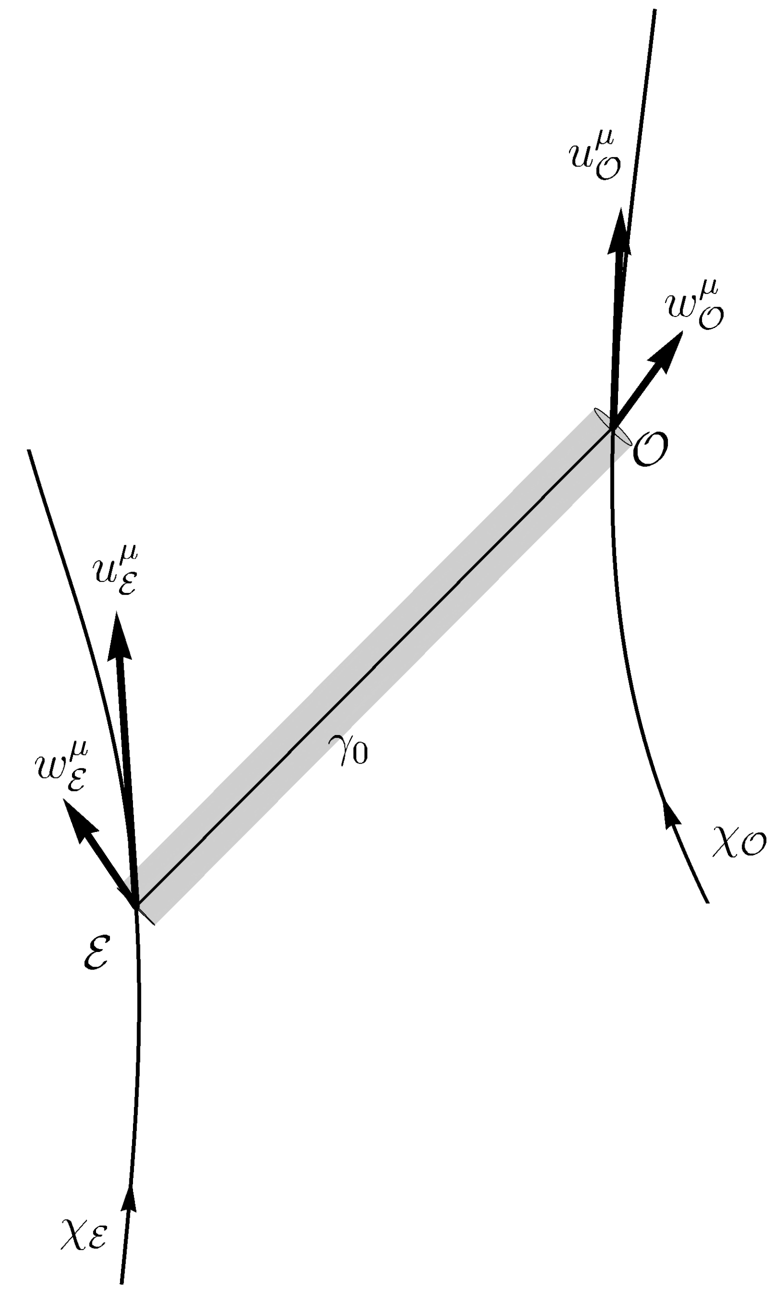

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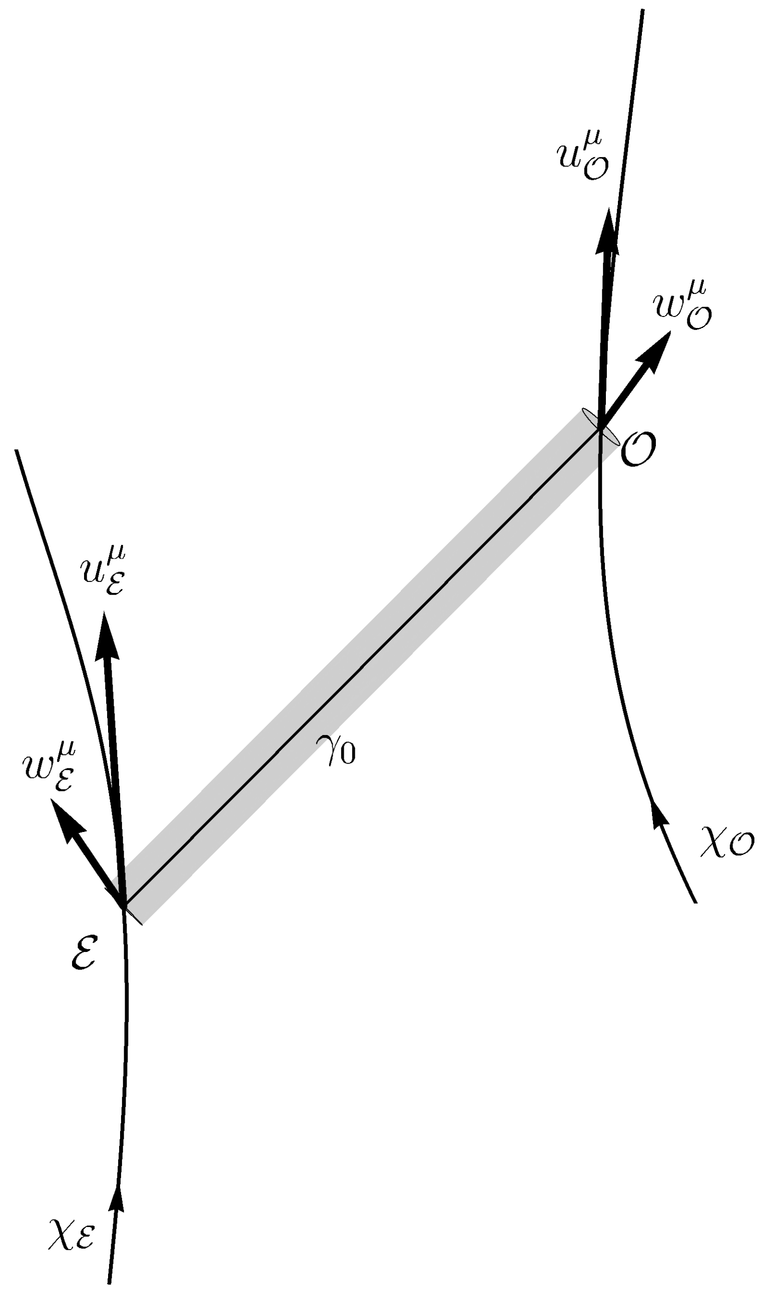
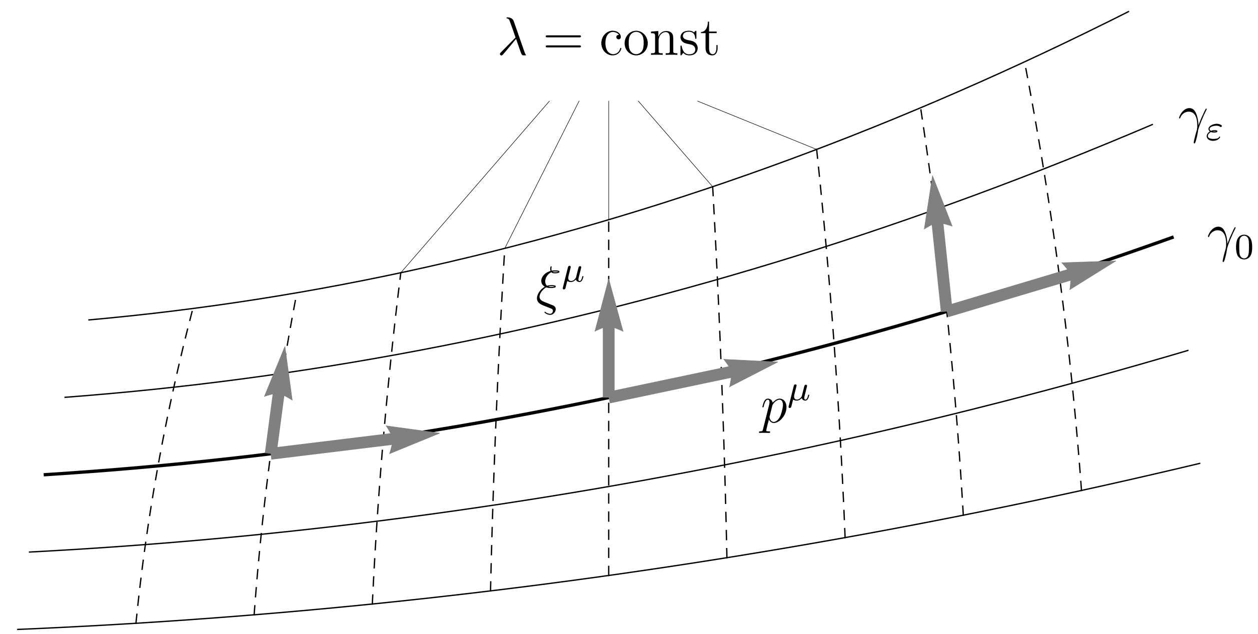


- Exact (all GR effects, all spacetimes)
- Separate the light propagation effects from motion effects

Geometry

Geodesic deviation equation

$$\mathcal{G}[\xi]^\mu \equiv \nabla_p \nabla_p \xi^\mu - R^\mu{}_{\nu\alpha\beta}\, p^\nu\, p^\alpha\, \xi^\beta = 0$$



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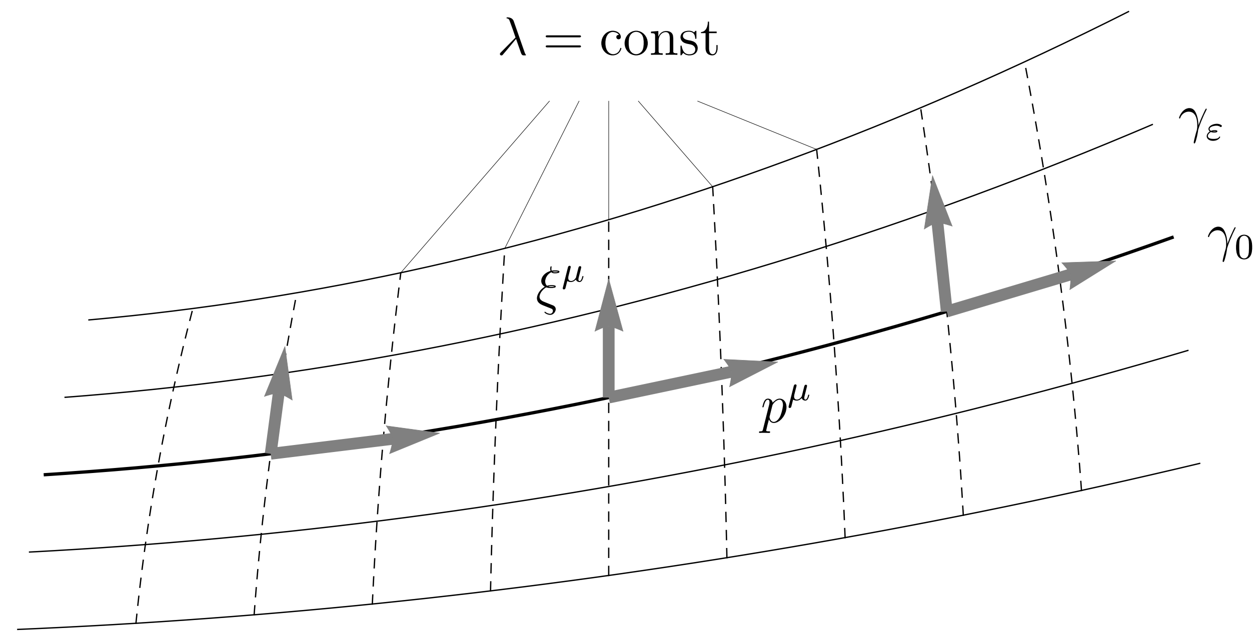

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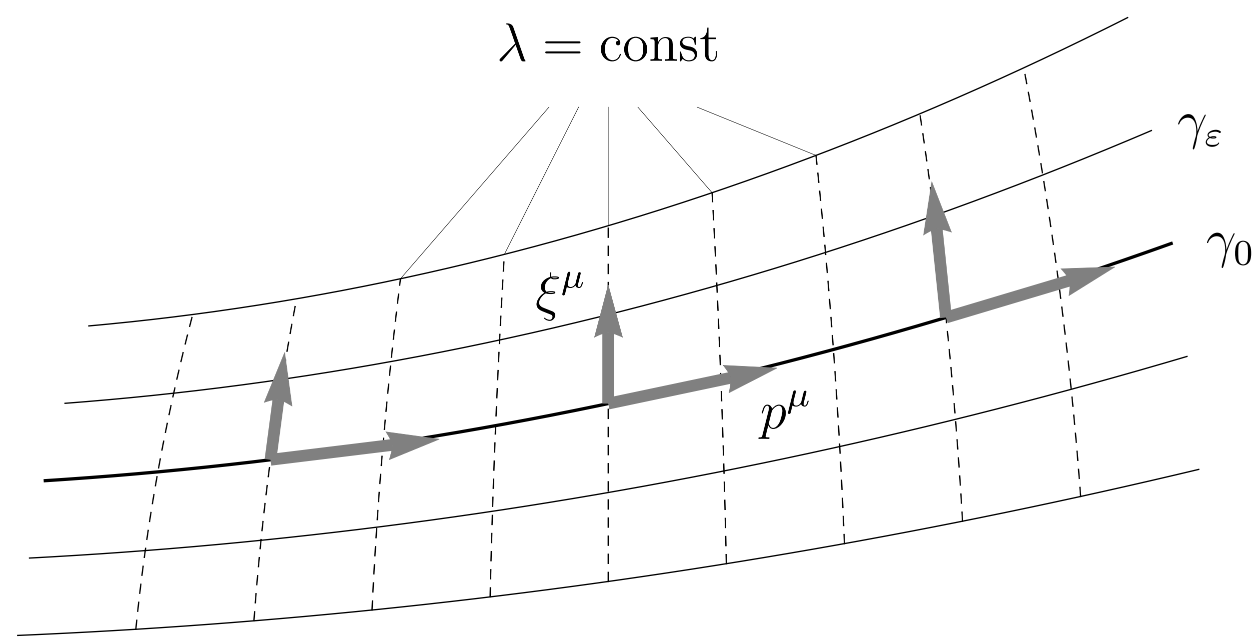

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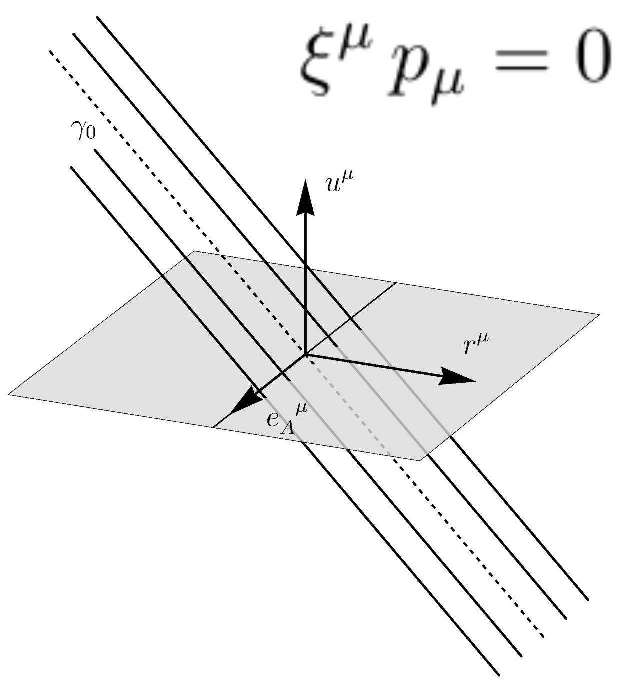

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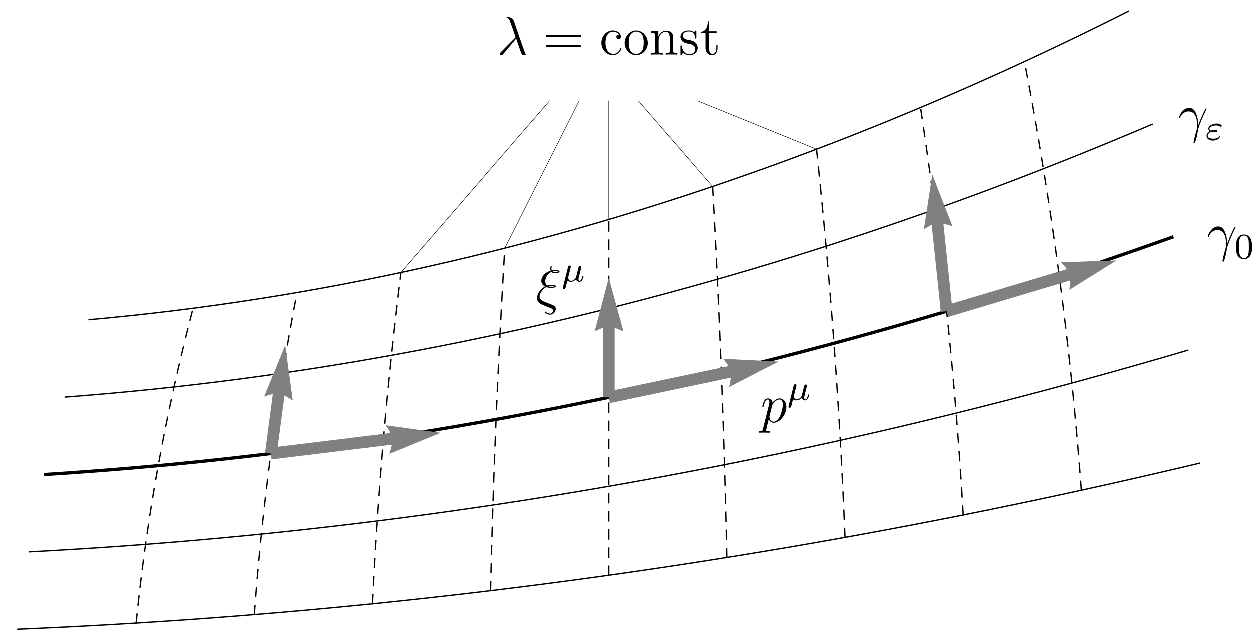

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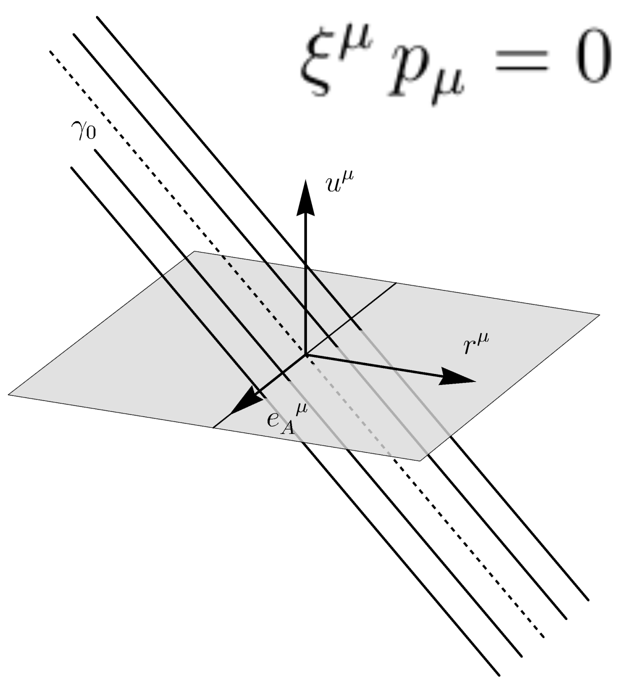

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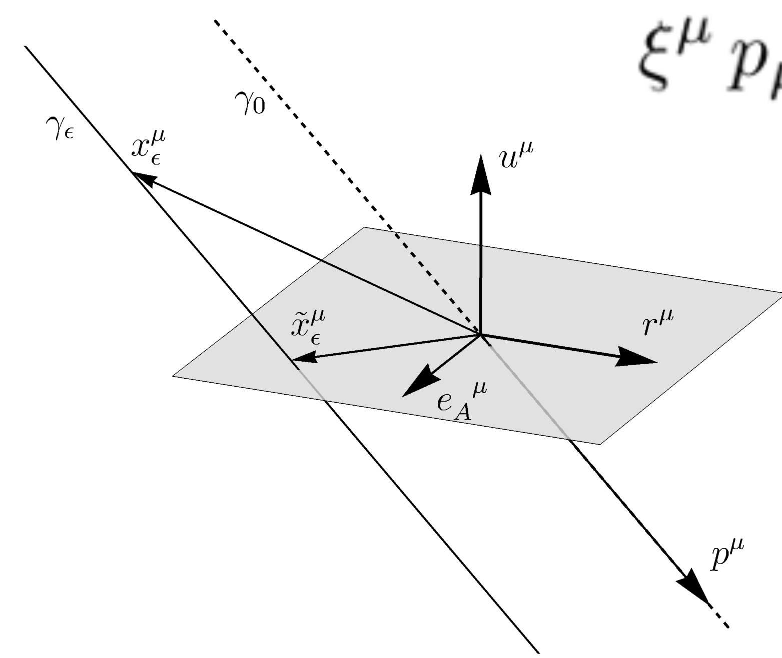

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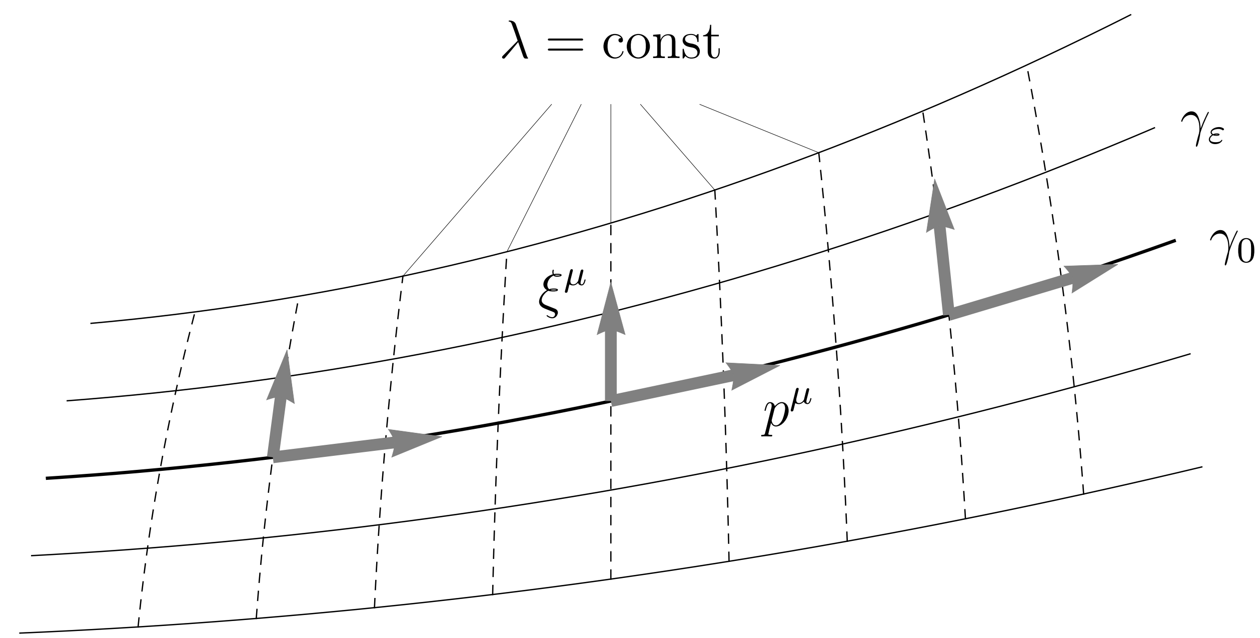
Moduli space, perpendicular space



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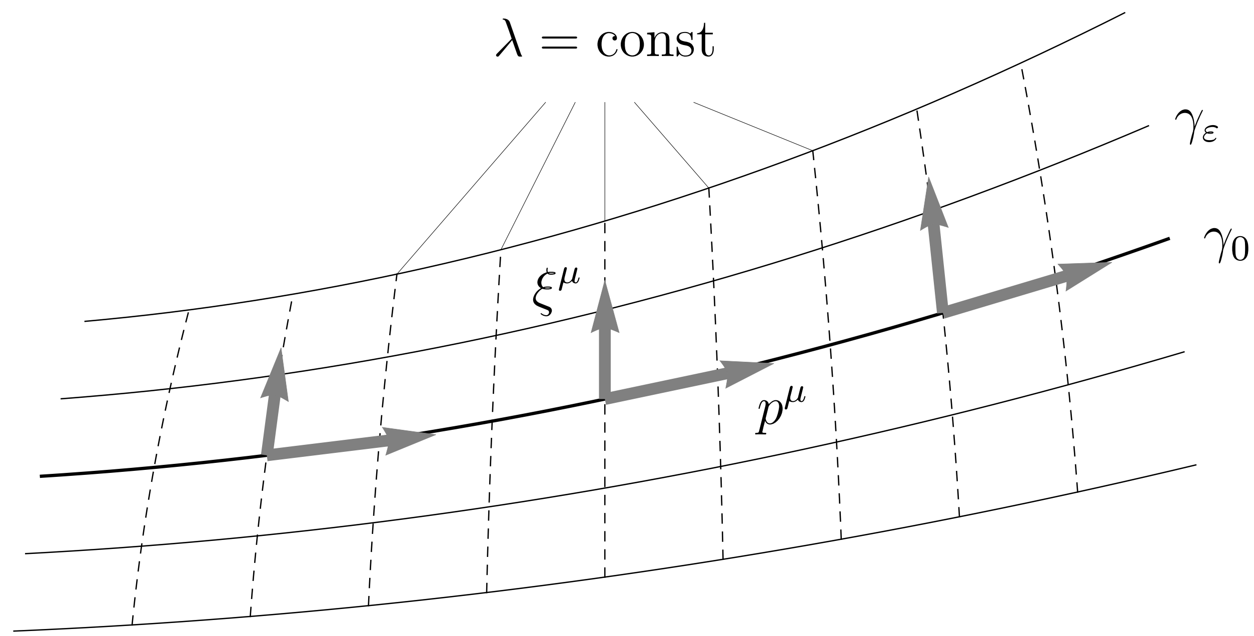
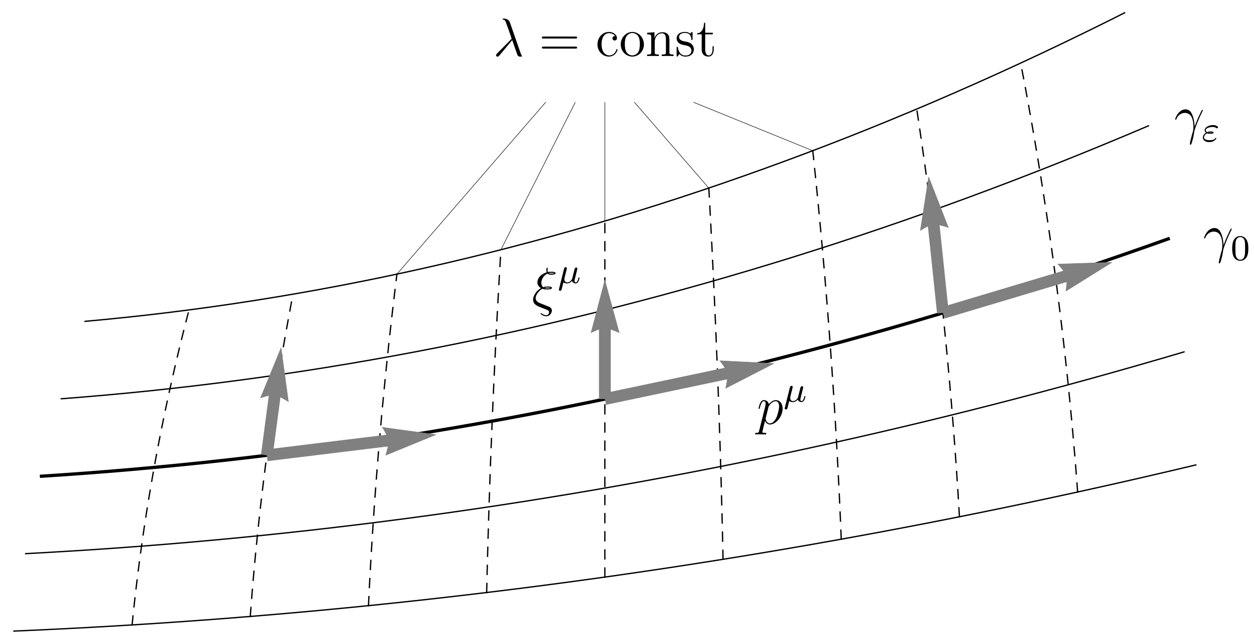
Moduli space, perpendicular space

- adding $C \cdot p^{\mu}$ is pure gauge



Geometry



Moduli space, perpendicular space

- adding $C \cdot p^\mu$ is pure gauge

Idea: consider the moduli space

Geometry

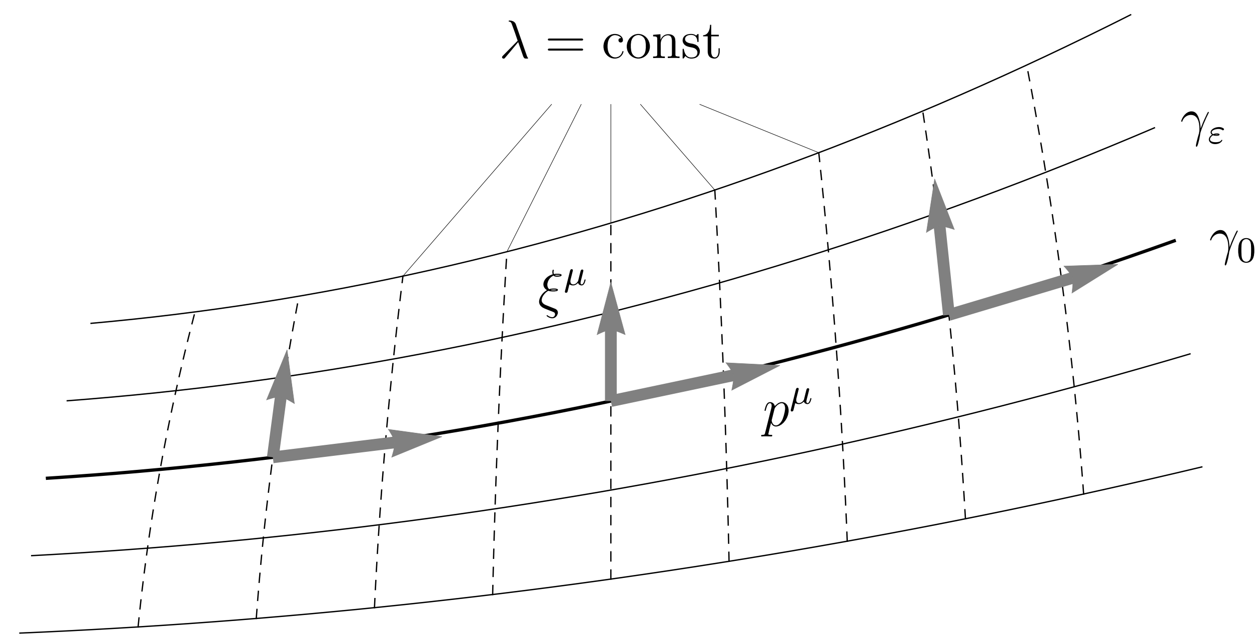
Moduli space, perpendicular space

- adding $C \cdot p^{\mu}$ is pure gauge

Idea: consider the moduli space

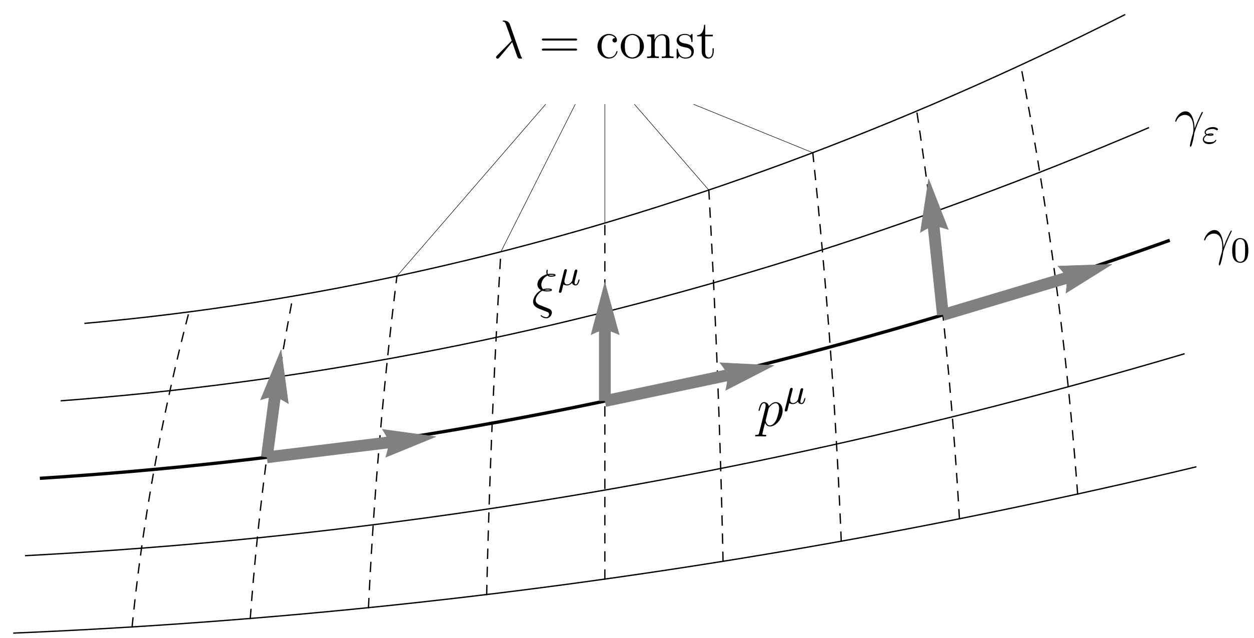
identifying vectors differing by $C \cdot p^{\mu}$

define $\mathrm{T}_{\mathcal{O}}M\,/\,p$



Geometry

Moduli space, perpendicular space



- adding $C \cdot p^\mu$ is pure gauge

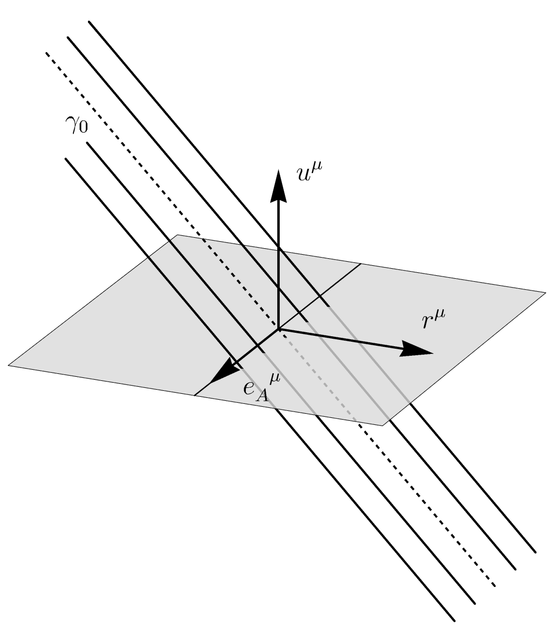
Idea: consider the moduli space

identifying vectors differing by $C \cdot p^\mu$

define $\mathrm{T}_{\mathcal{O}}M\,/\,p$

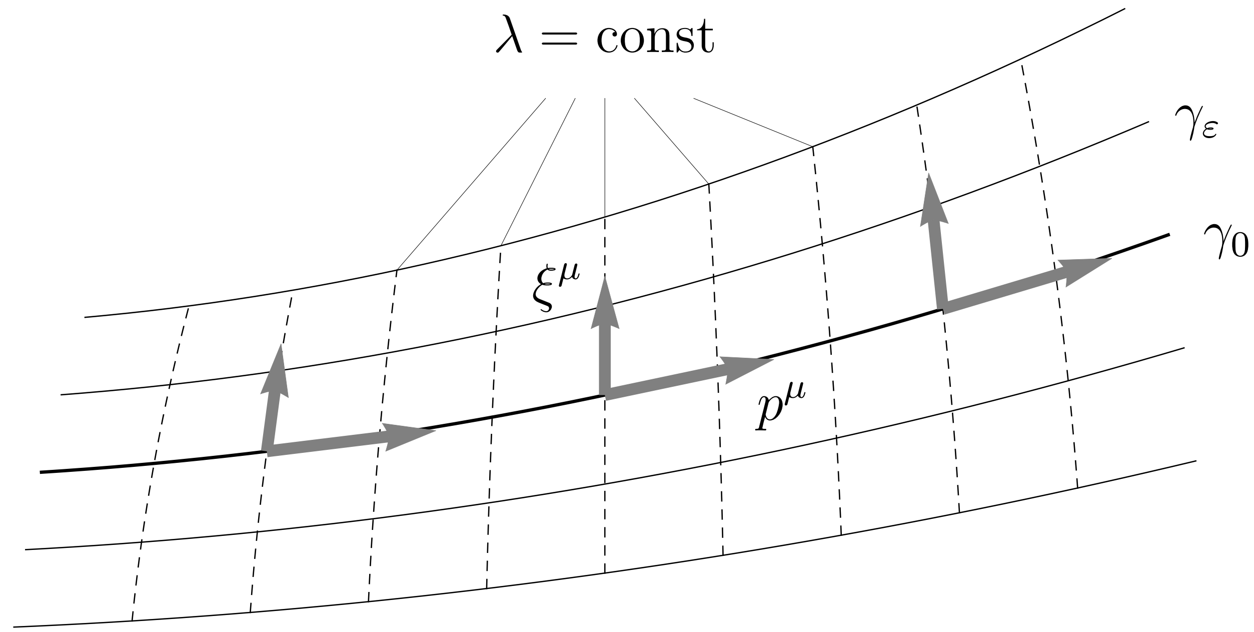
- orthogonally displaced null geodesics

$$\xi^\mu\, p_\mu = 0$$



Geometry

Moduli space, perpendicular space



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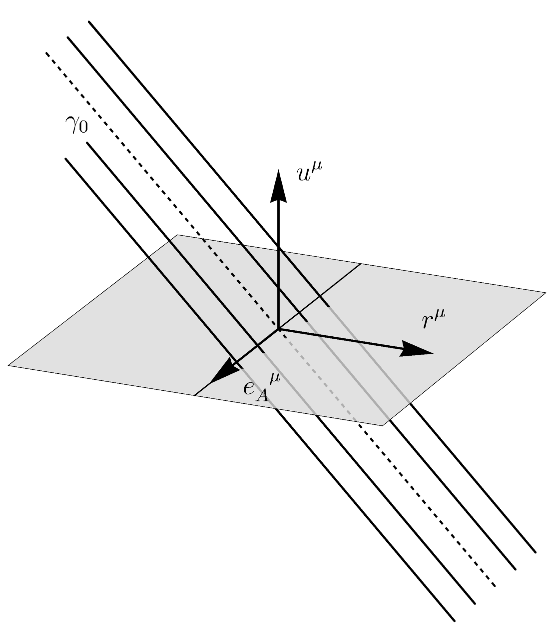
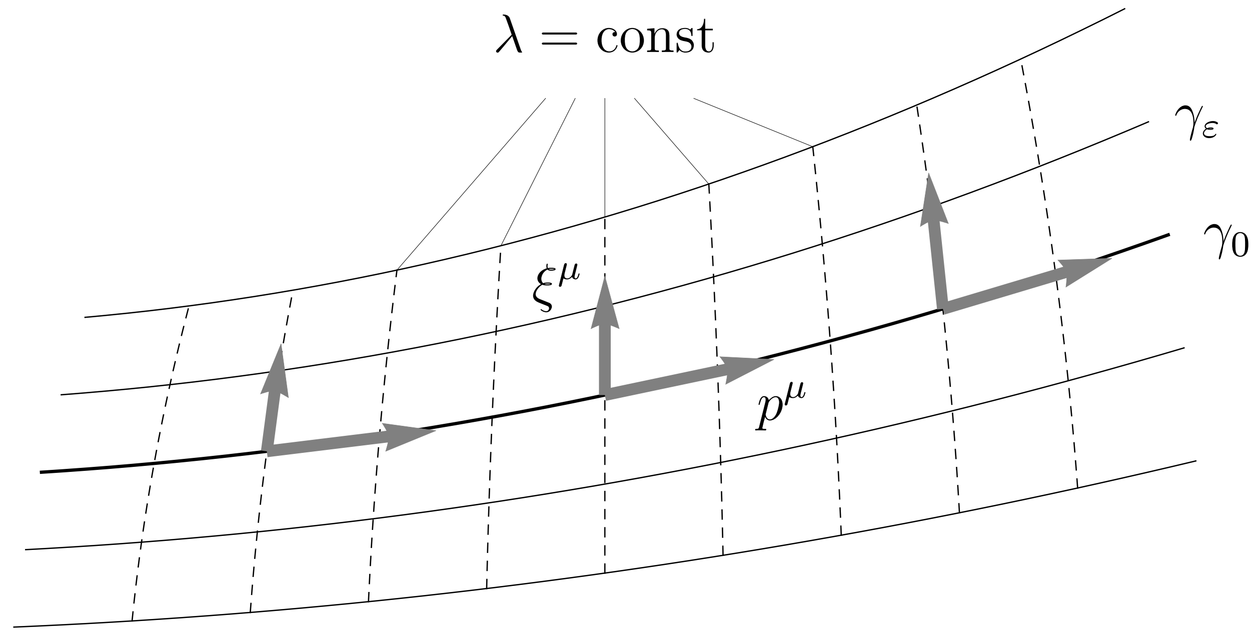


identifying vectors differing by $C \cdot p^\mu$

define *perpendicular space* $\mathscr{P} = p^\perp / p$

Geometry



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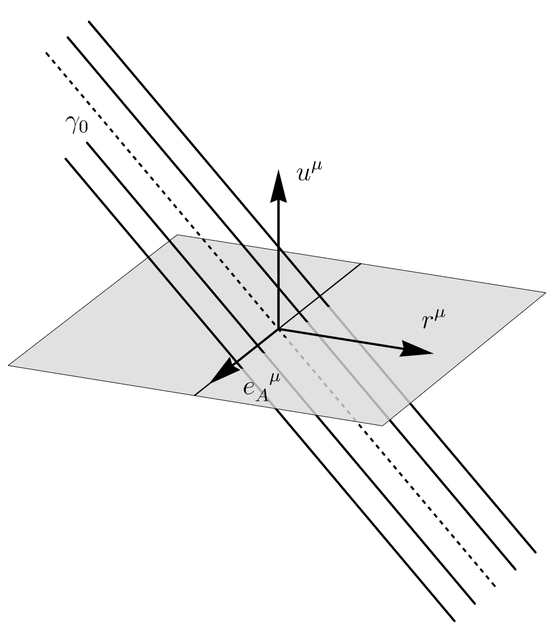


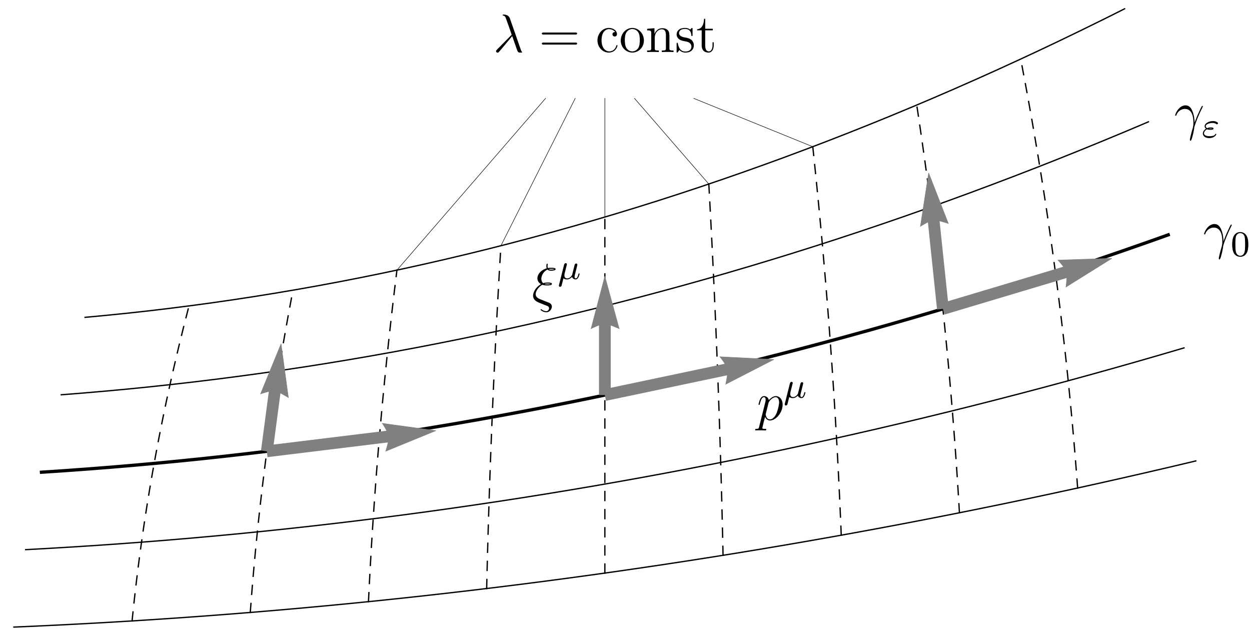
identifying vectors differing by $C \cdot p^\mu$

define *perpendicular space* $\mathscr{P} = p^\perp / p$

space of null geodesics displaced orthogonally to p^μ

Geometry

Moduli space, perpendicular space



- adding $C \cdot p^\mu$ is pure gauge

Idea: consider the moduli space

identifying vectors differing by $C \cdot p^\mu$

define $\mathrm{T}_{\mathcal{O}}M \,/\, p$

- orthogonally displaced null geodesics

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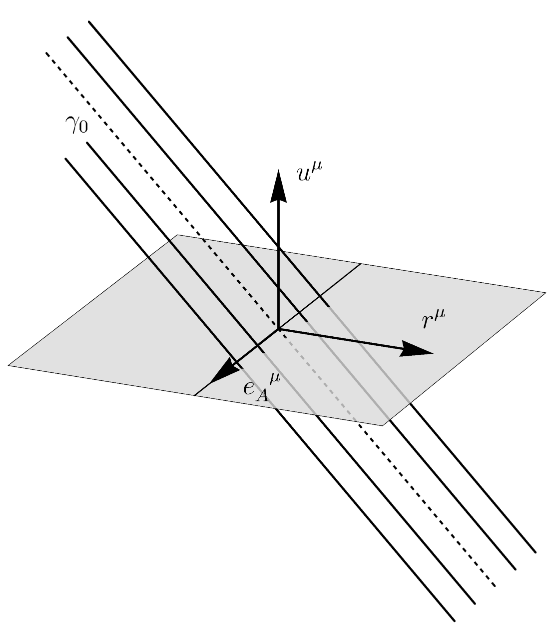


identifying vectors differing by $C \cdot p^\mu$

define *perpendicular space* $\mathscr{P} = p^\perp \,/\, p$

space of null geodesics displaced orthogonally to p^μ

$\mathscr{P}$ inherits an observer-independent metric

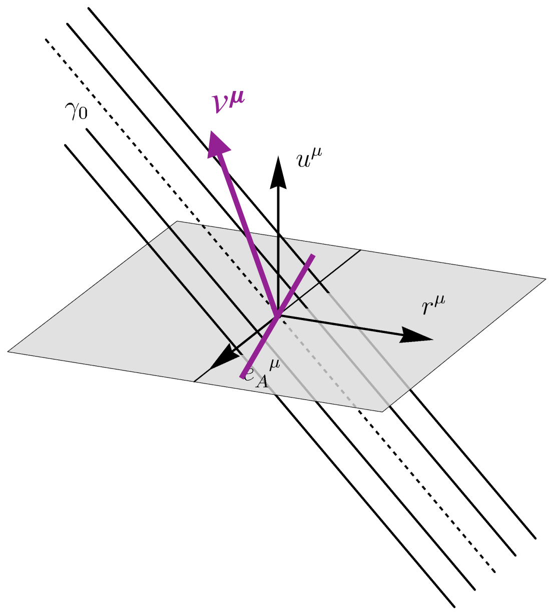
$\mathscr{P}$ makes the formalism observer-invariant

Geometry

Perpendicular space $\mathscr{P}$

- orthogonally displaced null geodesics

$$\xi^\mu\, p_\mu = 0$$



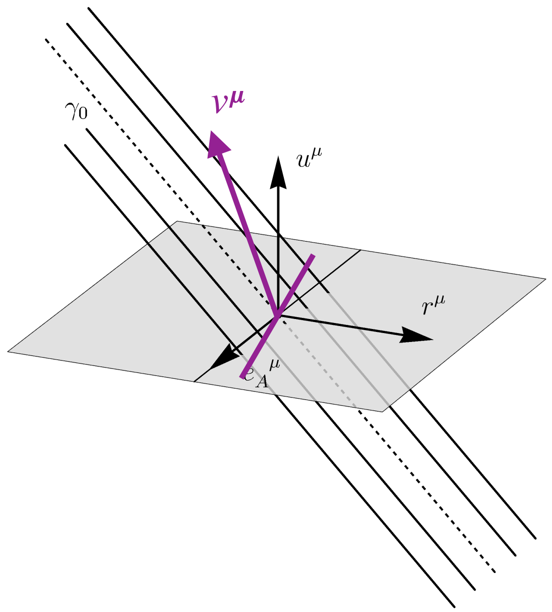
Geometry

Perpendicular space $\mathscr{P}$

Different observers u and v $\Rightarrow$ different notions of simultaneity

- orthogonally displaced null geodesics

$$\xi^{\mu}\, p_{\mu} = 0$$



Geometry

Perpendicular space $\mathscr{P}$

Different observers u and v ⇒ different notions of simultaneity

⇒ different screen spaces

- orthogonally displaced null geodesics

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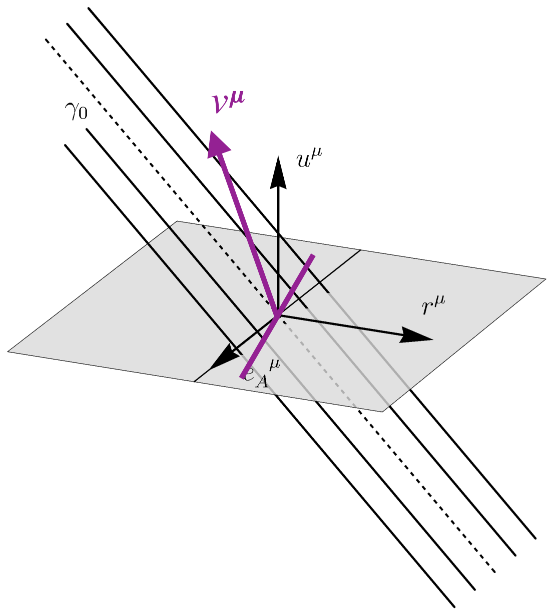


Geometry

Perpendicular space $\mathscr{P}$

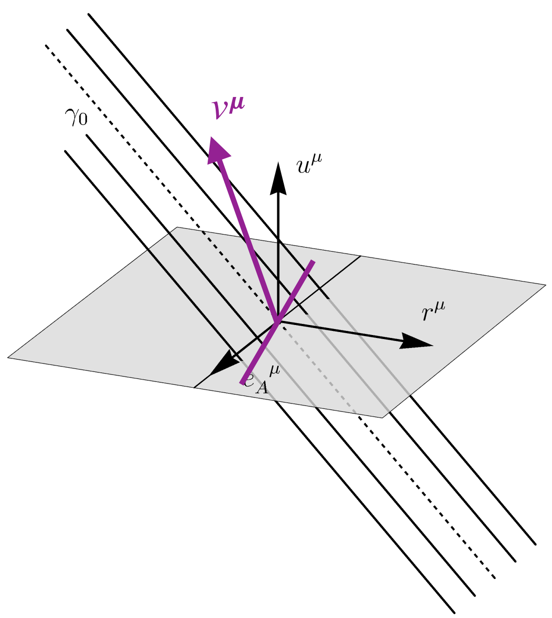
Different observers u and v ⇒ different notions of simultaneity

⇒ different screen spaces

⇒ screen spaces punctured by light rays at different points

- orthogonally displaced null geodesics

$$\xi^\mu p_\mu = 0$$

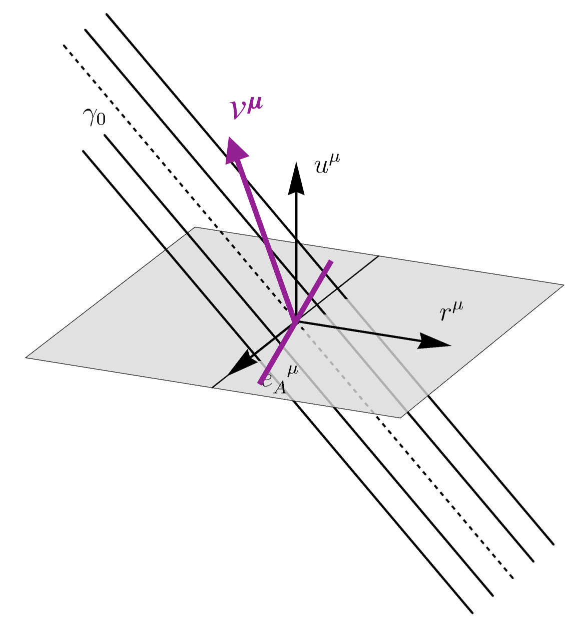


Geometry

Perpendicular space $\mathscr{P}$

- orthogonally displaced null geodesics

$$\xi^{\mu}\, p_{\mu} = 0$$



Different observers u and v ⇒ different notions of simultaneity

⇒ different screen spaces

⇒ screen spaces punctured by light rays at different points

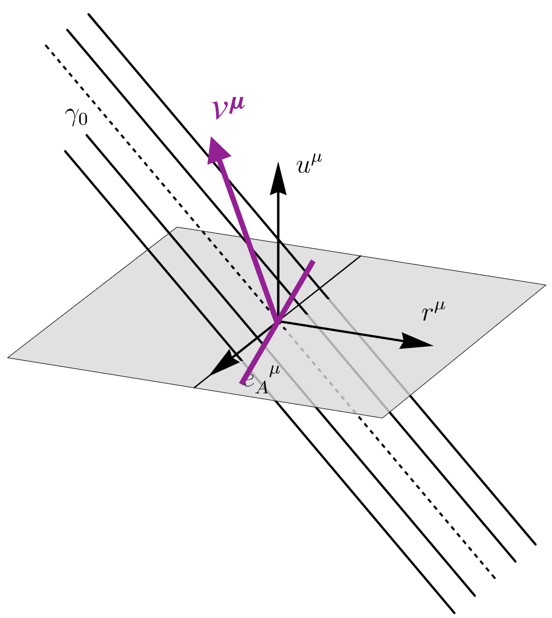
Yet, the distances measured to a given light rays (+ angles) are *the same*

Geometry

Perpendicular space $\mathscr{P}$

- orthogonally displaced null geodesics

$$\xi^\mu p_\mu = 0$$



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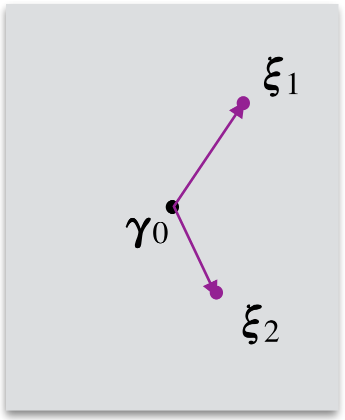
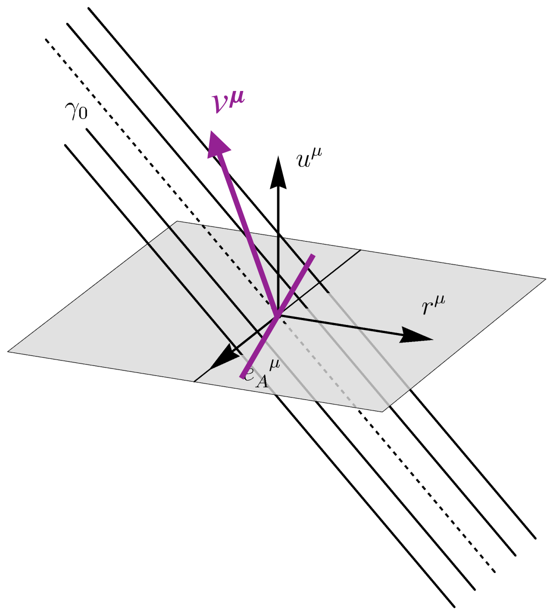


Geometry

Perpendicular space $\mathscr{P}$

- orthogonally displaced null geodesics

$$\xi^\mu p_\mu = 0$$



Different observers u and v ⇒ different notions of simultaneity

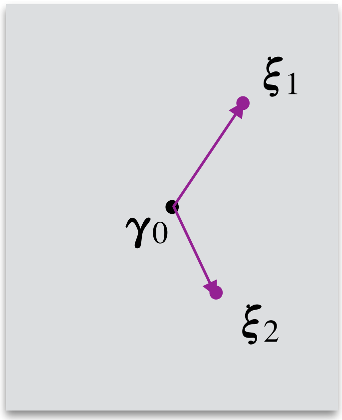
⇒ different screen spaces

⇒ screen spaces punctured by light rays at different points

Yet, the distances measured to a given light rays (+ angles) are *the same*

$$d(\xi) = g_{AB}\, \xi^A\, \xi^B$$

$\mathscr{P}$ $\mathscr{P}$ $\mathscr{P}$



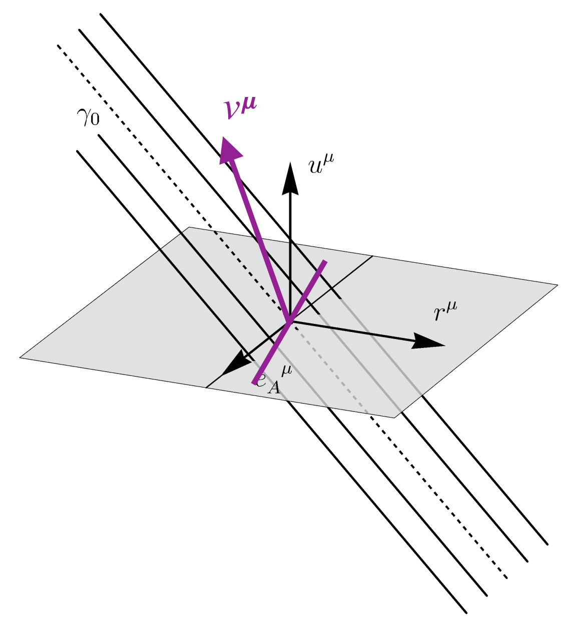
Geometry

Perpendicular space $\mathscr{P}$

- orthogonally displaced null geodesics

Different observers u and v ⇒ different notions of simultaneity

$$\xi^\mu p_\mu = 0$$



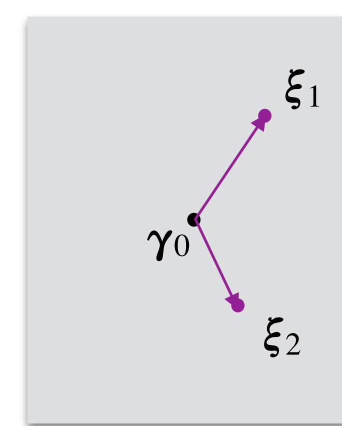
⇒ different screen spaces

⇒ screen spaces punctured by light rays at different points

Yet, the distances measured to a given light rays (+ angles) are *the same*

$$d(\xi) = g_{AB}\, \xi^A\, \xi^B$$

$$\mathscr{P} \qquad \mathscr{P} \qquad \mathscr{P}$$



$\mathscr{P}$ = identification of screen spaces of all observers

Geometry

Working in a parallel propagated frame

semi-null frame (u, e_1, e_2, p)

$g(u, u) = -1$

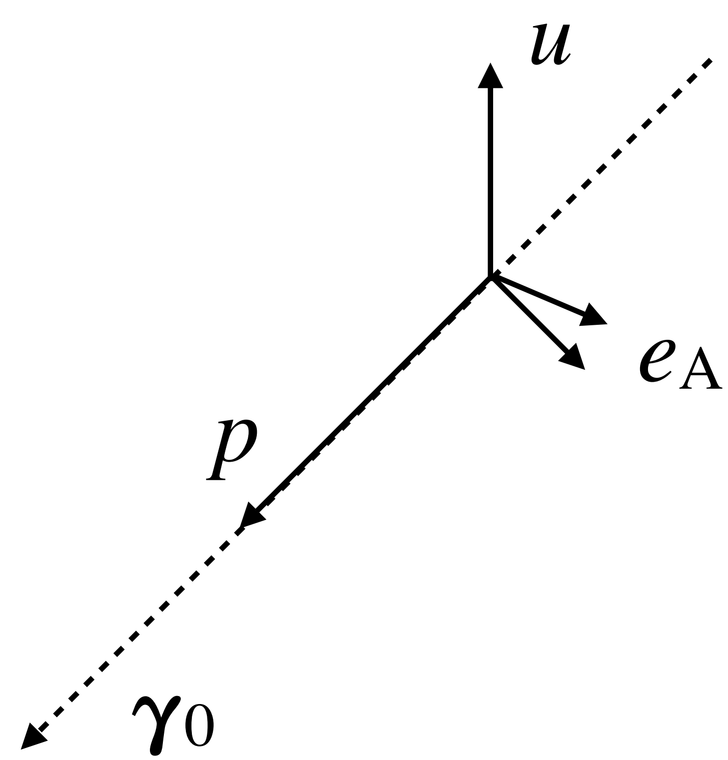
$g(p, p) = 0$

$g(u, p) = const$

$g(e_A, e_B) = \delta_{AB}$

$g(e_A, u) = 0$

$g(e_A, p) = 0$



Geometry

Working in a parallel propagated frame

semi-null frame (u, e_1, e_2, p)

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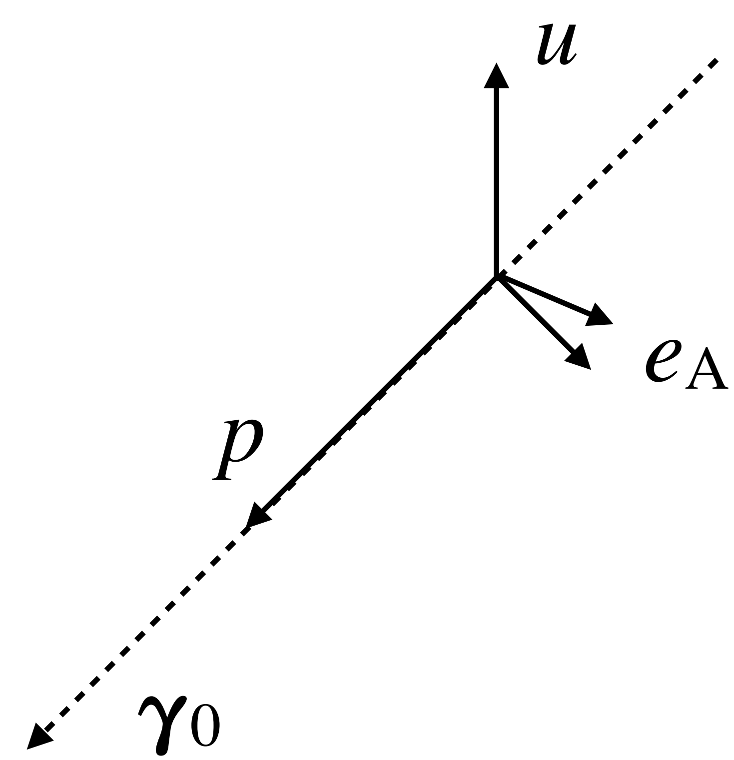
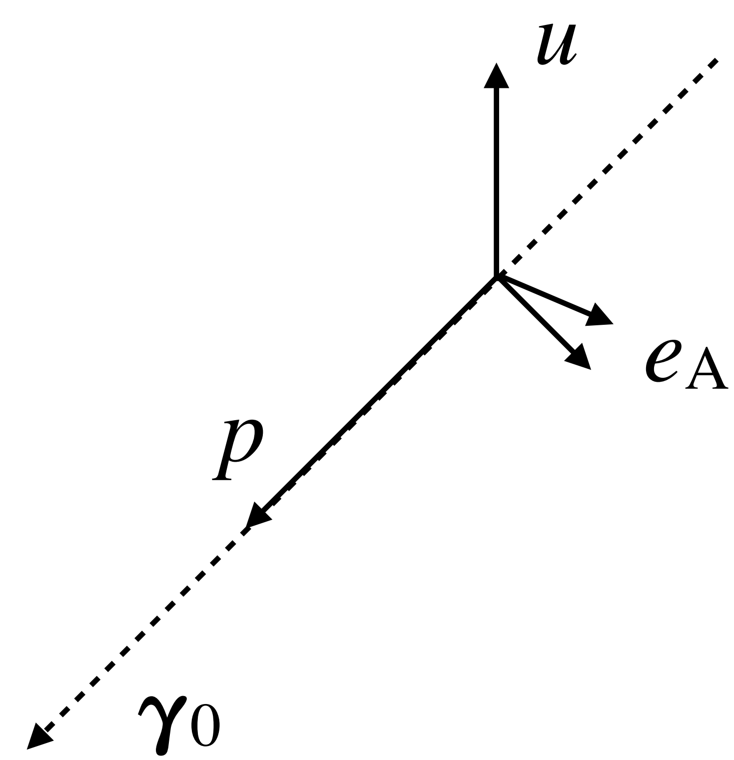
$g(u, p) = const$ $\qquad g(e_A, p) = 0$



$$\ddot{\xi}^\mu - R^\mu{}_{\alpha\beta\nu}\, p^\alpha\, p^\beta\, \xi^\nu = 0$$

Geometry

Working in a parallel propagated frame



semi-null frame (u, e_1, e_2, p)

$g(u, u) = -1$

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$g(e_A, u) = 0$

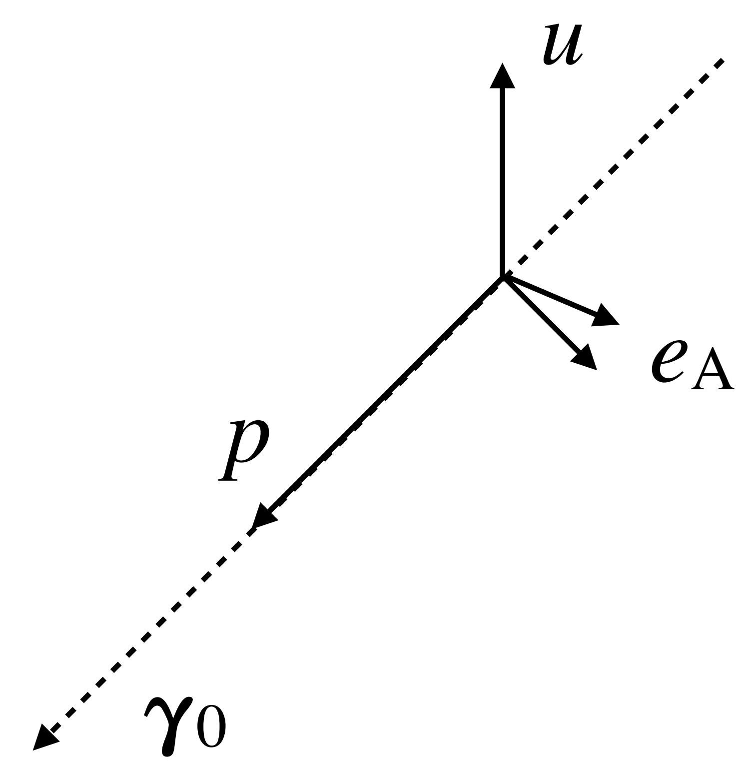
$g(e_A, p) = 0$

$$\ddot{\xi}^\mu - R^\mu{}_{\alpha\beta\nu}\, p^\alpha\, p^\beta\, \xi^\nu = 0$$

$$\begin{pmatrix} \xi^0 \\ \xi^1 \\ \xi^2 \\ \xi^3 \end{pmatrix}$$

Geometry

Working in a parallel propagated frame



semi-null frame (u, e_1, e_2, p)

$g(u, u) = -1$	$g(e_A, e_B) = \delta_{AB}$
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$g(u, p) = const$	$g(e_A, p) = 0$

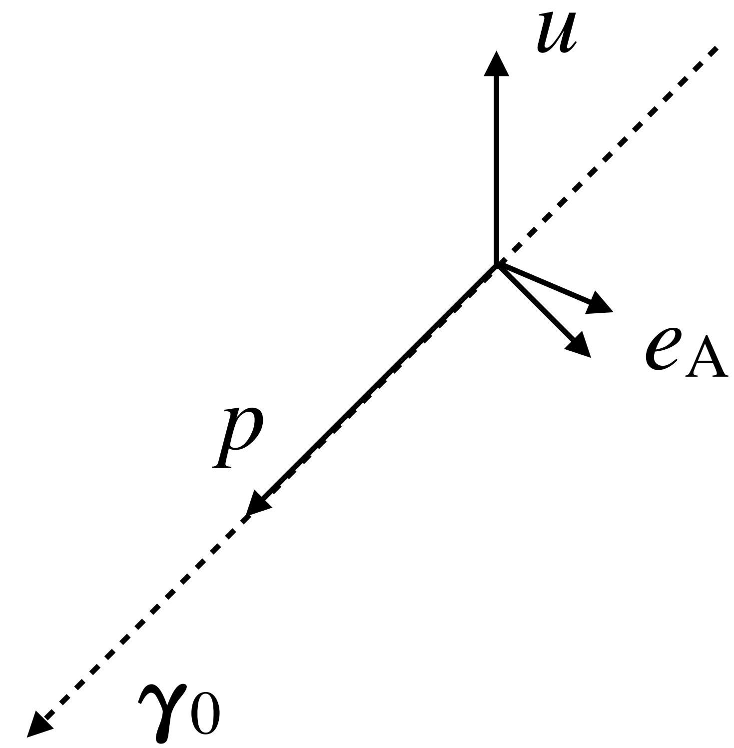
$$\ddot{\xi}^\mu - R^\mu{}_{\alpha\beta\nu}\, p^\alpha\, p^\beta\, \xi^\nu = 0$$

$$\begin{pmatrix} \xi^0 \\ \xi^1 \\ \xi^2 \\ \cancel{\xi^3} \end{pmatrix} \longleftarrow \text{irrelevant}$$

passing to $T_{\mathcal{O}}M \,/\, p$

Geometry

Working in a parallel propagated frame



semi-null frame (u, e_1, e_2, p)

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$$\ddot{\xi}^\mu - R^\mu{}_{\alpha\beta\nu}\, p^\alpha\, p^\beta\, \xi^\nu = 0$$

$$\begin{pmatrix} \xi^0 \\ \xi^1 \\ \xi^2 \\ \cancel{\xi^3} \end{pmatrix}$$

ξ^0 = 0 — passing to $\mathscr{P}$

$\cancel{\xi^3}$ irrelevant — passing to $T_{\odot}M / p$

Geometry

Working in a parallel propagated frame

semi-null frame (u, e_1, e_2, p)

$g(u, u) = -1$ $\quad$ $g(e_A, e_B) = \delta_{AB}$

$g(p, p) = 0$ $\quad$ $g(e_A, u) = 0$

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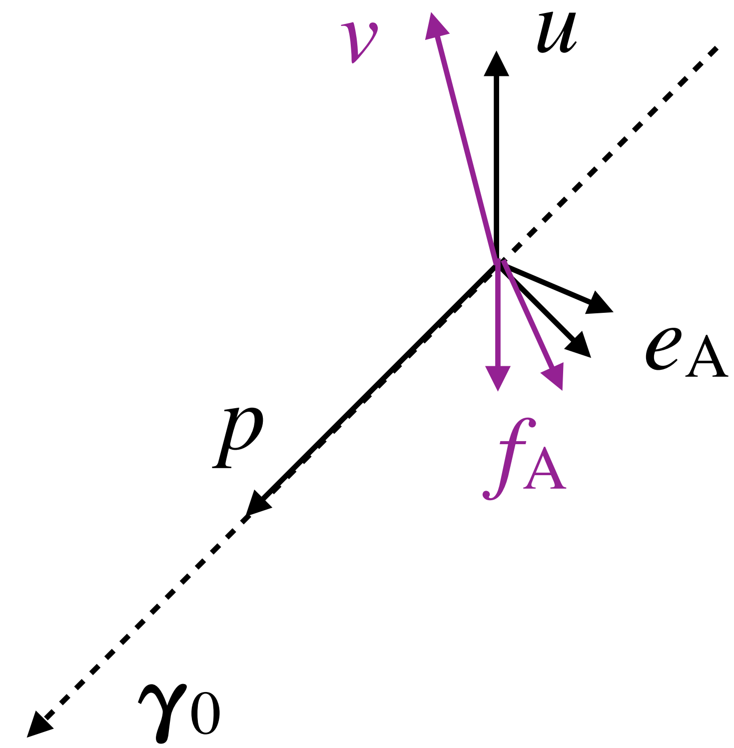


$$\ddot{\xi}^\mu - R^\mu{}_{\alpha\beta\nu}\, p^\alpha\, p^\beta\, \xi^\nu = 0$$

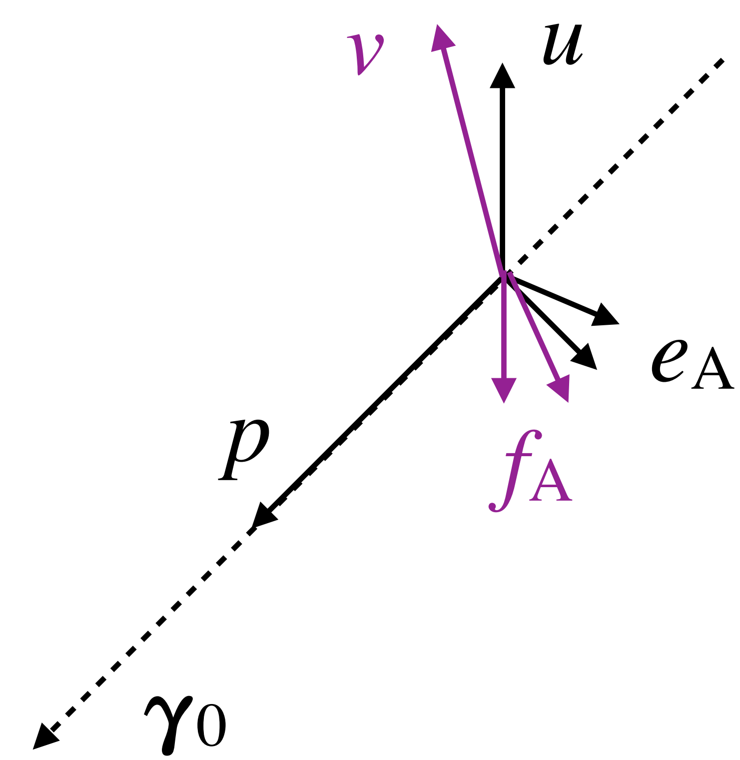
$$\begin{pmatrix} \xi^0 \\ \xi^1 \\ \xi^2 \\ \cancel{\xi^3} \end{pmatrix}$$

$\xi^0 \longleftarrow$ = 0 $\quad$ passing to $\mathscr{P}$

$\xi^3 \longleftarrow$ irrelevant $\quad$ passing to $T_{\odot}M\,/\,p$

Geometry

Working in a parallel propagated frame



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$g(u, p) = const$ $\qquad$ $g(e_A, p) = 0$

$$\ddot{\xi}^\mu - R^\mu{}_{\alpha\beta\nu}\, p^\alpha\, p^\beta\, \xi^\nu = 0$$

$$\begin{pmatrix} \xi^0 \\ \xi^1 \\ \xi^2 \\ \cancel{\xi^3} \end{pmatrix} \begin{array}{l} \longleftarrow \; = 0 \\ \\ \\ \longleftarrow \; \text{irrelevant} \end{array}$$

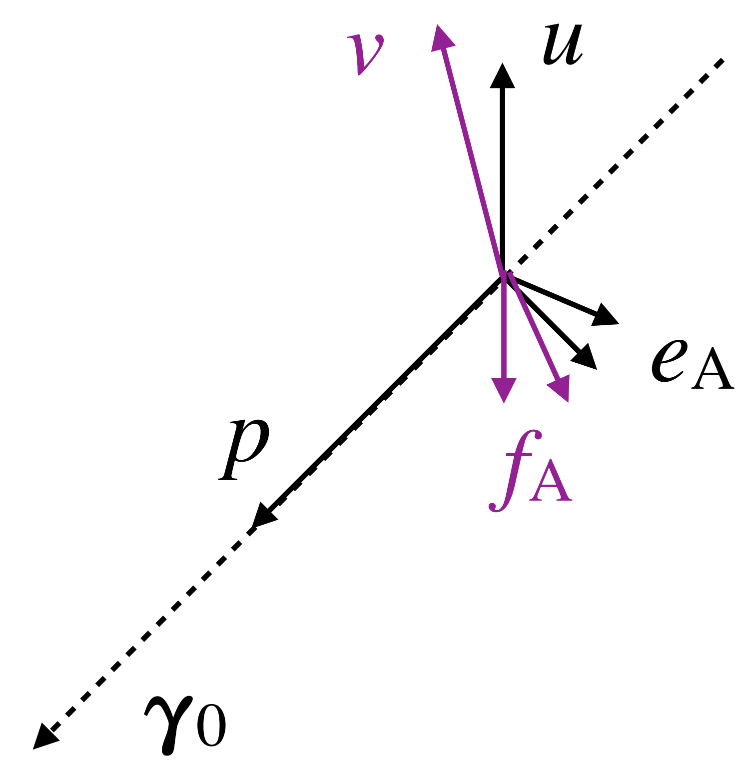
passing to $\mathscr{P}$

passing to $T_{\mathcal{O}}M \,/\, p$

$$\begin{pmatrix} \tilde{\xi}^1 \\ \tilde{\xi}^2 \end{pmatrix} = \begin{pmatrix} \cos\varphi & \sin\varphi \\ -\sin\varphi & \cos\varphi \end{pmatrix} \begin{pmatrix} \xi^1 \\ \xi^2 \end{pmatrix}$$

Geometry

Working in a parallel propagated frame



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$$\ddot{\xi}^\mu - R^\mu{}_{\alpha\beta\nu}\, p^\alpha\, p^\beta\, \xi^\nu = 0$$

$$\begin{pmatrix} \xi^0 \\ \xi^1 \\ \xi^2 \\ \cancel{\xi^3} \end{pmatrix}$$

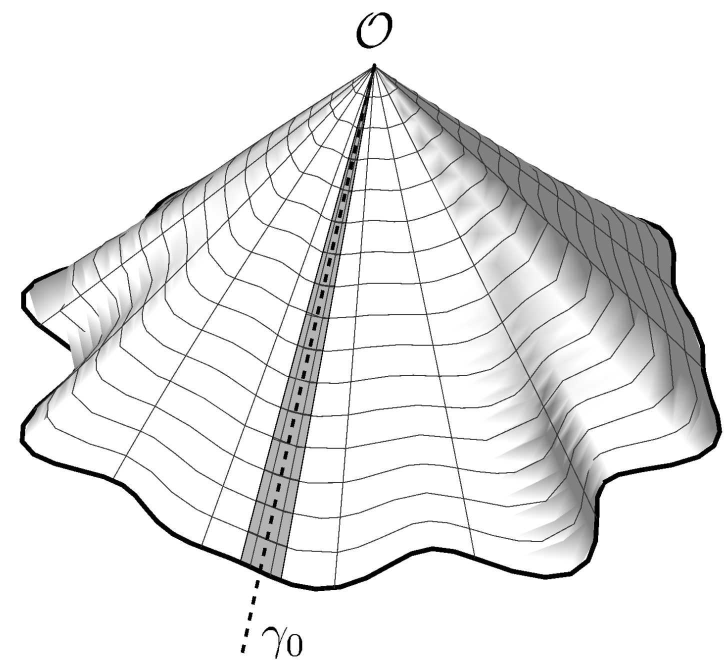
$\xi^0 \longleftarrow$ = 0 — passing to $\mathscr{P}$

$\cancel{\xi^3} \longleftarrow$ irrelevant — passing to $T_{\mathcal{O}}M \,/\, p$

$$\begin{pmatrix} \tilde{\xi}^1 \\ \tilde{\xi}^2 \end{pmatrix} = \begin{pmatrix} \xi^1 \\ \xi^2 \end{pmatrix}$$

Geometry

Jacobi operator $\mathcal{D}^A{}_B$

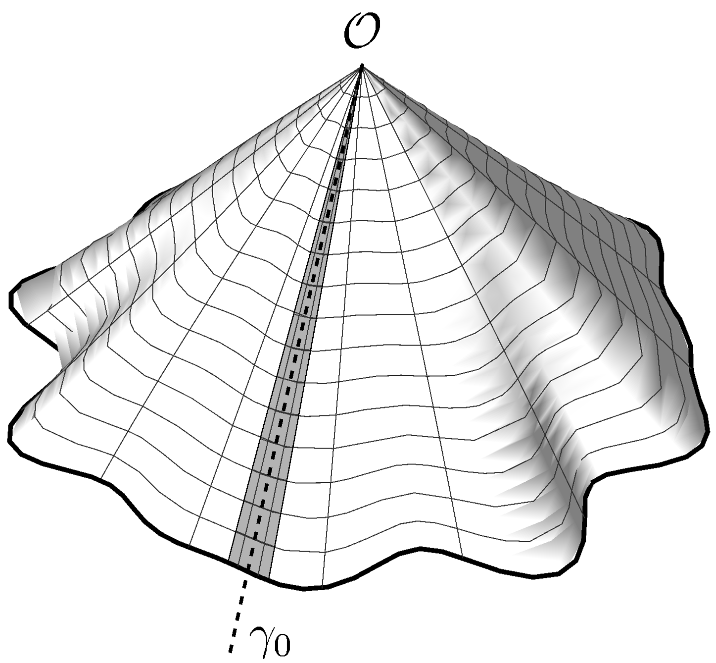


Geometry

Jacobi operator $\mathcal{D}^A{}_B$

- $\mathscr{D} : \mathscr{P}_{\mathscr{O}} \to \mathscr{P}_{\mathscr{E}}$

  $\xi^A(\lambda) = \mathcal{D}^A{}_B(\lambda)\, \nabla_p \xi^A(\lambda_{\mathcal{O}})$



Geometry

Jacobi operator $\mathcal{D}^A{}_B$

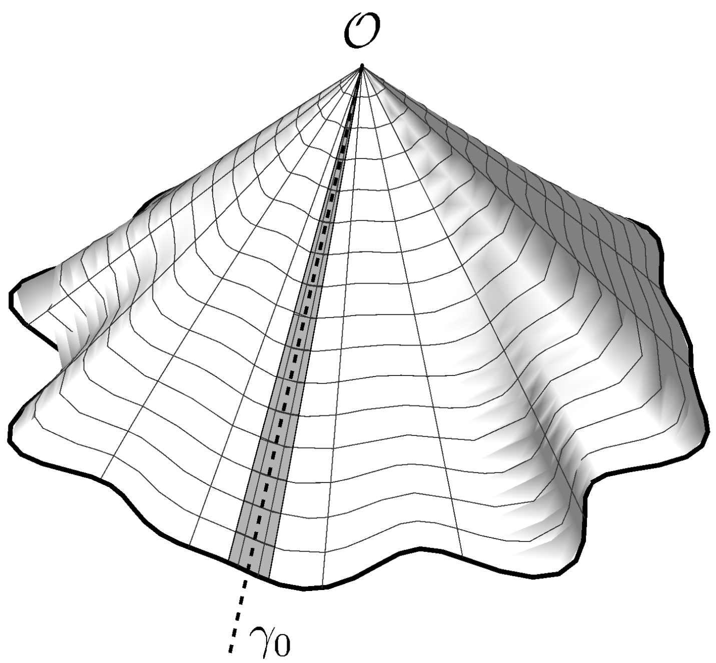
- $\mathcal{D} : \mathcal{P}_\mathcal{O} \to \mathcal{P}_\mathcal{E}$

  $\xi^A(\lambda) = \mathcal{D}^A{}_B(\lambda)\,\nabla_p\xi^A(\lambda_\mathcal{O})$
- given by ODE's

$$\frac{\mathrm{d}^2}{\mathrm{d}\lambda^2}\,\mathcal{D}^A{}_B - R^A{}_{\nu\alpha C}\,p^\nu\,p^\alpha\,\mathcal{D}^C{}_B = 0$$

$$\mathcal{D}^A{}_B\,(\lambda_\mathcal{O}) \;=\; 0$$

$$\frac{\mathrm{d}}{\mathrm{d}\lambda}\,\mathcal{D}^A{}_B\,(\lambda_\mathcal{O}) \;=\; \delta^A{}_B$$



Geometry

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- $\mathscr{D} : \mathscr{P}_{\mathscr{O}} \to \mathscr{P}_{\mathscr{E}}$

  $\xi^A(\lambda) = \mathcal{D}^A{}_B(\lambda)\,\nabla_p \xi^A(\lambda_\mathcal{O})$
- given by ODE's

$$\frac{\mathrm{d}^2}{\mathrm{d}\lambda^2}\mathcal{D}^A{}_B - R^A{}_{\nu\alpha C}\,p^\nu\,p^\alpha\,\mathcal{D}^C{}_B = 0$$

$$\mathcal{D}^A{}_B\,(\lambda_\mathcal{O}) = 0$$

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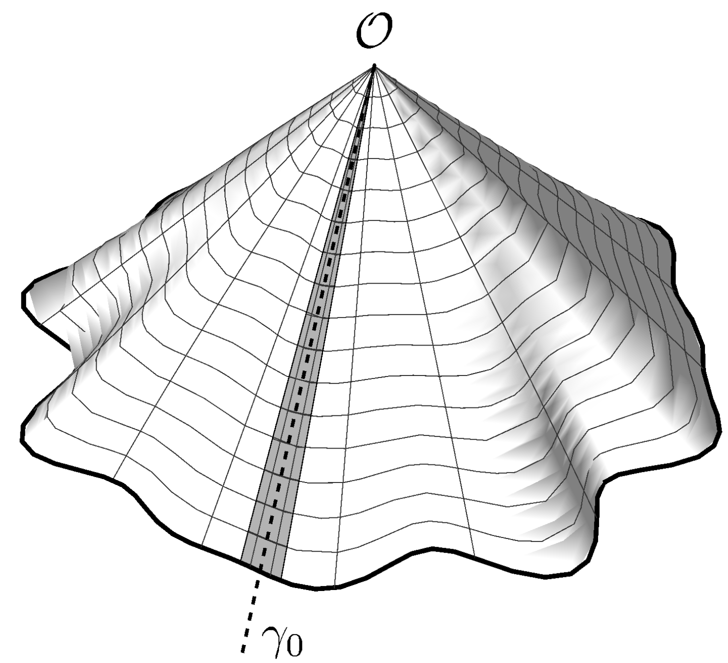


- defined by spacetime geometry (observer-independent)

Geometry

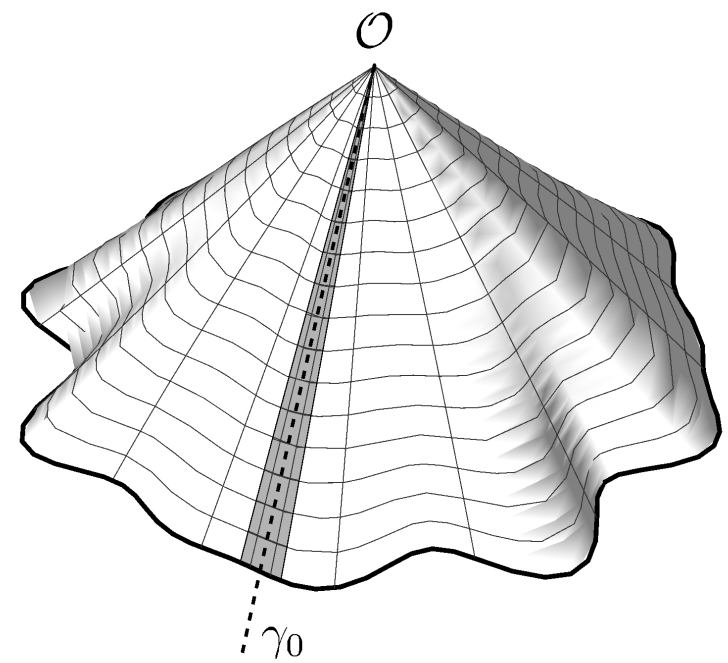
Jacobi operator $\mathcal{D}^A{}_B$

- $\mathscr{D} : \mathscr{P}_{\mathcal{O}} \to \mathscr{P}_{\mathcal{E}}$

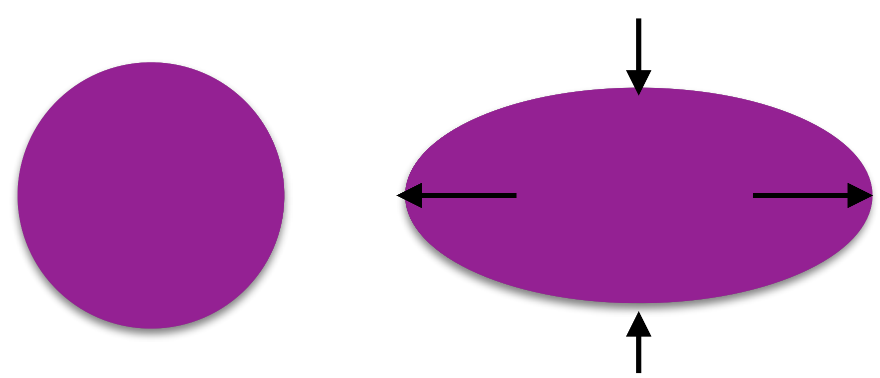
  $\xi^A(\lambda) = \mathcal{D}^A{}_B(\lambda)\, \nabla_p \xi^A(\lambda_{\mathcal{O}})$
- given by ODE's

$$\frac{\mathrm{d}^2}{\mathrm{d}\lambda^2}\mathcal{D}^A{}_B - R^A{}_{\nu\alpha C}\, p^\nu\, p^\alpha\, \mathcal{D}^C{}_B = 0$$

$$\begin{aligned} \mathcal{D}^A{}_B(\lambda_{\mathcal{O}}) &= 0 \\ \frac{\mathrm{d}}{\mathrm{d}\lambda}\mathcal{D}^A{}_B(\lambda_{\mathcal{O}}) &= \delta^A{}_B \end{aligned}$$



- defined by spacetime geometry (observer-independent)
- gravitational lensing



Geometry

Jacobi operator $\mathcal{D}^A{}_B$

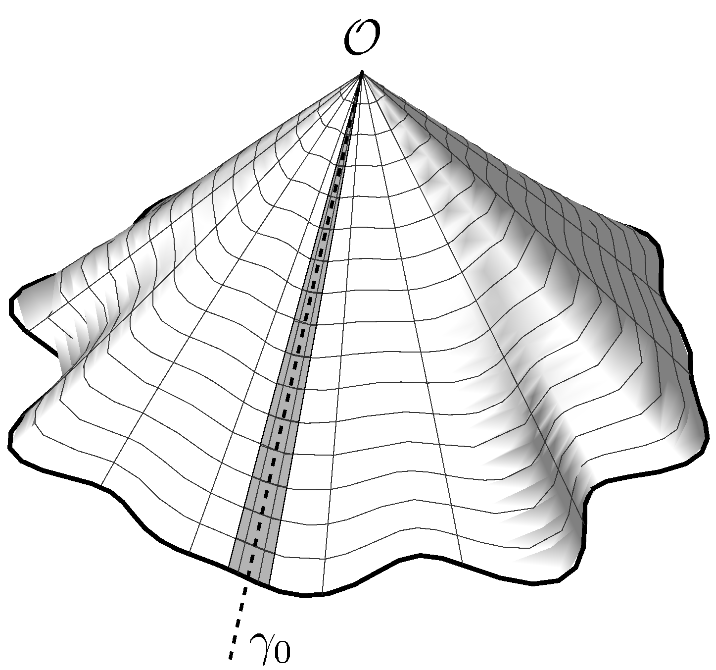
- $\mathcal{D} : \mathcal{P}_{\mathcal{O}} \to \mathcal{P}_{\mathcal{E}}$

  $\xi^A(\lambda) = \mathcal{D}^A{}_B(\lambda)\,\nabla_p \xi^A(\lambda_{\mathcal{O}})$
- given by ODE's

$$\frac{\mathrm{d}^2}{\mathrm{d}\lambda^2}\mathcal{D}^A{}_B - R^A{}_{\nu\alpha C}\,p^\nu\,p^\alpha\,\mathcal{D}^C{}_B = 0$$

$$\mathcal{D}^A{}_B(\lambda_{\mathcal{O}}) = 0$$

$$\frac{\mathrm{d}}{\mathrm{d}\lambda}\mathcal{D}^A{}_B(\lambda_{\mathcal{O}}) = \delta^A{}_B$$



- defined by spacetime geometry (observer-independent)
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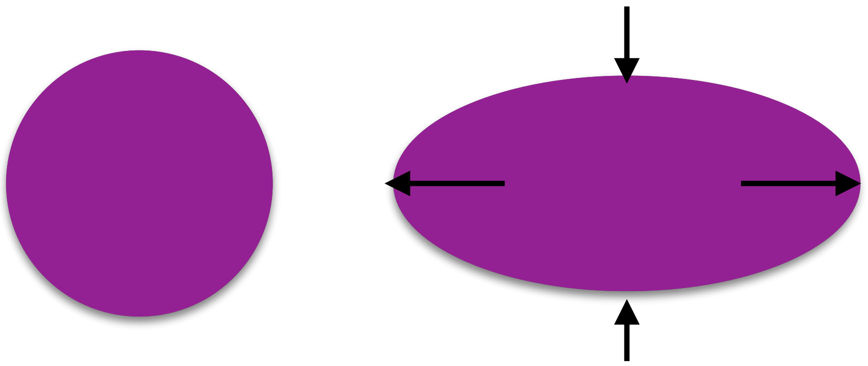


- area, luminosity distances

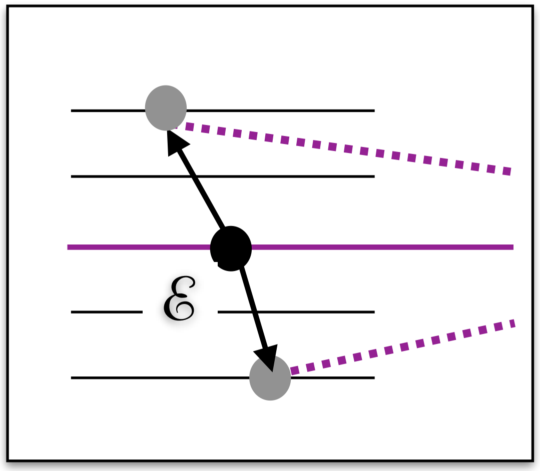
$$D_{ang} = (p_\mu\,u^\mu_{\mathcal{O}})\;\left|\det \mathcal{D}^A{}_B(\lambda_{\mathcal{E}})\right|^{1/2} \qquad D_{lum} = D_{ang}(1+z)^2$$

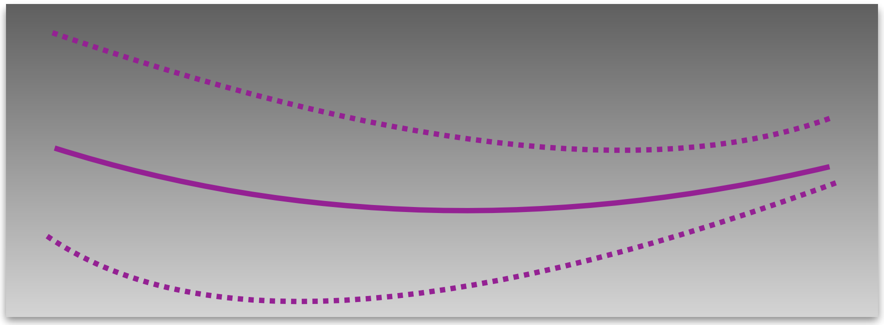
Geometry

Emitter-observer asymmetry operator $m^A{}_\mu$

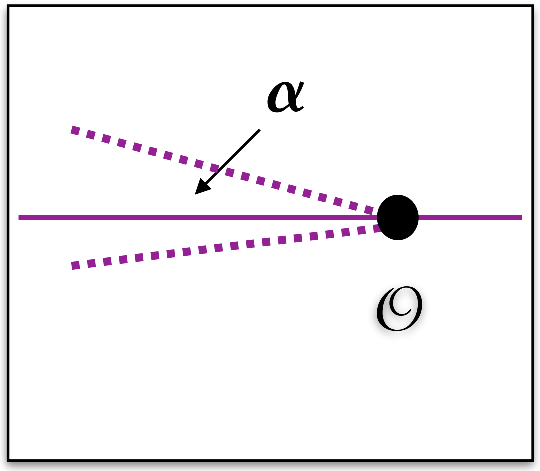
Geometry

Emitter-observer asymmetry operator $m^A{}_\mu$





$$\mathcal{D}^A{}_B\,\delta\dot{x}^B_{\mathcal{O}} \quad = \quad \delta x^A_{\mathcal{E}}$$



Geometry

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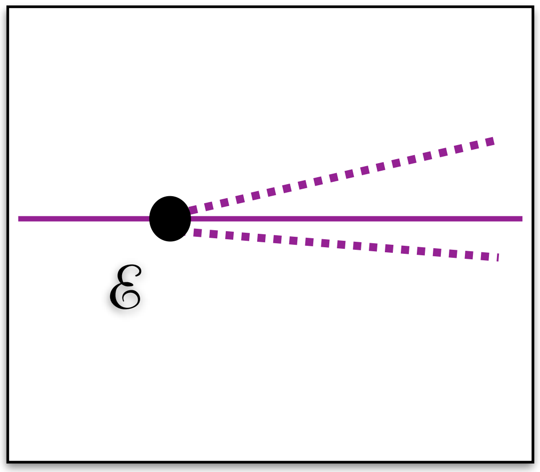
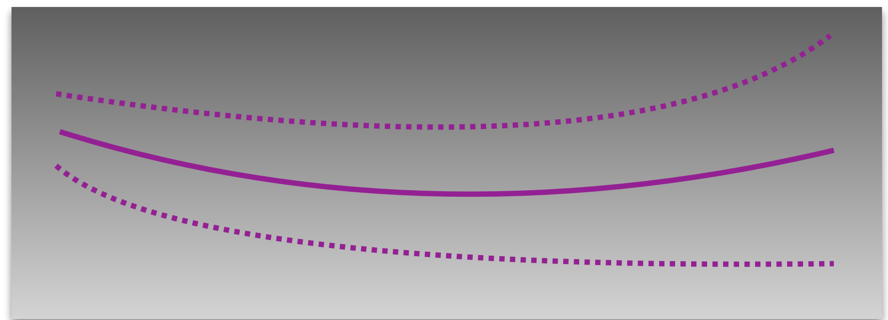
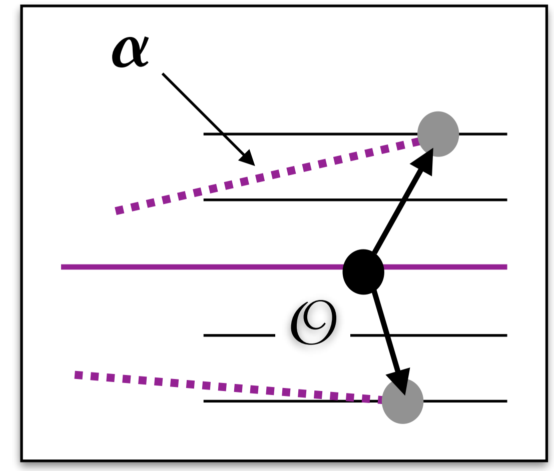




$$\mathcal{D}^A{}_B\,\delta\dot{x}^B_{\mathcal{O}} = -\delta\hat{x}^A_{\mathcal{O}} - m^A{}_B\,\delta x^B_{\mathcal{O}}$$



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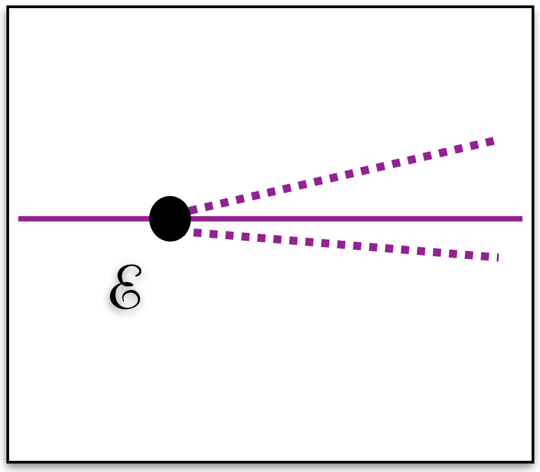

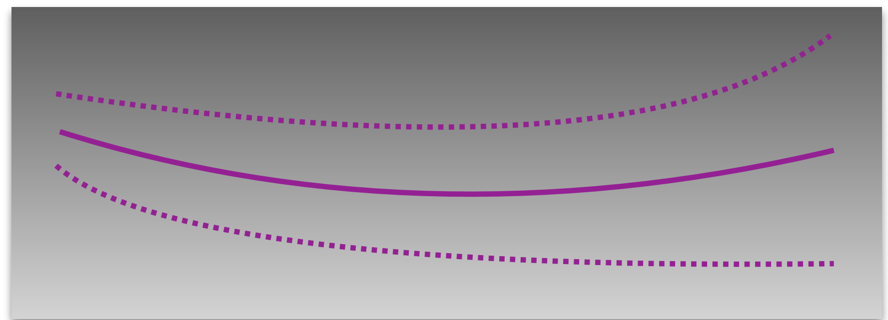

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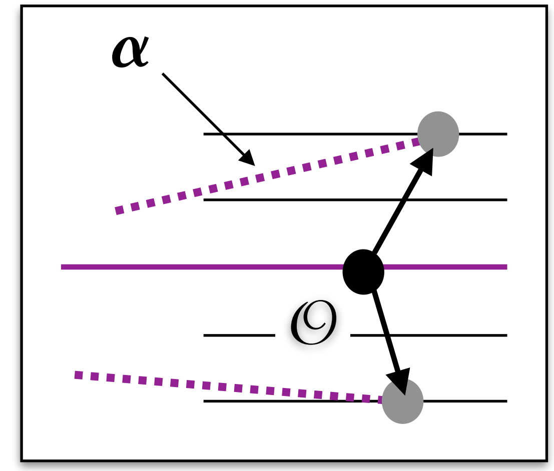
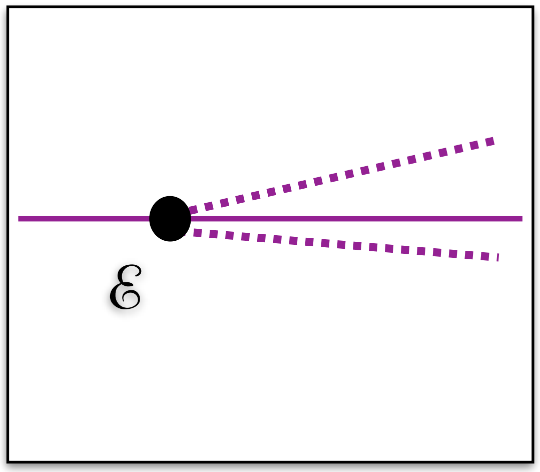
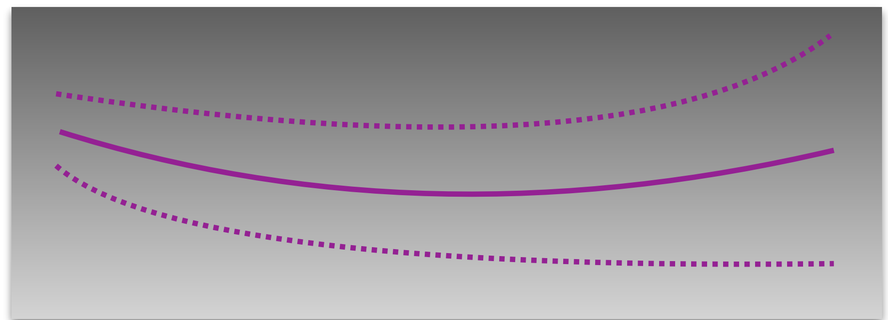


- mapping $m : \mathrm{T}_{\mathcal{O}}M \,/\, p \to \mathscr{P}_{\mathcal{E}}$

Geometry

Emitter-observer asymmetry operator $m^A{}_\mu$





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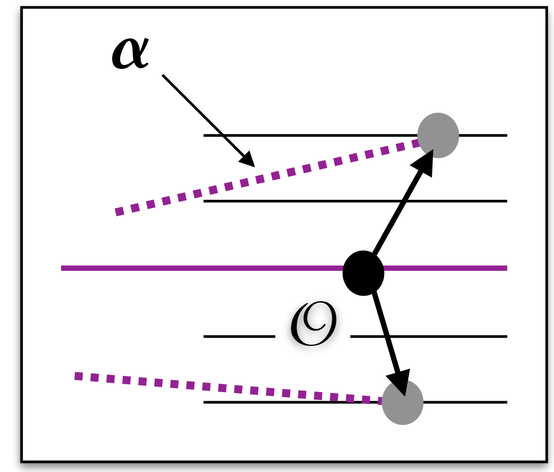
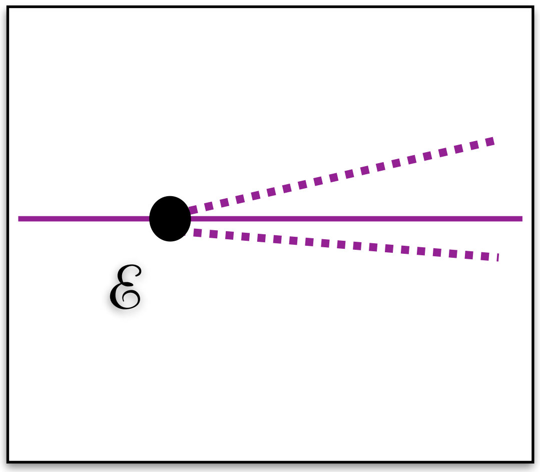


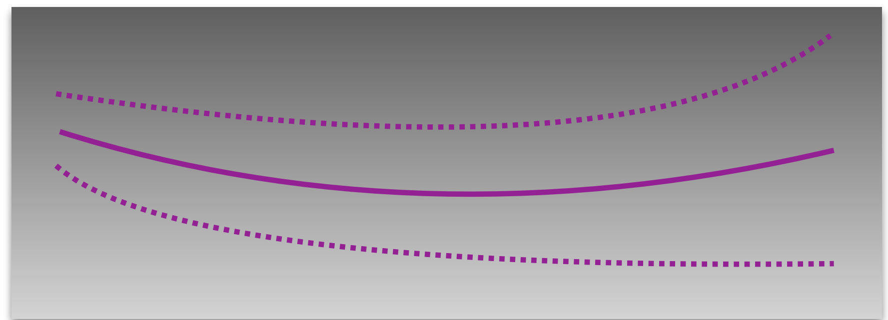
- mapping $m : T_{\mathcal{O}}M\,/\,p \to \mathcal{P}_{\mathcal{E}}$
- given by ODE's

$$\frac{\mathrm{d}^2}{\mathrm{d}\lambda^2}\,m^A{}_\mu - R^A{}_{\alpha\beta B}\,p^\alpha\,p^\beta\,m^B{}_\mu = R^A{}_{\alpha\beta\mu}\,p^\alpha\,p^\beta$$
$$m^A{}_\mu(\lambda_{\mathcal{O}}) = 0$$
$$\frac{\mathrm{d}}{\mathrm{d}\lambda}\,m^A{}_\mu(\lambda_{\mathcal{O}}) = 0.$$

Geometry

Emitter-observer asymmetry operator $m^A{}_\mu$





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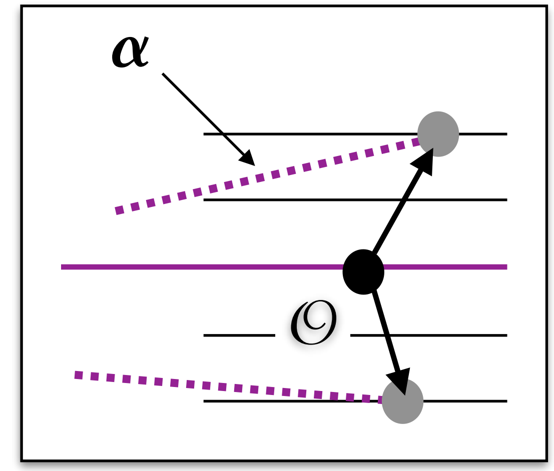

- mapping $m : \mathrm{T}_{\mathcal{O}}M \,/\, p \to \mathcal{P}_{\mathcal{E}}$
- given by ODE's
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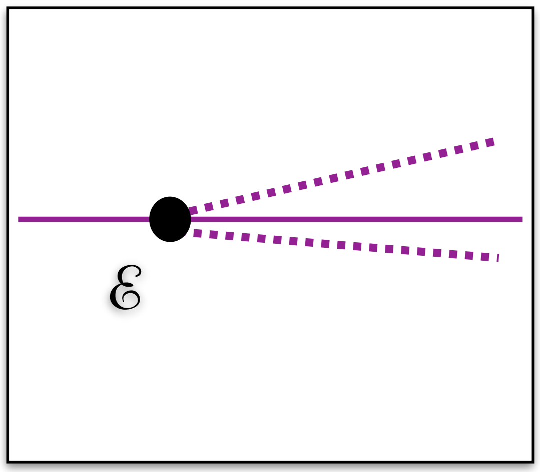
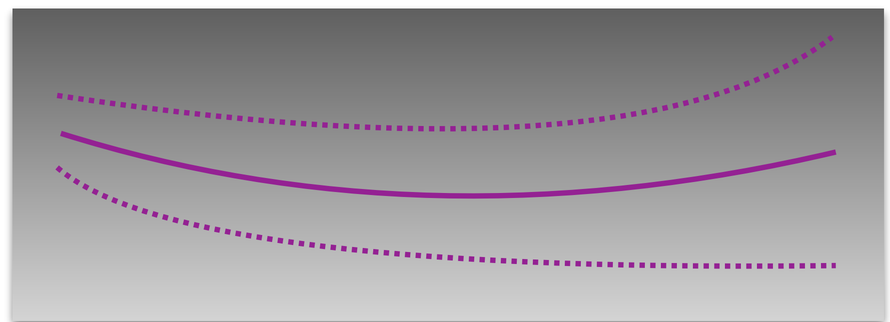
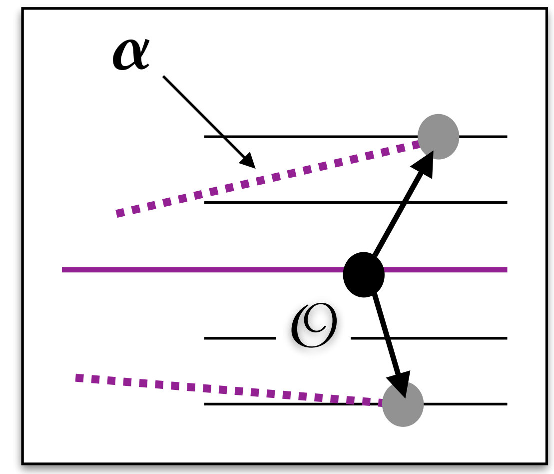
- defined by spacetime geometry (observer-independent)

Geometry

Emitter-observer asymmetry operator $m^A{}_\mu$







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$$\frac{\mathrm{d}}{\mathrm{d}\lambda} m^A{}_\mu(\lambda_{\mathcal{O}}) = 0.$$
- defined by spacetime geometry (observer-independent)
- parallax

$$D_{par} = D_{ang}\left|\det\left(\delta^A{}_B + m^A{}_B\right)\right|^{-1/2}$$

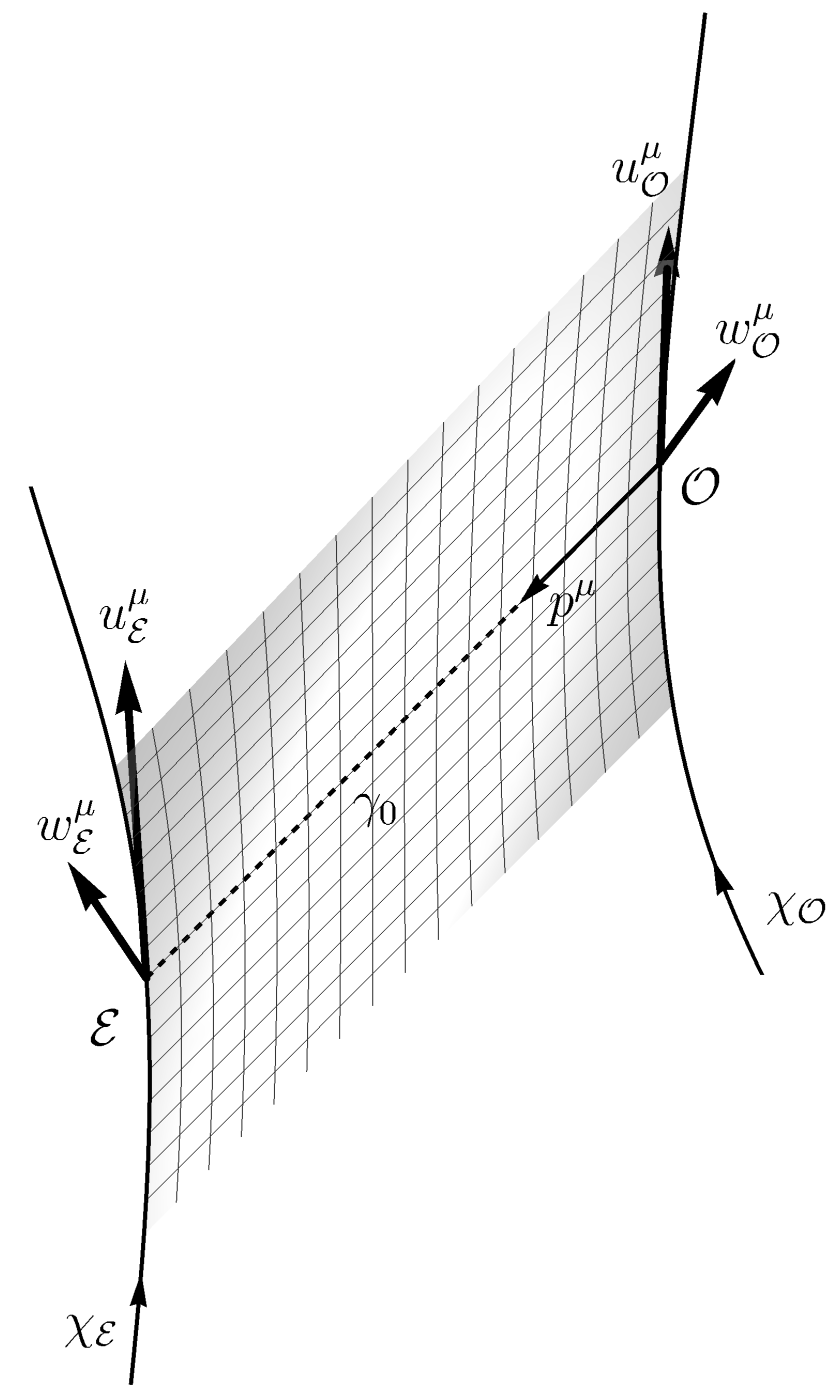
Geometry

Geometric setup

Geometry

Geometric setup

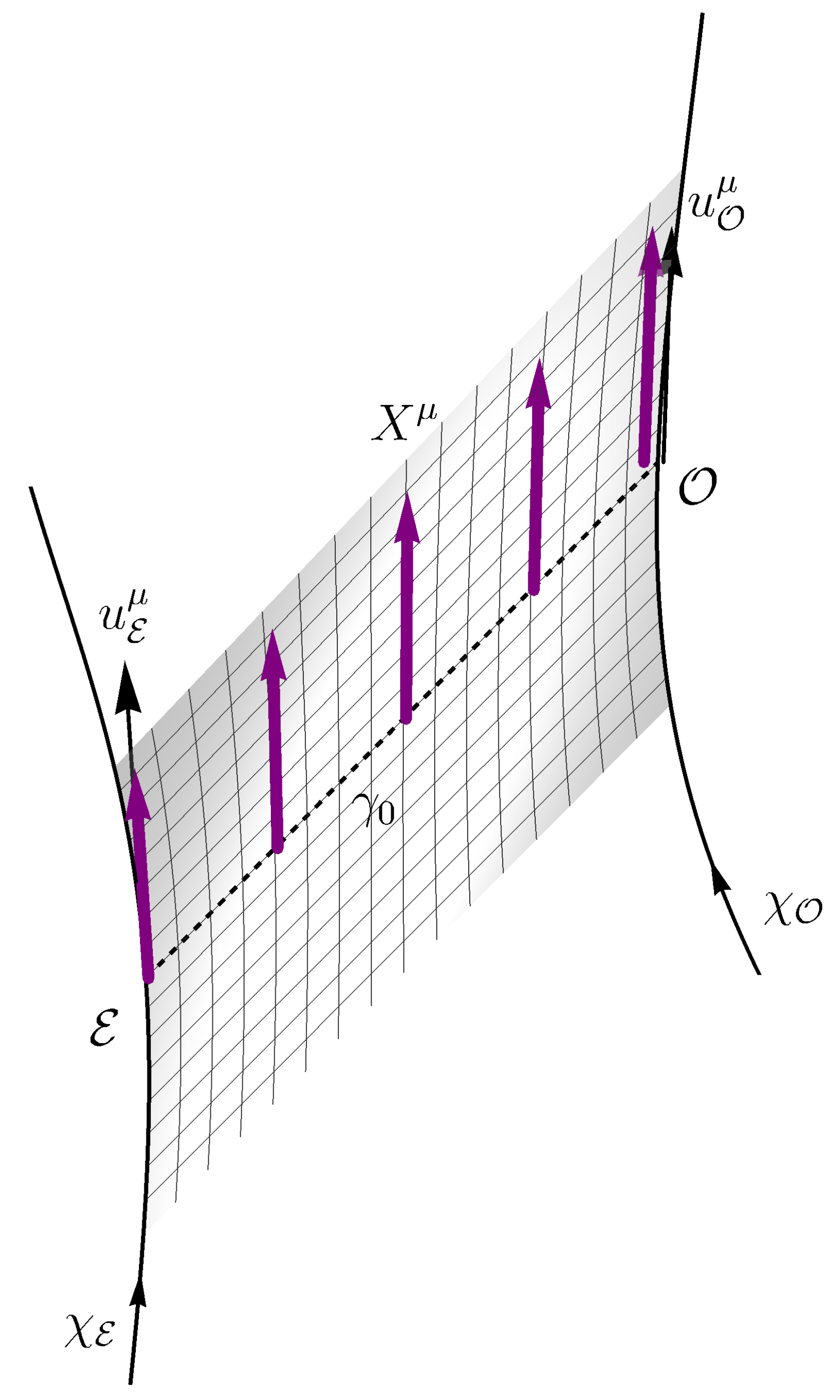
- null surface spanned by connecting null geodesics



Geometry

Geometric setup

- null surface spanned by connecting null geodesics
- main tool: observation time vector X^μ



Geometry

Geometric setup

- null surface spanned by connecting null geodesics
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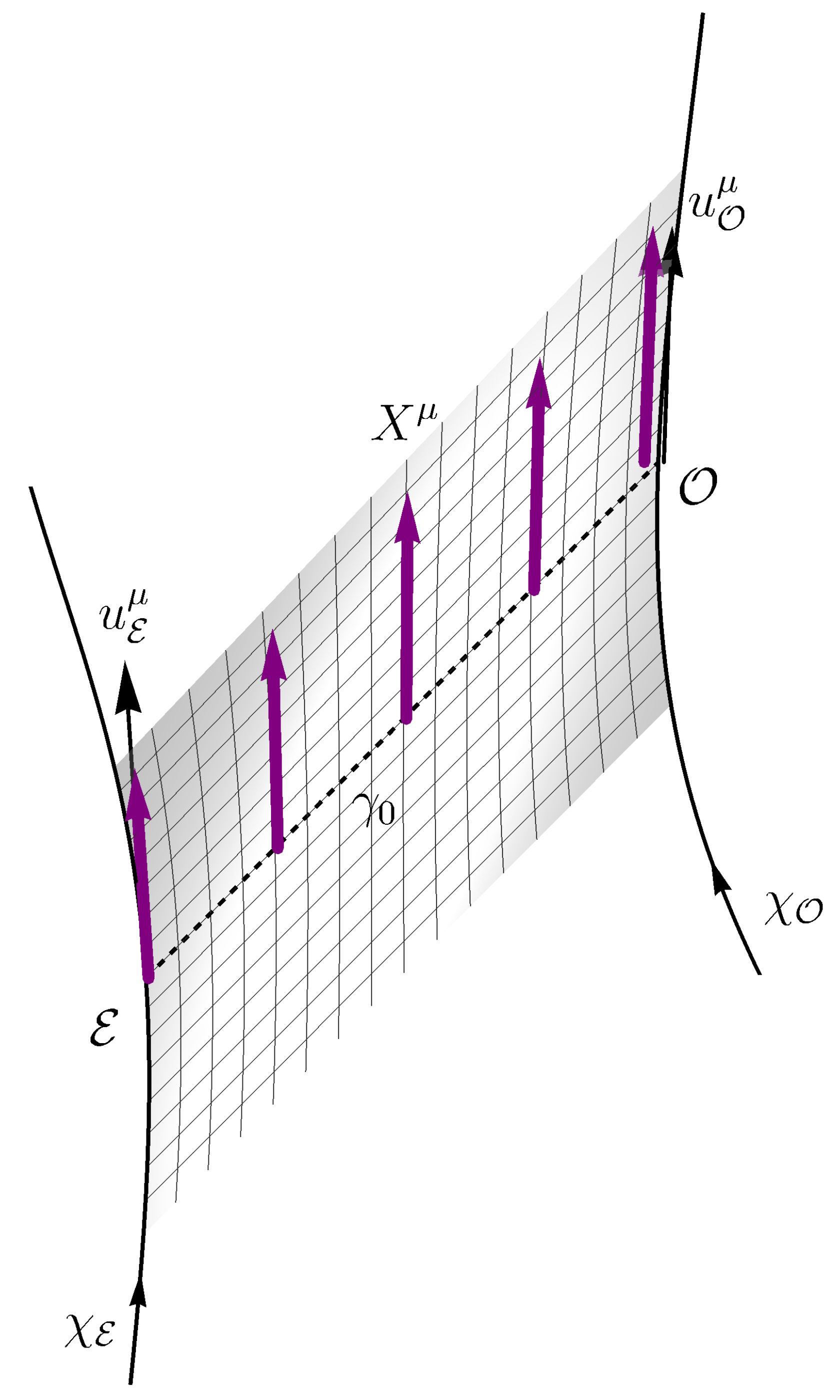
$$\mathcal{G}[X]^\mu = 0$$

$$X^\mu(\lambda_\mathcal{O}) = u^\mu_\mathcal{O}$$

$$X^\mu(\lambda_\mathcal{E}) = \frac{1}{1+z}\, u^\mu_\mathcal{E}$$



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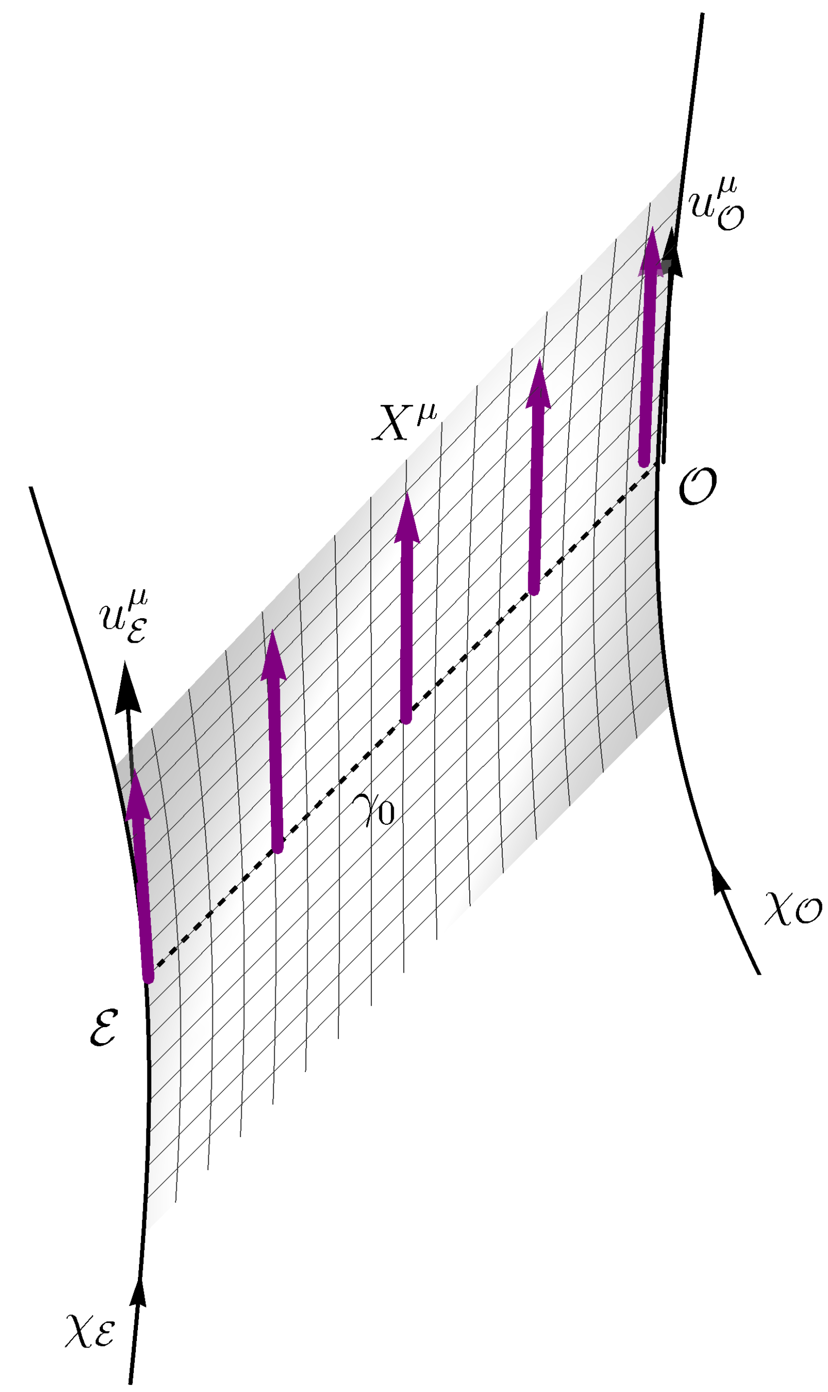

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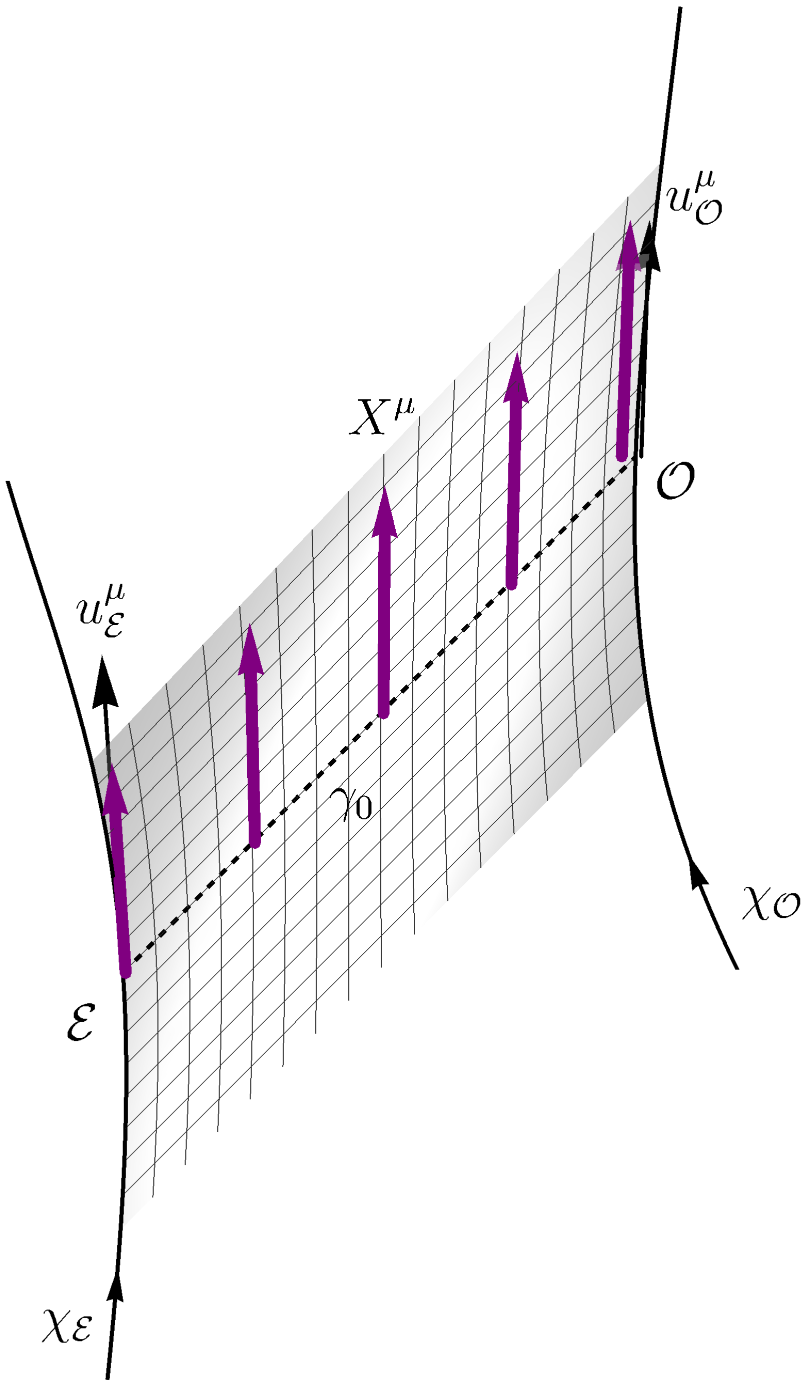
- ∇_X gives the drift effects



Geometry

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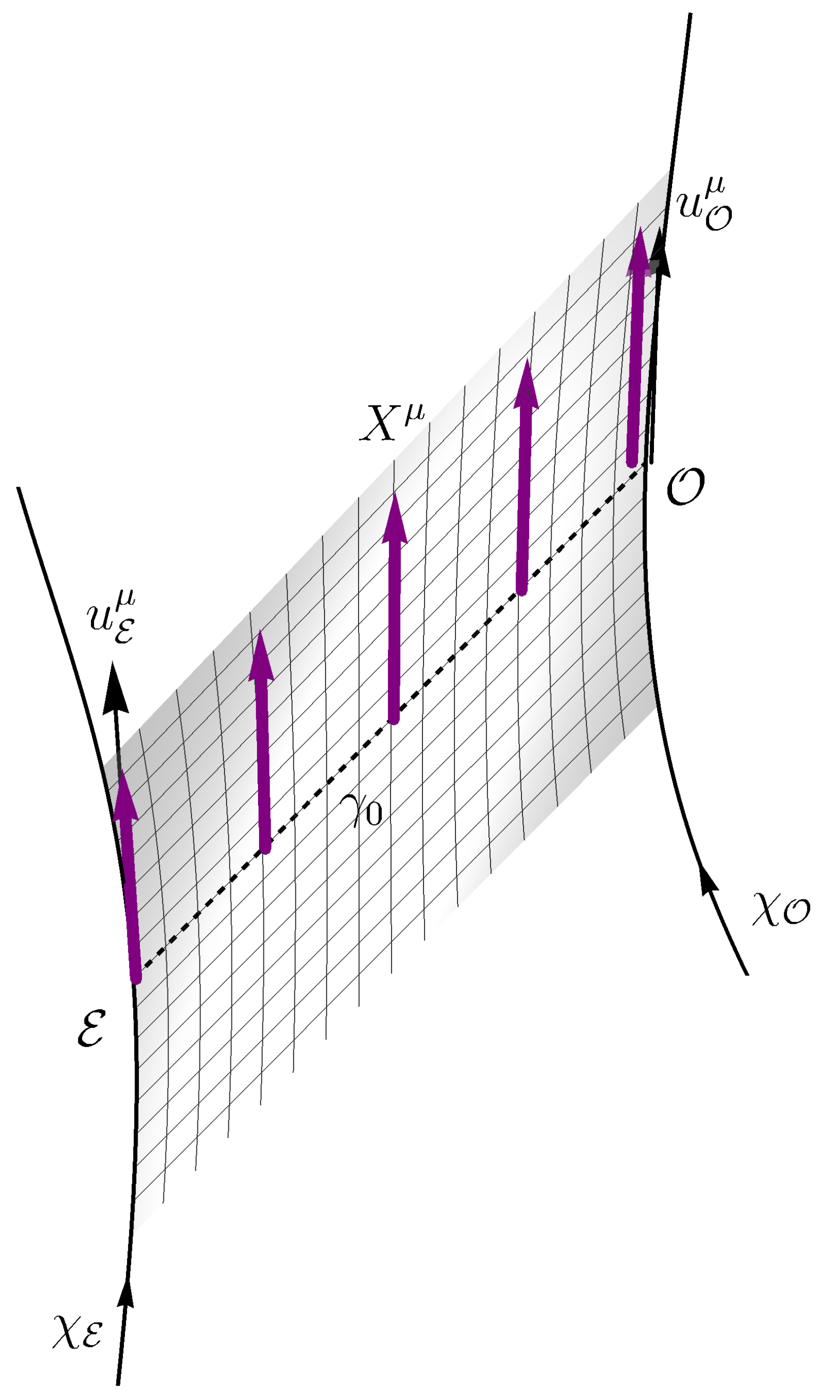
- ∇_X gives the drift effects

$\nabla_X p^\mu$ - position drift



Geometry

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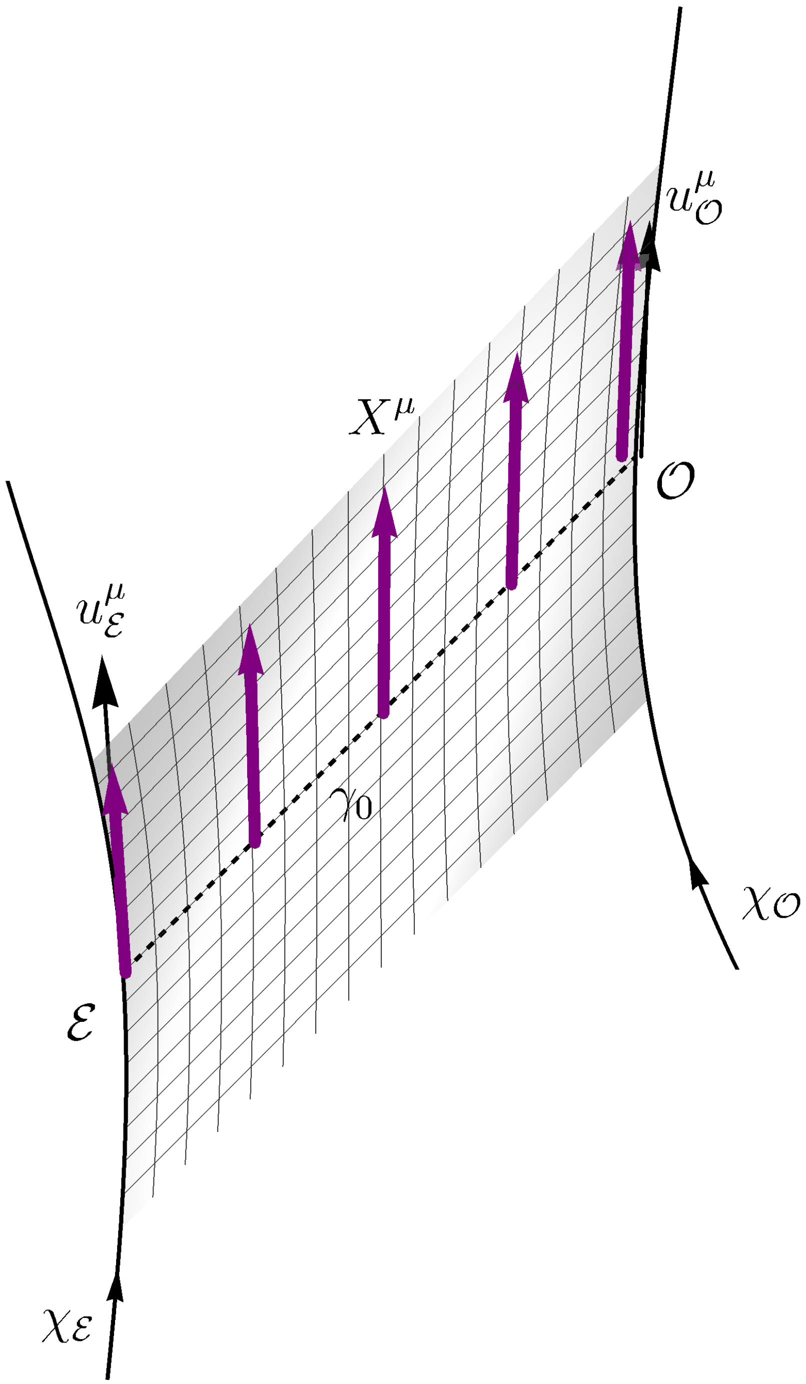
  $\nabla_X p^\mu$ - position drift

  $\nabla_X (p_\mu u^\mu)$ - redshift drift



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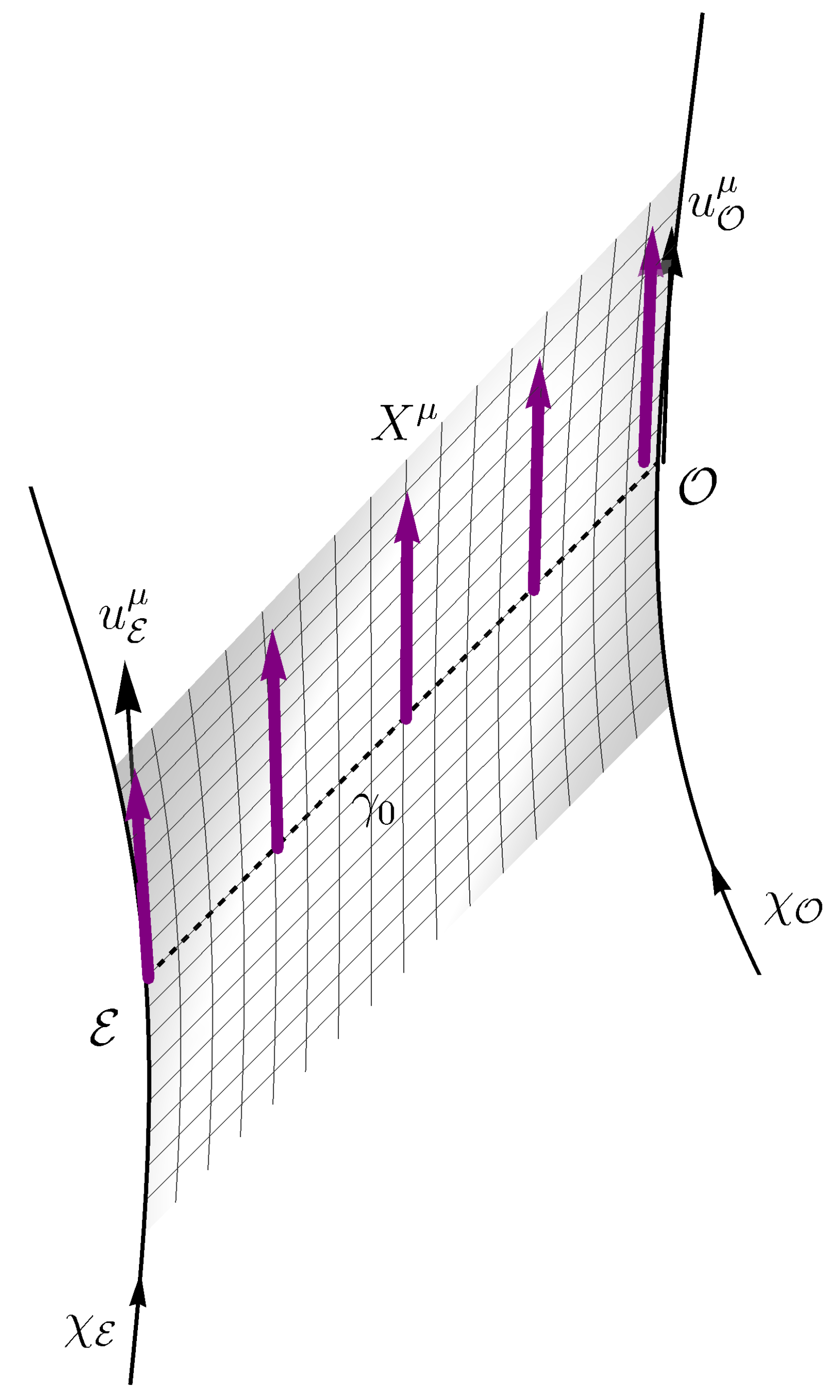
- ∇_X gives the drift effects

  $\nabla_X p^\mu$ - position drift

  $\nabla_X (p_\mu u^\mu)$ - redshift drift

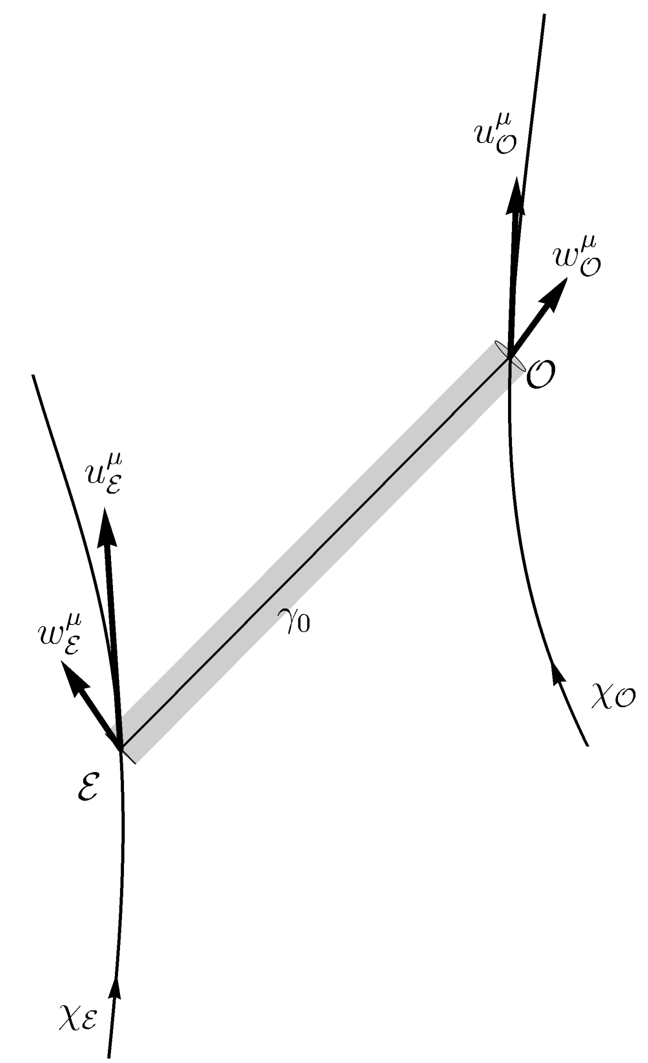
  $\nabla_X \mathcal{D}^A{}_B$ - distances drift



Drift effects

Position drift

- parallel propagation of $\hat{u}^{\mu}_{\mathcal{O}}$
- Jacobi operator
- emitter-observer asymmetry operator



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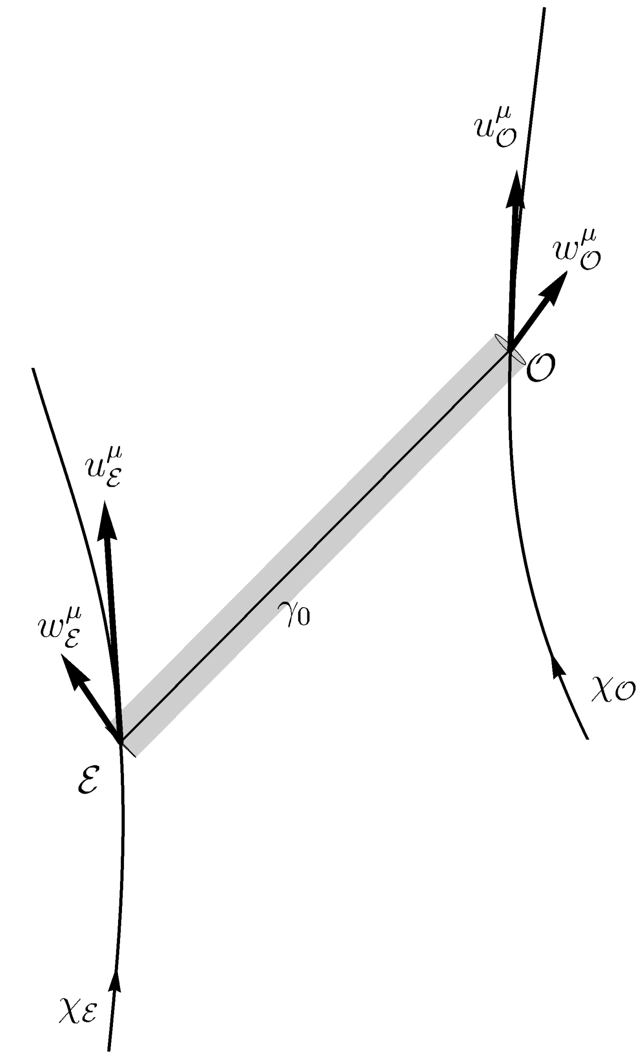

$$\delta_{\mathcal{O}} r^A = w^A_{\mathcal{O}} + \frac{1}{p_\sigma\, u^\sigma_{\mathcal{O}}}\, {\mathcal{D}^{-1}}^A{}_B \left(\left(\frac{1}{1+z} u_{\mathcal{E}} - \hat{u}_{\mathcal{O}} \right)^B - m^B{}_\mu\, u^\mu_{\mathcal{O}} \right)$$

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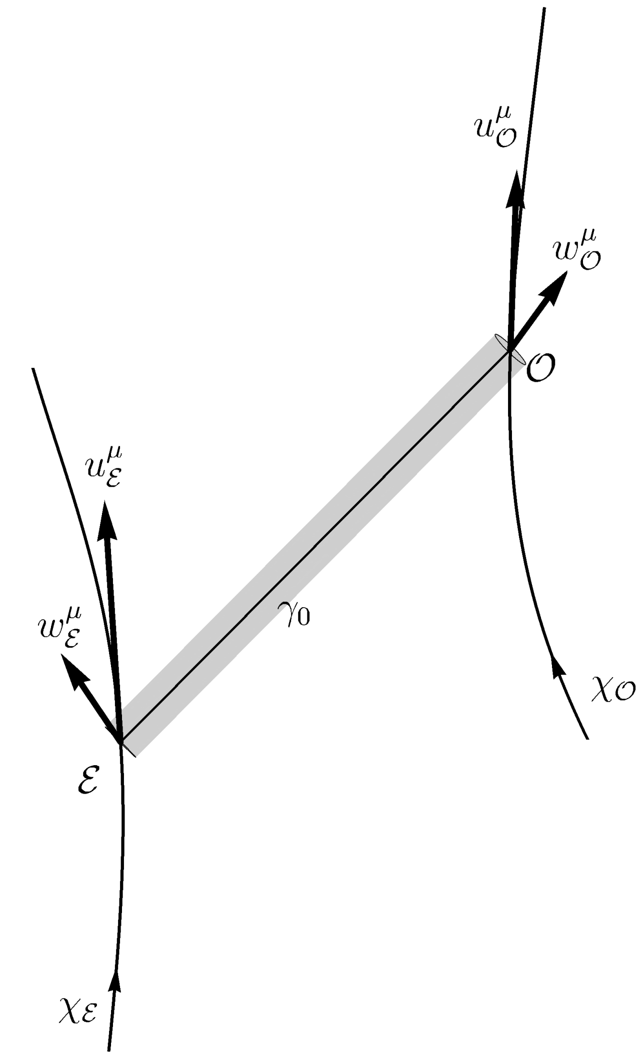

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Fermi-Walker derivative (observer)

Drift effects

Position drift

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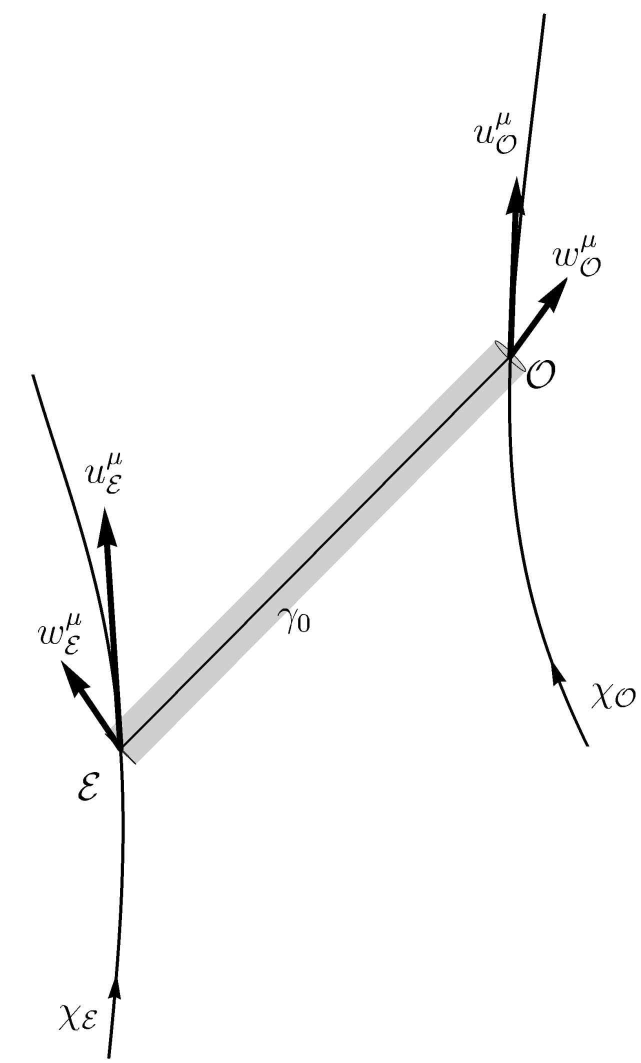
- emitter-observer asymmetry operator



aberration

$$\delta_{\mathcal{O}} r^A = w^A_{\mathcal{O}} + \frac{1}{p_\sigma\, u^\sigma_{\mathcal{O}}}\, {\mathcal{D}^{-1}}^A{}_B \left(\left(\frac{1}{1+z} u_{\mathcal{E}} - \hat{u}_{\mathcal{O}} \right)^B - m^B{}_\mu\, u^\mu_{\mathcal{O}} \right)$$

Fermi-Walker derivative (observer)

Drift effects

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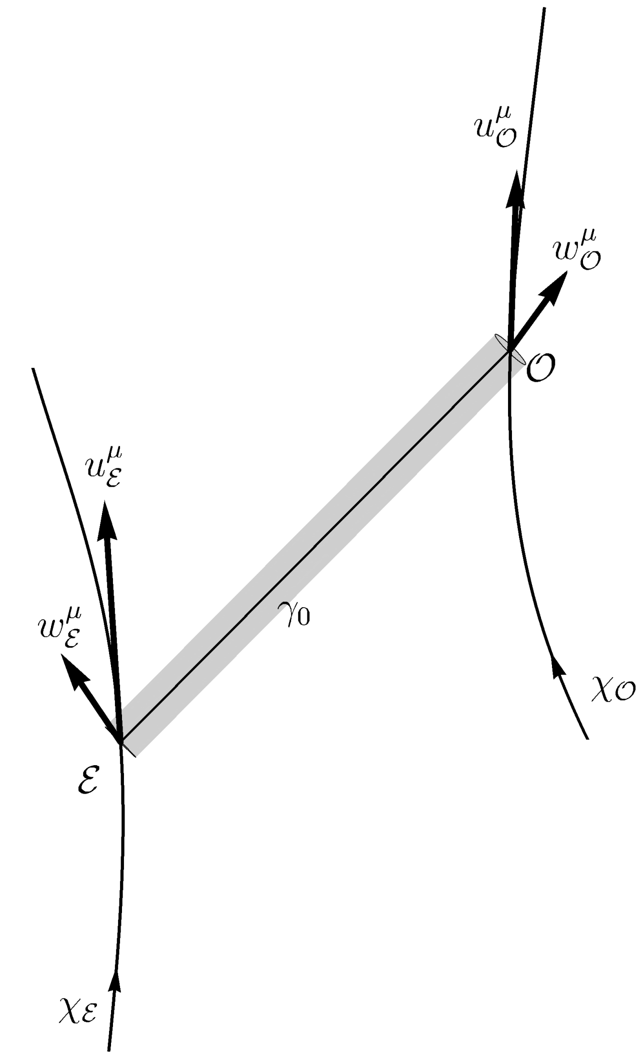

aberration

4-velocity difference

$$\delta_{\mathcal{O}} r^A = w^A_{\mathcal{O}} + \frac{1}{p_\sigma\, u^\sigma_{\mathcal{O}}}\, {\mathcal{D}^{-1}}^A{}_B \left(\left(\frac{1}{1+z} u_{\mathcal{E}} - \hat{u}_{\mathcal{O}} \right)^B - m^B{}_\mu\, u^\mu_{\mathcal{O}} \right)$$

Fermi-Walker derivative (observer)

Drift effects

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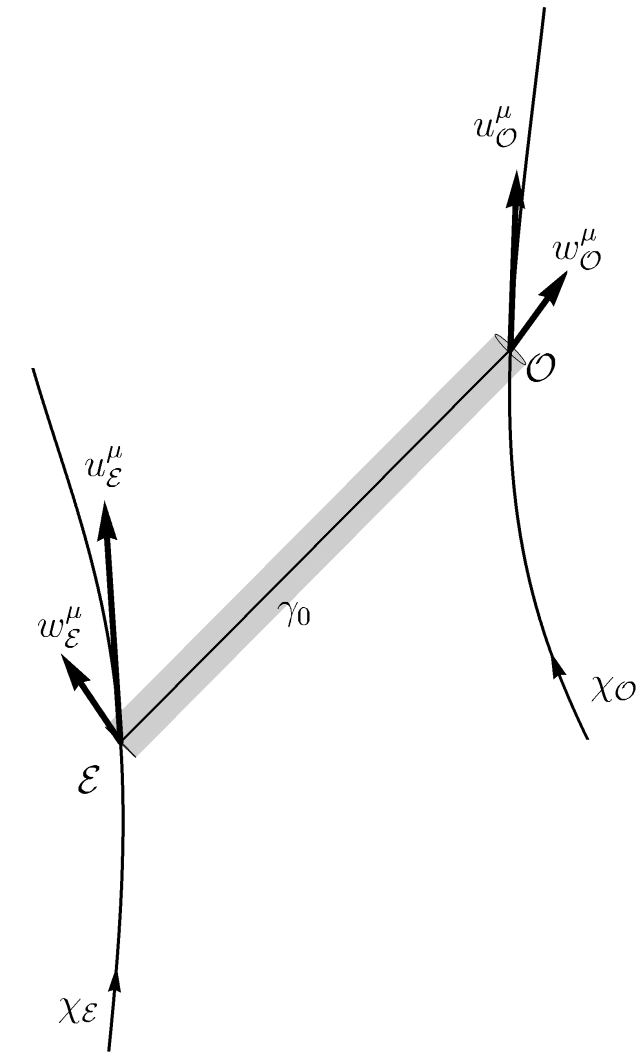

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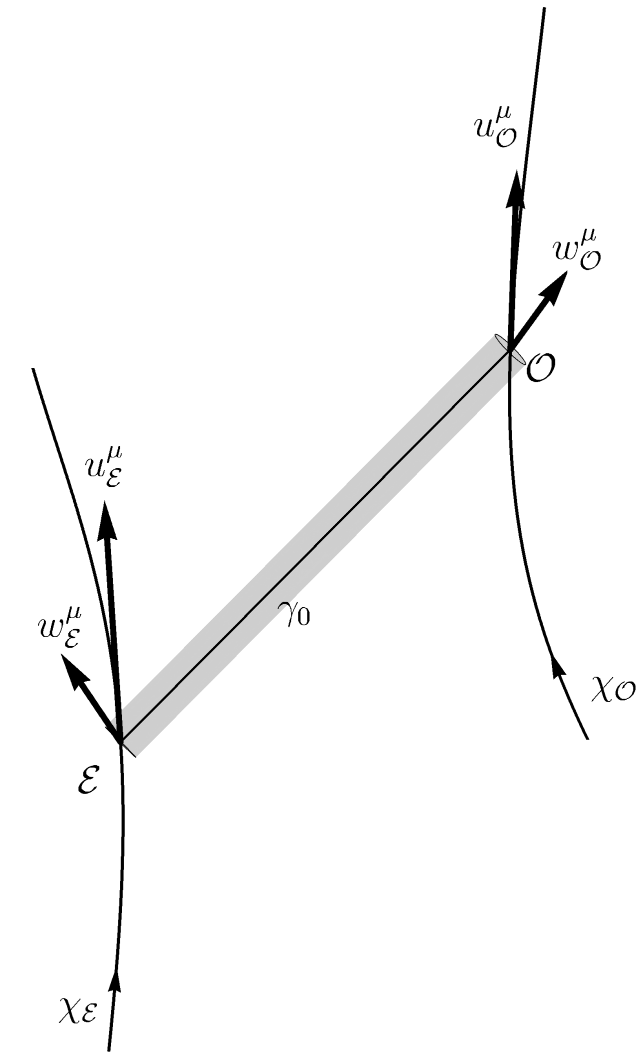
Fermi-Walker derivative (observer)

E-O asymmery operator

Drift effects

Position drift

- parallel propagation of $\hat{u}^{\mu}_{\mathcal{O}}$
- Jacobi operator
- emitter-observer asymmetry operator



aberration

4-velocity difference

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Fermi-Walker derivative (observer)

lensing

E-O asymmery operator

Drift effects

Position drift

$$\delta_{\mathcal{O}} r^A = w_{\mathcal{O}}^A + \frac{1}{p_\sigma\, u_{\mathcal{O}}^\sigma}\, {\mathcal{D}^{-1}}^A{}_B \left(\left(\frac{1}{1+z} u_{\mathcal{E}} - \hat{u}_{\mathcal{O}} \right)^B - m^B{}_\mu\, u_{\mathcal{O}}^\mu \right)$$

Drift effects

Position drift

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Physical consequences

Drift effects

Position drift

$$\delta_{\mathcal{O}} r^A = w_{\mathcal{O}}^A + \frac{1}{p_\sigma\, u_{\mathcal{O}}^\sigma}\, {\mathcal{D}^{-1}}^A{}_B \left(\left(\frac{1}{1+z} u_{\mathcal{E}} - \hat{u}_{\mathcal{O}} \right)^B - m^B{}_\mu\, u_{\mathcal{O}}^\mu \right)$$

Physical consequences

- Exact relation between lensing and drift

Drift effects

Position drift

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Physical consequences

- Exact relation between lensing and drift
  - Strong image magnification ⇒ large drift

Drift effects

Position drift

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Physical consequences

- Exact relation between lensing and drift
  - Strong image magnification ⇒ large drift
  - Caustics = powerful position drift amplifiers (formally $\delta_{\mathcal{O}}\, r^A \to \infty$)

Drift effects

Position drift

$$\delta_{\mathcal{O}} r^A = w_{\mathcal{O}}^A + \frac{1}{p_\sigma\, u_{\mathcal{O}}^\sigma}\, {\mathcal{D}^{-1}}^A{}_B \left(\left(\frac{1}{1+z} u_{\mathcal{E}} - \hat{u}_{\mathcal{O}} \right)^B - m^B{}_\mu\, u_{\mathcal{O}}^\mu \right)$$

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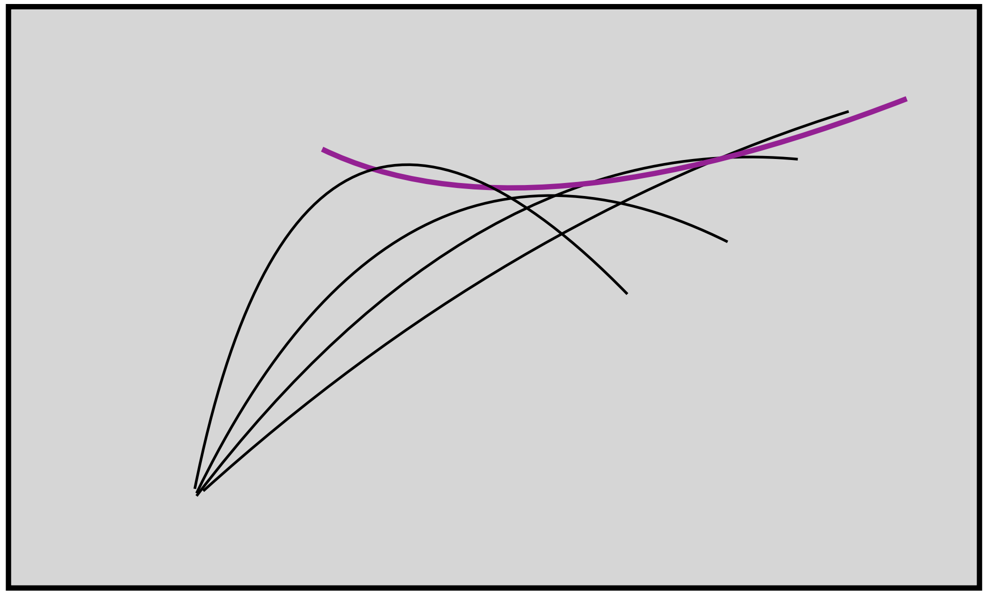

Drift effects

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  - Strong image magnification ⇒ large drift
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  - Lensing of small objects near caustics may be used to probe transversal velocities of galaxy clusters [Kelly *et al, Nature Astronomy* 2 (2018) 334-342, *"Extreme magnification of an individual star…"*]

Drift effects

Position drift

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- Asymmetry between $u_{\mathcal{E}}$ and $u_{\mathcal{O}}$ due to m

Drift effects

Position drift

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  - Strong image magnification ⇒ large drift
  - Caustics = powerful position drift amplifiers (formally $\delta_{\mathcal{O}}\, r^A \to \infty$)
  - Lensing of small objects near caustics may be used to probe transversal velocities of galaxy clusters [Kelly *et al*, *Nature Astronomy* 2 (2018) 334-342, *"Extreme magnification of an individual star…"*]
- Asymmetry between $u_{\mathcal{E}}$ and $u_{\mathcal{O}}$ due to m
  - Boosting of the emitter inequivalent to boosting of the observer

Example

Perfectly homogeneous cosmological model (FLRW)

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$$S_k(\chi) \quad = \quad \begin{cases} \frac{1}{\sqrt{k}} \sin(\sqrt{k}\,\chi) & \text{if } k > 0 \\ \chi & \text{if } k = 0 \\ \frac{1}{\sqrt{|k|}} \sinh(\sqrt{|k|}\,\chi) & \text{if } k < 0 \end{cases}$$

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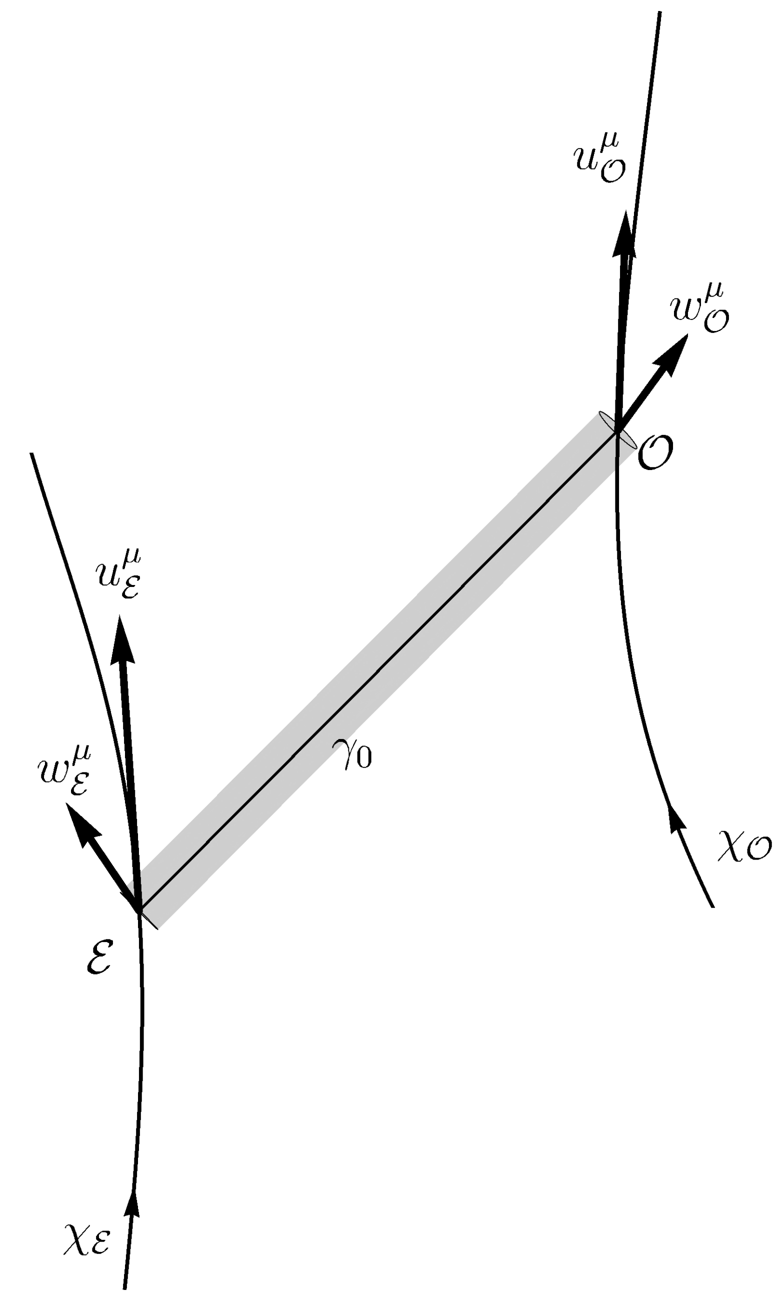
$$m^A{}_\mu \partial_t^\mu = 0$$

$$m^A{}_B = \delta^A{}_B\left(\frac{a_{\mathcal{E}}}{a_{\mathcal{O}}}\,C_k(\chi(a_{\mathcal{E}}, a_{\mathcal{O}})) + a_{\mathcal{E}}\,S_k(\chi(a_{\mathcal{E}}, a_{\mathcal{O}}))\,H(t_{\mathcal{O}}) - 1\right)$$

$$C_k(\chi) \;\equiv\; \frac{\mathrm{d}S_k}{\mathrm{d}\chi} = \begin{cases} \cos(\sqrt{k}\,\chi) & \text{if } k>0 \\ 1 & \text{if } k=0 \\ \cosh(\sqrt{|k|}\,\chi) & \text{if } k<0 \end{cases}$$

Drift effects

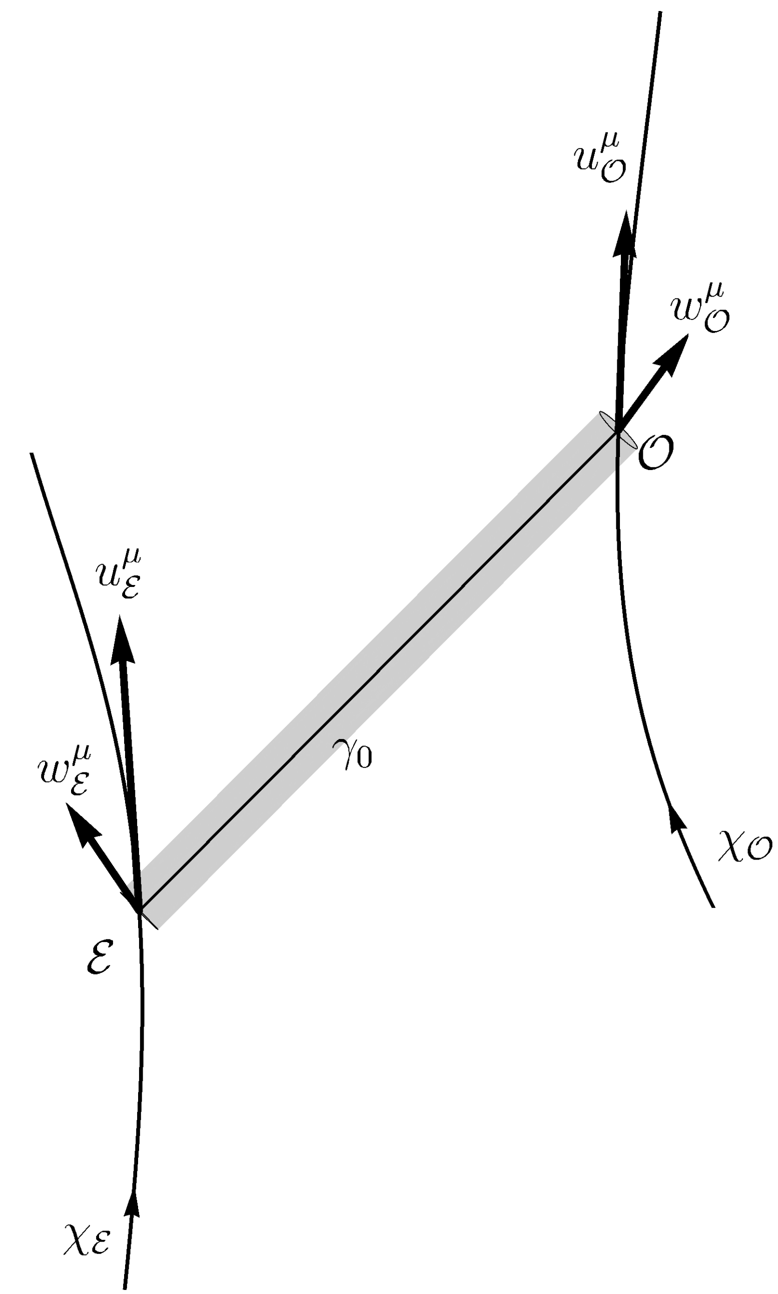
Redshift drift



Drift effects

Redshift drift

- parallel propagation of $\hat{w}^{\mu}_{\mathcal{E}}$, $\hat{u}^{\beta}_{\mathcal{E}}$



Drift effects

Redshift drift

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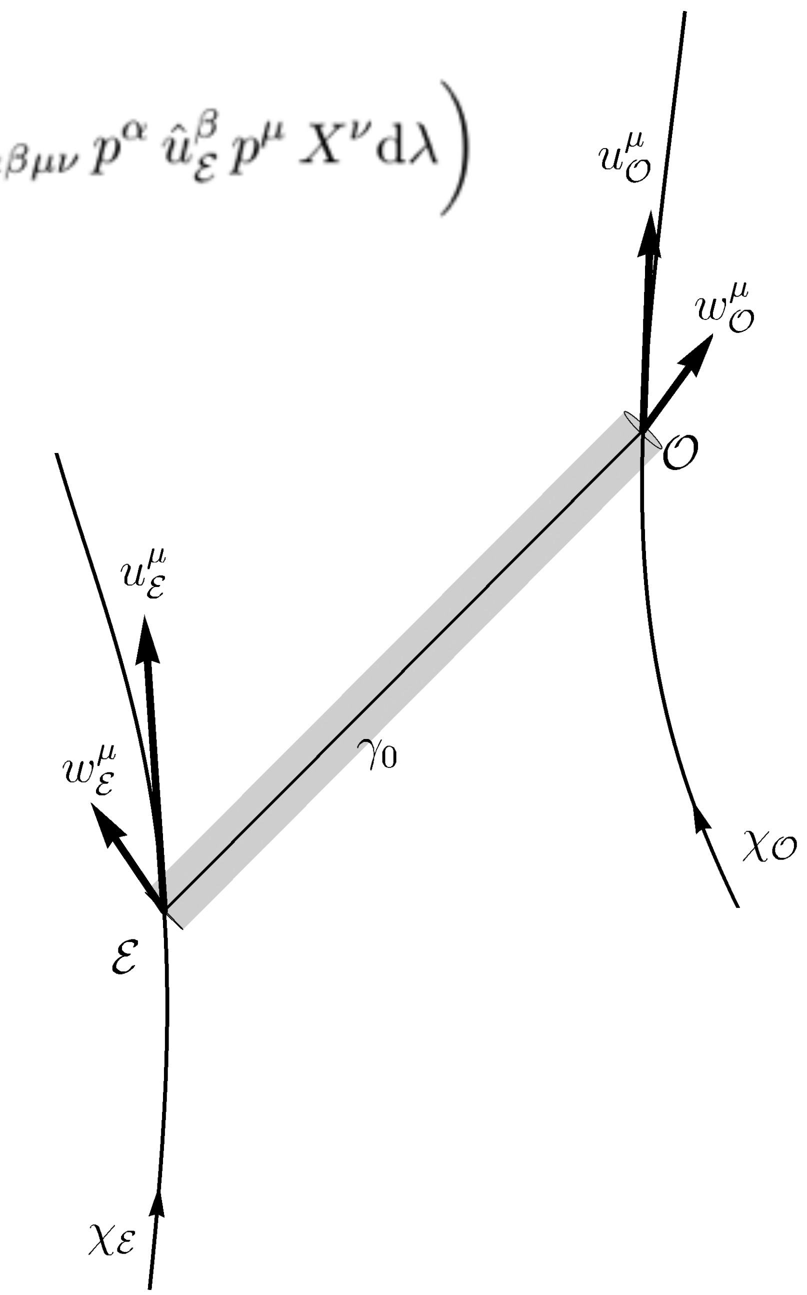
$$
\begin{aligned}
\nabla_X \ln(1+z) \; = \; & \frac{1}{p_\sigma\, u^\sigma_{\mathcal{O}}} \left(\left(\frac{1}{(1+z)^2} \hat{w}^\mu_{\mathcal{E}} - w^\mu_{\mathcal{O}} \right) p_\mu + \frac{1}{1+z} \int_{\lambda_{\mathcal{E}}}^{\lambda_{\mathcal{O}}} R_{\alpha\beta\mu\nu}\, p^\alpha\, \hat{u}^\beta_{\mathcal{E}}\, p^\mu\, X^\nu \mathrm{d}\lambda \right) \\
& + \left(\frac{\mathrm{D}^{F-W}}{\mathrm{d}\tau} r^A - w^A_{\mathcal{O}} \right) \left(\frac{1}{1+z} \hat{u}_{\mathcal{E}} - u_{\mathcal{O}} \right)_A \Bigg|_{\lambda=\lambda_{\mathcal{O}}}
\end{aligned}
$$



Drift effects

Redshift drift

line-of-sight
4-acceleration difference

- parallel propagation of $\hat{w}^{\mu}_{\mathcal{E}}$, $\hat{u}^{\beta}_{\mathcal{E}}$

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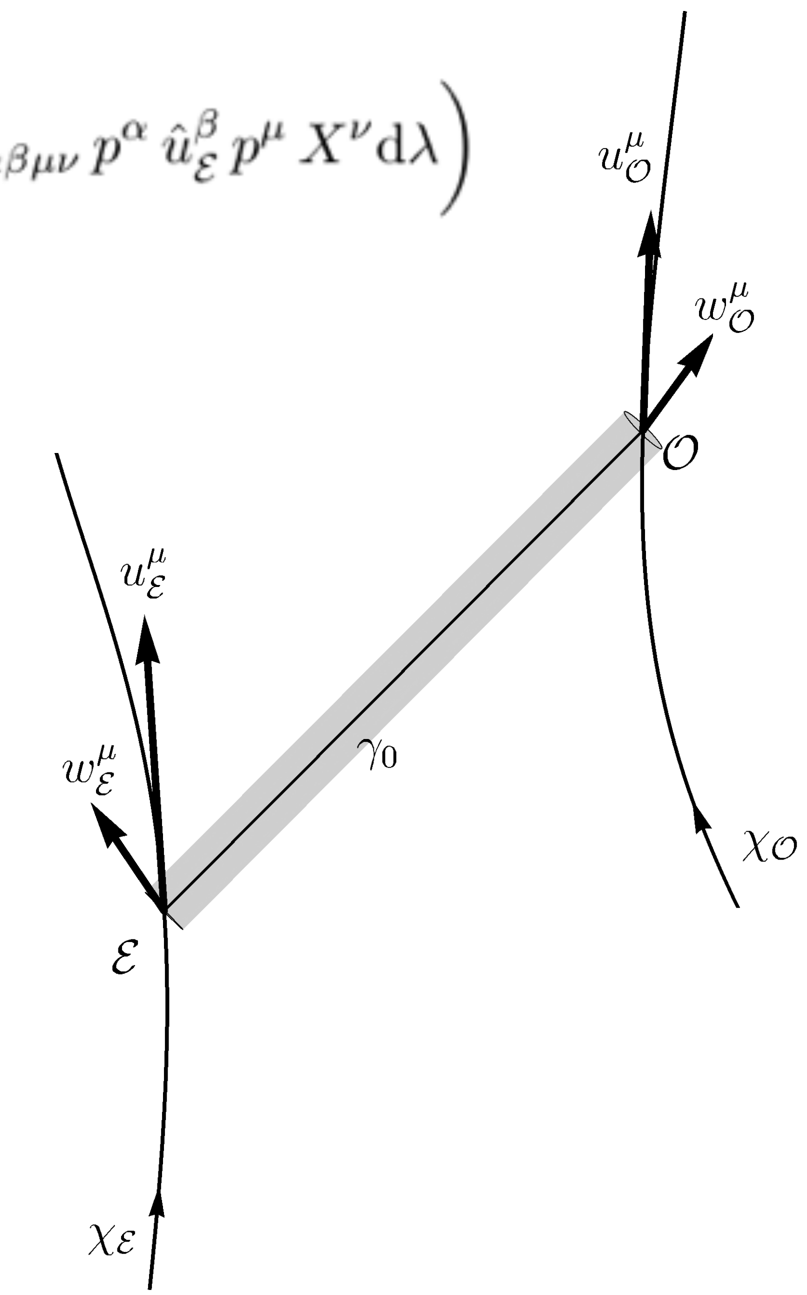

Drift effects

Redshift drift

line-of-sight 4-acceleration difference

integral of Riemann

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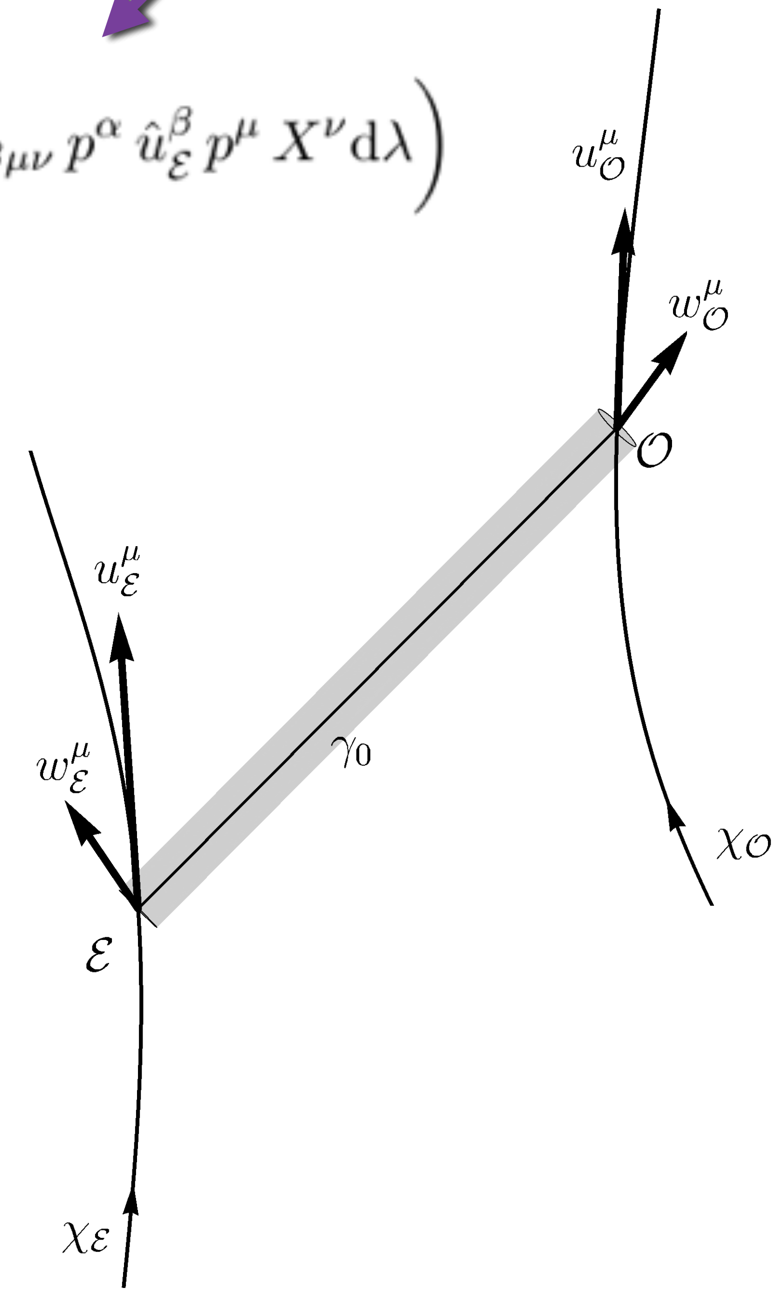

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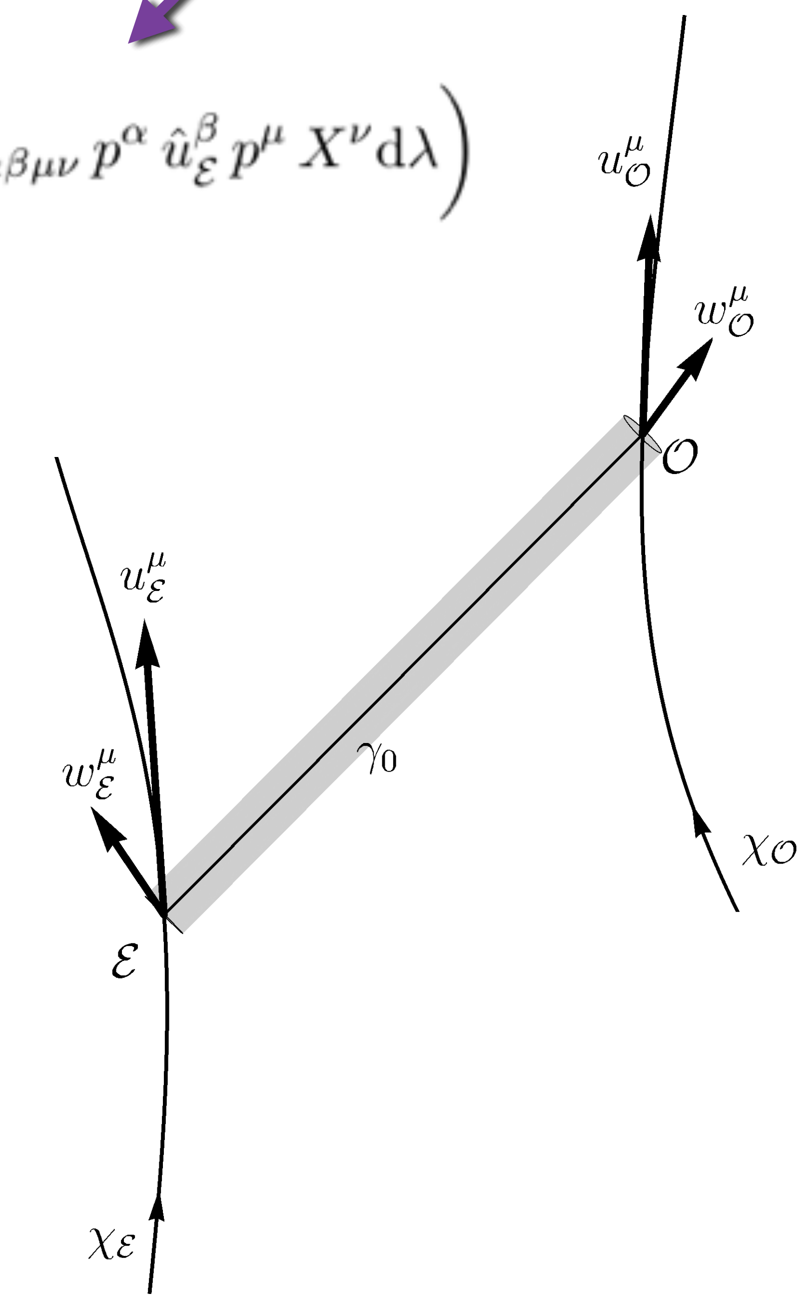

$$\frac{\mathrm{D}^{F-W}}{\mathrm{d}\tau}r^A = w^A_\mathcal{O} + \frac{1}{p_\sigma\, u^\sigma_\mathcal{O}}\mathcal{D}^{-1}(\lambda_\mathcal{E})^A{}_B\left(\left(\frac{1}{1+z}u_\mathcal{E} - \hat{u}_\mathcal{O}\right)^B - m^B\right)$$

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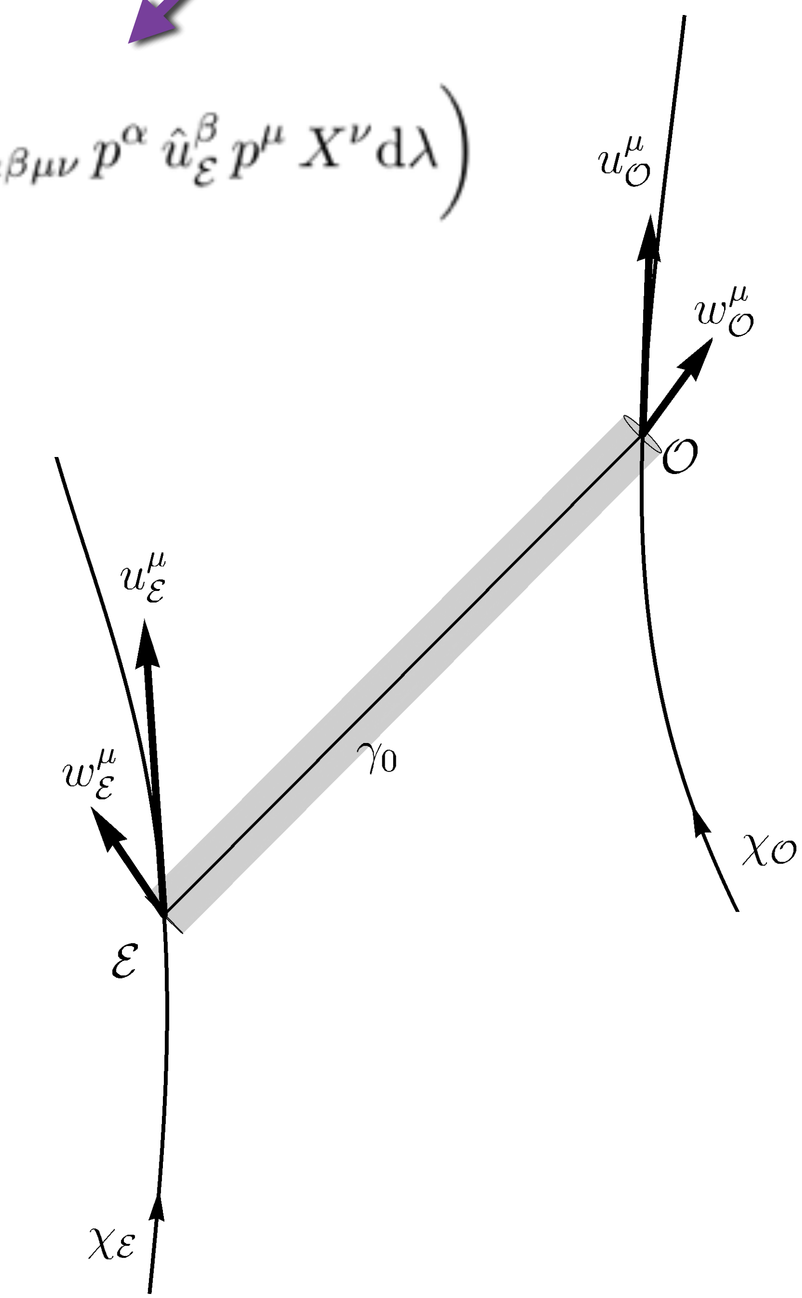
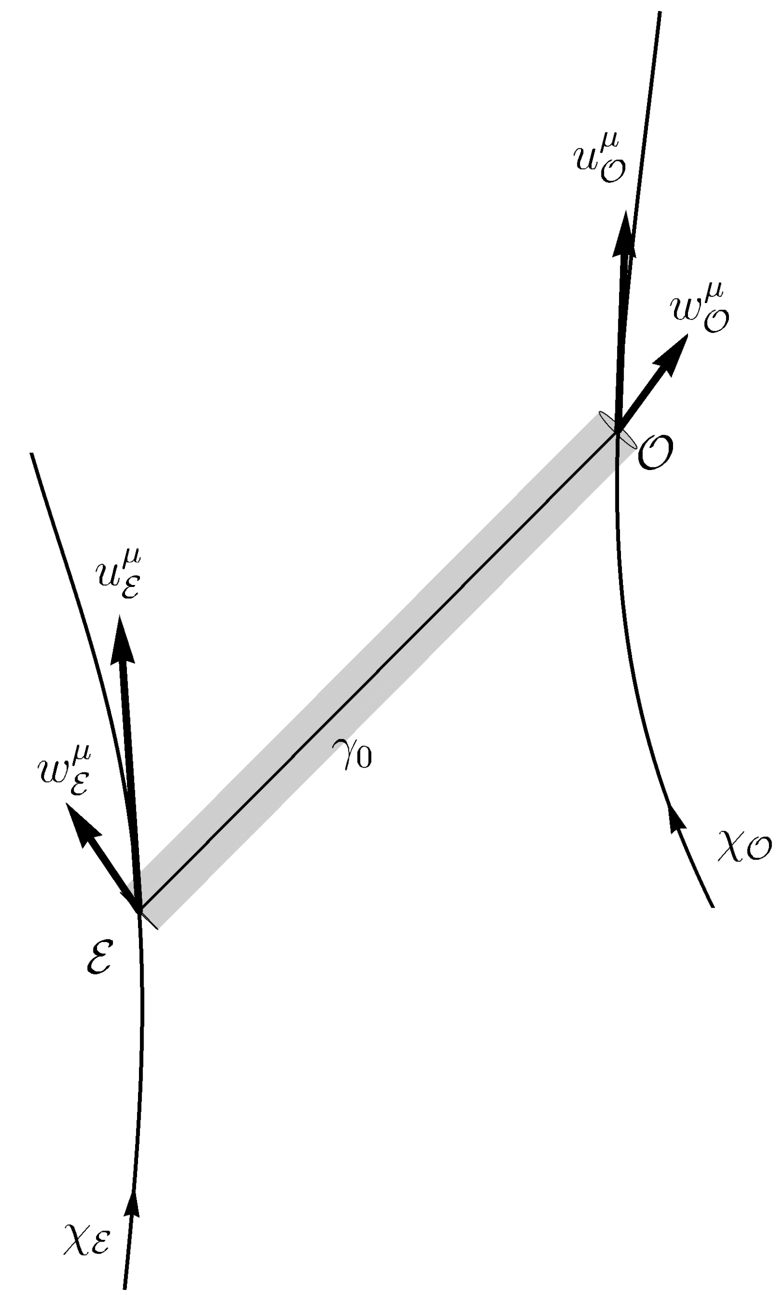
transversal 4-velocity difference, Jacobi matrix

$$\frac{\mathrm{D}^{F-W}}{\mathrm{d}\tau} r^A = w^A_{\mathcal{O}} + \frac{1}{p_\sigma\, u^\sigma_{\mathcal{O}}}\, \mathcal{D}^{-1}(\lambda_{\mathcal{E}})^A{}_B \left(\left(\frac{1}{1+z}\, u_{\mathcal{E}} - \hat{u}_{\mathcal{O}}\right)^B - m^B\right)$$



Drift effects

Jacobi matrix drift

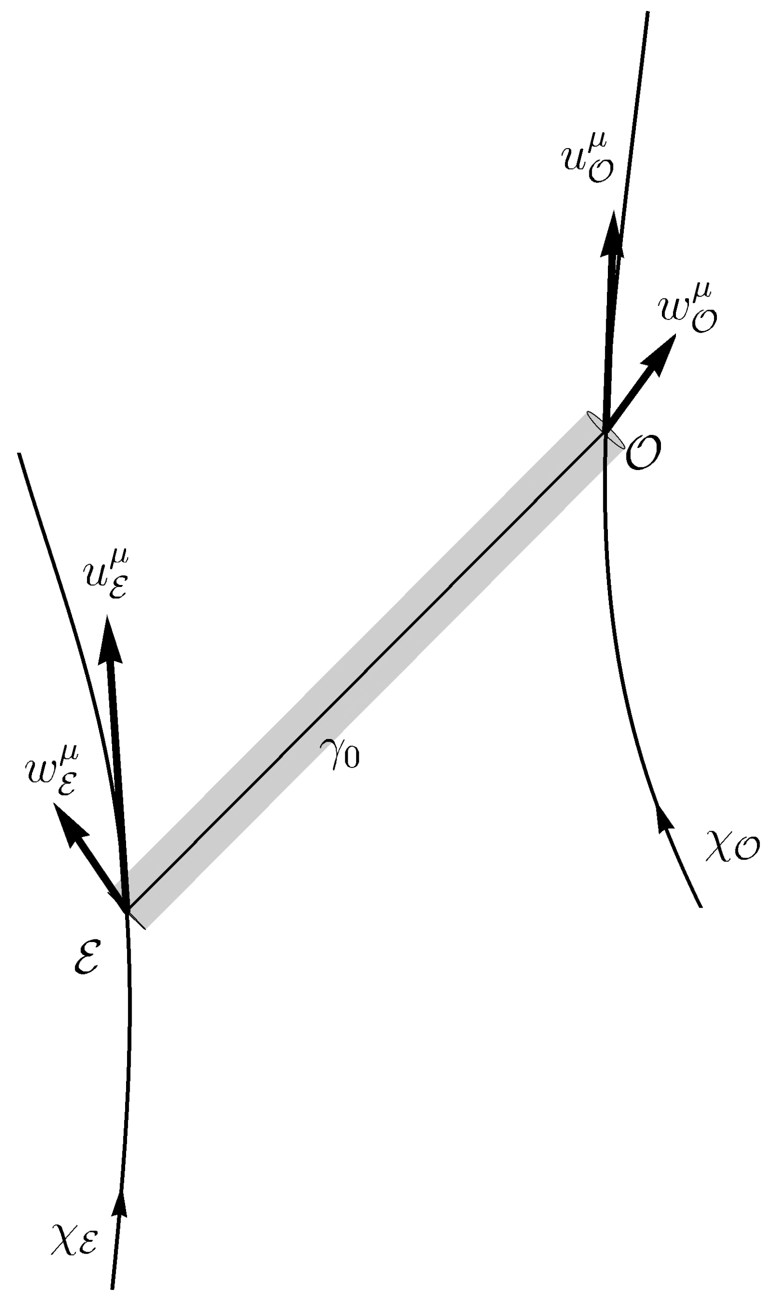


Drift effects

Jacobi matrix drift

- need to differentiate the GDE

$$\mathcal{G}[\xi]^\mu \equiv \nabla_p \nabla_p \xi^\mu - R^\mu{}_{\nu\alpha\beta}\, p^\nu\, p^\alpha\, \xi^\beta = 0$$



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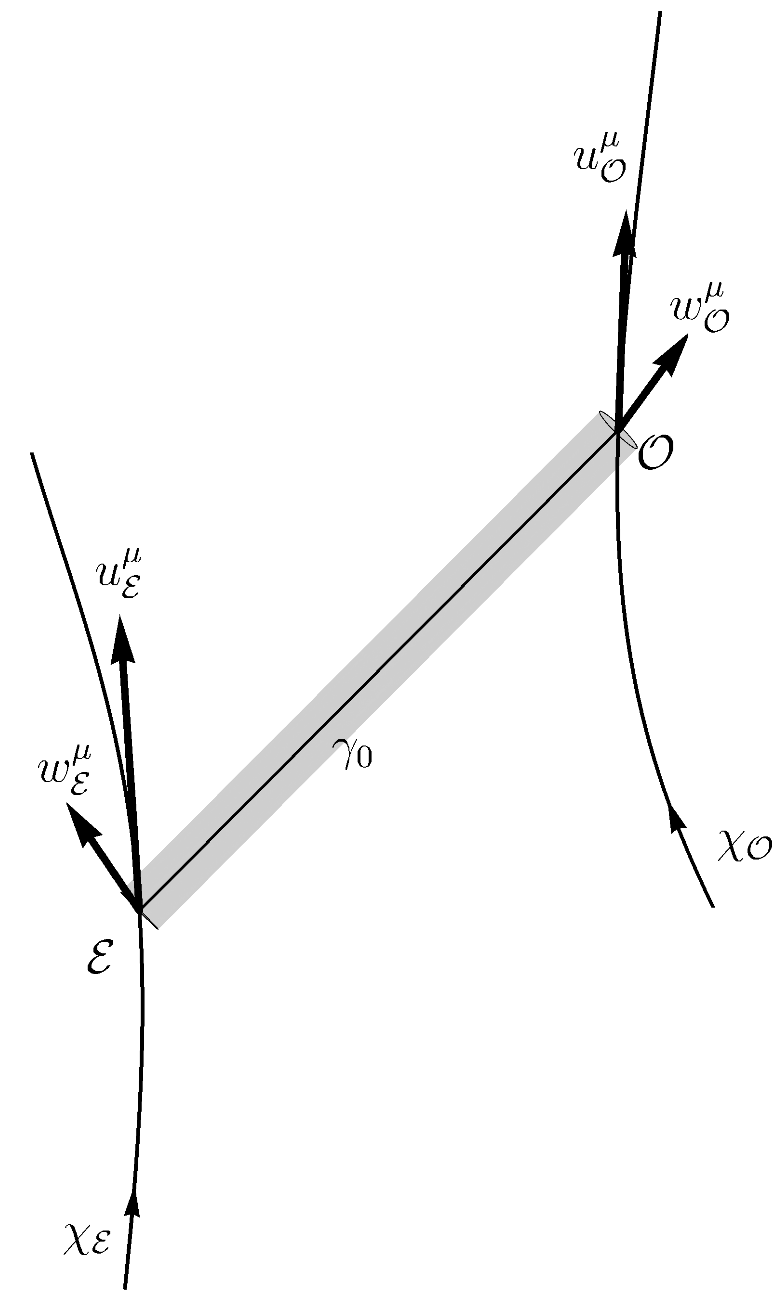

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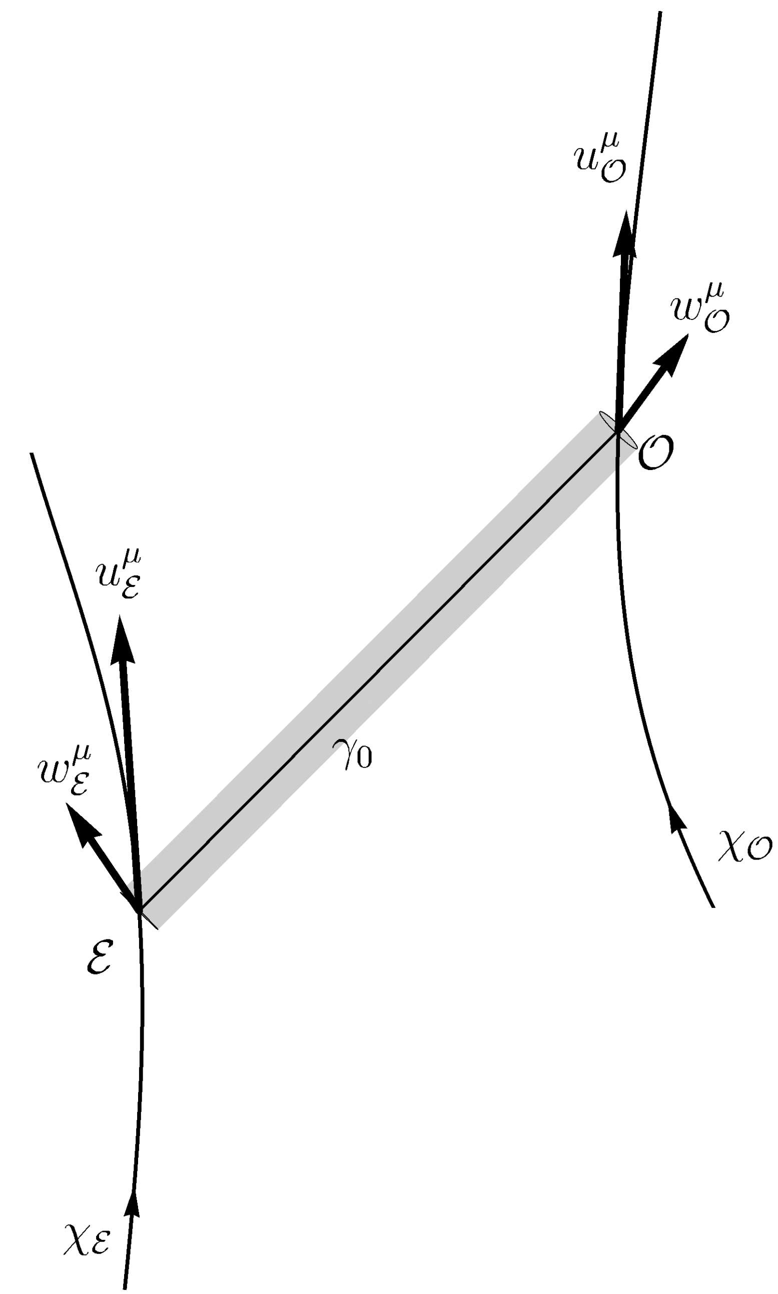
- 2nd order GDE (= inhomogeneous GDE)



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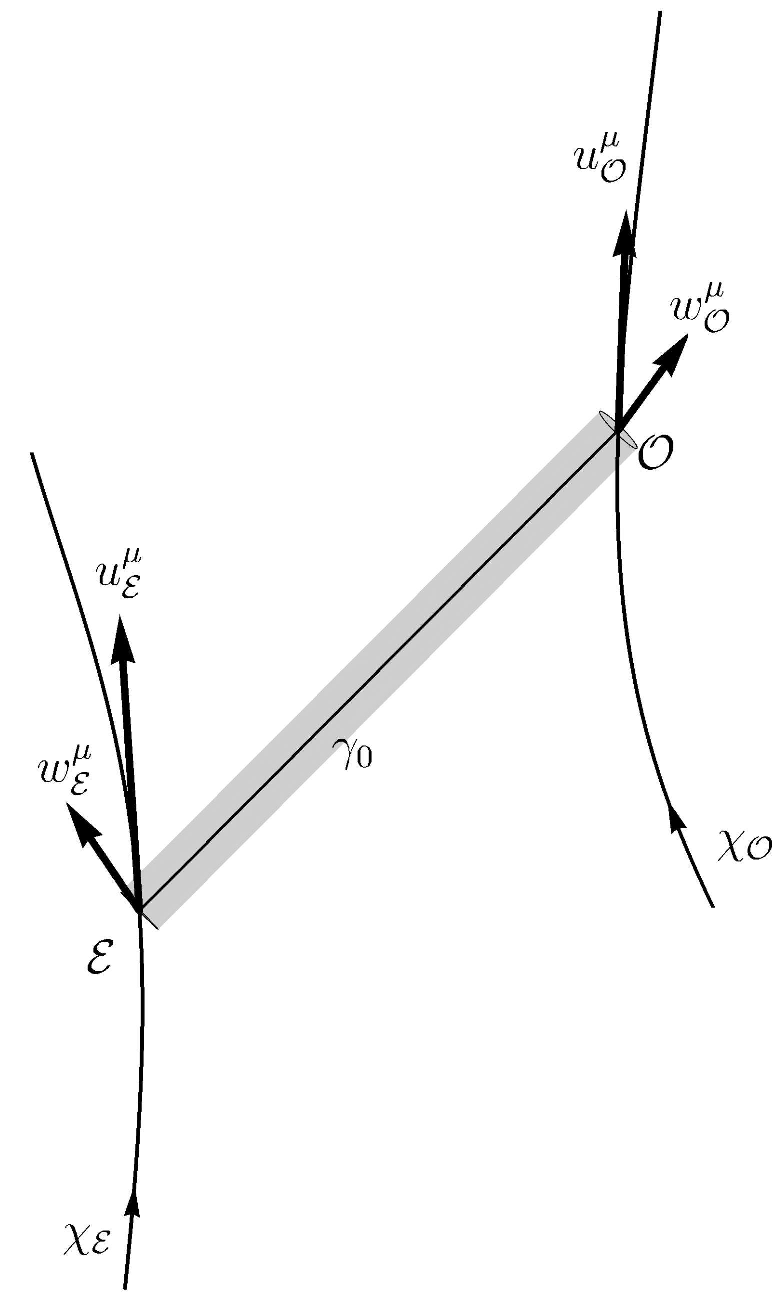

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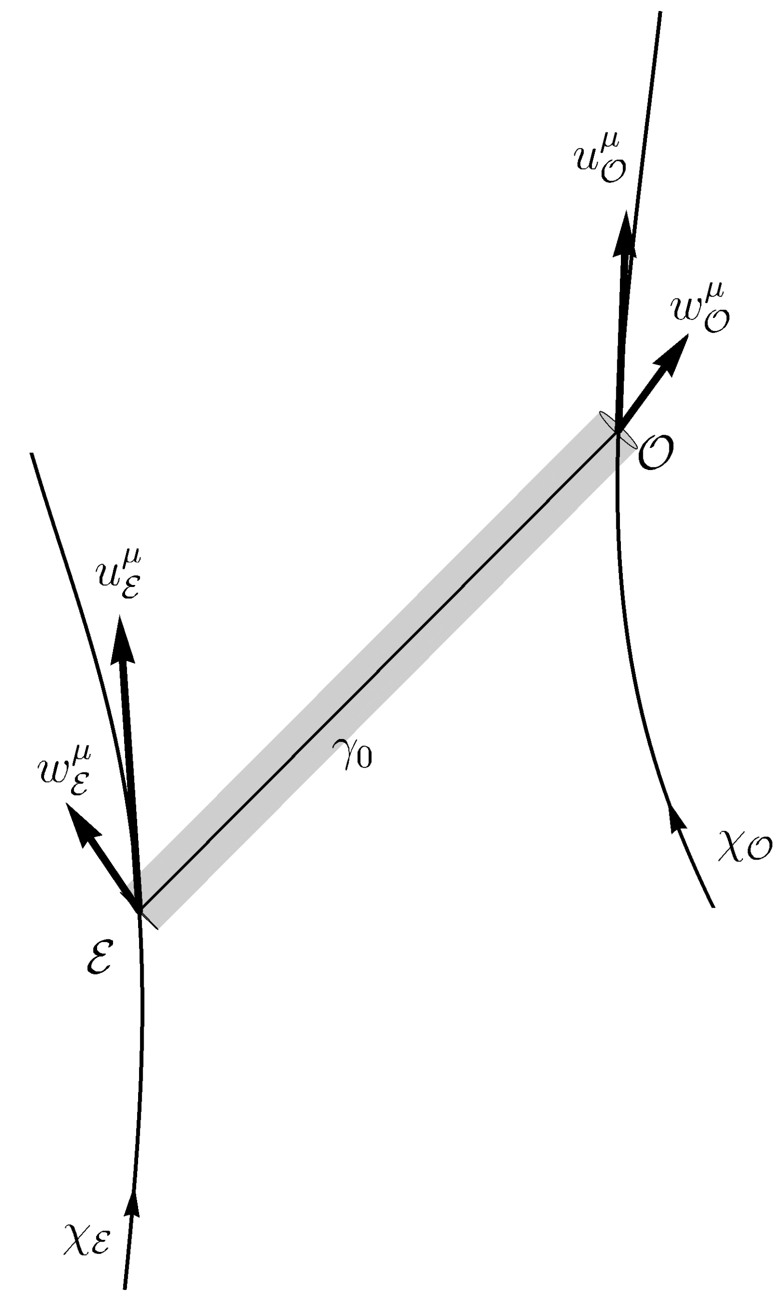

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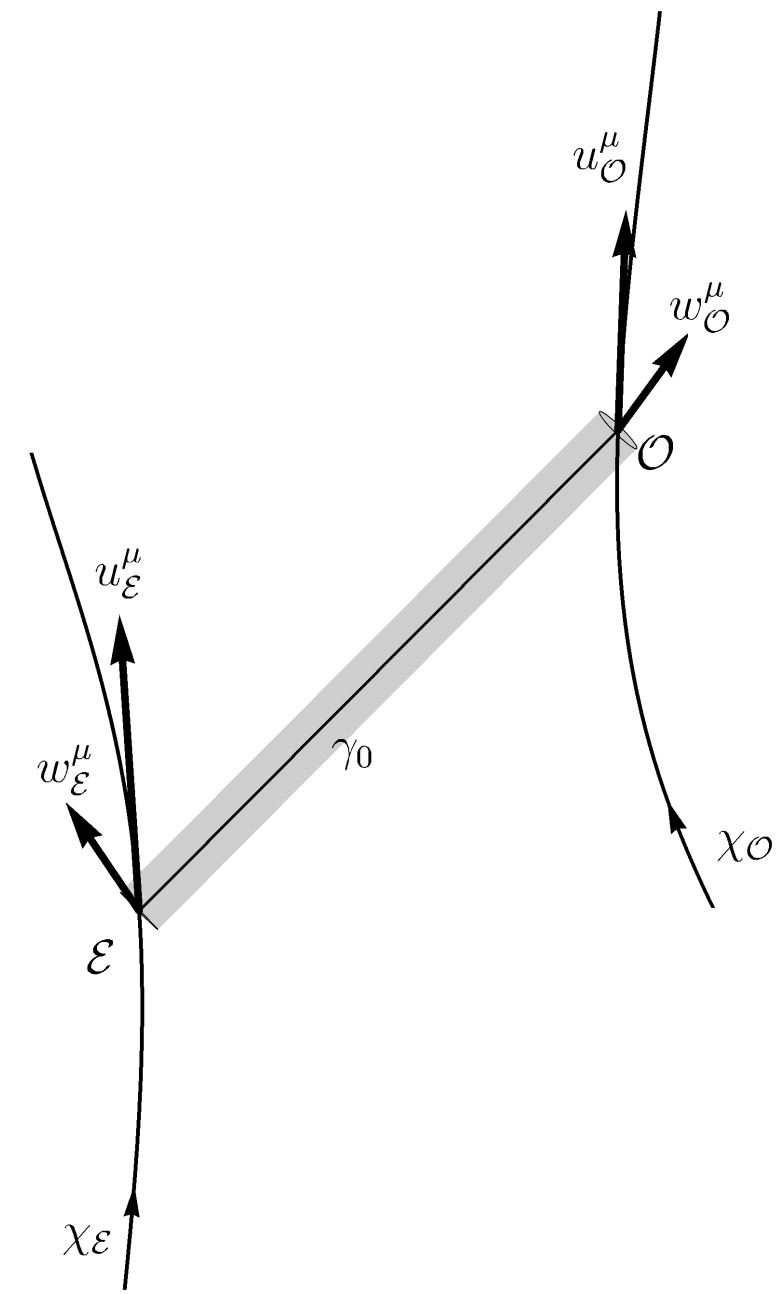
- very cumbersome algebra, complicated expressions result



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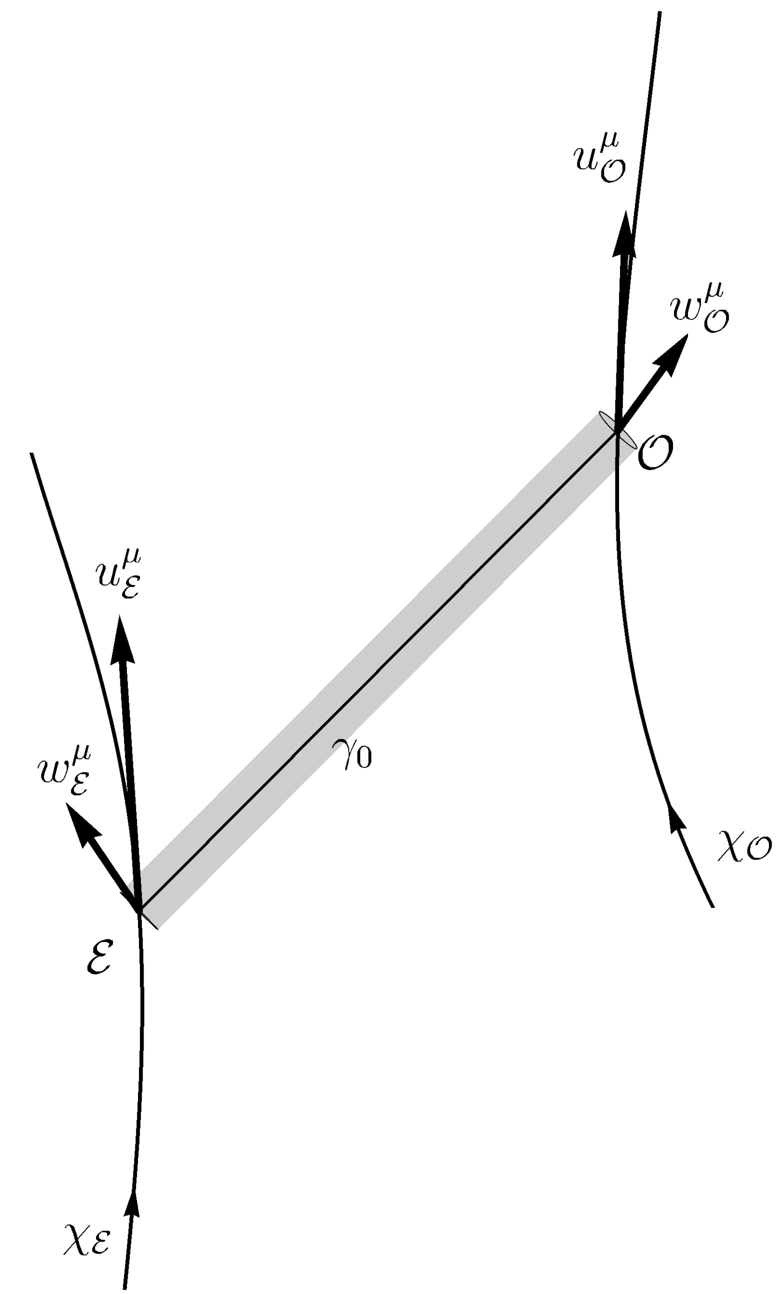
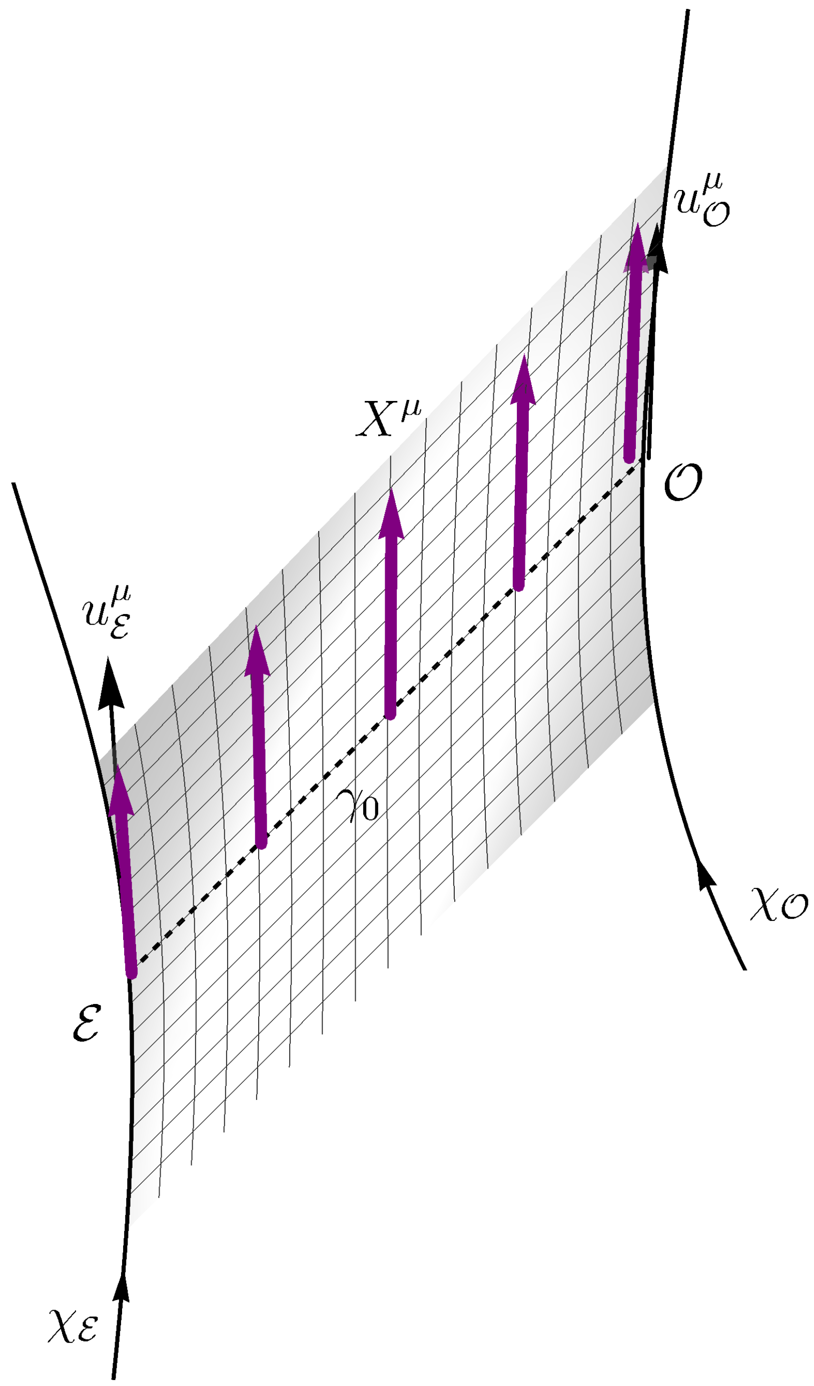
- D_{ang}, D_{lum} drifts



Drift effects

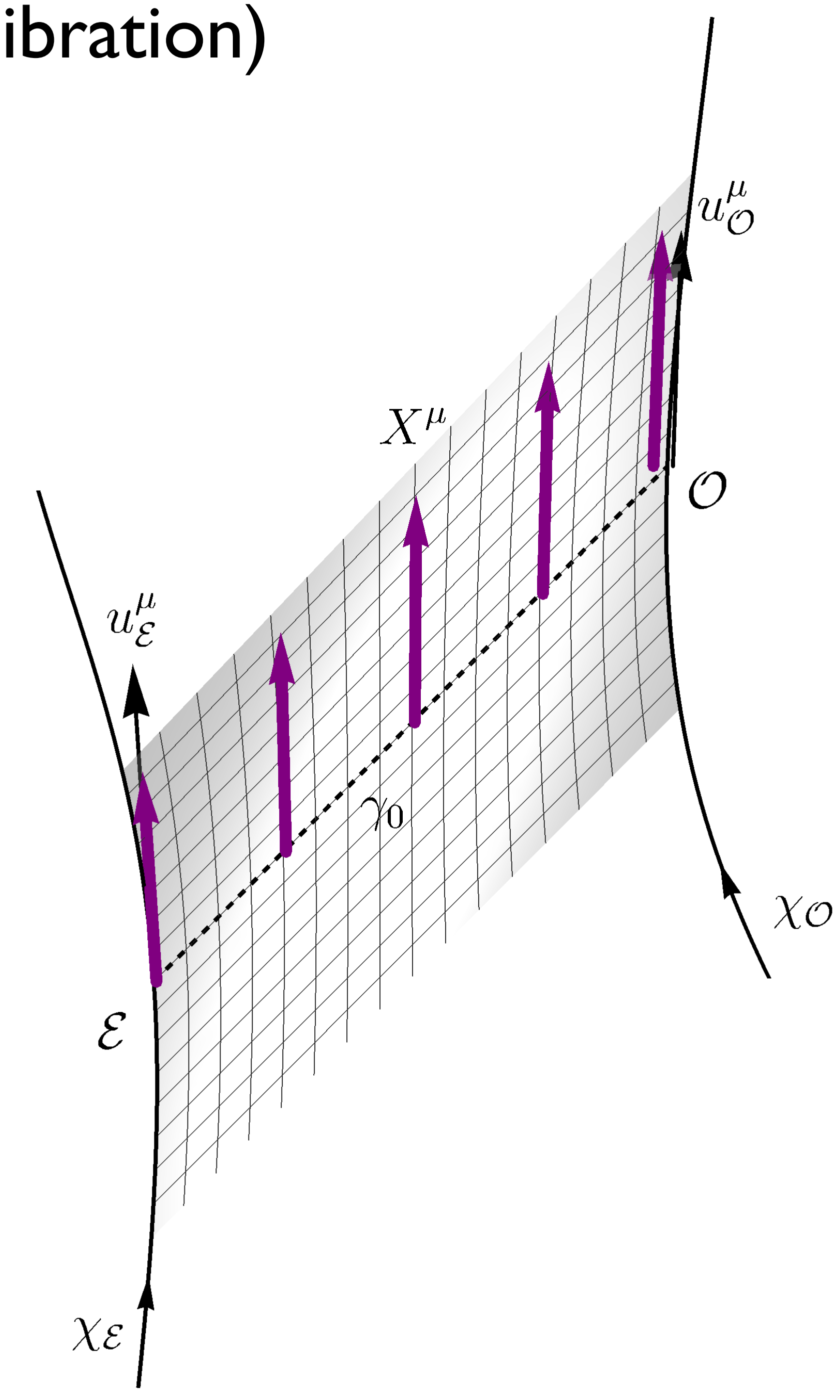
Remarks



Drift effects

Remarks

- $\nabla_X p^\mu$ at $\mathcal{E}$ gives the viewing angle drift (apparent libration)



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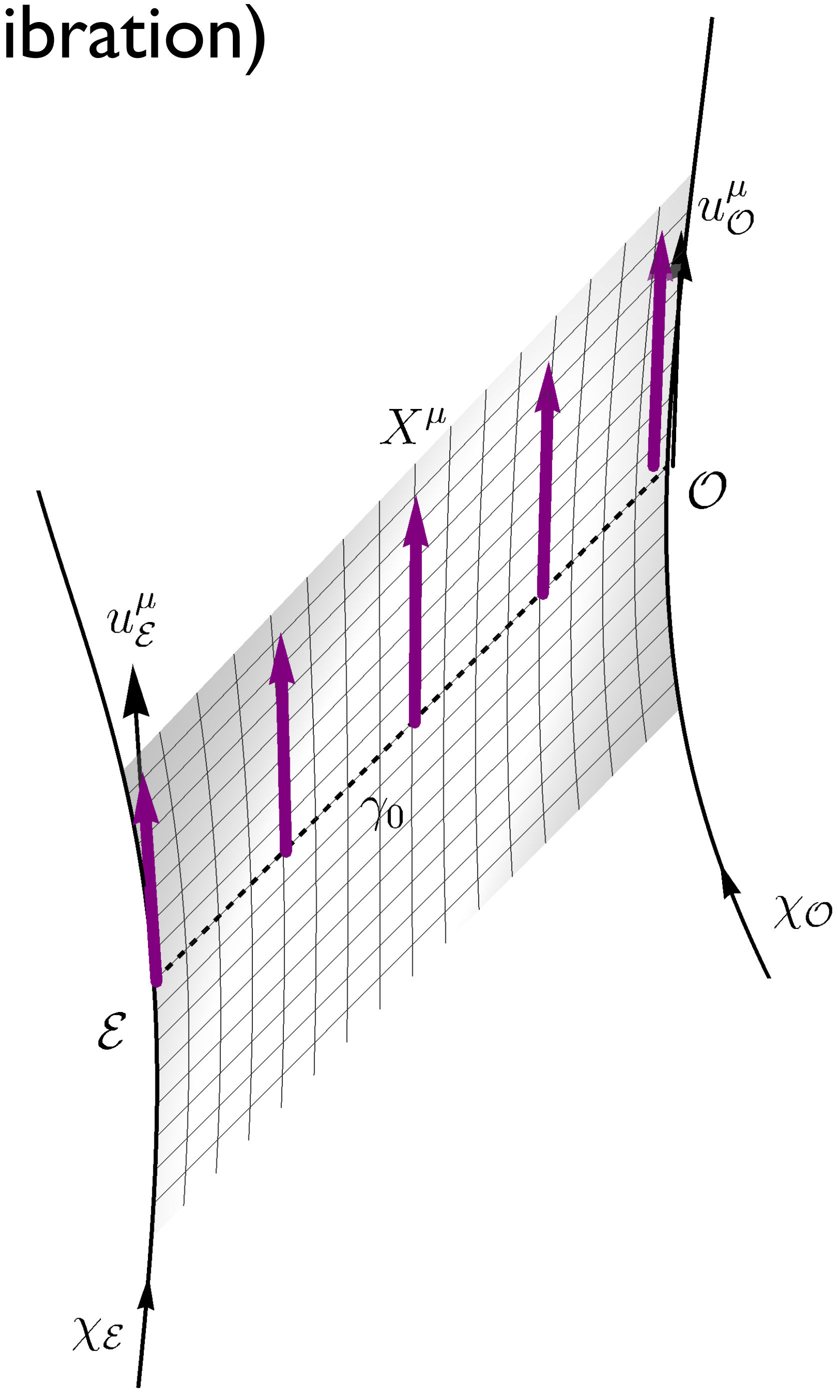

Drift effects

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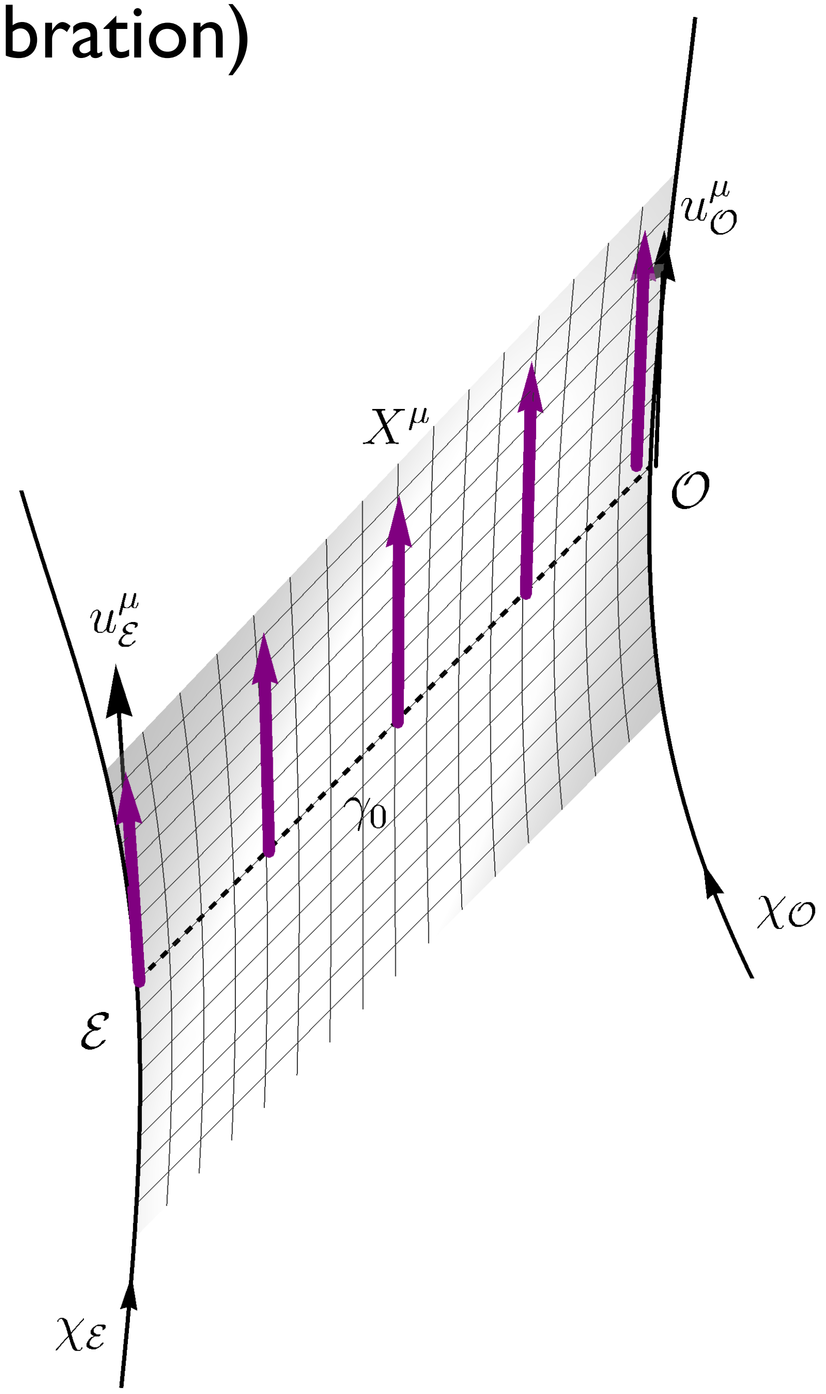
- dependence on observer's and emitter's motion:



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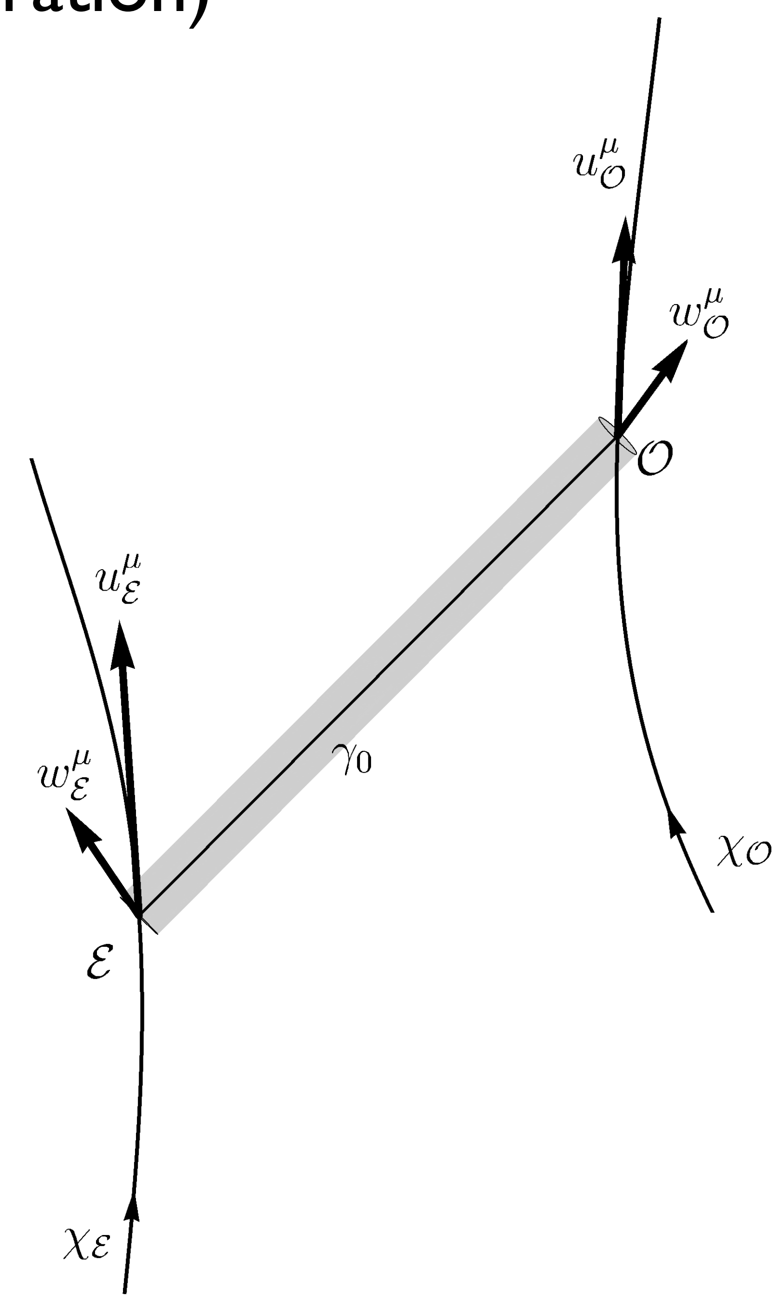

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$$\theta^A \equiv \theta^A\left(\mathcal{O},\mathcal{E},u^\mu_\mathcal{O},u^\mu_\mathcal{E}\right)$$
$$z \equiv z\left(\mathcal{O},\mathcal{E},u^\mu_\mathcal{O},u^\mu_\mathcal{E}\right)$$
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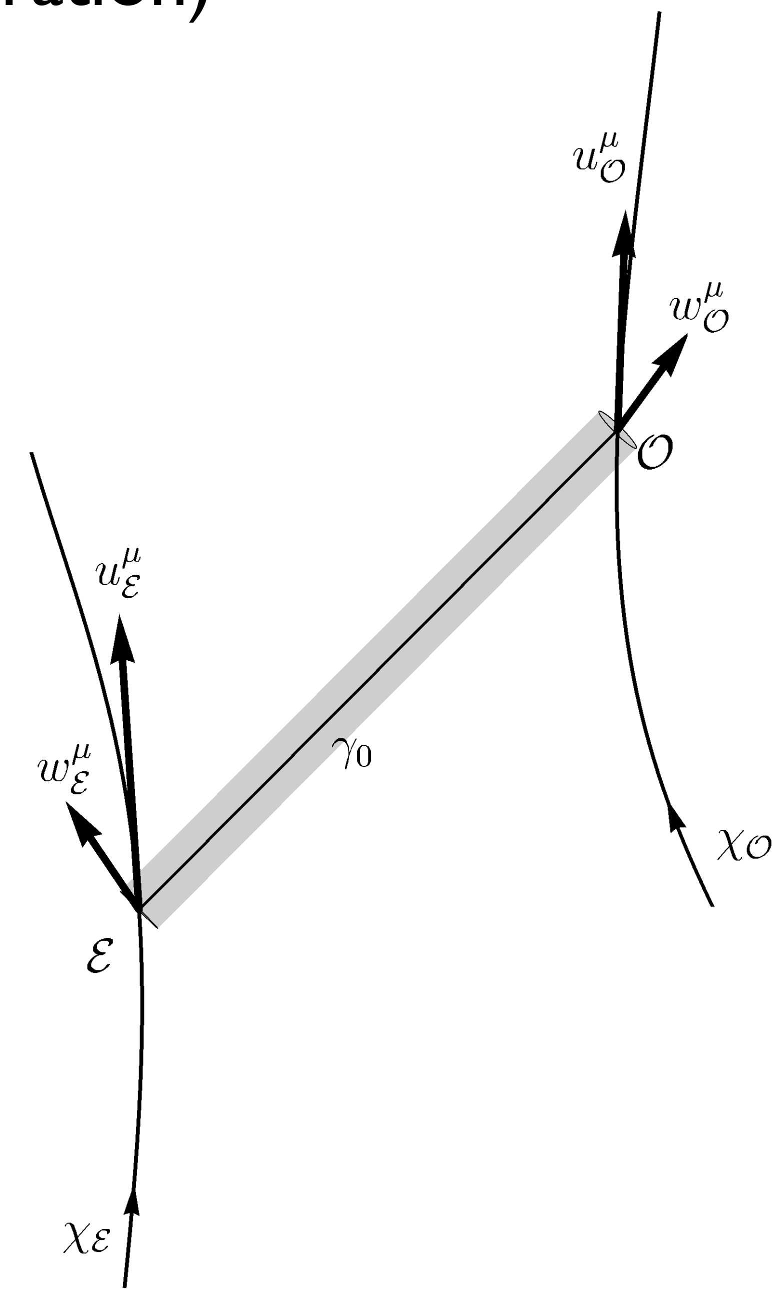

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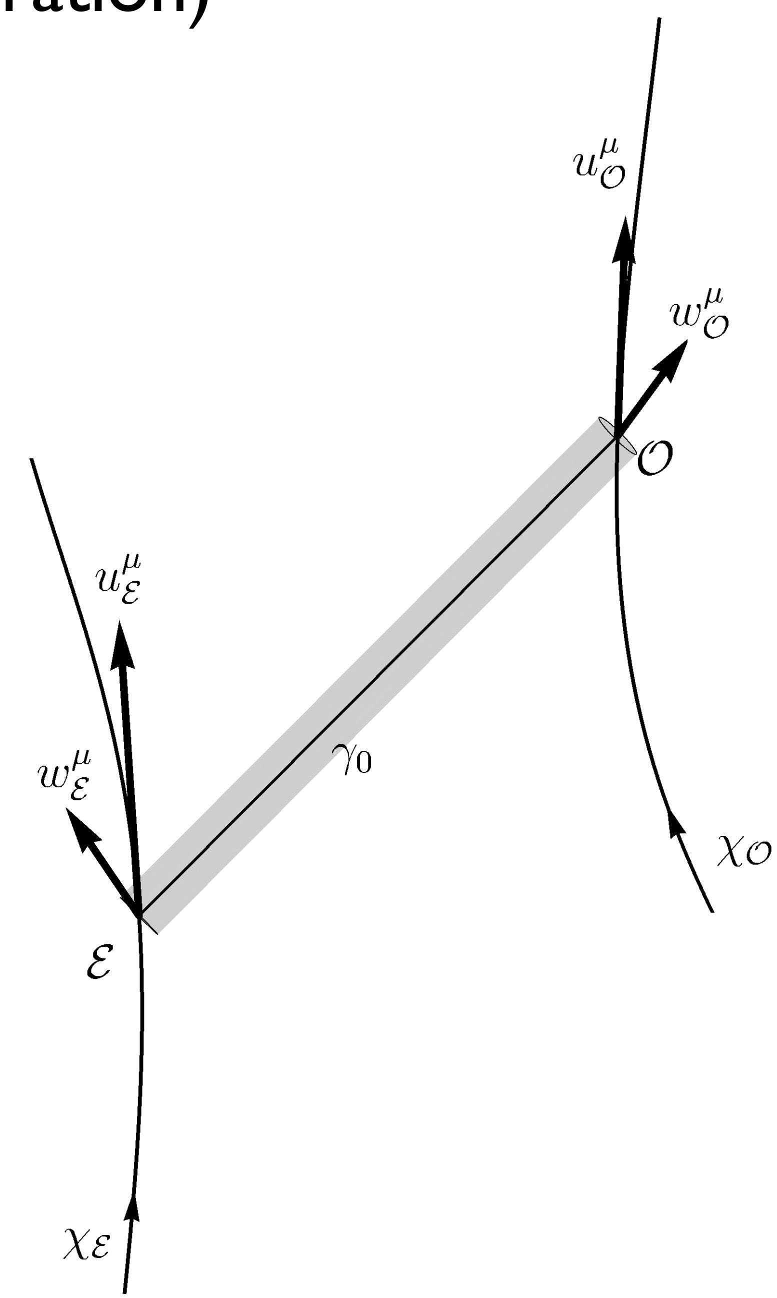
$$\frac{\mathrm{d}}{\mathrm{d}\tau}\ln D_{lum} \equiv (\ln D_{lum})^{\cdot}\left(\mathcal{O}, \mathcal{E}, u^\mu_{\mathcal{O}}, u^\mu_{\mathcal{E}}, w^\mu_{\mathcal{O}}, w^\mu_{\mathcal{E}}, R^\mu{}_{\nu\alpha\beta}, \nabla_\gamma R^\mu{}_{\nu\alpha\beta}\right)$$



Drift effects

Remarks

- $\nabla_X p^\mu$ at $\mathcal{E}$ gives the viewing angle drift (apparent libration)

- dependence on observer's and emitter's motion:

$$\theta^A \equiv \theta^A\left(\mathcal{O},\mathcal{E},u^\mu_\mathcal{O},\cancel{u^\mu_\mathcal{E}}\right)$$
$$z \equiv z\left(\mathcal{O},\mathcal{E},u^\mu_\mathcal{O},u^\mu_\mathcal{E}\right)$$
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$$\frac{\mathrm{d}}{\mathrm{d}\tau}\ln(1+z) \equiv (1+z)^{-1}\dot z\left(\mathcal{O},\mathcal{E},u^\mu_\mathcal{O},u^\mu_\mathcal{E},w^\mu_\mathcal{O},w^\mu_\mathcal{E},R^\mu{}_{\nu\alpha\beta}\right)$$
$$\frac{\mathrm{d}}{\mathrm{d}\tau}\mathcal{D}^A{}_B \equiv \dot{\mathcal{D}}^A{}_B\left(\mathcal{O},\mathcal{E},u^\mu_\mathcal{O},u^\mu_\mathcal{E},w^\mu_\mathcal{O},\cancel{w^\mu_\mathcal{E}},R^\mu{}_{\nu\alpha\beta},\nabla_\gamma R^\mu{}_{\nu\alpha\beta}\right)$$
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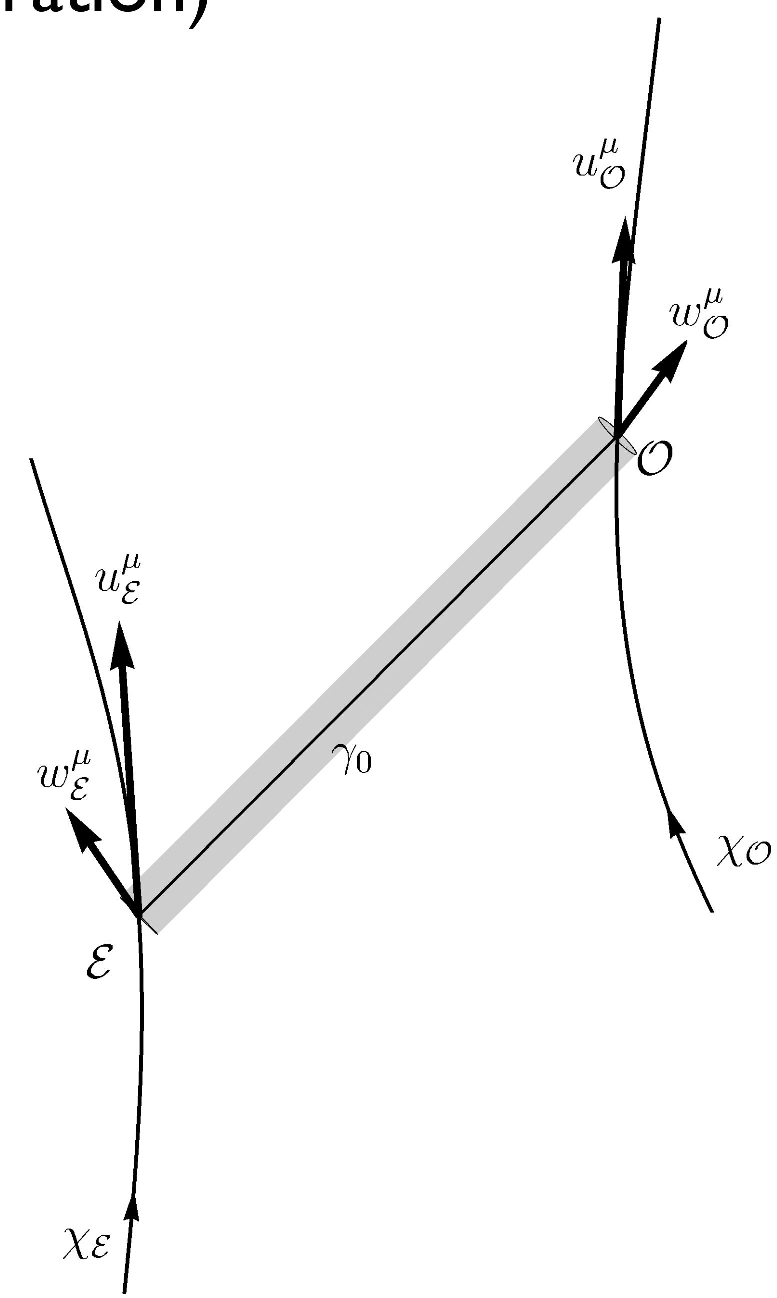

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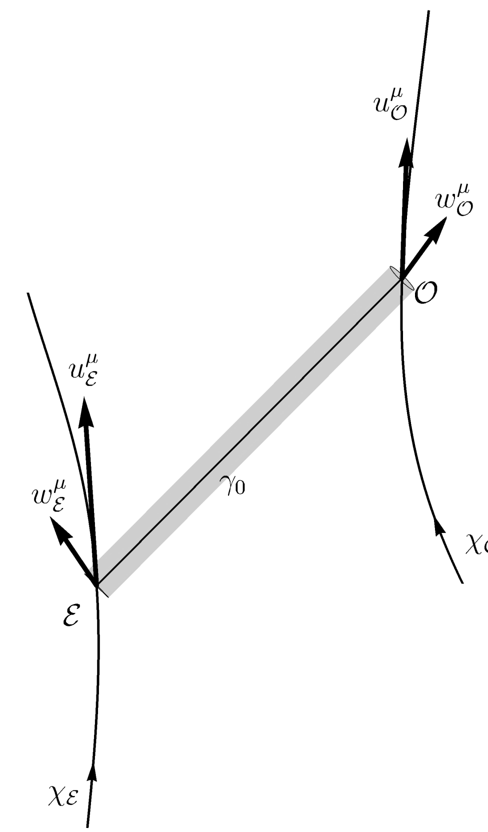

$u^{\mu}_{\mathcal{O}}$
$w^{\mu}_{\mathcal{O}}$
$\mathcal{O}$
$u^{\mu}_{\mathcal{E}}$
$w^{\mu}_{\mathcal{E}}$
γ_0
$\chi_{\mathcal{O}}$
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$\chi_{\mathcal{E}}$

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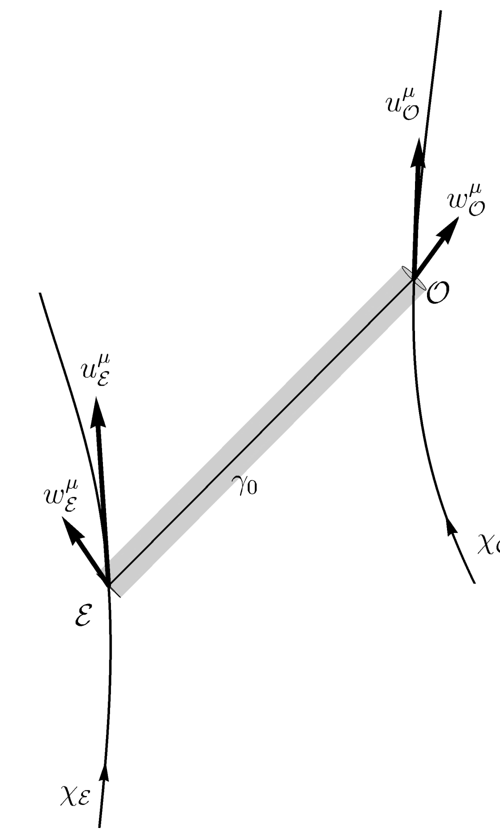

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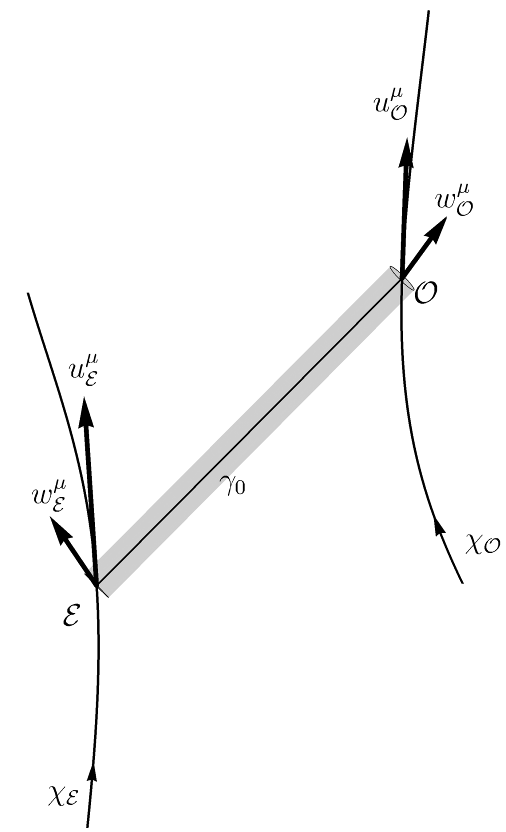

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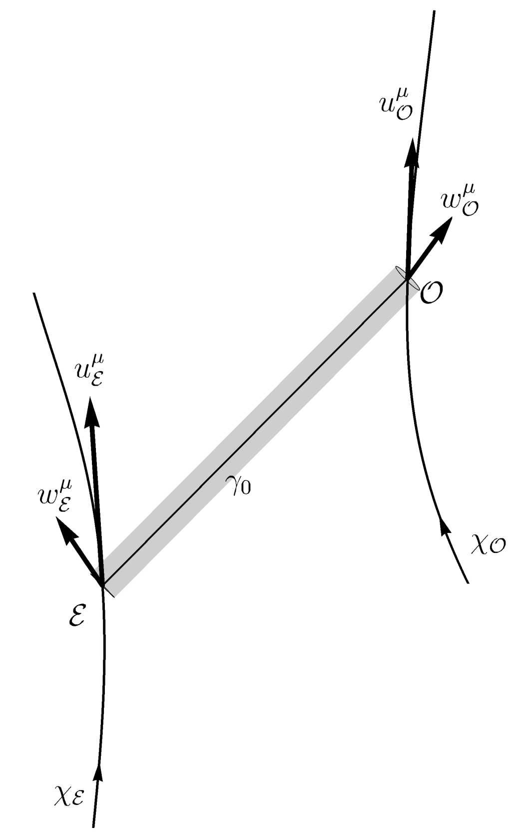

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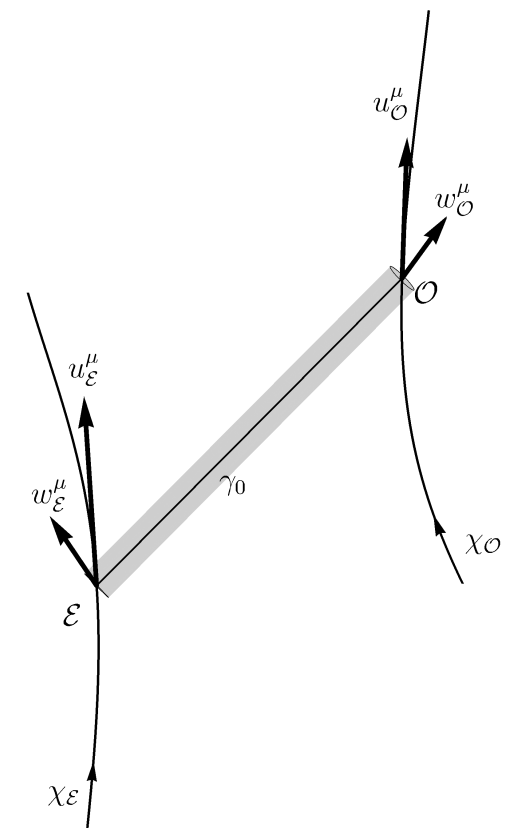

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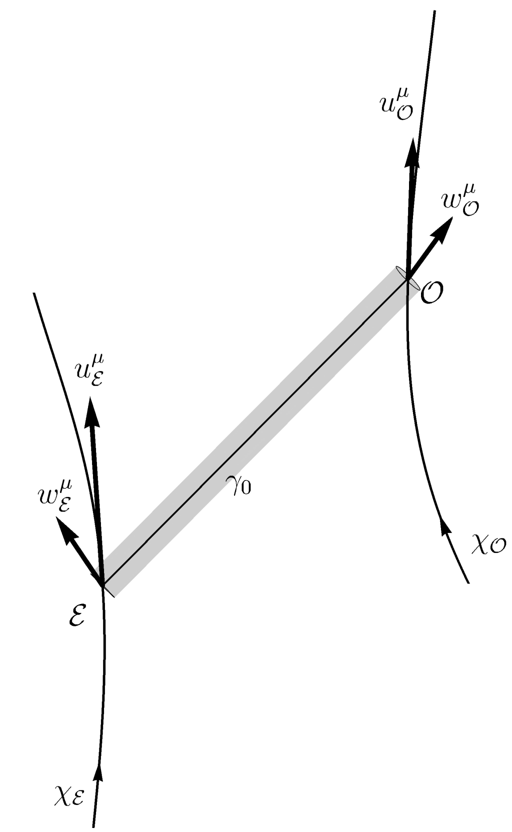

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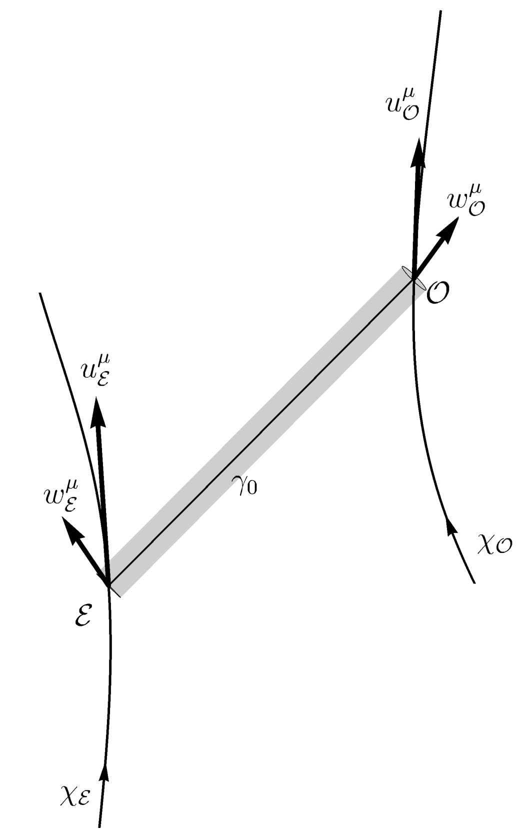

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  ⇒ separating the light propagation effects from peculiar motions

- Formalism works also for the parallax, drifts of $\mathcal{D}^A{}_B$, angular distance, luminosity distance (algebraically complicated)

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- Outside cosmology: methods direct measurements of curvature using optical properties of the spacetime