AC CONDUCTANCE AND NON-SYMMETRIZED NOISE AT FINITE FREQUENCY IN QUANTUM WIRE AND CARBON NANOTUBE

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CURRENT FLUCTUATIONS

ORIGINS OF NOISE
- High temperature: Johnson-Nyquist noise
- High voltage: Shot noise
- High frequency: Quantum noise
- We neglect the 1/f noise

ZERO FREQUENCY AND ZERO TEMPERATURE: SHOT NOISE

⇒ Schottky relation \( S^+(\omega = 0) = e^* |\langle \delta j \rangle| \)

where \( S^+(\omega) = \text{FT} \left\{ \frac{1}{2} \left[ \langle \delta j(0) \delta j(t) \rangle + \langle \delta j(t) \delta j(0) \rangle \right] \right\} \)

symmetrized noise

HIGH FREQUENCY NOISE MEASUREMENTS
- High-frequency measurement in a diffusive wire 1–20 GHz
- On-chip detection using SIS junction → 100 GHz
- Direct measurement in a QPC 4–8 GHz

What is measured is \( S(\omega) = \int dt e^{i \omega x} \langle \delta j(0) \delta j(t) \rangle \)

non-symmetrized noise
The system comprises a wire with an impurity at position $x_i$, modulated by gates $V_1$, $V_2$, and $V_3$, and a backscattering amplitude $L$.

**Model**

\[ H = H_0 + H_B + H_V \]

\[
H_0 = \frac{\hbar v_F}{2} \int_{-\infty}^{\infty} dx \left[ \Pi^2 + \frac{1}{g^2(x)} \left( \partial_x \Phi \right)^2 \right]
\]

\[
H_B = \lambda \cos \sqrt{4 \pi} \Phi(x_i, t) + 2 k_F x_i
\]

\[
H_V = -\int_{-\infty}^{\infty} \frac{dx}{\sqrt{\pi}} eV(x) \partial_x \Phi(x, t)
\]

**WE CALCULATE PERTURBATIVELY**

- The non-symmetrized noise:
  \[ S_{nm}(\omega) = \int dt e^{i\alpha} \langle \delta m(0) \delta n(t) \rangle \]
  where \( n,m = 1,2,3 \)

- The AC conductance:
  \[ G_{nm}(t-t') = \left. \frac{\partial \delta n(t)}{\partial V_m(t')} \right|_{V_m=0} \]

**RESULT**

\[ S_{nm}(\omega) = S_{nm}^+(\omega) - \frac{\hbar}{i} \omega \text{Re}[G_{nm}(\omega)] \]

with \[ S_{nm}^+(\omega) = \frac{1}{2} \int dt e^{i\alpha} \langle \delta m(0) \delta n(t) + \delta m(t) \delta n(0) \rangle \]

**Symmetrized noise**

**Generalized Kubo-type formula**

\[ S_{nm}(\omega) = S_{nm}(\omega) - S_{nm}(-\omega) = -2 \hbar \omega \text{Re}[G_{nm}(\omega)] \]

Safi and Sukhorukov (unpublished)
AC CONDUCTANCE

WEAK-BACKSCATTERING LIMIT OF THE EXCESS AC CONDUCTANCE

\[ \Delta G_{11}(\omega) = G_{11}(\omega) - G_{11}(\omega) \] _\big|_ \omega = 0 \]

where \( V = V_2 - V_1 \)  

source-drain voltage

FOR \( g=1 \): \[ \Delta G_{11}(\omega) = 0 \] because of the linearity of the I-V characteristic

FOR \( g \neq 1 \): oscillations with frequency with a pseudo-period related to the wire frequency \( \omega_L = v_F / gL \)

The pseudo-period depends on \( L \) and \( g \):

\[ \Omega_g = \frac{hv_F}{gL\lambda} \]
ZERO-FREQUENCY NOISE

IN THE WEAK-BACKSCATTERING LIMIT

\[ S_{nm}(\omega = 0) = eI_B \coth\left( \frac{eV}{2k_B T} \right) + 2k_BT \left[ \frac{e^2}{\hbar} - 2 \frac{\partial I_B}{\partial V} \right] \]

where \( I_B \) is the backscattering current

- **REGION A:** short-wire limit \( eV < \hbar \omega_L \)
  \( \Rightarrow \) linear variation with voltage
  \( \Rightarrow \) qualitative agreement with experiments on carbon nanotubes

  \textit{WU et al., PRL 99, 156803 (2007)}
  \textit{HERRMANN et al., PRL 99, 156804 (2007)}

- **REGION B:** long-wire limit \( eV > \hbar \omega_L \)
  \( \Rightarrow \) oscillations whose envelope has a power-law dependence

- **REGION C:** high temperature limit \( k_B T > \hbar \omega_L \)
  \( \Rightarrow \) behaves like the noise of an infinite length interacting wire: power-law variation
FINITE-FREQUENCY NON-SYMMETRIZED NOISE

WE CALCULATE \[ \Delta S_{nm}(\omega) = S_{nm}(\omega) - S_{nm}(\omega) \big|_{V=0} \]

FOR \( g=1 \): the non-symmetrized excess noise is symmetric

\[ S_{nm}(\omega) = S_{nm}^+(\omega) - \hbar \omega \text{Re}[G_{nm}(\omega)] \Rightarrow \Delta S_{nm}(\omega) = \Delta S_{nm}^+(\omega) \] because \( \Delta G_{11}(\omega) = 0 \) when \( g = 1 \)

FOR \( g \neq 1 \): the non-symmetrized excess noise becomes asymmetric

SHORT-WIRE LIMIT \( g \hbar \omega_L / eV = 1 \)

\[ \Delta S_{11}(\omega) / eB \]

\[ x_i = 0 \]
\[ T = 0 \]
\[ \lambda / eV = 0.01 \]
FINITE-FREQUENCY NON-SYMMETRIZED NOISE

LONG-WIRE LIMIT

\( g = 0.25 \)
\( x_i = 0 \)
\( T = 0 \)
\( \lambda / eV = 0.01 \)
\( g \hbar \omega_L / eV = 0.01 \)

CARBON NANOTUBE

\[ \Delta S_{11}(\omega) / eB \]
\[ \text{infinite length wire limit} \]

\[ \Delta S_{11}(\omega) / eB \]
\[ \text{infinite length nanotube limit} \]

four channels of conduction

\{ charge sector with \( g < 1 \)
3 others sectors with \( g = 1 \) \}
WE CALCULATE THE AVERAGE OVER THE FIRST HALF PERIOD

\[
\langle F \rangle_{\pi \omega_L} = \frac{\langle \Delta S_{11}(\omega) \rangle_{\pi \omega_L}}{e I_B}
\]

where

\[
\langle \Delta S_{11}(\omega) \rangle_{\pi \omega_L} = \frac{1}{\pi \omega_L} \int_0^{\pi \omega_L} \Delta S_{11}(\omega) d\omega
\]

⇒ we obtain \( \langle F \rangle_{\pi \omega_L} \approx g \)

⇒ it should be possible to extract the value of the interaction parameter \( g \)
Simple relation between the AC conductance the non-symmetrized noise

\[ S_{nm}(\omega) = S_{nm}^+(\omega) - \hbar \omega \text{Re}[G_{nm}(\omega)] \]

In the presence of Coulomb interactions, the non-symmetrized noise is asymmetric:

Emission noise (\( \omega > 0 \)) ≠ Absorption noise (\( \omega < 0 \))

At low-temperature and for a long wire or a long nanotube, we obtain oscillations with a period related to \( L \) and \( g \)

The average non-symmetrized excess noise over the first half period gives the value of \( g \)

\[ \langle \Delta S_{11}(\omega) \rangle_{\pi \omega_L} \approx g \]

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