Lecture on

LUTTINGER LIQUID THEORY

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Contents

1 Introduction

Electrons in one-dimensional systems form a quantum liquid which can not be described with the Fermi-liquid theory, for the reason that there are no single electron quasi-particles: the quasi-particles δ-function peak in the spectral function disappears and is replaced by a power-law divergence. It is due to the presence of the Coulomb electron-electron interactions. To describe the dynamic of electrons in one-dimensional systems, we must use Luttinger liquid theory for which the low energy excitations are collective (density waves), rather than single-particle like. As a consequence, one-dimensional systems have specific properties, in particular:

- absence of quasi-particles carrying electron quantum number in the vicinity of Fermi surface ;
- importance of low energy collective excitations ;
- spin-charge separation ;
- power-law dependence of the density of states, electrical current and conductance ;
- possibility to evaluate exactly all correlation functions.

In condensed matter systems, there are several systems which behave as a Luttinger liquid:

- semiconductor wire where spin-charge separation has been experimentally observed[1].
- quasi-1D system: organic crystals with very large anisotropies have a conduction along one crystal direction which is much larger that along the others. Optical response of such systems can be described by power laws that are characteristic of Luttinger liquid[2].
- edge states in the fractional quantum Hall effect.
- metallic wire and carbon nanotube.

The aim of this lecture is to present the Luttinger liquid theory (section 2) and to applied it first, in the fractional quantum Hall regime (section 3) and second, for cabon nanotubes (section 4).

2 Luttinger liquid theory

2.1 Non-interacting Hamiltonian

We consider a one-dimensional metallic wire with parabolic dispersion band as shown on Fig. 1. In this section, we do not include spin degree of freedom. Thus, the kinetic part of the Hamiltonian reads:

$$H_0 = \sum_k E(k) a^\dagger(k) a(k) = \sum_k \frac{\hbar^2 k^2}{2m} a^\dagger(k) a(k) \ , \tag{1}$$

where $a^\dagger(k)$ is the creation operator and $a(k)$ is the destruction operator of one electron in a k-state.

The approximation done in the Tomonaga model[3] is to linearize the dispersion relation $E(k)$ in the vicinity of Fermi energy E_F: $E_+(k) = E_F + \hbar v_F(k - k_F)$ for $k \approx k_F$ and $E_-(k) = E_F - \hbar v_F(k + k_F)$ for $k \approx -k_F$. As a consequence, there is a finite energy (or momentum) range where the linear dispersion relations are applicable.

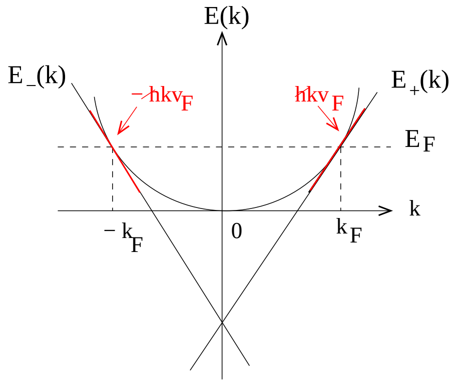

Figure 1: *Parabolic band structure of metallic wire and linearization.*

In the Luttinger model[4], the linearization is extended to all values of k, including negative values. An infinite number of fictive states are added which have no physical signification. However, Luttinger model has an important advantage over Tomonaga model: it is exactly solvable using a bosonization technique as seen below.
After linearization, we have two kinds of electron: the right moving electrons with dispersion relation $E_+(k)$ and the left moving ones with dispersion relation $E_-(k)$. We rewrite the electron creation operator as:

$$a^\dagger(k) = a^\dagger_+(k)\Theta(k) + a^\dagger_-(k)\Theta(-k) \ , \tag{2}$$

where Θ is the Heaviside function and $a^\dagger_r(k)$ is the creation operator for right $(r = +)$ or left $(r = -)$ moving electrons which obeys to the anti-commutation relations:

$$\{a_r(k), a_{r'}(k')\} = 0 \ , \tag{3}$$
$$\{a^\dagger_r(k), a^\dagger_{r'}(k')\} = 0 \ , \tag{4}$$
$$\{a^\dagger_r(k), a_{r'}(k')\} = \delta_{r,r'}\delta_{k,k'} \ . \tag{5}$$

Neglecting the constant terms in energy, we obtain after linearization[5] and extension to all values of k:

$$H_0 = \hbar v_F \sum_k k \left(a^\dagger_+(k)a_+(k) - a^\dagger_-(k)a_-(k) \right) \ , \tag{6}$$

which can be simply written under the form $H_0 = \hbar v_F \sum_{rk} rka^\dagger_r(k)a_r(k)$.
We want now to rewrite this non-interacting Hamiltonian in term of electron density operator $\rho_r(x) = \Psi^\dagger_r(x)\Psi_r(x)$, where the fermionic operators $\Psi^\dagger_r(x)$ and $\Psi_r(x)$ are the Fourier transform of $a^\dagger_r(k)$ and $a_r(k)$:

$$\Psi^\dagger_r(x) = \frac{1}{\sqrt{L}} \sum_k e^{-ikx} a^\dagger_r(k) \ , \tag{7}$$

$$\Psi_r(x) = \frac{1}{\sqrt{L}} \sum_k e^{ikx} a_r(k) \ , \tag{8}$$

where L is the length of the wire. The operators $\Psi_r^\dagger$ and Ψ_r obey to the anti-commutation relation $\{\Psi_r^\dagger(x), \Psi_{r'}(x')\} = \delta_{r,r'}\delta(x-x')$ [**EXERCISE 1**].

The commutation relation between the fermionic operator and the density operator is [**EXERCISE 2**]:

$$[\Psi_r(x), \rho_{r'}(x')] = \delta_{r,r'}\delta(x-x')\Psi_r(x) \ . \tag{9}$$

We will first express the electron density operator $\rho_r(k)$ in term of $a_r(k)$ operator, and next, calculate the commutation relation between H_0 and the density operator $[H_0, \rho_r(k)]$. We have:

$$\begin{aligned} \rho_r(k) &= \frac{1}{\sqrt{L}}\int_{-L/2}^{L/2} dx e^{ikx}\Psi_r^\dagger(x)\Psi_r(x) \\ &= \frac{1}{L\sqrt{L}}\sum_{k',k''}\int_{-L/2}^{L/2} dx e^{i(k-k'+k'')x} a_r^\dagger(k')a_r(k'') \\ &= \frac{1}{\sqrt{L}}\sum_{k'} a_r^\dagger(k'+k)a_r(k') \ , \end{aligned} \tag{10}$$

and, it can be shown that:

$$[\rho_r(k), \rho_{r'}(k')] = -\frac{rk}{2\pi}\delta_{r,r'}\delta_{k,-k'} \ . \tag{11}$$

The commutator we have to evaluate is:

$$[H_0, \rho_r(k)] = \frac{\hbar v_F}{\sqrt{L}}\sum_{r'k'k''} r'k'[a_{r'}^\dagger(k')a_{r'}(k'), a_r^\dagger(k''+k)a_r(k'')] \ . \tag{12}$$

After some lines of calculation, and with the help of Eqs. (3,4,5), we obtain [**EXERCISE 3**]:

$$[H_0, \rho_r(k)] = \hbar v_F rk\rho_r(k) \ . \tag{13}$$

From this result, we conclude that the operator $\rho_r(k)$ put the system in an eigenstate of H_0 with eigenvalue equals to $\hbar v_F rk$. As a consequence, we have[5]:

$$H_0 = \pi\hbar v_F\sum_r\int_{-L/2}^{L/2} dx\rho_r^2(x) \ . \tag{14}$$

From this expression, it is possible to check Eq. (13) with the help of Eq. (11) [**EXERCISE 4**].

2.2 Bosonization

We define the non-chiral bosonic field:

$$\phi_r(x) = \frac{i}{\sqrt{L}}\sum_{(rk)>0}\frac{2\pi}{k}(\rho_r(k)e^{-ikx} - \rho_r(-k)e^{ikx})e^{-a|k|/2} \ . \tag{15}$$

where $a \to 0$ is a distance cutoff which is introduced in every Luttinger liquid theory to insure the convergence of the integrals. This convergence cutoff has been introduced due to the negative energy states that have been added in the model. With the help of Eq. (11), we calculate the commutation relations between the density operator ρ_r and the bosonic field ϕ_r:

$$[\phi_r(x), \rho_r(k)] = -\frac{ir}{\sqrt{L}} e^{ikx} \ . \tag{16}$$

In real space, it leads to:

$$[\phi_r(x), \rho_r(x')] = -ir\delta(x - x') \ , \tag{17}$$

which allows to conclude that $r\phi_r$ and $-\rho_r$ are canonical conjugate fields. When two fields q and p are canonical conjugate: $[q(x), p(x')] = i\delta(x - x')$, we have the identity: $[p(x), e^{iq(x')}] = \delta(x-x')e^{iq(x)}$. By using this identity and comparing Eqs. (9) and (17), we deduce the bosonized form of the fermionic operator Ψ_r:

$$\Psi_r(x) = \frac{F_r}{\sqrt{2\pi a}} e^{irk_F x + ir\phi_r(x)} \ . \tag{18}$$

The factor $F_r/\sqrt{2\pi a}$ allows to obtain the adequate anticommutation relations, and the k_F term in the exponential is needed to described correctly the energy band.
Taking the derivative of Eq. (15), we show that:

$$\partial_x \phi_r(x) = 2\pi\rho_r(x) \ . \tag{19}$$

Thus, the kinetic part of the Hamiltonian given by Eq. (14) reduces to:

$$H_0 = \frac{\hbar v_F}{4\pi} \sum_r \int_{-L/2}^{L/2} dx \left(\partial_x \phi_r(x)\right)^2 \ . \tag{20}$$

We define the total fields:

$$\phi(x) = \phi_+(x) + \phi_-(x) \tag{21}$$

$$\theta(x) = \phi_+(x) - \phi_-(x) \tag{22}$$

Thus,

$$H_0 = \frac{\hbar v_F}{8\pi} \int_{-L/2}^{L/2} dx \left((\partial_x \phi(x))^2 + (\partial_x \theta(x))^2 \right) \ . \tag{23}$$

2.3 Interacting Hamiltonian

We shall now include the Coulomb interactions between the electrons. The total Hamiltonian becomes $H = H_0 + H_{int}$ with:

$$H_{\text{int}} = \int_{-L/2}^{L/2} dx \int_{-L/2}^{L/2} dx' \rho(x) U(x - x') \rho(x') \ , \tag{24}$$

where $\rho(x) = \rho_+(x) + \rho_-(x)$ is the total electron density operator and U is the Coulomb interactions potential. From Eq. (19), we deduces:

$$\rho(x) = \frac{1}{2\pi}\partial_x(\phi_+(x) + \phi_-(x)) = \frac{1}{2\pi}\partial_x\phi(x) \ . \tag{25}$$

We assume a short range Coulomb potential $U(x - x') = U_0\delta(x - x')$, it leads to:

$$H_{\text{int}} = \frac{U_0}{4\pi^2}\int_{-L/2}^{L/2} dx(\partial_x\phi(x))^2 \ . \tag{26}$$

Finally, the total Hamiltonian reads:

$$H \ = \ \frac{\hbar v}{8\pi}\int_{-L/2}^{L/2} dx \left(\frac{1}{g}(\partial_x\phi(x))^2 + g(\partial_x\theta(x))^2\right) \ , \tag{27}$$

where $g = 1/\sqrt{(1 + 2U_0/\pi\hbar v_F)}$ is the Coulomb interaction parameter and $v = v_F/g$ is the velocity of the collective excitations in the wire. The parameter g depends of Coulomb interactions:

- for $U_0 = 0$: $g = 1$ (non-interacting system) ;
- for $U_0 > 0$: $g < 1$ (repulsive interactions) ;
- for $U_0 < 0$: $g > 1$ (attractive interactions).

Under this quadratic form of Eq. (27), the total Hamiltonian is exactly solvable and its eigenstates are similar to the eigenstates of harmonic oscillator.

3 Application to the FQHE

3.1 Introduction to fractional quantum Hall effect

The fractional quantum Hall effect has been discovered in 1981 by Tsui, Stormer and Gossard[6] in GaAs/AlGaAs junction at high magnetic field: they observed plateaus in the Hall resistivity associated with fractional filling factor $\nu = n_s/n_\phi$ where n_s is the electronic density and n_ϕ is the quantum flux. This effect can not be explained with a single electron theory. It is a many-body effect dues to Coulomb interactions between electrons[7] which propagate along the edges of the sample (see Fig. 2). These edge states can be modeled with the help of chiral Luttinger liquid theory.

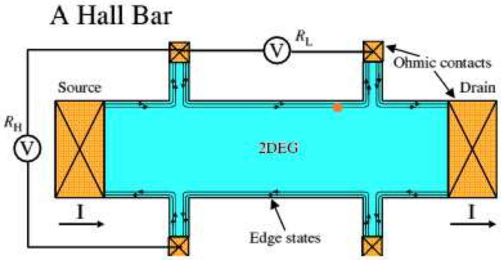

Figure 2: *A quantum Hall bar for which the electrons flow along the edges.*

3.2 Chiral Luttinger liquid

The Hamiltonian which describes one propagating edge mode is simply an electrostatic term[8]:

$$H = \frac{1}{2}\int_0^L V(x)e\rho(x)dx \ , \tag{28}$$

where x a curvilinear coordinate along the edge, L is the length of the edge and $V(x)$ the confining potential related to the applied magnetic field[9]: $V(x) = v_F B\rho(x)/n_s$. Thus, the Hamiltonian can be rewrite as:

$$H = \frac{hv_F}{2\nu}\int_0^L \rho^2(x)dx \ , \tag{29}$$

where $\nu = n_s/n_\phi$ with the quantum flux defines as $n_\phi = Be/h$. We introduce the chiral bosonic field:

$$\phi(x) = \frac{i}{\sqrt{L}}\sum_k \frac{2\pi}{k\sqrt{\nu}}\rho(k)e^{-ikx}e^{-a|k|/2} \ , \tag{30}$$

which leads to $\partial_x\phi(x) = 2\pi\rho(x)/\sqrt{\nu}$. Thus[12]:

$$H = \frac{\hbar v_F}{4\pi}\int_0^L (\partial_x\phi(x))^2 dx \ . \tag{31}$$

The form of the electron operator $\Psi(x)$ is found by an analogy with the properties of canonical conjugate variables p and q:

$$[q(x), p(x')] = i\delta(x-x') \to [p(x), e^{iq(x')}] = \delta(x-x')e^{iq(x)} \ . \tag{32}$$

The density operator $\rho(x) = \Psi^\dagger(x)\Psi(x)$ obeys to the commutation relation:

$$[\Psi(x), \rho(x')] = \delta(x-x')\Psi(x) \ . \tag{33}$$

As we have $[\phi(x), \rho(x')] = i\sqrt{\nu}\delta(x-x')$ because $[\rho(k), \rho(k')] = -\nu k\delta_{k,-k'}/2\pi$, we can identify p as ρ and q as $\phi/\sqrt{\nu}$. Next, comparing Eqs. (33) and (32), we obtain:

$$\Psi(x) = \frac{F}{\sqrt{2\pi a}}e^{i\phi(x)/\sqrt{\nu}} \ . \tag{34}$$

In order to obtain information on the dynamics of fractional quasiparticles, one needs to specify the Green's function. For this, we follow Ref. [9] and first write the Lagrangian which is defined as $\mathcal{L} = -i\int_0^L dxp\partial_\tau q - H$ where $p = \rho = \sqrt{\nu}\partial_x\phi/\pi$ and $q = \phi/\sqrt{\nu}$ are canonical conjugate variables, thus:

$$\mathcal{L} = -\frac{1}{\pi}\int_0^L (\partial_x\phi(x,\tau)(v\partial_x + i\partial_\tau)\phi(x,\tau))\, dx \ , \tag{35}$$

and the action reads:

$$S = \int_0^\beta d\tau\mathcal{L} = -\frac{1}{\pi}\int d\tau\int_0^L (\partial_x\phi(x,\tau)(v\partial_x + i\partial_\tau)\phi(x,\tau))\, dx \ . \tag{36}$$

We perform integrations by parts using periodic conditions, thus:

$$S = \frac{1}{\pi}\int_0^\beta d\tau \int_0^L \phi(x,\tau)\partial_x(v\partial_x + i\partial_\tau)\phi(x,\tau)dx = \int_0^\beta d\tau \int_0^L \phi(x,\tau)G(x,\tau)\phi(x,\tau)dx \ , \quad (37)$$

where $G(x,\tau) = \langle T_\tau \phi(x,\tau)\phi(0,0)\rangle$ is the bosonic Green's function which obeys to the differential equation:

$$(v\partial_x + i\partial_\tau)\partial_x G(x,\tau) = \pi\delta(x)\delta(\tau) \ . \quad (38)$$

The solution of this equation is obtained by setting $-\partial_x G = f$, and using the complex variables $z = x/v + i\tau$. The equation for f becomes:

$$\partial_{\bar{z}} f(z) = v\pi\delta(z) \ . \quad (39)$$

For two dimensional electrostatics, it can be justified that $f(z) = 1/z$. Yet, one is dealing here with the thermal Green's function, which must be periodic function of τ with period β, so the periodic extension of $f(z)$ is given by:

$$f(z+i\beta) = f(z) = \frac{1}{z} + \sum_{n\neq 0}\frac{1}{z - in\beta} = \frac{\pi}{v\beta}\coth\left(\frac{\pi z}{\beta}\right) \ . \quad (40)$$

The thermal Green's function is subsequently obtained by integrating over x:

$$G(x,\tau) = -\ln\left(\sinh\left(\pi\frac{x/v + i\tau}{\beta}\right)\right) \ . \quad (41)$$

3.3 Back-scattering current and noise

We consider a 2D-electron gas in a high perpendicular magnetic field in the fractional quantum Hall regime. When a constriction is added, it acts as an impurity which couples left edge state (L) and right edge state (R) as depicted on Fig. 3. In this section, we calculate the back-scattering current between the two edge states and the noise (fluctuations of back-scattering current) by considering the back-scattering perturbatively.

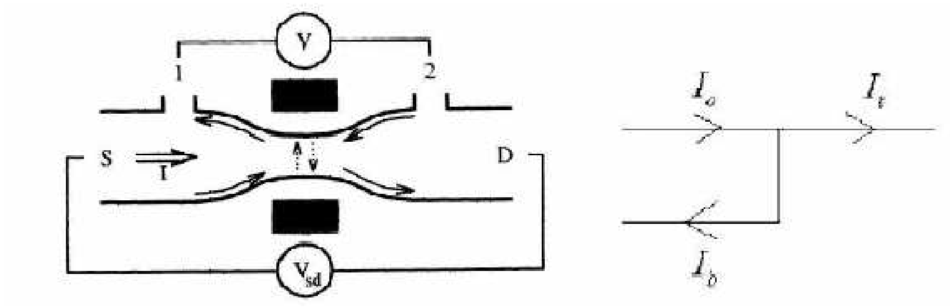

Figure 3: *Constriction in a 2D electron gas and definition of back-scattering current.*

The Hamiltonian which describes this system is $H = H_L + H_R + H_B$ with:

$$H_L = \frac{\hbar v_F}{4\pi}\int_0^L (\partial_x\phi_L(x,t))^2 dx \ , \quad (42)$$

$$H_R = \frac{\hbar v_F}{4\pi}\int_0^L (\partial_x\phi_R(x,t))^2 dx \ , \quad (43)$$

$$H_B = \Gamma(t)\Psi_R^\dagger(t)\Psi_L(t) + \Gamma^*(t)\Psi_L^\dagger(t)\Psi_R(t) \ , \quad (44)$$

where ϕ_R and ϕ_L are the chiral bosonic fields for excitations with charge νe, and:

$$\Psi_L(t) = \frac{F_L}{\sqrt{2\pi a}} e^{i\sqrt{\nu}\phi_L(0,t)} \ , \tag{45}$$

$$\Psi_R(t) = \frac{F_R}{\sqrt{2\pi a}} e^{i\sqrt{\nu}\phi_R(0,t)} \ . \tag{46}$$

$\Gamma(t) = \Gamma_0 \exp(-ie\nu\chi(t)/\hbar c)$ is the tunneling amplitude between the edge states with $\chi(t) = -c\int V(t)$. For $V(t) = V_0$, it leads to $\Gamma(t) = \Gamma_0 \exp(ie\nu V_0 t/\hbar) = \Gamma_0 \exp(i\omega_0 t)$ which has a time dependence due to the voltage which is applied to the constriction. We have introduced the frequency $\omega_0 = e\nu V_0/\hbar$.

The back-scattering current operator can be derived from the Heisenberg equation of motion for the density operator, or alternatively by calculating $I_B = -c\partial H_B/\partial\chi$, then:

$$I_B(t) \ = \ ie^* \left(\Gamma(t)\Psi_R^\dagger(t)\Psi_L(t) - \Gamma^*(t)\Psi_L^\dagger(t)\Psi_R(t)\right) \ , \tag{47}$$

with $e^* = \nu e$ is the fractional charge. The average back-scattering current is expressed using Keldysh contour (see Fig. 4) which allows to treat non-equilibrium situation:

$$\langle I_B(t)\rangle = \frac{1}{2}\sum_\eta \langle T_K\{I_B(t^\eta)e^{-i\int_K dt_1 H_B(t_1)}\}\rangle \ . \tag{48}$$

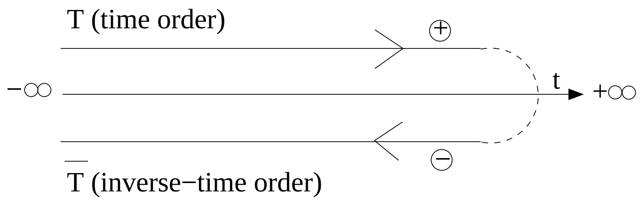

Figure 4: *Keldysh contour: the η index refers to the upper branch ($\eta = +$) or to the lower branch ($\eta = -1$) [10].*

To the lowest order with the back-scattering amplitude $\Gamma(t)$, it gives:

$$\begin{aligned}\langle I_B(t)\rangle &= -\frac{i}{2}\sum_{\eta\eta_1}\eta_1\int_{-\infty}^{+\infty} dt_1 \langle T_K\{I_B(t^\eta)H_B(t_1^{\eta_1})\}\rangle \\ &= \frac{e^*}{2}\sum_{\eta\eta_1\varepsilon\varepsilon_1}\varepsilon\eta_1\int_{-\infty}^{+\infty} dt_1 \Gamma^{(\varepsilon)}(t)\Gamma^{(\varepsilon_1)}(t_1)\langle T_K\{[\Psi_R^\dagger(t^\eta)\Psi_L(t^\eta)]^{(\varepsilon)}[\Psi_R^\dagger(t_1^{\eta_1})\Psi_L(t_1^{\eta_1})]^{(\varepsilon_1)}\}\rangle \ ,\end{aligned} \tag{49}$$

where the superscript ε leaves either the operators in bracket unchanged ($\varepsilon = +$) or transform them into their hermitian conjugate ($\varepsilon = -$). The correlator in Eq. (49) is different from zero only when $\varepsilon_1 = -\varepsilon$. The sum over ε gives:

$$\begin{aligned}\langle I_B(t)\rangle = \frac{e^*}{2}\sum_{\eta\eta_1}\eta_1\int_{-\infty}^{+\infty} dt_1 \quad &\Big(\Gamma(t)\Gamma^*(t_1)\langle T_K\{\Psi_R^\dagger(t^\eta)\Psi_L(t^\eta)\Psi_L^\dagger(t_1^{\eta_1})\Psi_R(t_1^{\eta_1})\}\rangle \\ &- \Gamma^*(t)\Gamma(t_1)\langle T_K\{\Psi_L^\dagger(t^\eta)\Psi_R(t^\eta)\Psi_R^\dagger(t_1^{\eta_1})\Psi_L(t_1^{\eta_1})\}\rangle\Big) \ .\end{aligned} \tag{50}$$

We insert the chiral bosonic fields ϕ_R and ϕ_L (in order to simplify the notations, only time-dependence is indicated):

$$\langle I_B(t)\rangle = \frac{e^* F_R^2 F_L^2}{8\pi^2 a^2}\sum_{\eta\eta_1}\eta_1\int_{-\infty}^{+\infty} dt_1 \Big(\Gamma(t)\Gamma^*(t_1)\langle T_K\{e^{-i\sqrt{\nu}\phi_R(t^\eta)}e^{i\sqrt{\nu}\phi_L(t^\eta)}e^{-i\sqrt{\nu}\phi_L(t_1^{\eta_1})}e^{i\sqrt{\nu}\phi_R(t_1^{\eta_1})}\}\rangle - \Gamma^*(t)\Gamma(t_1)\langle T_K\{e^{-i\sqrt{\nu}\phi_L(t^\eta)}e^{i\sqrt{\nu}\phi_R(t^\eta)}e^{-i\sqrt{\nu}\phi_R(t_1^{\eta_1})}e^{i\sqrt{\nu}\phi_L(t_1^{\eta_1})}\}\rangle\Big) . \tag{51}$$

We use $F_L^2 = F_R^2 = 1$ and, when the condition $\sum_n \varepsilon_n = 0$ applies, the relation:

$$\prod_n e^{i\varepsilon_n\phi_n(t_n)} = e^{-\sum_n\sum_{n'>n}\varepsilon_n\varepsilon_{n'}(\langle T_K\{\phi_n(t_n)\phi_{n'}(t_{n'})\}\rangle - \langle T_K\{\phi_n^2(t_n)\}\rangle/2 - \langle T_K\{\phi_{n'}^2(t_{n'})\}\rangle/2)} . \tag{52}$$

It allows to introduce the chiral Green's function:

$$G^{\eta\eta'}(t-t') = \langle T_K\{\phi_r(t^\eta)\phi_r(t'^{\eta'})\}\rangle - \frac{1}{2}\langle T_K\{\phi_r^2(t^\eta)\}\rangle - \frac{1}{2}\langle T_K\{\phi_r^2(t'^{\eta'})\}\rangle , \tag{53}$$

which does not depend of the chirality $r = R, L$. We get the final expression for the backscattering current:

$$\langle I_B(t)\rangle = \frac{e^*}{8\pi^2 a^2}\sum_{\eta\eta_1}\eta_1\int_{-\infty}^{+\infty} dt_1 e^{2\nu G^{\eta\eta_1}(t-t_1)}\left(\Gamma(t)\Gamma^*(t_1) - \Gamma^*(t)\Gamma(t_1)\right) . \tag{54}$$

We have:

$$\Gamma(t)\Gamma^*(t_1) - \Gamma^*(t)\Gamma(t_1) = 2i\Gamma_0^2\sin(\omega_0(t-t_1)) . \tag{55}$$

Inserting this in Eq. (54), it gives:

$$\langle I_B(t)\rangle = \frac{e^* i\Gamma_0^2}{4\pi^2 a^2}\sum_{\eta\eta_1}\eta_1\int_{-\infty}^{+\infty} d\tau e^{2\nu G^{\eta\eta_1}(\tau)}\sin(\omega_0\tau) . \tag{56}$$

The Keldysh Green's functions are given by:

$$\begin{pmatrix} G^{++}(\tau) & G^{+-}(\tau) \\ G^{-+}(\tau) & G^{--}(\tau) \end{pmatrix} = \begin{pmatrix} g(|\tau|) & g(-\tau) \\ g(\tau) & g(-|\tau|) \end{pmatrix} \tag{57}$$

where at finite temperature, $g(\tau)$ is given by Ref. [9]:

$$g(\tau) = -\ln\left(\frac{\sinh\left(\frac{\pi}{\beta}(-\tau + i\tau_0)\right)}{\sinh\left(\frac{i\pi\tau_0}{\beta}\right)}\right) , \tag{58}$$

where $\tau_0 = a/v_F$ and $\beta = 1/k_B T$. We remark that $G^{\eta-\eta}(\tau) = g(-\eta\tau)$.
Only the terms $\eta_1 = -\eta$ contribute to the integral because, for $\eta_1 = \eta$, the Keldysh Green's functions are even according to the time dependence. Thus:

$$\begin{aligned}\langle I_B(t)\rangle &= -\frac{e^* i\Gamma_0^2}{4\pi^2 a^2}\sum_\eta \eta\int_{-\infty}^{+\infty} d\tau e^{2\nu g(-\eta\tau)}\sin(\omega_0\tau) \\ &= -\frac{e^* i\Gamma_0^2}{4\pi^2 a^2}\sum_\eta \eta\int_{-\infty}^{+\infty} d\tau \left(\frac{\sinh\left(\frac{i\pi\tau_0}{\beta}\right)}{\sinh\left(\frac{\pi}{\beta}(\eta\tau + i\tau_0)\right)}\right)^{2\nu}\sin(\omega_0\tau) .\end{aligned} \tag{59}$$

Using the identity $\sinh(a) = -i\sin(ia)$, we obtain:

$$\langle I_B(t)\rangle = -\frac{e^* i\Gamma_0^2}{4\pi^2 a^2}\sum_\eta \eta \int_{-\infty}^{+\infty} d\tau \left(\frac{\sin\left(\frac{\pi\tau_0}{\beta}\right)}{-\sin\left(\frac{\pi}{\beta}(i\eta\tau-\tau_0)\right)}\right)^{2\nu} \sin(\omega_0\tau) \ . \tag{60}$$

We perform the change of variables: $t = -\tau - i\eta\tau_0 + i\eta\beta/2$ with $dt = -d\tau$, thus:

$$\langle I_B(t)\rangle = \frac{e^* i\Gamma_0^2}{4\pi^2 a^2}\sum_\eta \eta \int_{-\infty-i\eta\tau_0+i\eta\beta/2}^{+\infty-i\eta\tau_0+i\eta\beta/2} dt \sin\left(\omega_0\left(t+i\eta\tau_0-i\eta\frac{\beta}{2}\right)\right)\left(\frac{\sin\left(\frac{\pi\tau_0}{\beta}\right)}{\sin\left(\frac{\pi}{\beta}\left(i\eta t+\frac{\beta}{2}\right)\right)}\right)^{2\nu} . \tag{61}$$

We remark that: $\sin\left(\frac{\pi}{\beta}\left(i\eta t+\frac{\beta}{2}\right)\right) = \cos\left(\frac{\pi}{\beta}i\eta t\right) = \cosh\left(\frac{\pi}{\beta}t\right)$ and we take the limit $\tau_0 \to 0$ in the function $\sin\left((\omega_0+n\omega)\left(t+i\eta\tau_0-i\eta\frac{\beta}{2}\right)\right)$, then:

$$\begin{aligned}\langle I_B(t)\rangle &= \frac{e^* i\Gamma_0^2}{4\pi^2 a^2}\sin^{2\nu}\left(\frac{\pi\tau_0}{\beta}\right)\sum_\eta \eta \\ &\times \int_{-\infty-i\eta\tau_0+i\eta\beta/2}^{+\infty-i\eta\tau_0+i\eta\beta/2} dt \frac{\sin(\omega_0 t)\cos(i\eta\omega_0\beta/2) - \cos(\omega_0 t)\sin(i\eta\omega_0\beta/2)}{\cosh^{2\nu}\left(\frac{\pi t}{\beta}\right)} \ .\end{aligned} \tag{62}$$

As the integration contour in the complex plane does not contain any poles of the function $1/\cosh^{2\nu}\left(\frac{\pi t}{\beta}\right)$, we can remove the imaginary part $-i\eta\tau_0 + i\eta\beta/2$ in the integral boundaries. The only non-vanishing contribution comes from the $\cos(\omega_0 t)$ factor:

$$\langle I_B(t)\rangle = \frac{e^*\Gamma_0^2}{2\pi^2 a^2}\left(\frac{\pi\tau_0}{\beta}\right)^{2\nu}\sinh\left(\frac{\omega_0\beta}{2}\right)\int_{-\infty}^{+\infty} dt \frac{\cos(\omega_0 t)}{\cosh^{2\nu}\left(\frac{\pi t}{\beta}\right)} \ , \tag{63}$$

where we have expanded the function $\sin^{2\nu}\left(\frac{\pi\tau_0}{\beta}\right)$ in the limit $\tau_0 \to 0$. For $\mathrm{Re}[b] > 0$ and $\mathrm{Re}[\mu] > 0$, we have:

$$\int_{-\infty}^{+\infty} dt \frac{\cos(at)}{\cosh^\mu(bt)} = \frac{2^{\mu-1}}{b\mathbf{\Gamma}(\mu)}\left|\mathbf{\Gamma}\left(\frac{\mu}{2}+i\frac{a}{2b}\right)\right|^2 \ , \tag{64}$$

where $\mathbf{\Gamma}$ is the Gamma function. With the help of this integral, we obtain the final result:

$$\langle I_B(t)\rangle = \frac{e^*\Gamma_0^2}{2\pi^2 a^2\mathbf{\Gamma}(2\nu)}\left(\frac{a}{v_F}\right)^{2\nu}\left(\frac{2\pi}{\beta}\right)^{2\nu-1}\sinh\left(\frac{\omega_0\beta}{2}\right)\left|\mathbf{\Gamma}\left(\nu+i\frac{\omega_0\beta}{2\pi}\right)\right|^2 \ . \tag{65}$$

Next, we calculate the symmetrized noise:

$$S(t,t') = \frac{1}{2}\sum_\eta \langle T_K\{I_B(t^\eta)I_B(t'^{-\eta})e^{-i\int_K dt_1 H_B(t_1)}\}\rangle \ . \tag{66}$$

To the lowest order with the back-scattering amplitude Γ, we have:

$$\begin{aligned} S(t,t') &= \frac{1}{2}\sum_{\eta}\langle T_K\{I_B(t^\eta)I_B(t'^{-\eta})\}\rangle \\ &= -\frac{(e^*)^2}{2}\sum_{\eta\varepsilon\varepsilon'}\varepsilon\varepsilon'\Gamma^{(\varepsilon)}(t)\Gamma^{(\varepsilon')}(t')\langle T_K\{[\Psi_R^\dagger(t^\eta)\Psi_L(t^\eta)]^{(\varepsilon)}[\Psi_R^\dagger(t'^{-\eta})\Psi_L(t'^{-\eta})]^{(\varepsilon')}\}\rangle \ . \end{aligned} \quad (67)$$

The correlator is different from zero only when $\varepsilon' = -\varepsilon$. The sum over ε gives:

$$S(t,t') = \frac{(e^*)^2}{2}\sum_{\eta} \Big(\Gamma(t)\Gamma^*(t')\langle T_K\{\Psi_R^\dagger(t^\eta)\Psi_L(t^\eta)\Psi_L^\dagger(t'^{-\eta})\Psi_R(t'^{-\eta})\}\rangle + \Gamma^*(t)\Gamma(t')\langle T_K\{\Psi_L^\dagger(t^\eta)\Psi_R(t^\eta)\Psi_R^\dagger(t'^{-\eta})\Psi_L(t'^{-\eta})\}\rangle\Big) \ . \quad (68)$$

Such correlators have already been calculated, thus:

$$S(t,t') = \frac{(e^*)^2}{8\pi^2a^2}\sum_{\eta} e^{2\nu G^{\eta-\eta}(t-t')}\left(\Gamma(t)\Gamma^*(t')+\Gamma^*(t)\Gamma(t')\right) \ . \quad (69)$$

We have:

$$\Gamma(t)\Gamma^*(t')+\Gamma^*(t)\Gamma(t') = 2\Gamma_0^2\cos(\omega_0(t-t')) \ . \quad (70)$$

Then,

$$S(t-t') = \frac{(e^*)^2\Gamma_0^2}{4\pi^2a^2}\sum_{\eta} e^{2\nu G^{\eta-\eta}(t-t')}\cos(\omega_0(t-t')) \ . \quad (71)$$

The Fourier transform $S(\Omega) = \int \tau e^{i\Omega\tau}S(\tau)$ is:

$$S(\Omega) = \frac{(e^*)^2\Gamma_0^2}{4\pi^2a^2}\sum_{\eta}\int d\tau e^{i\Omega\tau}e^{2\nu G^{\eta-\eta}(\tau)}\cos\left(\omega_0\tau\right) \ . \quad (72)$$

The consider only zero-frequency noise:

$$S(\Omega=0) = \frac{(e^*)^2\Gamma_0^2}{4\pi^2a^2}\sum_{\eta}\int_{-\infty}^{+\infty} d\tau e^{2\nu g(-\eta\tau)}\cos\left(\omega_0\tau\right) \ . \quad (73)$$

We report Eq. (58), use the identity $\sinh(a) = -i\sin(ia)$ and perform the change of variables: $t = -\tau - i\eta\tau_0 + i\eta\beta/2$ with $dt = -d\tau$, then we obtain:

$$S(\Omega=0) = \frac{(e^*)^2\Gamma_0^2}{4\pi^2a^2}\sum_{\eta}\int_{-\infty-i\eta\tau_0+i\eta\beta/2}^{+\infty-i\eta\tau_0+i\eta\beta/2} dt\cos\left(\omega_0\left(t+i\eta\tau_0-i\eta\frac{\beta}{2}\right)\right)\left(\frac{\sin\left(\frac{\pi\tau_0}{\beta}\right)}{\sin\left(\frac{\pi}{\beta}\left(i\eta t+\frac{\beta}{2}\right)\right)}\right)^{2\nu} . (74)$$

We remark that: $\sin\left(\frac{\pi}{\beta}\left(i\eta t+\frac{\beta}{2}\right)\right) = \cos\left(\frac{\pi}{\beta}i\eta t\right) = \cosh\left(\frac{\pi}{\beta}t\right)$ and we take the limit $\tau_0 \to 0$ in the function $\cos\left(\omega_0\left(t+i\eta\tau_0-i\eta\frac{\beta}{2}\right)\right)$, then:

$$S(\Omega=0) = \frac{(e^*)^2\Gamma_0^2}{4\pi^2a^2}\sin^{2\nu}\left(\frac{\pi\tau_0}{\beta}\right)\sum_\eta\int_{-\infty-i\eta\tau_0+i\eta\beta/2}^{+\infty-i\eta\tau_0+i\eta\beta/2} dt\frac{\cos(\omega_0 t)\cos(i\eta\omega_0\beta/2)+\sin(\omega_0 t)\sin(i\eta\omega_0\beta/2)}{\cosh^{2\nu}\left(\frac{\pi t}{\beta}\right)} . \quad (75)$$

As the integration contour in the complex plane does not contain any poles of the function $1/\cosh^{2\nu}\left(\frac{\pi t}{\beta}\right)$, we can remove the imaginary part $-i\eta\tau_0+i\eta\beta/2$ in the integral boundaries. The only non-vanishing contribution comes from the $\cos(\omega_0 t)$ factor:

$$S(\Omega=0) = \frac{(e^*)^2\Gamma_0^2}{2\pi^2a^2}\left(\frac{\pi\tau_0}{\beta}\right)^{2\nu}\cosh\left(\frac{\omega_0\beta}{2}\right)\int_{-\infty}^{+\infty} dt\frac{\cos(\omega_0 t)}{\cosh^{2\nu}\left(\frac{\pi t}{\beta}\right)} , \quad (76)$$

where we have expanded the function $\sin^{2\nu}\left(\frac{\pi\tau_0}{\beta}\right)$ in the limit $\tau_0 \to 0$. The summation over η gives a factor 2. Using the integral given by Eq. (64), we obtain the final result:

$$S(\Omega=0) = \frac{(e^*)^2\Gamma_0^2}{2\pi^2a^2\mathbf{\Gamma}(2\nu)}\left(\frac{a}{v_F}\right)^{2\nu}\left(\frac{2\pi}{\beta}\right)^{2\nu-1}\cosh\left(\frac{\omega_0\beta}{2}\right)\left|\mathbf{\Gamma}\left(\nu+i\frac{\omega_0\beta}{2\pi}\right)\right|^2 . \quad (77)$$

In conclusion, we have:

$$S(\Omega=0) = e^*|\langle I_B(t)\rangle|\coth\left(\frac{\omega_0\beta}{2}\right) . \quad (78)$$

In the limit $T=0$, i.e. $\beta \to +\infty$, we simply get: $S(\Omega=0) = e^*|\langle I_B(t)\rangle|$, in full agreement with experiments of Ref. [11] depicted on Fig. 5.

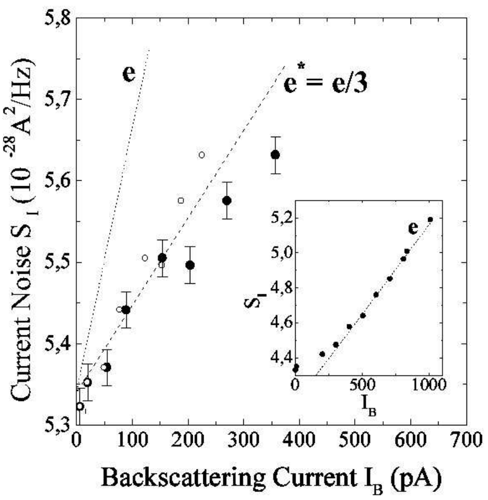

Figure 5: *Measured noise in a quantum Hall constriction as a function of back-scattering current at 25 mK and 45 mK (inset)[11].*

4 Application to metallic carbon nanotubes

In this section, I will start from the linear band structure of a metallic carbon nanotubes, write its Hamiltonian and apply the bosonization procedure in order to obtain the bosonized Hamiltonian in the presence of Coulomb interactions.

4.1 Band structure for metallic carbon nanotube

In Ref. [13], it is shown that metallic carbon nanotube exhibits a linear dispersion curves at low energies as depicted on Fig. 6: the index r represents the direction of propagation ($r = +$ for right and $r = -$ for left), the index α represents the mode and σ the spin [14].

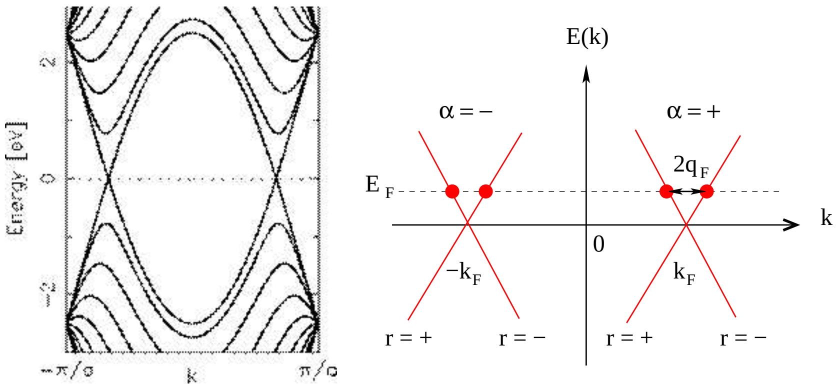

Figure 6: *Band structure of metallic carbon nanotube.*

The dispersion relations for the eight branches ($r = \pm$, $\alpha = \pm$, $\sigma = \pm$) are:

$$E_{r\alpha\sigma}(k) = \hbar v_F r(k - \alpha k_F) \ . \tag{79}$$

The kinetic part of the nanotube Hamiltonian can be written as:

$$\begin{aligned} H_0 &= \sum_{r\alpha\sigma k} E_{r\alpha\sigma}(k) a^\dagger_{r\alpha\sigma}(k) a_{r\alpha\sigma}(k) \\ &= \hbar v_F \sum_{r\alpha\sigma k} r k a^\dagger_{r\alpha\sigma}(k) a_{r\alpha\sigma}(k) \ , \end{aligned} \tag{80}$$

where we have dropped the constant terms. The fermionic operators $a^\dagger_{r\alpha\sigma}(k)$ and $a_{r\alpha\sigma}(k)$ correspond to the creation or annihilation of one electron with wave vector k in the $\{r\alpha\sigma\}$ channel, and obey to the anti-commutation relations:

$$\{a_{r\alpha\sigma}(k), a_{r'\alpha'\sigma'}(k')\} = 0 \ , \tag{81}$$

$$\{a^\dagger_{r\alpha\sigma}(k), a^\dagger_{r'\alpha'\sigma'}(k')\} = 0 \ , \tag{82}$$

$$\{a^\dagger_{r\alpha\sigma}(k), a_{r'\alpha'\sigma'}(k')\} = \delta_{rr'}\delta_{\alpha\alpha'}\delta_{\sigma\sigma'}\delta_{kk'} \ . \tag{83}$$

We introduce the density operator per channel $\rho_{r\alpha\sigma}$:

$$\rho_{r\alpha\sigma}(x) = \Psi^\dagger_{r\alpha\sigma}(x)\Psi_{r\alpha\sigma}(x) \ , \tag{84}$$

where $\Psi^\dagger_{r\alpha\sigma}(x)$ and $\Psi_{r\alpha\sigma}(x)$ are the Fourier transforms of operators $a^\dagger_{r\alpha\sigma}(k)$ and $a_{r\alpha\sigma}(k)$:

$$\Psi^\dagger_{r\alpha\sigma}(k) = \frac{1}{\sqrt{L}}\int_{-L/2}^{L/2} dx e^{-ikx} a^\dagger_{r\alpha\sigma}(x) \ , \tag{85}$$

$$\Psi_{r\alpha\sigma}(k) = \frac{1}{\sqrt{L}}\int_{-L/2}^{L/2} dx e^{ikx} a_{r\alpha\sigma}(x) \ . \tag{86}$$

We have the commutation relation between the fermionic operator $\Psi_{r\alpha\sigma}$ and the channel density operator $\rho_{r\alpha\sigma}$:

$$[\Psi_{r\alpha\sigma}(x), \rho_{r\alpha\sigma}(x')] = \delta_{rr'}\delta_{\alpha\alpha'}\delta_{\sigma\sigma'}\delta(x-x')\Psi_{r\alpha\sigma}(x) \ . \tag{87}$$

The Fourier transform of the density operator is:

$$\begin{aligned}
\rho_{r\alpha\sigma}(k) &= \frac{1}{\sqrt{L}}\int_{-L/2}^{L/2} dx e^{ikx}\Psi^\dagger_{r\alpha\sigma}(x)\Psi_{r\alpha\sigma}(x) \\
&= \frac{1}{L\sqrt{L}}\sum_{k',k''}\int_{-L/2}^{L/2} dx e^{i(k-k'+k'')x} a^\dagger_{r\alpha\sigma}(k')a_{r\alpha\sigma}(k'') \\
&= \frac{1}{\sqrt{L}}\sum_{k'} a^\dagger_{r\alpha\sigma}(k'+k)a_{r\alpha\sigma}(k') \ .
\end{aligned} \tag{88}$$

It can be checked that the commutation relation for the density operator is:

$$[\rho_{r\alpha\sigma}(k), \rho_{r'\alpha'\sigma'}(k')] = -\frac{rk}{2\pi}\delta_{rr'}\delta_{\alpha\alpha'}\delta_{\sigma\sigma'}\delta_{kk'} \ . \tag{89}$$

4.2 Non-interacting Hamiltonian

To resolve this Hamiltonian, we need to transform it in order to obtain a quadratic form: it is the bosonization procedure which is described in this section. First, we calculate the commutation relation between H_0 given by Eq. (80) and the operator $\rho_{r\alpha\sigma}(k)$ given by Eq. (88):

$$[H_0, \rho_{r\alpha\sigma}(k)] = \frac{\hbar v_F}{L}\sum_{r'\alpha'\sigma'k'k''} r'k'[a^\dagger_{r'\alpha'\sigma'}(k')a_{r'\alpha'\sigma'}(k'), a^\dagger_{r\alpha\sigma}(k''+k)a_{r\alpha\sigma}(k'')] \ . \tag{90}$$

After some lines of calculation, and with the help of Eqs. (81,82,83), we obtain:

$$[H_0, \rho_{r\alpha\sigma}(k)] = \hbar v_F rk\rho_{r\alpha\sigma}(k) \ . \tag{91}$$

This result shows that the states generated by $\rho_{r\alpha\sigma}(k)$ are eigenstates of H_0 with eigenvalues equal to $\hbar v_F rk$. The kinetic part of the Hamiltonian can thus be written as[5]:

$$H_0 = \pi\hbar v_F\sum_{r\alpha\sigma}\int_{-L/2}^{L/2} dx\rho^2_{r\alpha\sigma}(x) \ . \tag{92}$$

We define the bosonic field:

$$\varphi_{r\alpha\sigma}(x) = \frac{i}{\sqrt{L}}\sum_{(rk)>0}\frac{2\pi}{k}(\rho_{r\alpha\sigma}(k)e^{-ikx} - \rho_{r\alpha\sigma}(-k)e^{ikx})e^{-a|k|/2} \ . \tag{93}$$

where $a \to 0$ is a distance cutoff which is introduced in every Luttinger liquid theory. With the help of Eq. (89), we calculate the commutation relations between the density operator $\rho_{r\alpha\sigma}$ and the bosonic field $\varphi_{r\alpha\sigma}$:

$$[\varphi_{r\alpha\sigma}(x), \rho_{r\alpha\sigma}(k)] = -\frac{ir}{\sqrt{L}} e^{ikx} \; . \tag{94}$$

In the real space, it leads to:

$$[\varphi_{r\alpha\sigma}(x), \rho_{r\alpha\sigma}(x')] = i\delta(x - x') \; , \tag{95}$$

which allows to conclude that $r\varphi_{r\alpha\sigma}$ and $-\rho_{r\alpha\sigma}$ are canonical conjugate fields. When two fields are conjugate: $[q(x), p(x')] = i\delta(x - x')$, we have the identity: $[p(x), e^{iq(x')}] = \delta(x - x')e^{iq(x)}$. By using this identity and comparing Eq. (87) and Eq. (95), we deduce the bosonized form of the fermionic operator $\Psi_{r\alpha\sigma}$:

$$\Psi_{r\alpha\sigma}(x) = \frac{F_{r\alpha\sigma}}{\sqrt{2\pi a}} e^{i\alpha k_F x + irq_F x + i\varphi_{r\alpha\sigma}(x)} \; . \tag{96}$$

The factor $F_{r\alpha\sigma}/\sqrt{2\pi a}$ allows to obtain the adequate anticommutation relations, and the k_F and q_F terms in the exponential are needed to described correctly the energy bands.
By taking the derivative with x of Eq. (93), we obtain:

$$\partial_x \varphi_{r\alpha\sigma}(x) = 2\pi \rho_{r\alpha\sigma}(x) \; . \tag{97}$$

Thus, the kinetic part of the nanotube Hamiltonian given by Eq. (92) reduces to:

$$H_0 = \frac{\hbar v_F}{4\pi} \sum_{r\alpha\sigma} \int_{-L/2}^{L/2} dx \left(\partial_x \varphi_{r\alpha\sigma}(x)\right)^2 \; . \tag{98}$$

4.3 Interacting Hamiltonian

Since the conduction of electrons is unidimensional in carbon nanotube, Coulomb interactions play an important role. The total Hamiltonian is $H_N = H_0 + H_{\rm int}$ with:

$$H_{\rm int} = \int_{-L/2}^{L/2} dx \int_{-L/2}^{L/2} dx' \rho(x) U(x - x') \rho(x') \; , \tag{99}$$

where U is the Coulomb interactions potential and the total electron density operator is:

$$\rho(x) = \sum_{r\alpha\sigma} \rho_{r\alpha\sigma}(x) \; . \tag{100}$$

We assume a short range Coulomb potential: $U(x - x') = U_0\delta(x - x')$, then:

$$\begin{aligned} H_{\rm int} &= U_0 \int_{-L/2}^{L/2} dx \rho^2(x) = U_0 \int_{-L/2}^{L/2} dx \left(\sum_{r\alpha\sigma} \rho_{r\alpha\sigma}(x) \right)^2 \\ &= \frac{U_0}{4\pi^2} \int_{-L/2}^{L/2} dx \left(\sum_{r\alpha\sigma} \partial_x \varphi_{r\alpha\sigma}(x) \right)^2 \; . \end{aligned} \tag{101}$$

Thus, the total Hamiltonian for the nanotube $H_N = H_0 + H_{\text{int}}$ reads:

$$H_N = \frac{\hbar v_F}{4\pi}\int_{-L/2}^{L/2} dx \sum_{r\alpha\sigma} (\partial_x \varphi_{r\alpha\sigma}(x))^2 + \frac{U_0}{4\pi^2}\int_{-L/2}^{L/2} dx \left(\sum_{r\alpha\sigma} \partial_x \varphi_{r\alpha\sigma}(x)\right)^2 . \quad (102)$$

It is convenient to express the bosonic field $\varphi_{r\alpha\sigma}$ in terms of conventional non-chiral Luttinger liquid fields $\theta_{j\delta}$ and $\phi_{j\delta}$, with $\{j\delta\}$ identifying the charge/spin and total/relative sectors:

$$\varphi_{r\alpha\sigma}(x) = \frac{\sqrt{\pi}}{2}\sum_{j\delta} h_{\alpha\sigma j\delta}\left(\phi_{j\delta}(x) + r\theta_{j\delta}(x)\right) , \quad (103)$$

where $h_{\alpha\sigma c+} = 1$, $h_{\alpha\sigma c-} = \alpha$, $h_{\alpha\sigma s+} = \sigma$ and $h_{\alpha\sigma s-} = \alpha\sigma$.

We have:

$$\sum_{r\alpha\sigma} (\partial_x \varphi_{r\alpha\sigma}(x))^2 = 2\pi \sum_{j\delta}\left((\partial_x \phi_{j\delta}(x))^2 + (\partial_x \theta_{j\delta}(x))^2\right) , \quad (104)$$

$$\left(\sum_{r\alpha\sigma} \partial_x \varphi_{r\alpha\sigma}(x)\right)^2 = 16\pi(\partial_x \phi_{c+}(x))^2 . \quad (105)$$

Thus,

$$H_N = \frac{\hbar}{2}\int_{-L/2}^{L/2} dx \sum_{j\delta}\left(v_{j\delta}K_{j\delta}(\partial_x \phi_{j\delta}(x))^2 + \frac{v_{j\delta}}{K_{j\delta}}(\partial_x \theta_{j\delta}(x))^2\right) , \quad (106)$$

with $v_{j\delta} = v_F/K_{j\delta}$, $K_{c-} = K_{s+} = K_{s-} = 1$ and $K_{c+} = 1/\sqrt{1 + 8U_0/\pi\hbar v_F}$. This Hamiltonian is quadratic with the bosonic fields and as a consequence, it can be exactly solvable.

4.4 Current and noise in the STM + Nanotube system

To calculate electric transport properties, see for example, Refs. [15] and [16] where the STM tip + carbone nanotube system depicted on Fig. 7 is considered. Due to the applied voltage, electrons can tunnel from the tip to the nanotube.

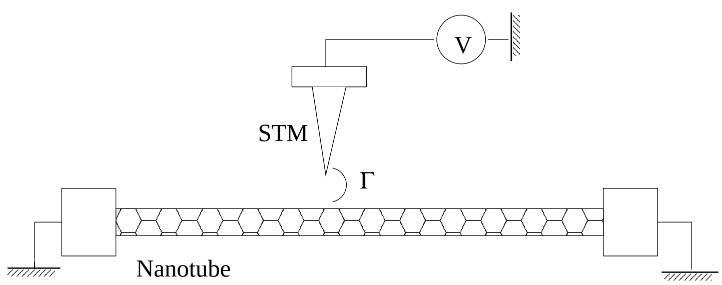

Figure 7: *Schematic configuration of the nanotube-STM device: electrons are injected from the tip to the nanotube at $x = 0$.*

The total Hamiltonian for this system is $H = H_N + H_{STM} + H_T$ where H_N is given by Eq. (106), H_{STM} is the Hamiltonian of the STM tip which is a Fermi liquid and H_T is the tunnel Hamiltonian:

$$H_T(t) = \sum_{r\alpha\sigma\epsilon} \Gamma^{(\epsilon)}(t)\left[\Psi^\dagger_{r\alpha\sigma}(0,t)c(t)\right]^{(\epsilon)} . \tag{107}$$

The average current and noise are defined by:

$$\langle I(x,t)\rangle = \frac{1}{2}\sum_\eta \langle T_K\{\hat{I}(x,t^\eta)e^{-i\int_K dt_1 H_T(t_1)}\}\rangle , \tag{108}$$

$$S(x,x',t,t') = \frac{1}{2}\sum_\eta \left\langle T_{\mathrm{K}}\left\{\hat{I}(x,t^\eta)\hat{I}(x',t'^{-\eta})e^{-i\int_{\mathrm{K}} dt_1 H_{\mathrm{T}}(t_1)}\right\}\right\rangle , \tag{109}$$

where the current operator is $\hat{I}(x,t) = 2ev_{\mathrm{F}}\,\partial_x\phi_{c+}(x,t)/\sqrt{\pi}$. This relation can be derived from the continuity equation.

We do not give here the details of the calculations, only the final results at order Γ^2 in the perturbative calculations at zero temperature:

$$\langle I(x,t)\rangle = \frac{2e\Gamma^2}{\pi a u_F}\left(\frac{a}{v_F}\right)^\nu \frac{|\omega_0|^\nu \mathrm{sgn}(\omega_0)}{\mathbf{\Gamma}(\nu+1)}\mathrm{sgn}(x) , \tag{110}$$

$$S(x,x',\omega=0) = \frac{K_{c+}^2 + \mathrm{sgn}(x)\mathrm{sgn}(x')}{2}e|\langle I(x)\rangle| , \tag{111}$$

where $\omega_0 = eV/\hbar$ and:

$$\nu = \frac{1}{8}\sum_{j\delta}\left(K_{j\delta} + \frac{1}{K_{j\delta}}\right) . \tag{112}$$

The current and noise are power laws with the applied voltage and the noise obeys to a Schottky relation with anomalous charges: $Q^{\pm}_{c+} = e(1 \pm K_{c+})/2$. These anomalous charges are the charges of the excitations which propagate in the nanotube[15].

5 Books

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- *Many-Body Quantum Theory in Condensed Matter Physics: an Introduction*, H. Bruus and K. Flensberg, Oxford University Press 2004 (Chapter 19).
- *Advanced Solid State Physics*, P. Phillips, Westview Press 2003 (Chapter 9).
- *Bosonization and Strongly Correlated Systems*, A.O. Gogolin, A.A. Nersesyan and A.M. Tsvelik, Cambridge University Press 1998 (Chapters 16-20).
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