Josephson effect in carbon nanotubes with spin-orbit coupling.

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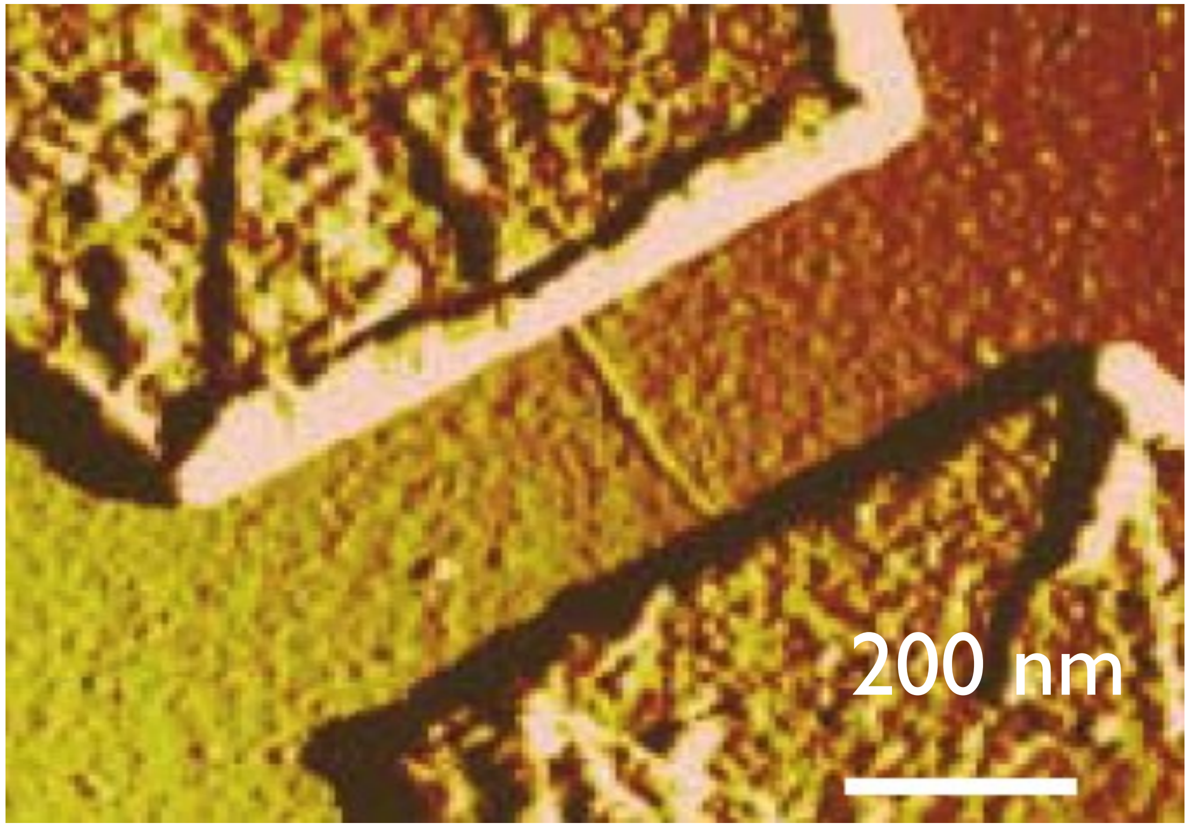
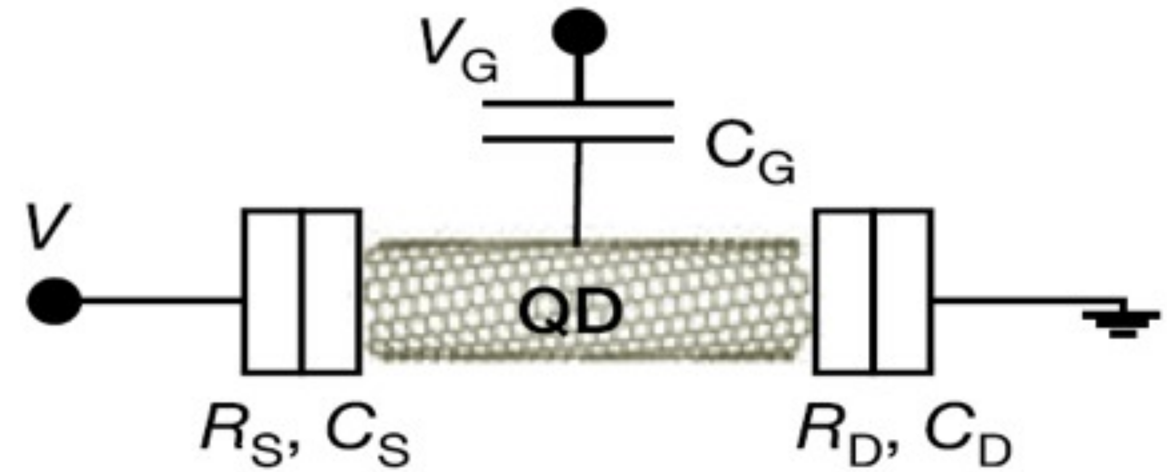
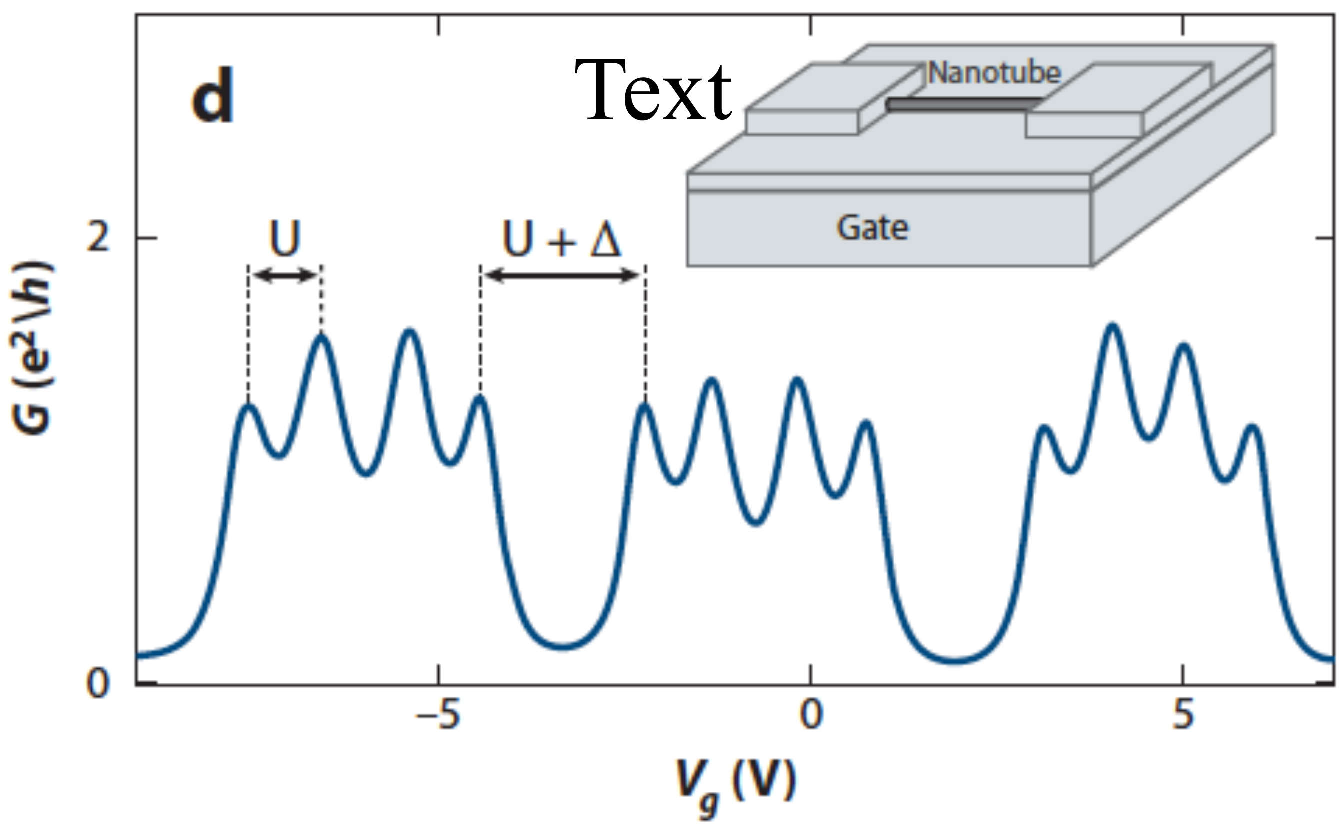
arXiv:1104.0513
Phys. Rev. Lett (2011)
in press





ORBITAL
+
SPIN



W. Liang, M. Bockrath and H. Park, PRL, **88**, 126801 (2002)
M. R. Buitelaar, A. Bachtold, T. Nussbaumer, M. Iqbal and C. Schönenberger, PRL, **88**, 156801 (2002)
Jarillo-Herrero *et al.*, PRL, 94, 156802, (2005)

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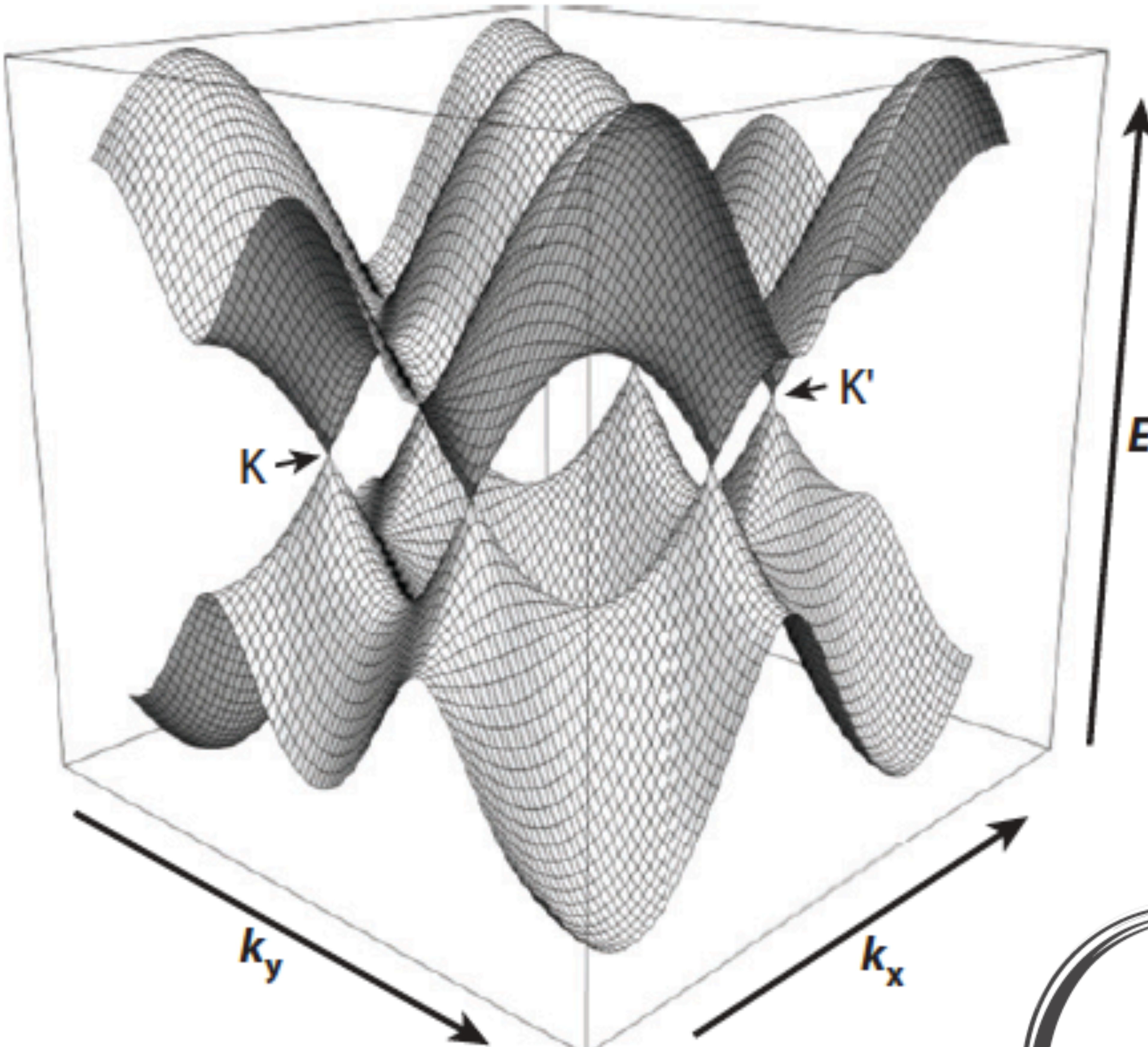
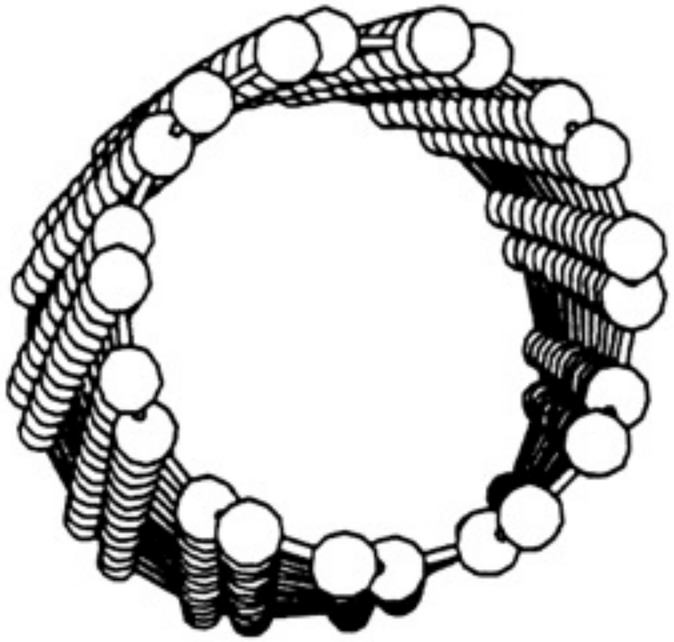

K'
K
E
k_y
k_x

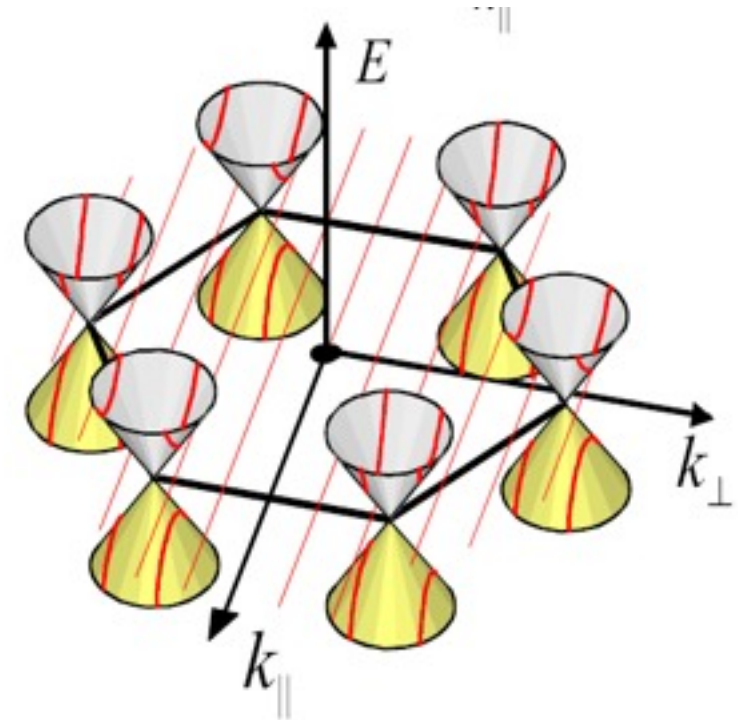
Nanotubes

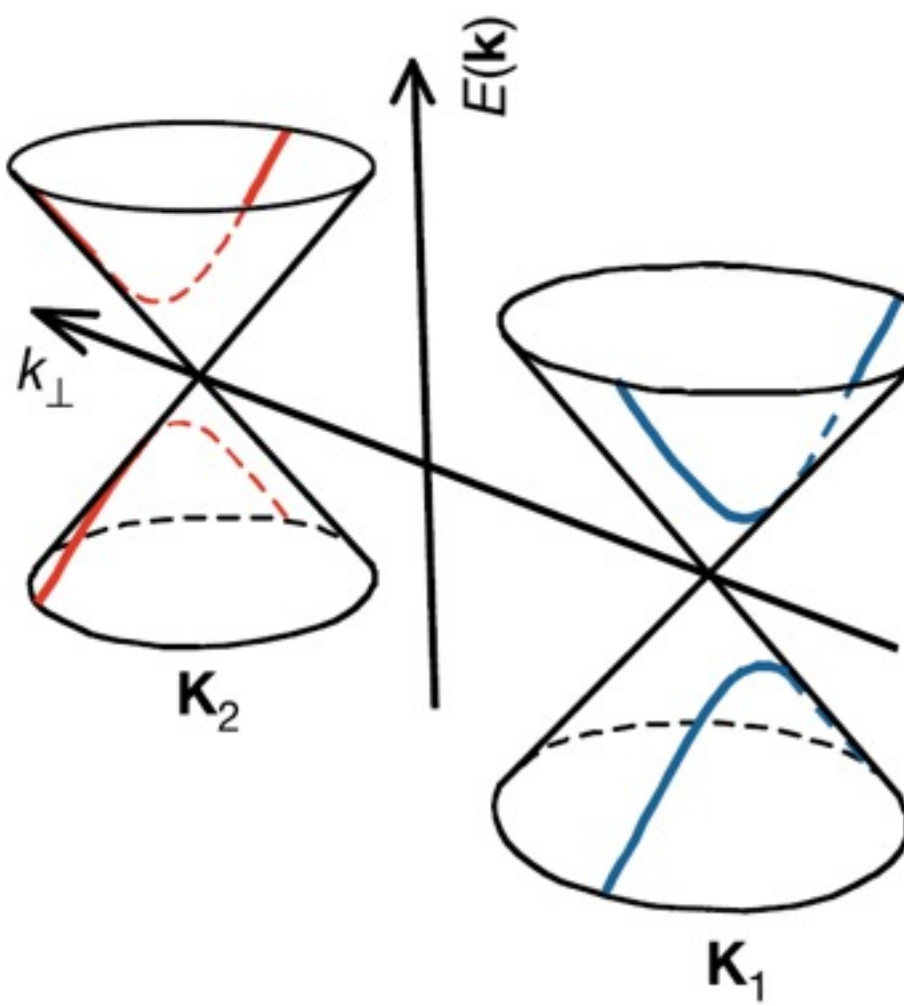


Periodic Boundary Conditions

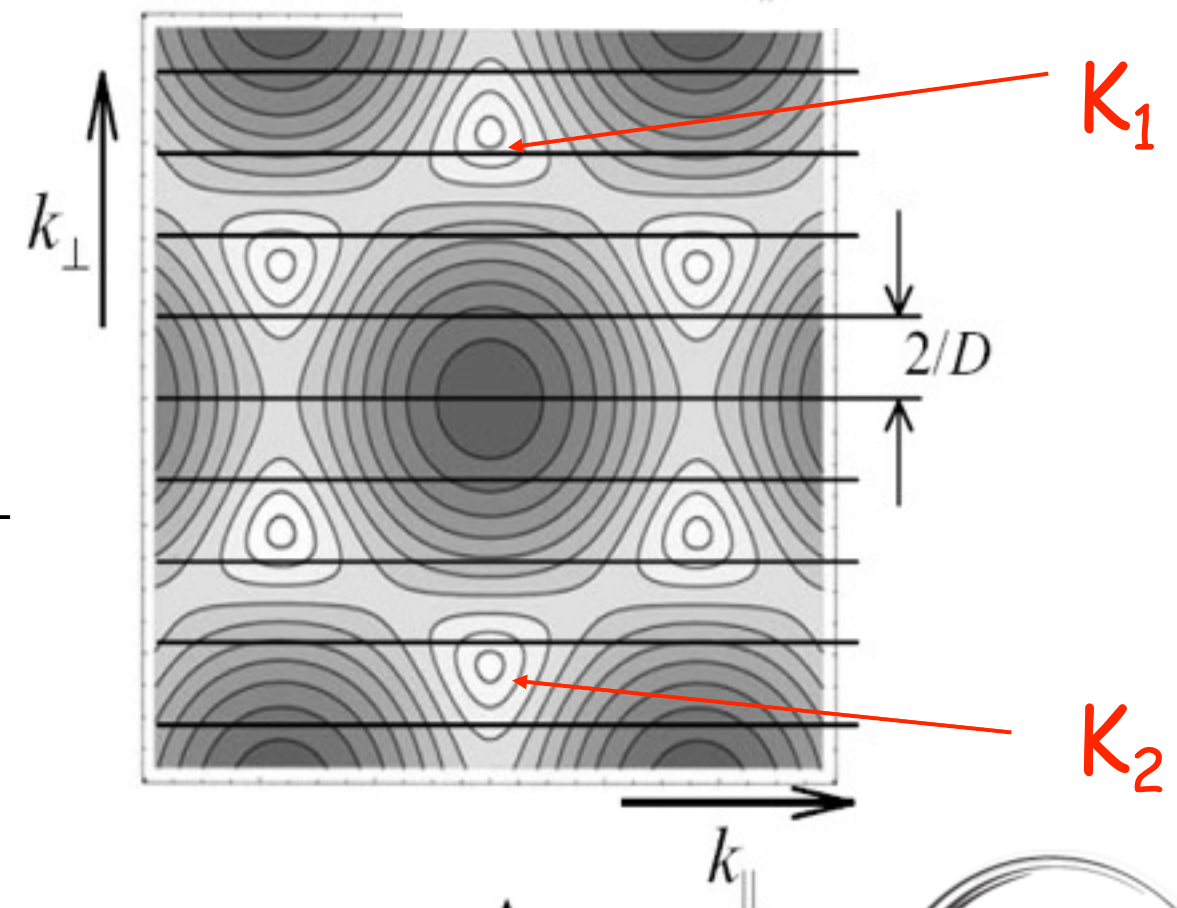
$$\pi D k_{\perp} = 2\pi j$$

$\Rightarrow$ Sub-bands in 1D.



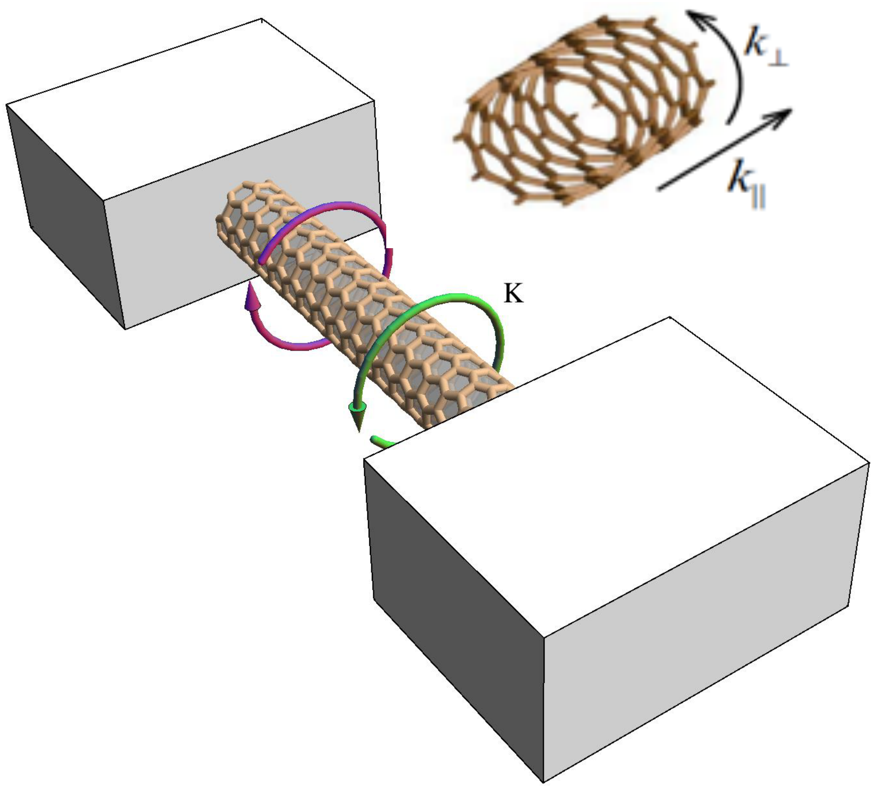


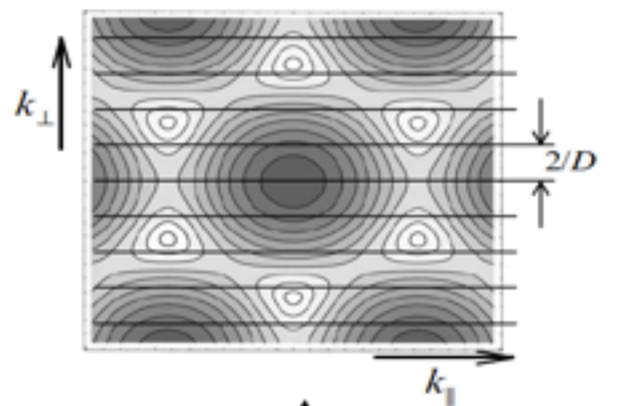
$$k_{\perp} = -k_{\perp}$$

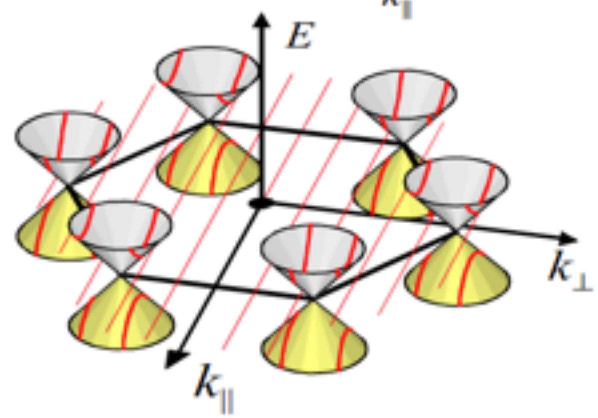


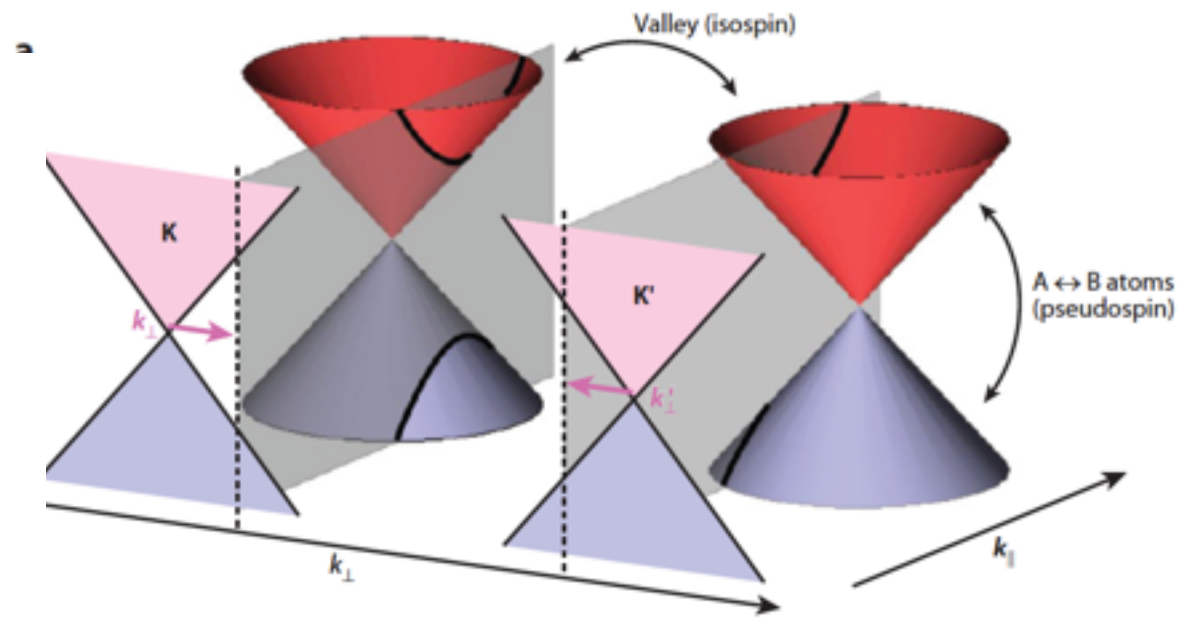
Ethan Minot, “Tuning the band structure of carbon nanotubes”, PhD Thesis, Cornell 2004.

Modelling Quantum Dot Carbon Nanotubes I









Small bandgap nanotubes $k_\perp = \tau_3 k_g$

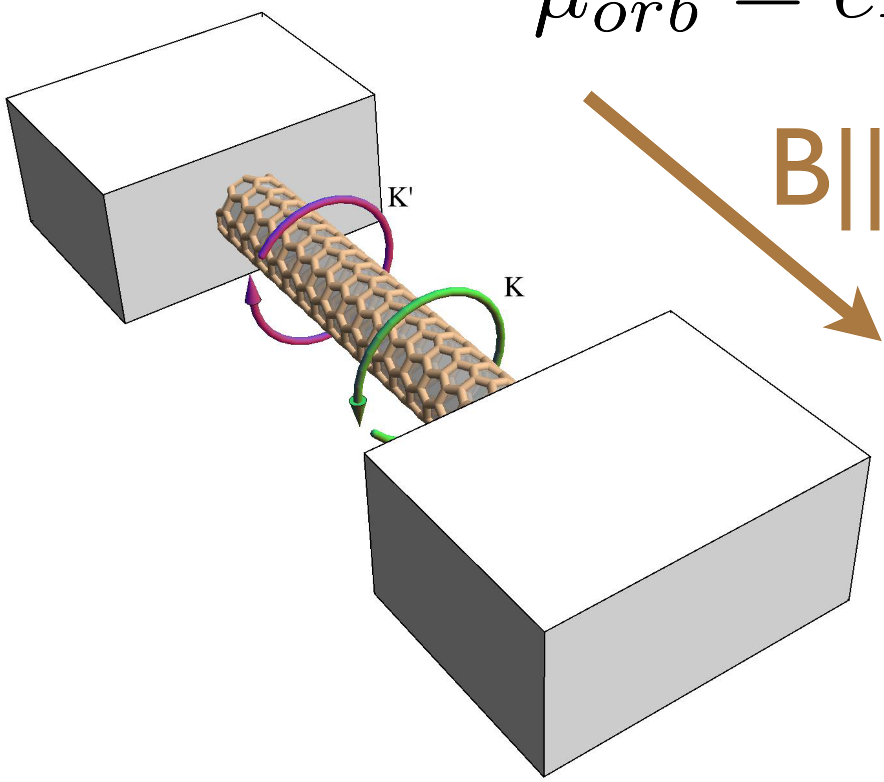
K and K' are degenerate owing to time-reversal symmetry (isospin)

$$\left.\begin{matrix}K\\K'\end{matrix}\right\}\tau_3 = \pm 1 \;\Rightarrow\; H_0 = \hbar v_F (k_\perp \tau_3 \otimes \sigma_1 + k_{||} \tau_0 \otimes \sigma_2)$$

$k_{||}$ also becomes quantized due to the finite length (quantum dot).

Modelling Quantum Dot Carbon Nanotubes II

$$\mu_{orb} = eD\nu_F/4$$

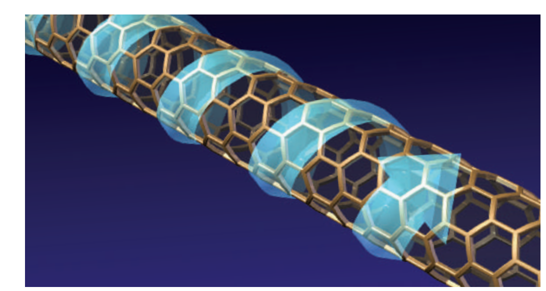


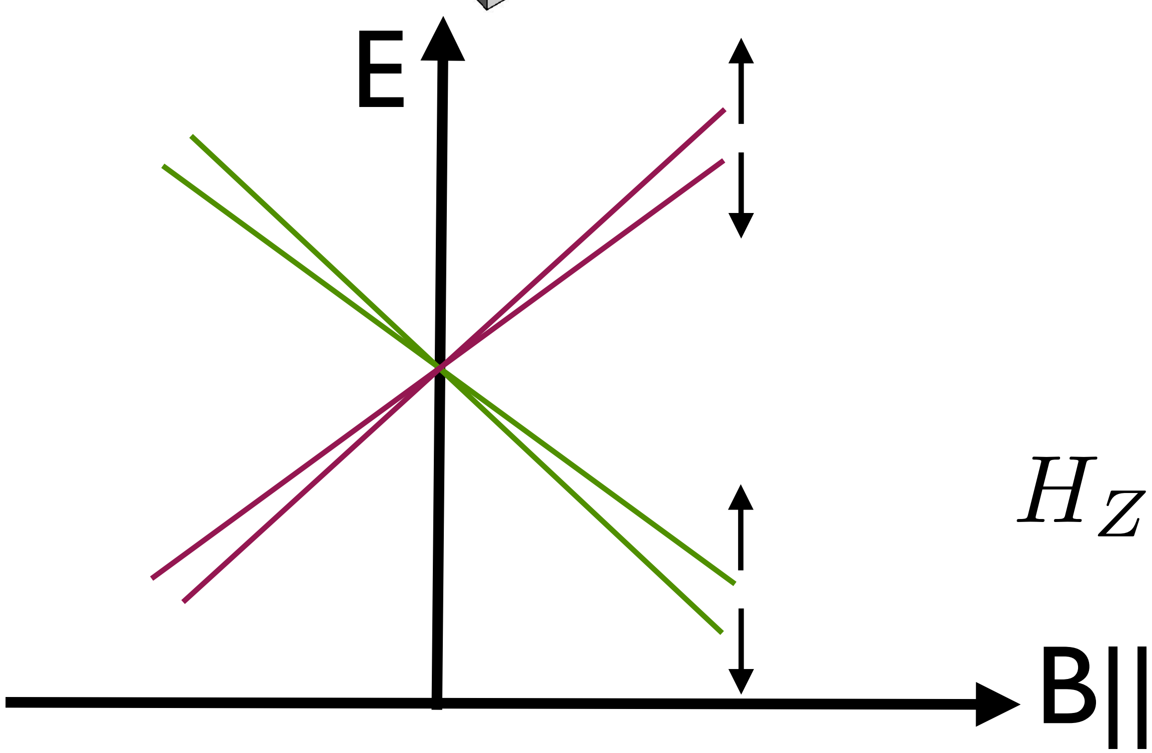
Large orbital moments couple to parallel magnetic fields

$$H_{orb} = \tau_3 \mu_{orb} B_{||} = \tau_3 \hbar \nu_F \pi B_{||} D / 2\Phi_0$$

$$k_{\perp} = \tau_3 k_g + \Phi_{AB}/D\Phi_0$$

$$\Phi_{AB} = B_{||} \pi D^2$$





$$H_Z = \frac{1}{2} g \mu_B B_{||} \tau_0 \otimes \sigma_0 \otimes s_3$$

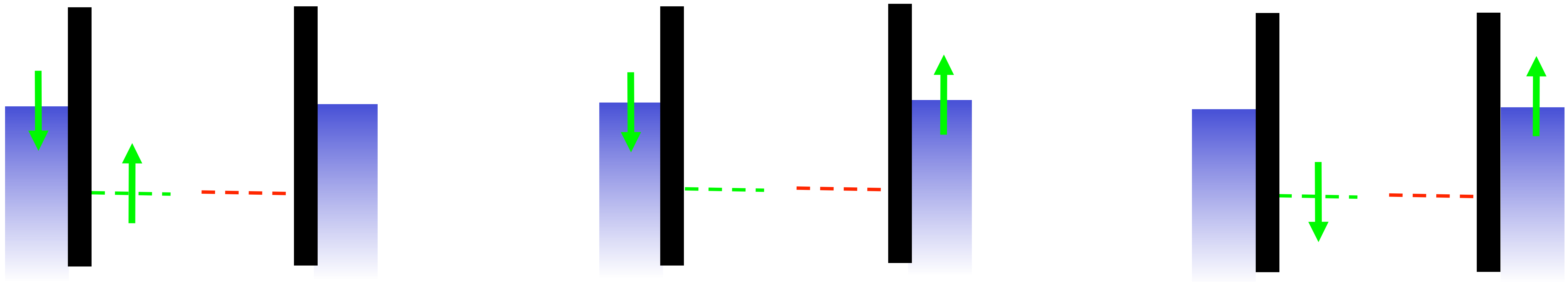
Determination of electron orbital magnetic moments in carbon nanotubes,
E. D. Minot, Yuval Yaish, Vera Sazonova & Paul L. McEuen, Nature, 428, 536 (2004)
See News&Views Nanoscale physics: M. Chiao, Big moment for nanotubes (2004)

SPIN KONDO EFFECT AND ISOSPIN KONDO EFFECT

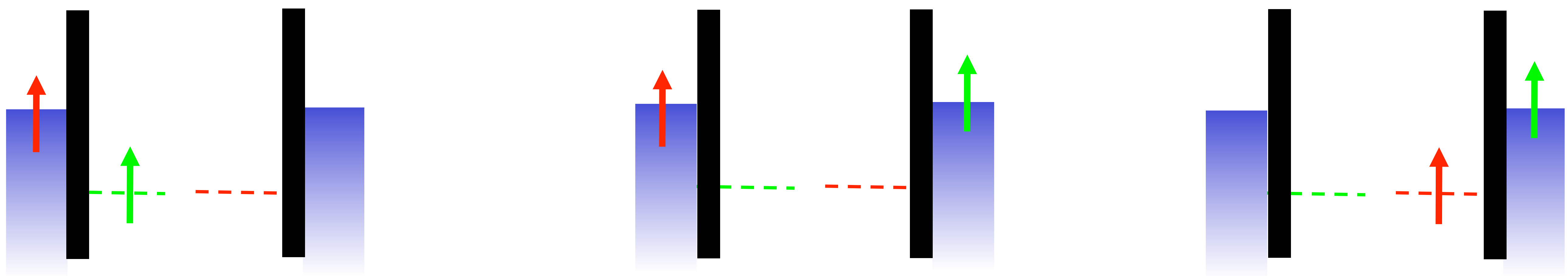
↓

SU(4) SYMMETRY

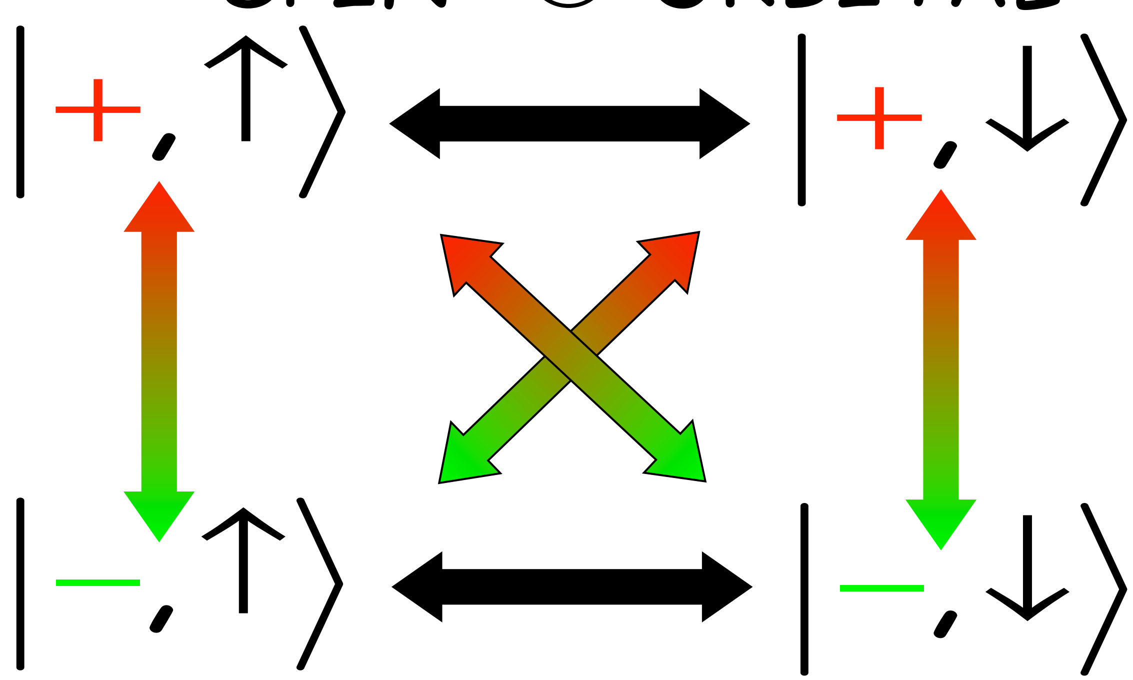
SPIN KONDO



ORBITAL KONDO



SPIN $\oplus$ ORBITAL



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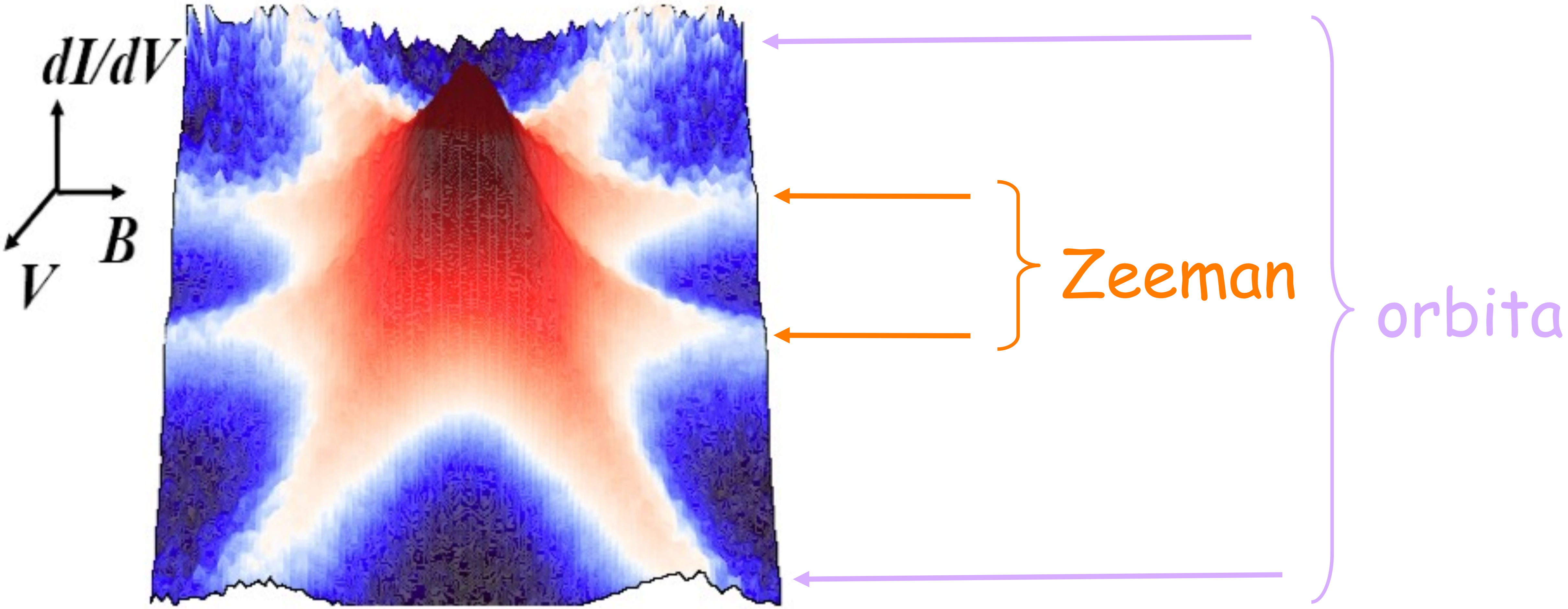
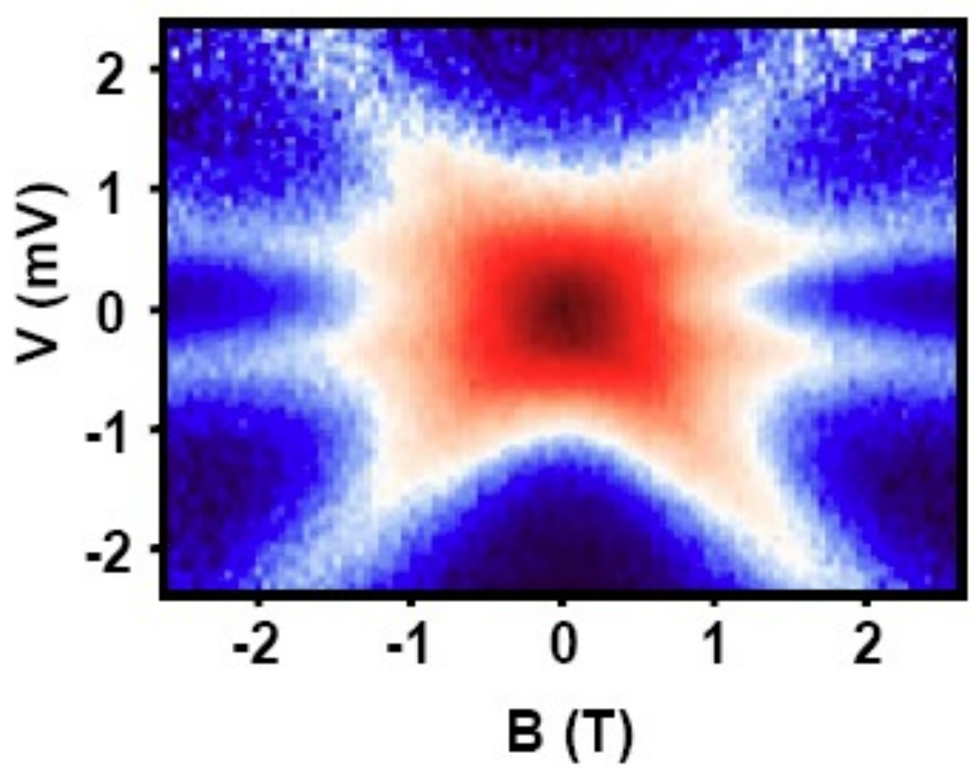
Low temperature transport: evidence of SU(4) symmetry

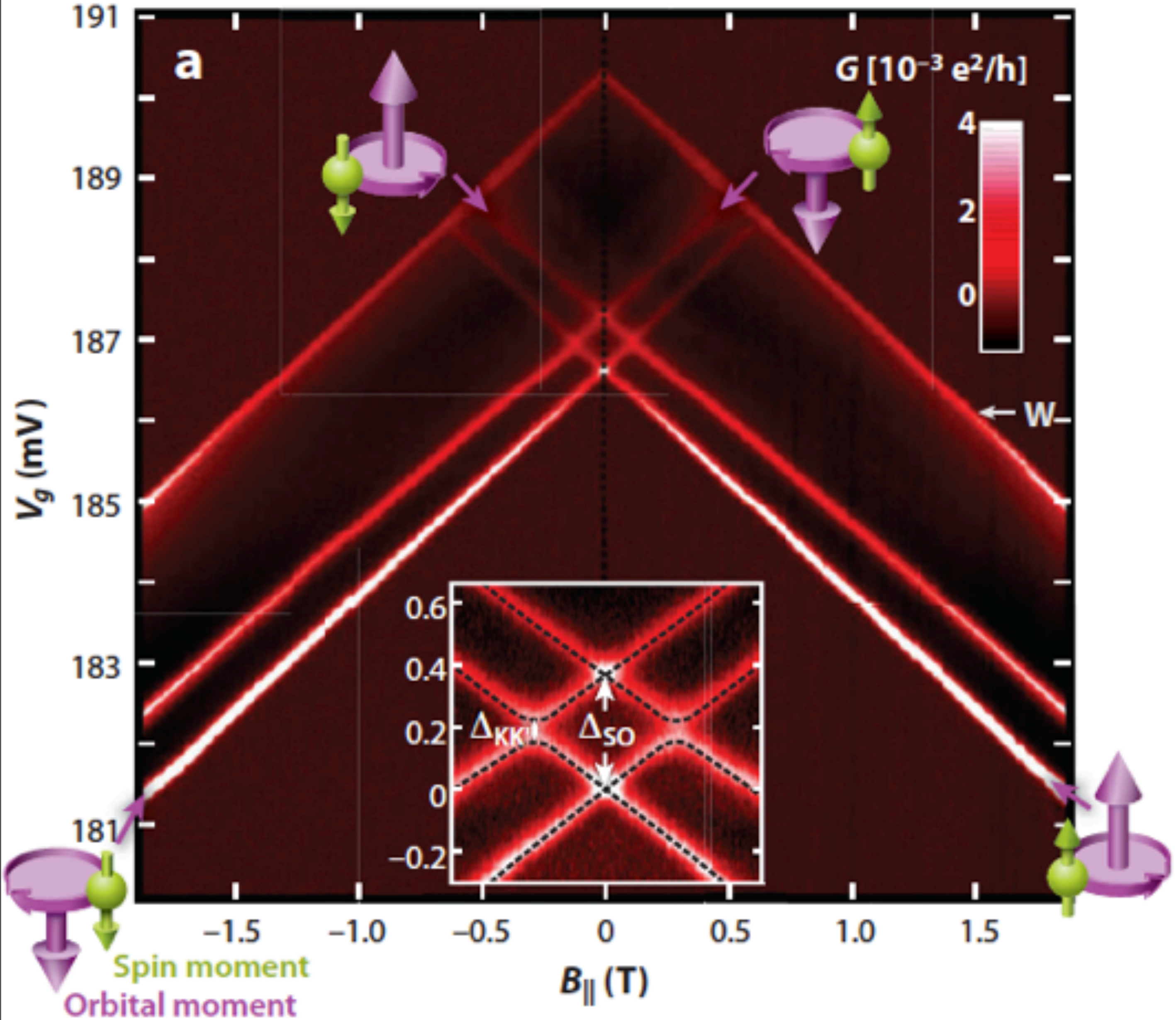




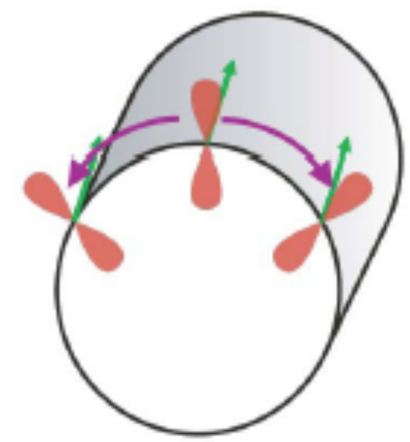
Experiment: "Orbital Kondo effect in Carbon Nanotubes", Pablo Jarillo-Herrero, Jing Kong, Herre S.J. van der Zant, Cees Dekker, Leo P. Kouwenhoven, Silvano De Franceschi, Nature, 434, 484 (2005).

Theory: "SU(4) Kondo effect in carbon nanotubes, Mahn-Soo Choi, Rosa López and Ramón Aguado, PRL, **95**,067204 (2005)

Carbon Nanotube with Spin-orbit coupling I



•The orbital motion of electrons also couples to a curvature-induced radial electric field. This creates an effective axial magnetic field which polarizes the spins along the NT axis and favors parallel alignment of the spin and orbital magnetic momenta or antiparallel depending on the sign of this spin-orbit coupling.



Coupling of spin and orbital motion of electrons in carbon nanotubes, F. Kuemmeth, S. Ilani, D. C. Ralph & P.L.McEuen, Nature, 452, 448, 2008.

$$\vec{B}_{SO} = \vec{v} \times \vec{E}$$

THEORY: T. Ando, J. Phys. Soc. Jpn. 69, 1757 (2000).
D. Huertas-hernando et al, Phys. Rev. B, 74, 155426 (2006).
D.V. Bulaev et al, Phys. Rev. B 77, 235301 (2008)
L. Chico et al, Phys. Rev. B, 79 , 235423 (2009).
J. Jeong and H. Lee, Phys. Rev. B, 80 , 075409 (2009).
W. Izumida, K. Sato and R. Saito, J. Phys. Soc. Jpn., 78 , 074707 (2009).

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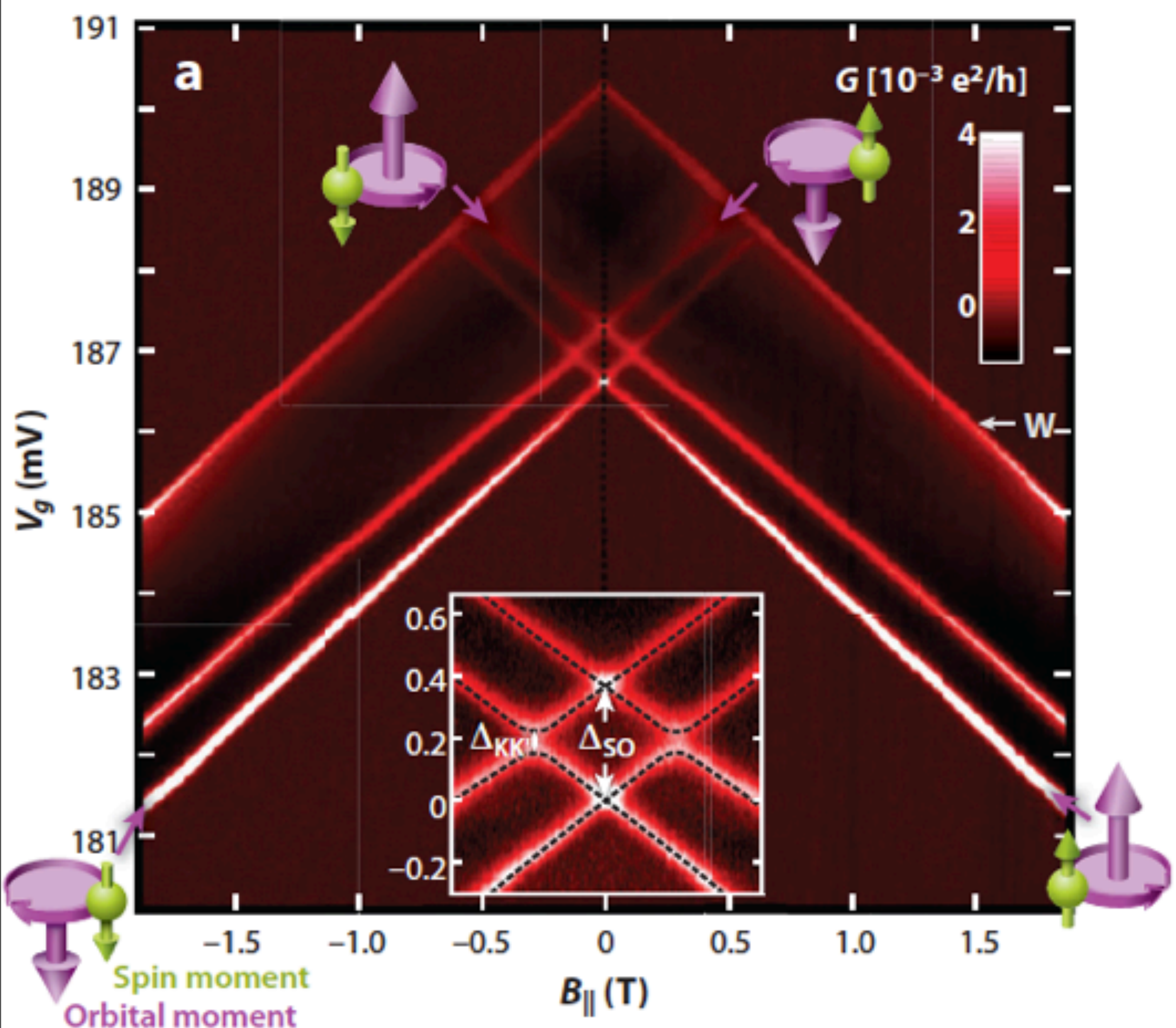
Carbon Nanotube with Spin-orbit coupling II



•The orbital motion of electrons also couples to a curvature-induced radial electric field. This creates an effective axial magnetic field which polarizes the spins along the NT axis and favors parallel alignment of the spin and orbital magnetic momenta or antiparallel depending on the sign of this spin-orbit coupling.

•As a result, SU(4) degeneracy breaks into two Kramers doublets (time-reversed electrons pairs). Spin-orbit interaction can be understood as an effective exchange field between these Kramers pairs (J-S Lim, R. López, G-L Giorgi, D. Sánchez Phys. Rev. B **83**, 155325 (2011))

$$H_{SO} = (\Delta^1_{SO}\tau_3 \otimes \sigma_1 \otimes s_3 + \Delta^0_{SO}\tau_3 \otimes \sigma_0 \otimes s_3)$$

Coupling of spin and orbital motion of electrons in carbon nanotubes, F. Kuemmeth, S. Ilani, D. C. Ralph & P.L.McEuen, Nature, 452, 448, 2008.

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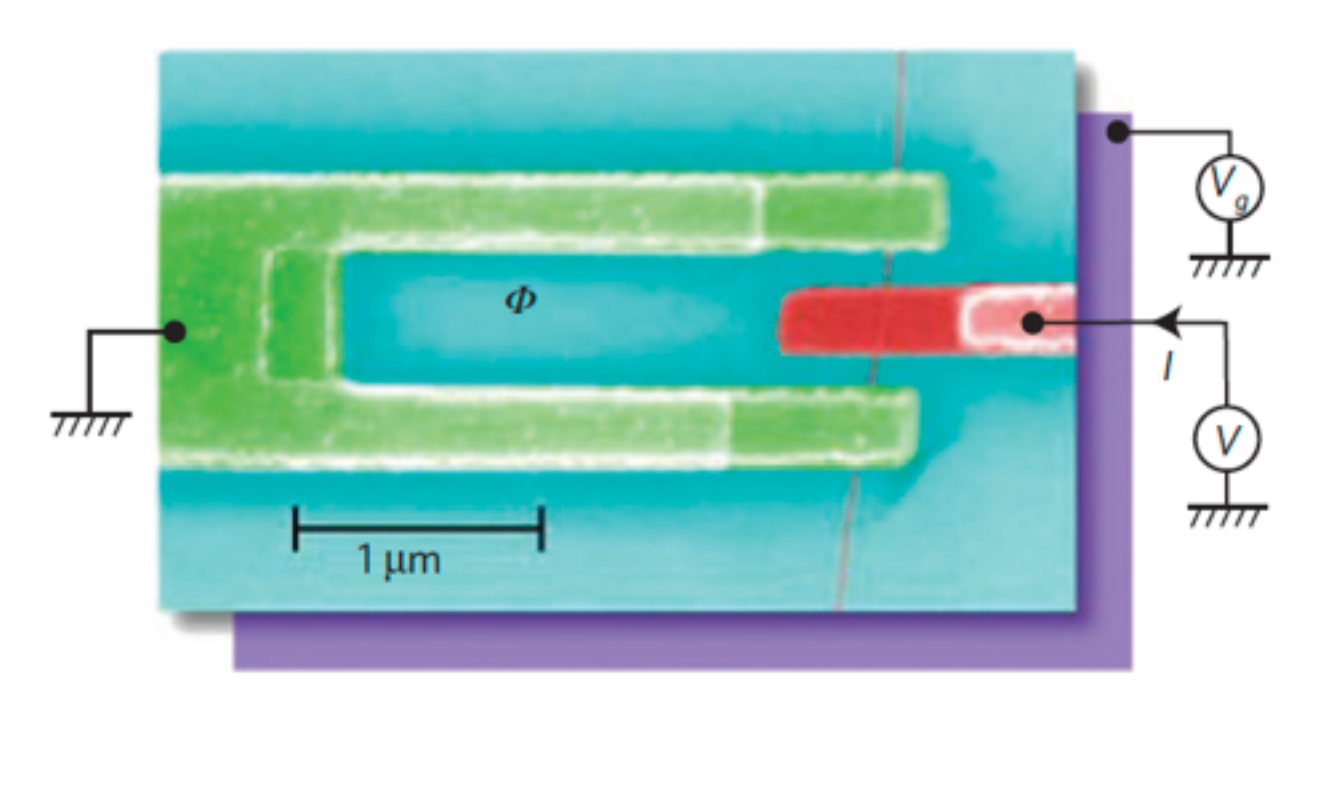
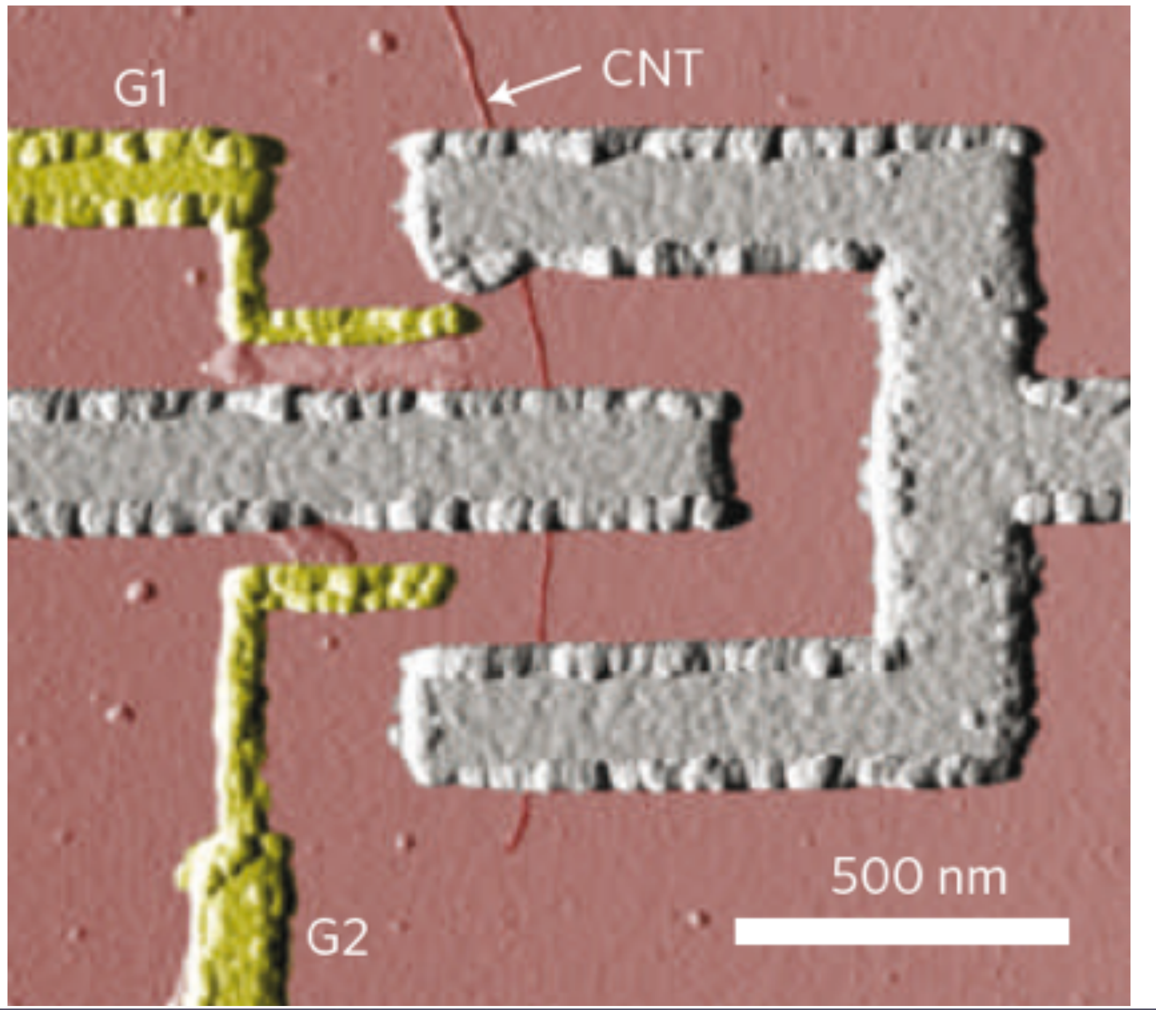
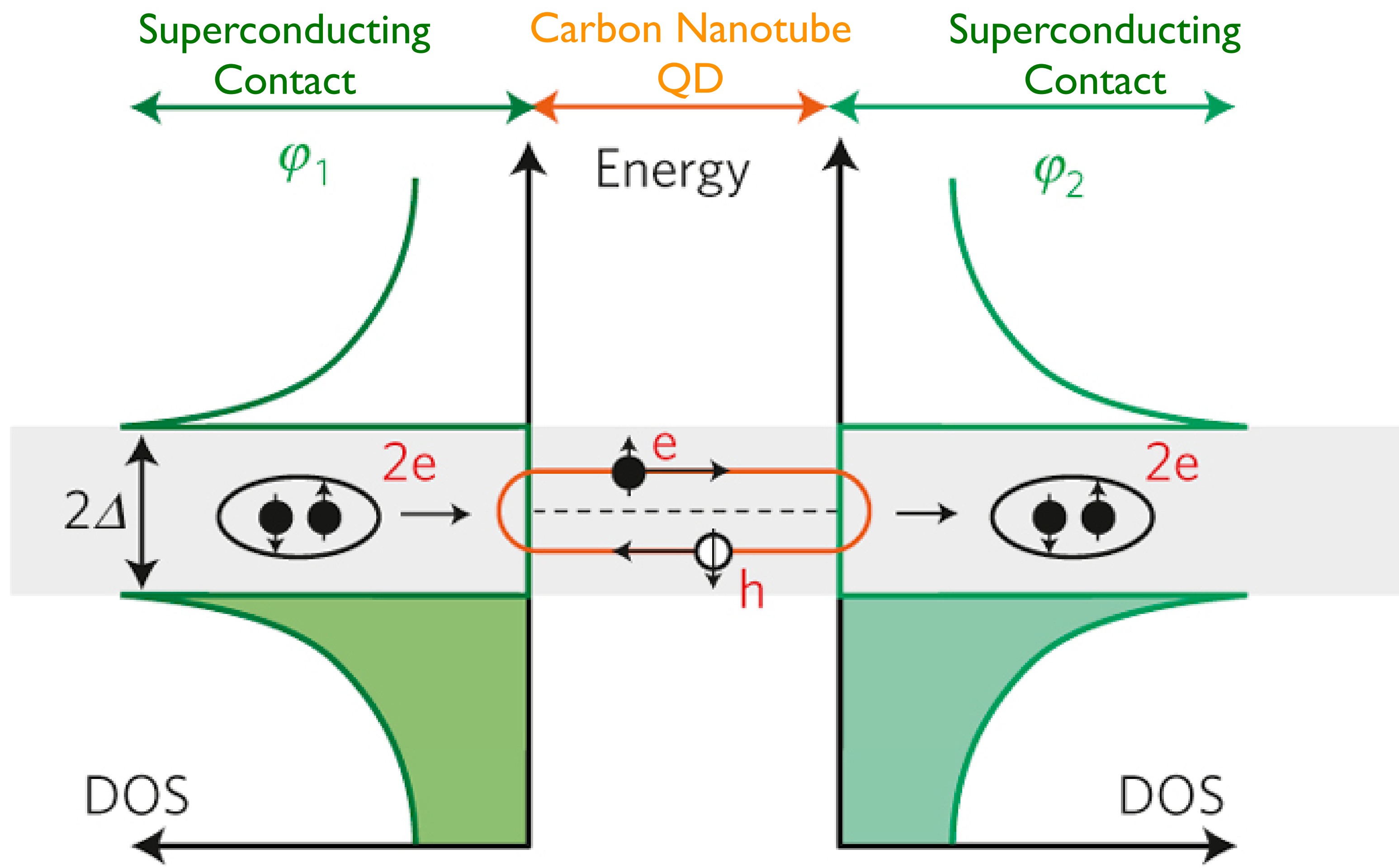
Nanotubes can be contacted with superconducting leads

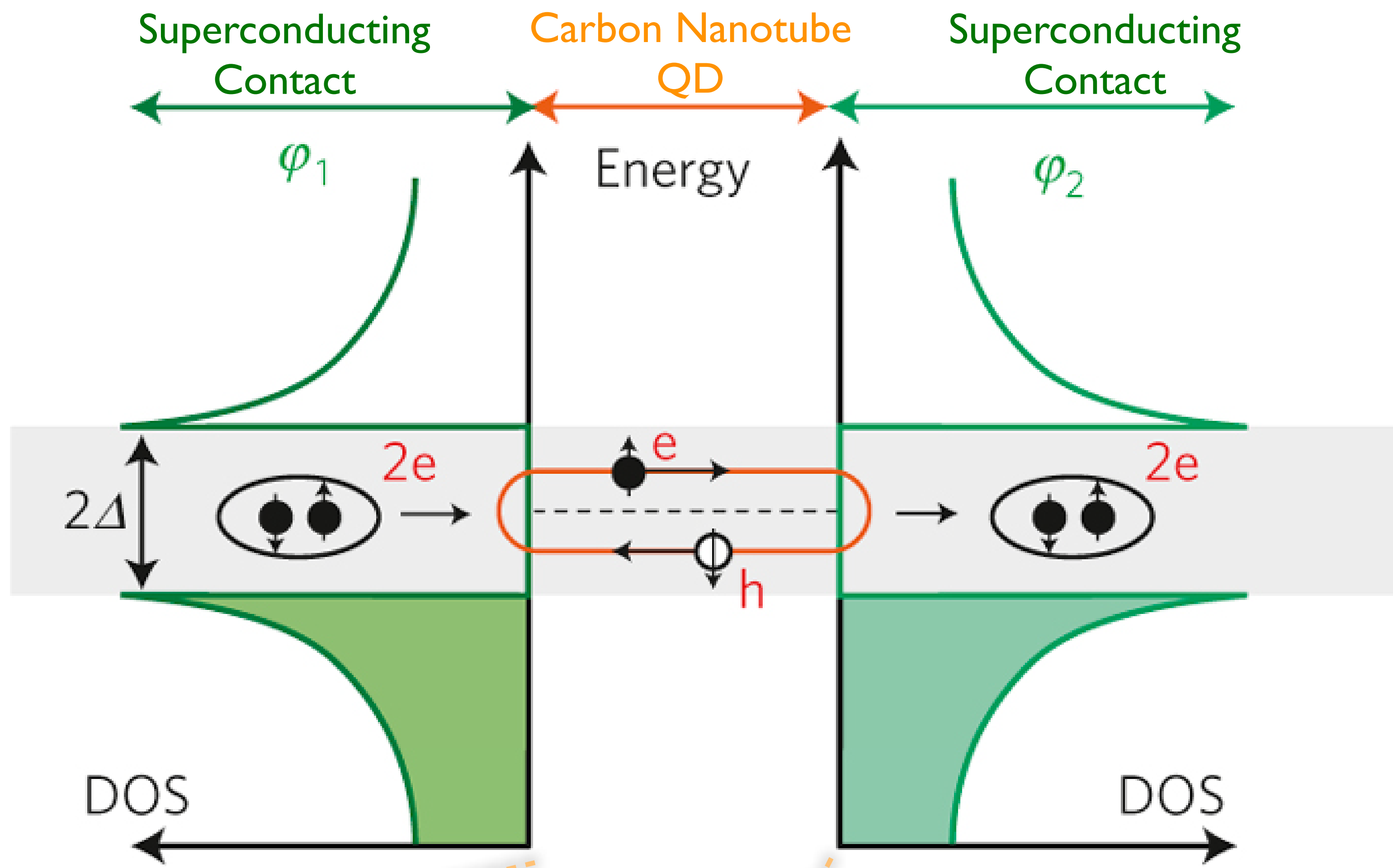


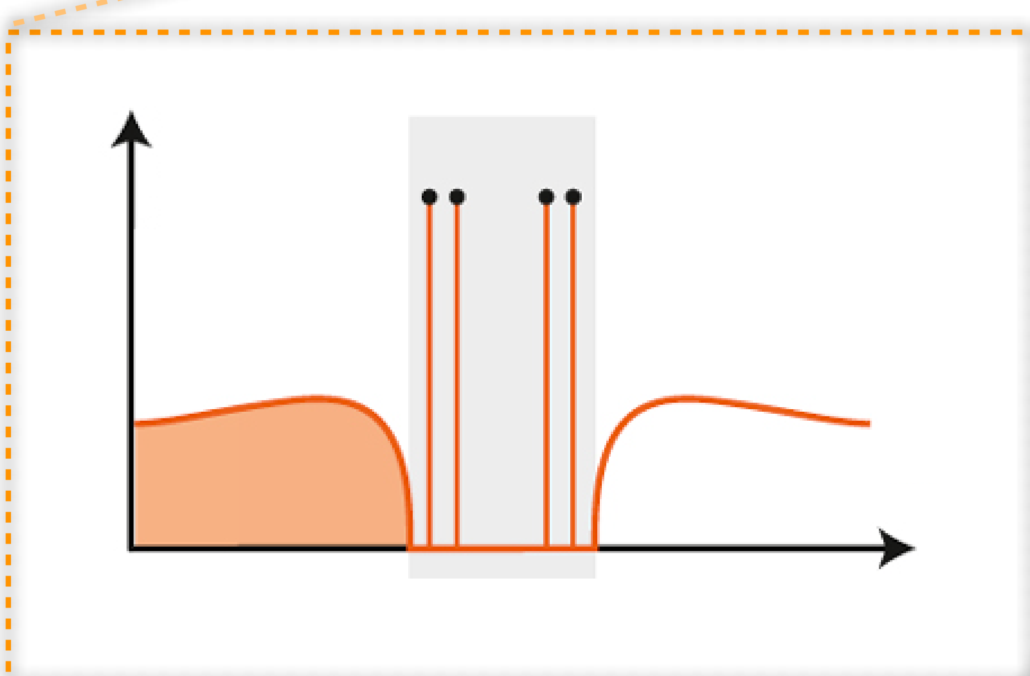
Pillet *et al*, Nature Physics, 6, 965, (2010).



J. P. Cleuziou *et al*, Nature Nanotech., 1, 53, (2006)

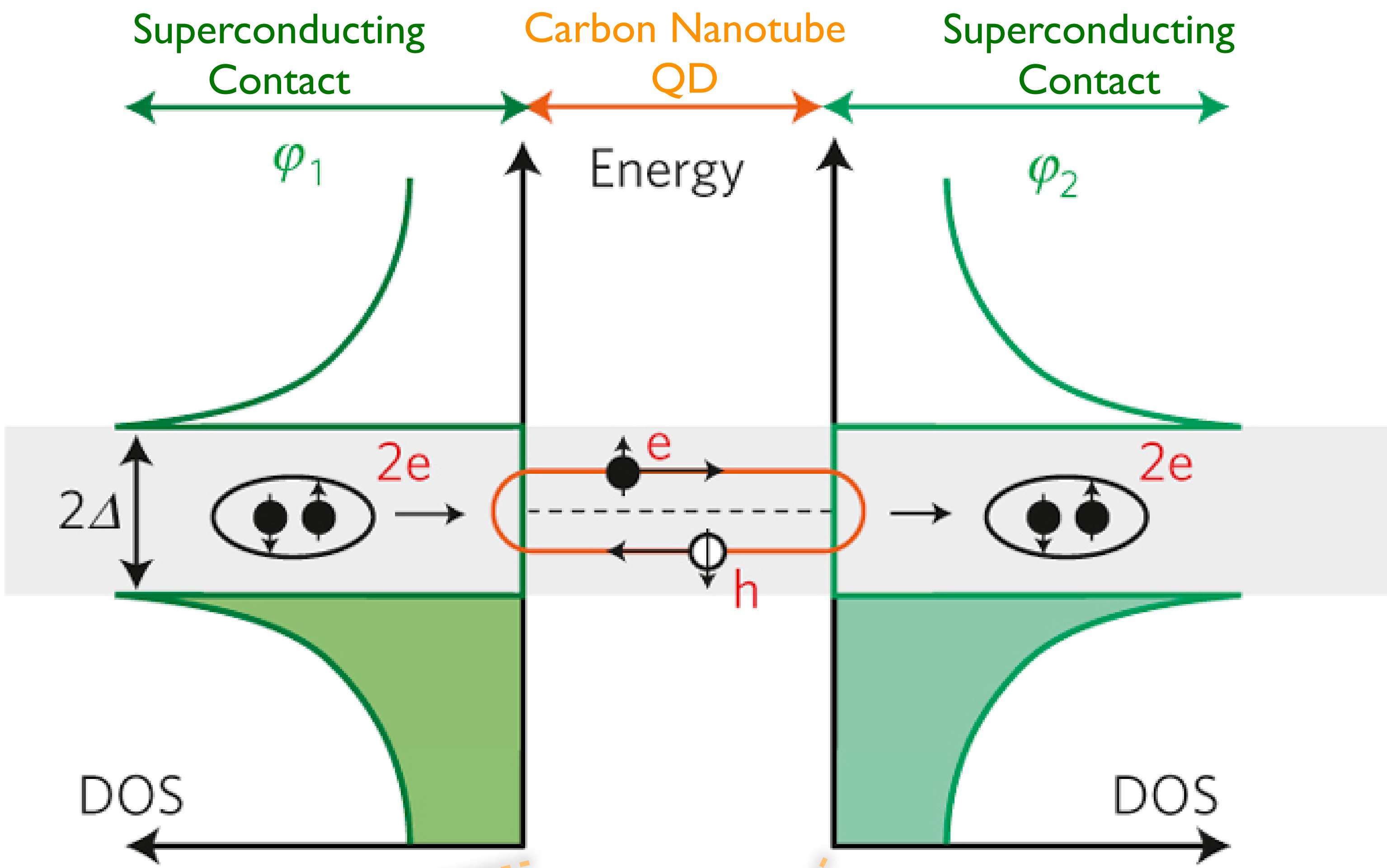

Superconducting Contact
Carbon Nanotube QD
Superconducting Contact
φ_1
Energy
φ_2
2Δ
2e
e
h
2e
DOS
DOS

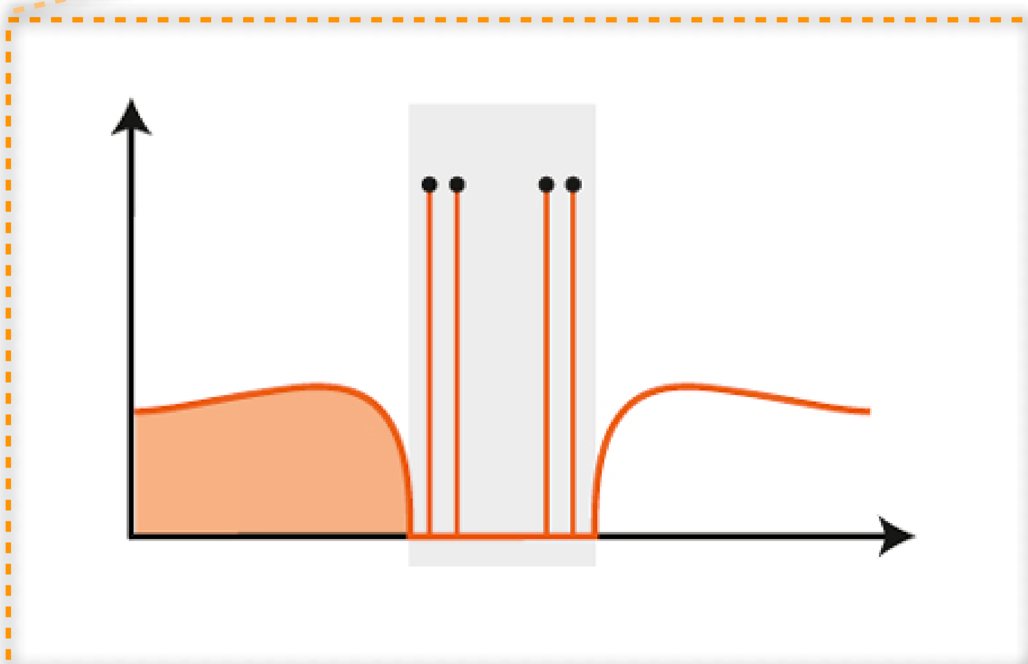




Andreev Bound states (ABS): entangled time-reversed electron-hole Kramers pairs.

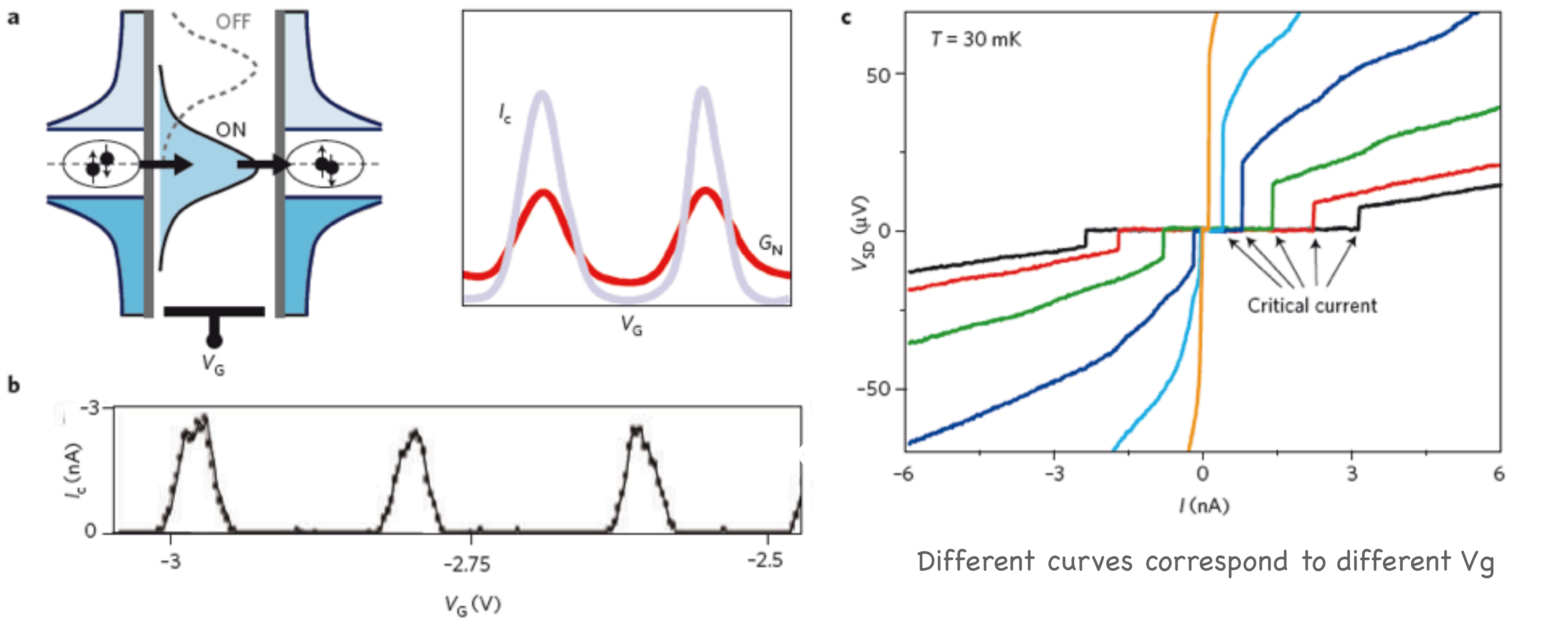
Recently measured in nanotubes and graphene
Pillet *et al*, Nature Physics, 6, 965, 2010 (nanotubes);
T. Dirks, *et al*, Nature Physics, 6 February 2011 (graphene)





The Josephson current is mainly given by resonant tunneling of Cooper pairs through these bound states

Nanotubes can be contacted with superconducting leads



Different curves correspond to different Vg

Quantum supercurrent transistors in carbon nanotubes,
Pablo Jarillo-Herrero, Jorden van Dam and Leo Kouwenhoven, Nature 439, 953 (2006).

For a review, see "Hybrid superconductor-quantum dot devices"
Silvano de Franceschi et al, Nature Nanotechnology, 5, 703 (2010

As both phenomena, spin-orbit and the formation of Andreev bound states, are related to time-reversed Kramers pairs, it is interesting to address the following question:
what happens to the Josephson effect in QD carbon nanotubes in the presence of spin-orbit?

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Anderson-like hamiltonian + BCS leads

$$H_C = \sum_{\alpha=L/R,k,\tau,s} \xi_k c^\dagger_{\alpha k\tau s} c_{\alpha k\tau s}$$

$$- \sum_{\alpha,k,\tau} \left[\Delta_\alpha e^{i\phi_\alpha} c^\dagger_{\alpha k\tau\uparrow} c^\dagger_{\alpha\bar{k}\bar{\tau}\downarrow} + h.c.\right]$$

$$H_D = \sum_{\tau,s} \varepsilon_{\tau s} d^\dagger_{\tau s} d_{\tau s} + U \sum_{(\tau,s)\neq(\tau',s')} n_{\tau s} n_{\tau' s'}$$

$$H_T = \sum_{\alpha=L/R,k,\tau,s} \left(V_\alpha c^\dagger_{\alpha k\tau s} d_{\tau s} + h.c.\right),$$

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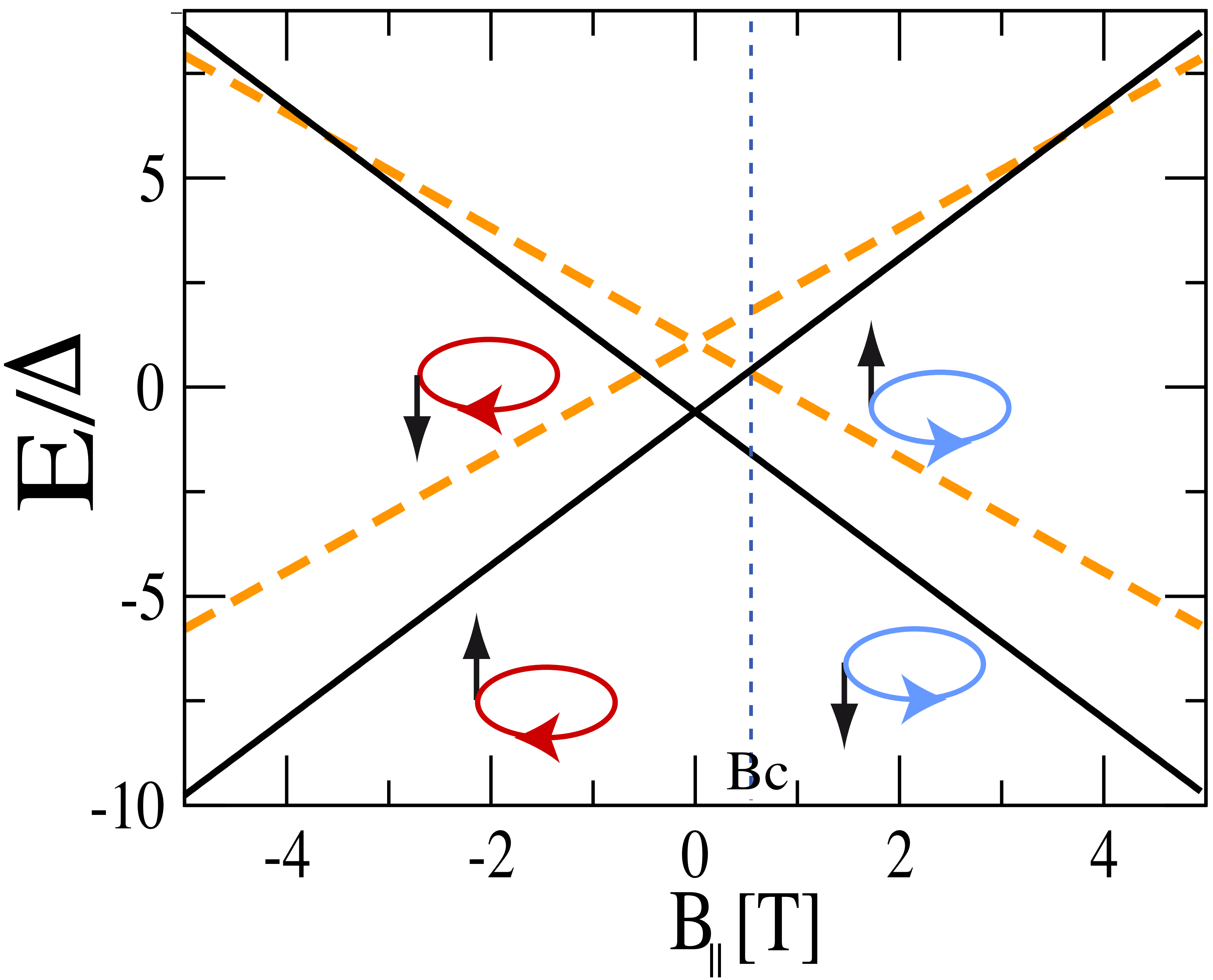


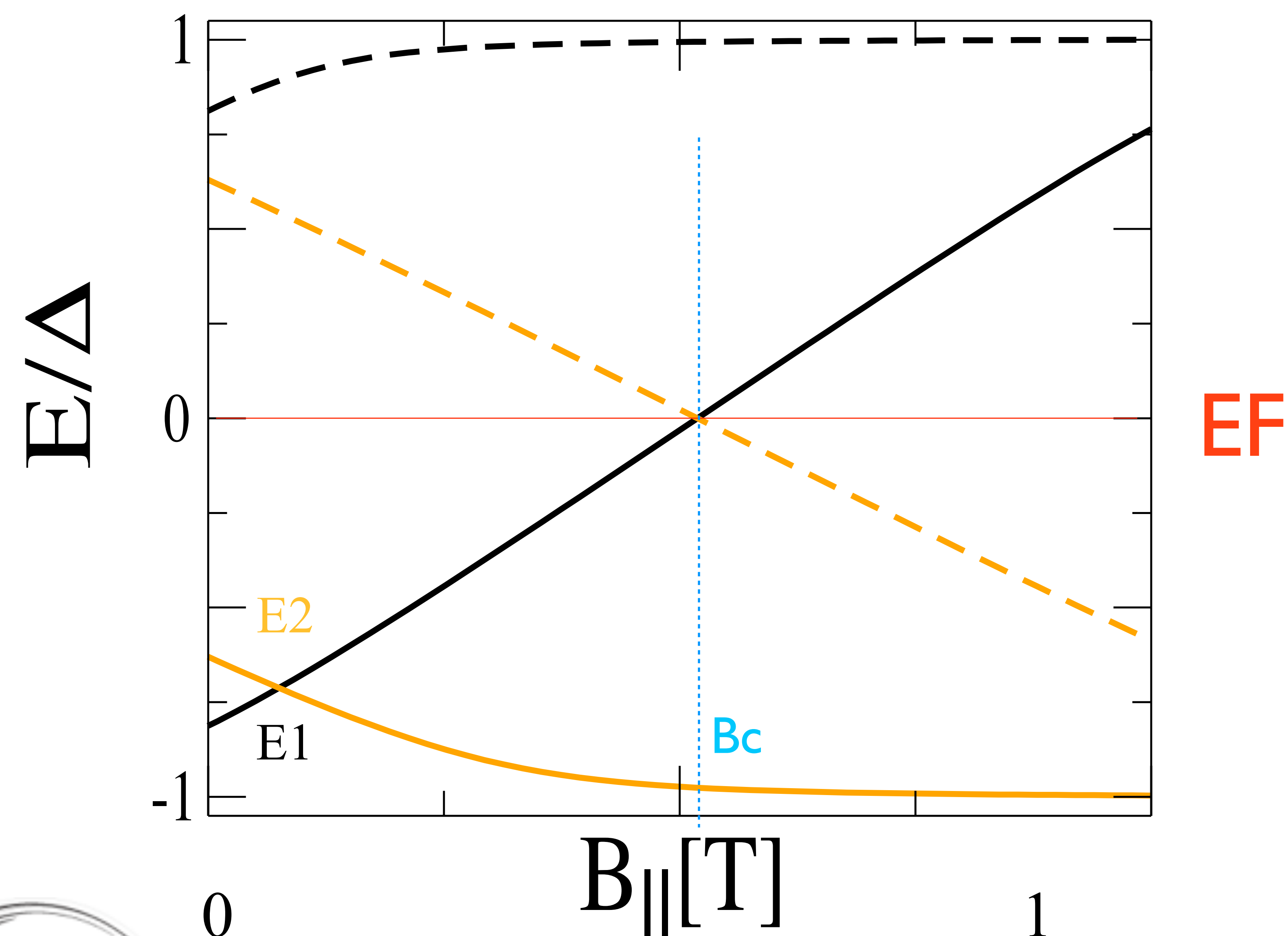
$$\varepsilon_{\tau,\sigma} = \varepsilon_0 + \sigma\tau\Delta_{SO} + \sigma\Delta_Z + \tau\Delta_{orb},$$

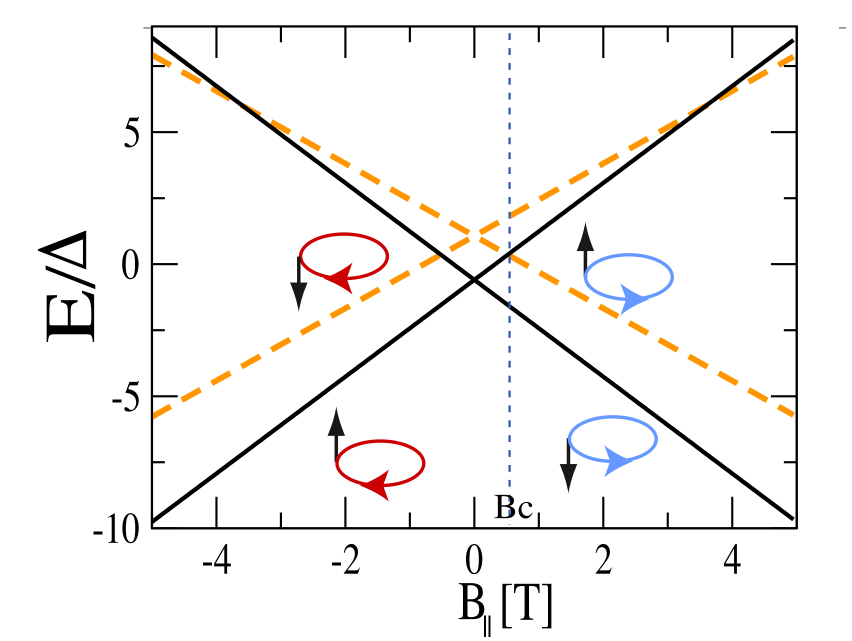
Calculation: Green's functions in Nambu space.
The poles of the retarded Green's function give the Andreev bound states

$$Det[G_d^r(\omega)^{-1}] = D_+ D_- = 0$$

$$\left(E_{1(2)} - \varepsilon_{\mp\uparrow} + \frac{\Gamma E_{1(2)}}{\sqrt{\Delta^2 - E_{1(2)}^2}}\right)\left(E_{1(2)} + \varepsilon_{\pm\downarrow} + \frac{\Gamma E_{1(2)}}{\sqrt{\Delta^2 - E_{1(2)}^2}}\right) - \frac{\Gamma^2 \Delta^2 \cos^2(\phi/2)}{\Delta^2 - E_{1(2)}^2} = 0$$

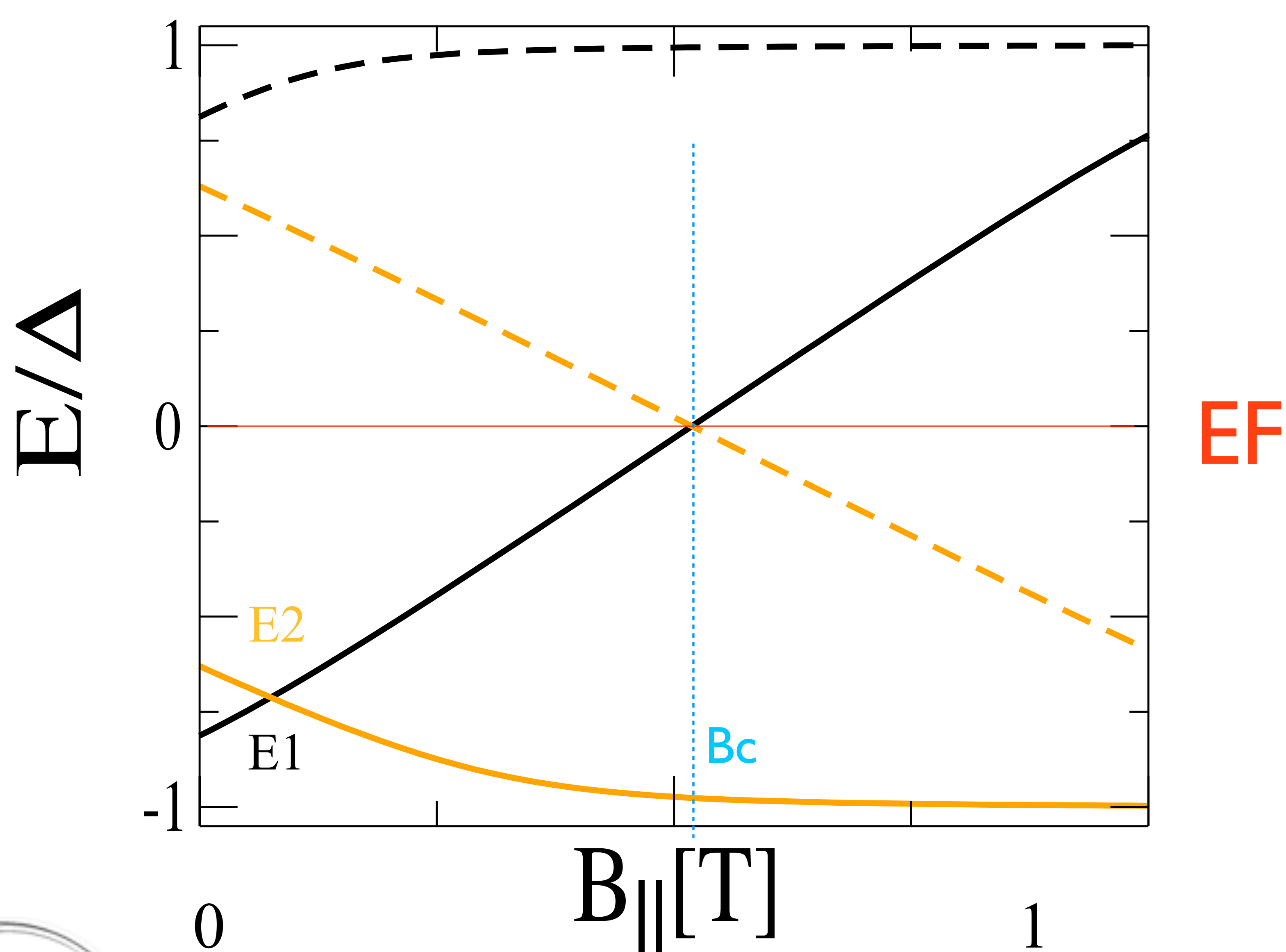
$$\left(E_{1(2)} - \varepsilon_{\mp\uparrow} + \frac{\Gamma E_{1(2)}}{\sqrt{\Delta^2 - E_{1(2)}^2}}\right)\left(E_{1(2)} + \varepsilon_{\pm\downarrow} + \frac{\Gamma E_{1(2)}}{\sqrt{\Delta^2 - E_{1(2)}^2}}\right) - \frac{\Gamma^2 \Delta^2 \cos^2(\phi/2)}{\Delta^2 - E_{1(2)}^2} = 0$$





$$2\mu_{orb} B_c = \Delta_{SO}$$

$$\left(E_{1(2)} - \varepsilon_{\mp\uparrow} + \frac{\Gamma E_{1(2)}}{\sqrt{\Delta^2 - E_{1(2)}^2}}\right)\left(E_{1(2)} + \varepsilon_{\pm\downarrow} + \frac{\Gamma E_{1(2)}}{\sqrt{\Delta^2 - E_{1(2)}^2}}\right) - \frac{\Gamma^2\Delta^2\cos^2(\phi/2)}{\Delta^2 - E_{1(2)}^2} = 0$$



- Each Kramer's doublet produces two ABS (four in total).
- At B=Bc, the ABS corresponding to different Kramers doublets cross. After the crossing, the two ABS below EF belong to the same Kramers doublet.

Josephson current in terms of Green's functions (both discrete and continuum contribution calculated on the same footing)

$$I = \frac{2e}{\hbar}\Re\int \frac{d\omega}{2\pi}\; Tr\left[\hat{\sigma}_3\left(\hat{\Sigma}_0^<\hat{G}^a(\omega) + \hat{\Sigma}_0^r\hat{G}^<(\omega)\right)\right] = I_{dis} + I_{con}$$

Discrete Josephson current (resonant Cooper pairs)

$$I_{dis} \quad = -\frac{e\Gamma^2}{\hbar}\sin(\phi)\left[\sum_{E_2}\frac{f(E_2)\Delta^2}{(\Delta^2-E_2^2)D'_+(E_2)} + \sum_{E_1}\frac{f(E_1)\Delta^2}{(\Delta^2-E_1^2)D'_-(E_1)}\right]$$

Charge and heat dynamics in nano-systems

Discrete Josephson current (resonant Cooper pairs)

$$I_{dis} = -\frac{e\Gamma^2}{\hbar}\sin(\phi)\left[\sum_{E_2}\frac{f(E_2)\Delta^2}{(\Delta^2-E_2^2)D'_+(E_2)}+\sum_{E_1}\frac{f(E_1)\Delta^2}{(\Delta^2-E_1^2)D'_-(E_1)}\right]$$

$$I_{dis} = \frac{2e}{\hbar}\sum_{E_{1(2)}} f(E_{1(2)})\frac{\partial E_{1(2)}}{\partial \phi}$$

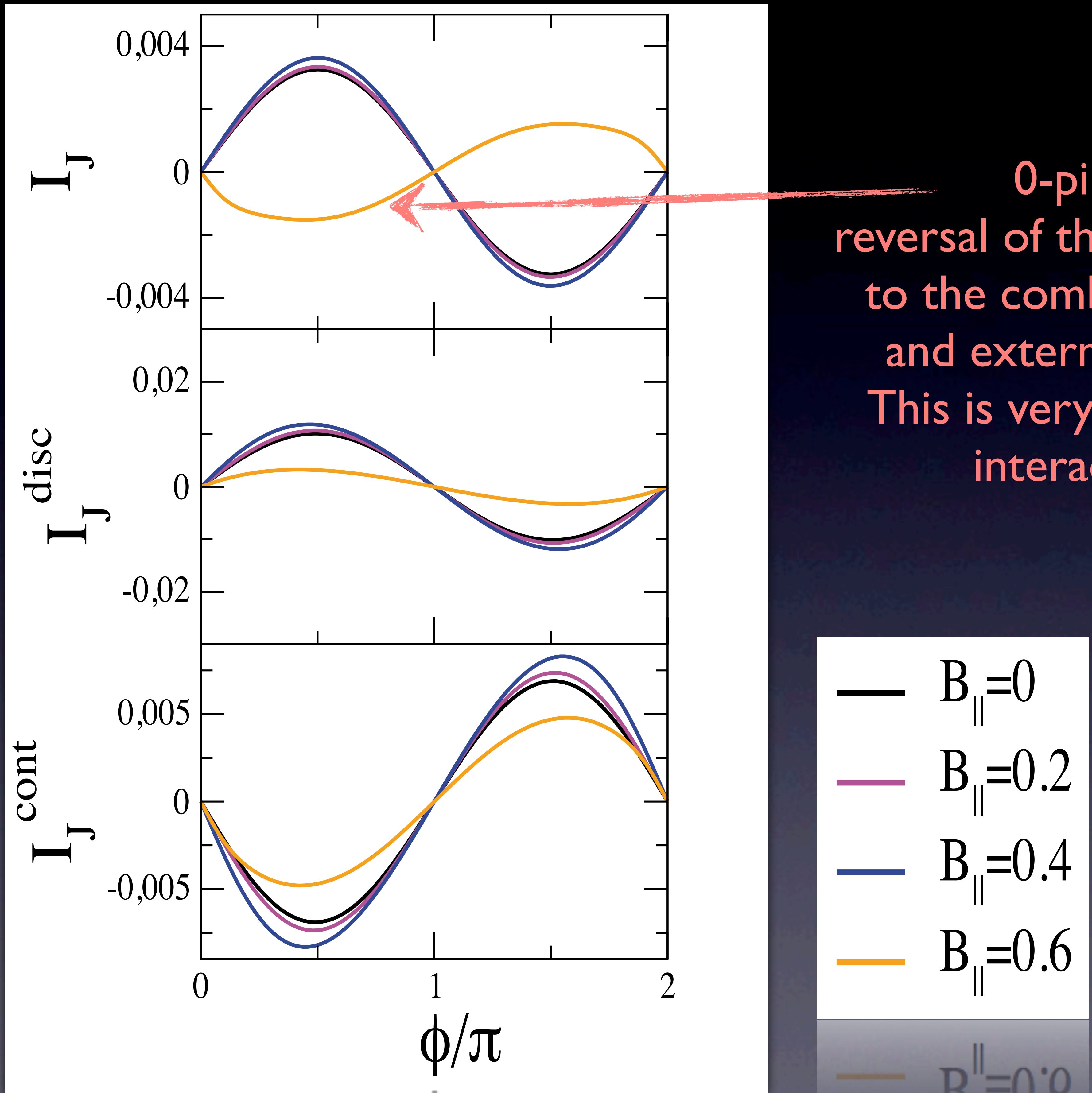
The discrete Josephson current is given by the derivative of the occupied (i. e. below EF) ABS with respect to phase

$$I_{dis} = -\frac{e\Gamma^2}{\hbar}\sin(\phi)\left[\sum_{E_2}\frac{f(E_2)\Delta^2}{(\Delta^2-E_2^2)D'_+(E_2)}+\sum_{E_1}\frac{f(E_1)\Delta^2}{(\Delta^2-E_1^2)D'_-(E_1)}\right]$$

Continuous Josephson current (quasiparticle states above the gap)

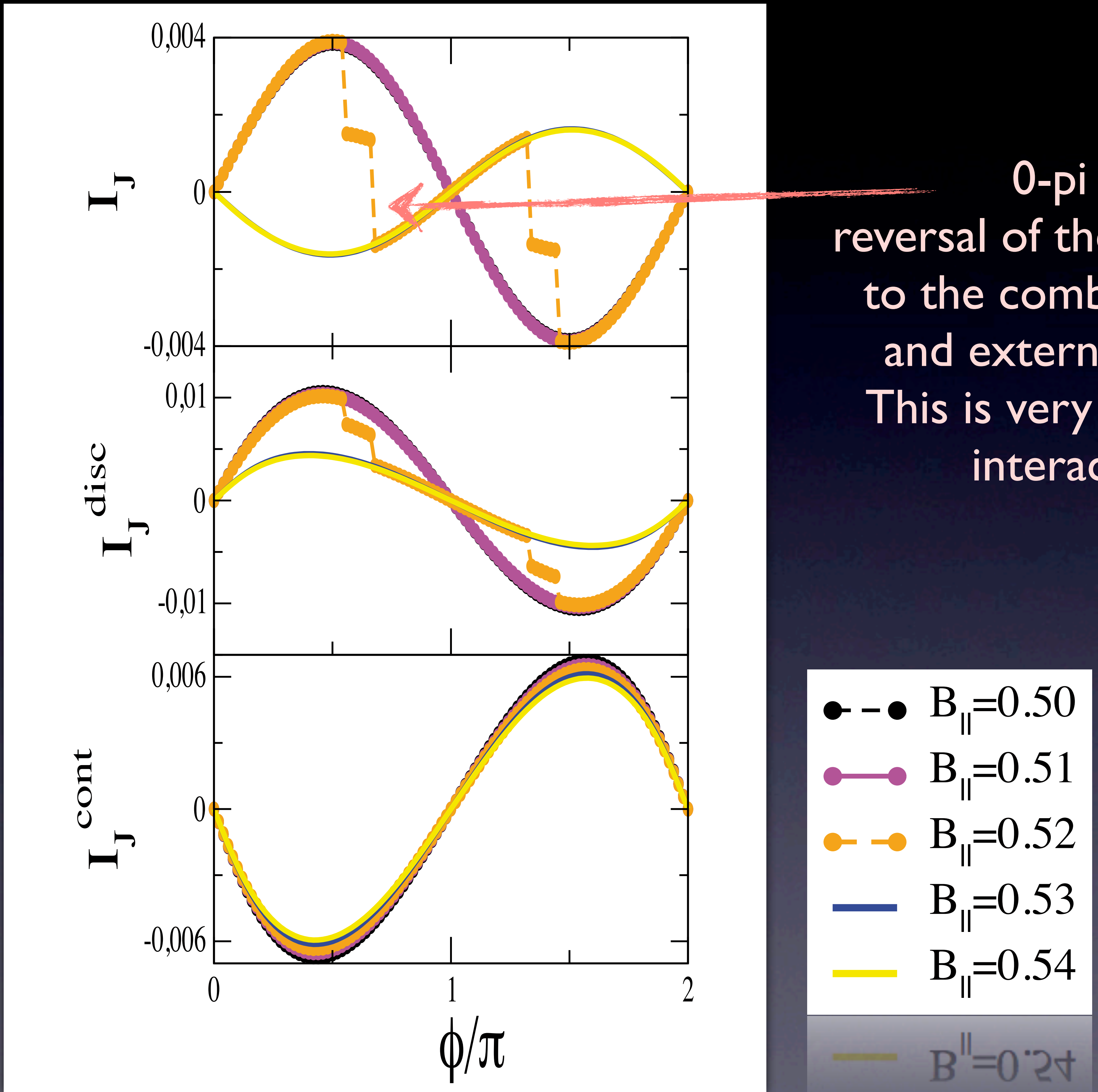
$$I_{con} = -\frac{e\Gamma^2}{\pi\hbar}\sin(\phi)\int d\omega\ \Theta(|\omega|-\Delta)\frac{f(\omega)\Delta^2}{(\omega^2-\Delta^2)}\Im\left[\frac{1}{D_+(\omega)}+\frac{1}{D_-(\omega)}\right]$$

Non-interacting regime



0-pi transition:
reversal of the supercurrent due to the combined effect of SO and external magnetic field.
This is very unusual in a non-interacting system.

Non-interacting regime II



0-pi transition:
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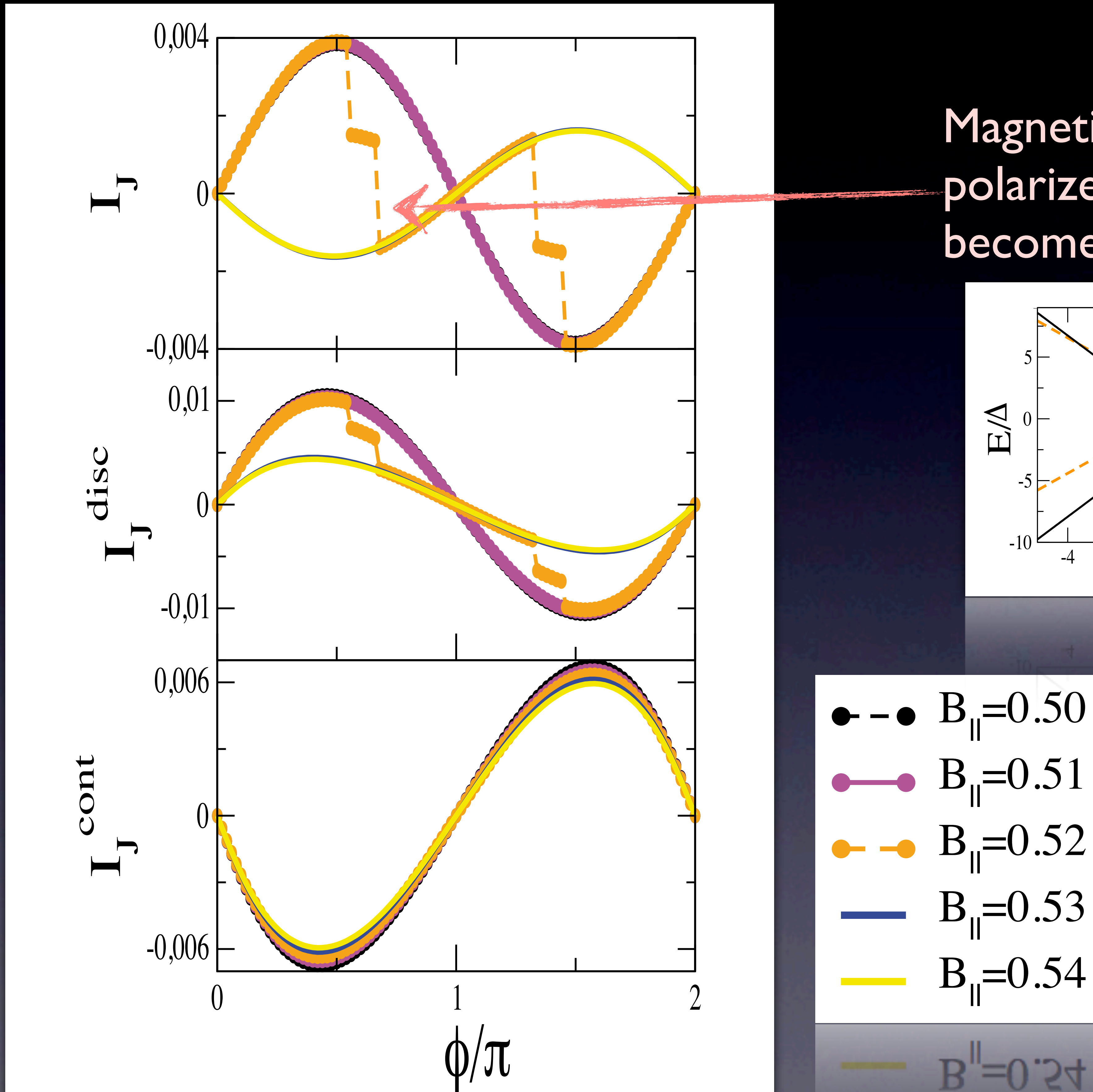
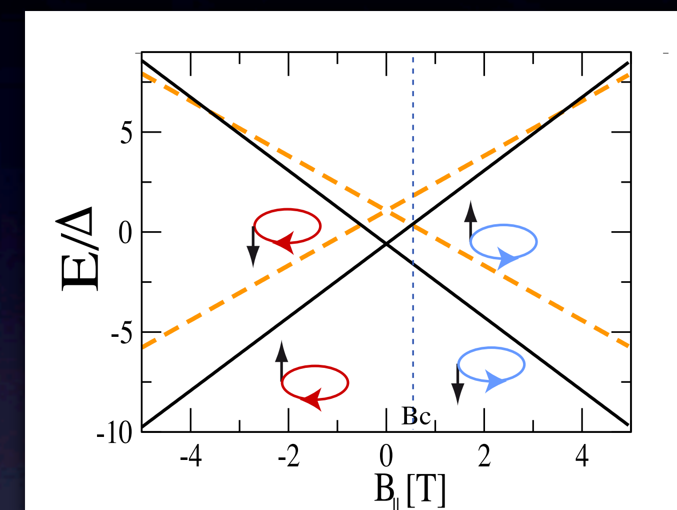
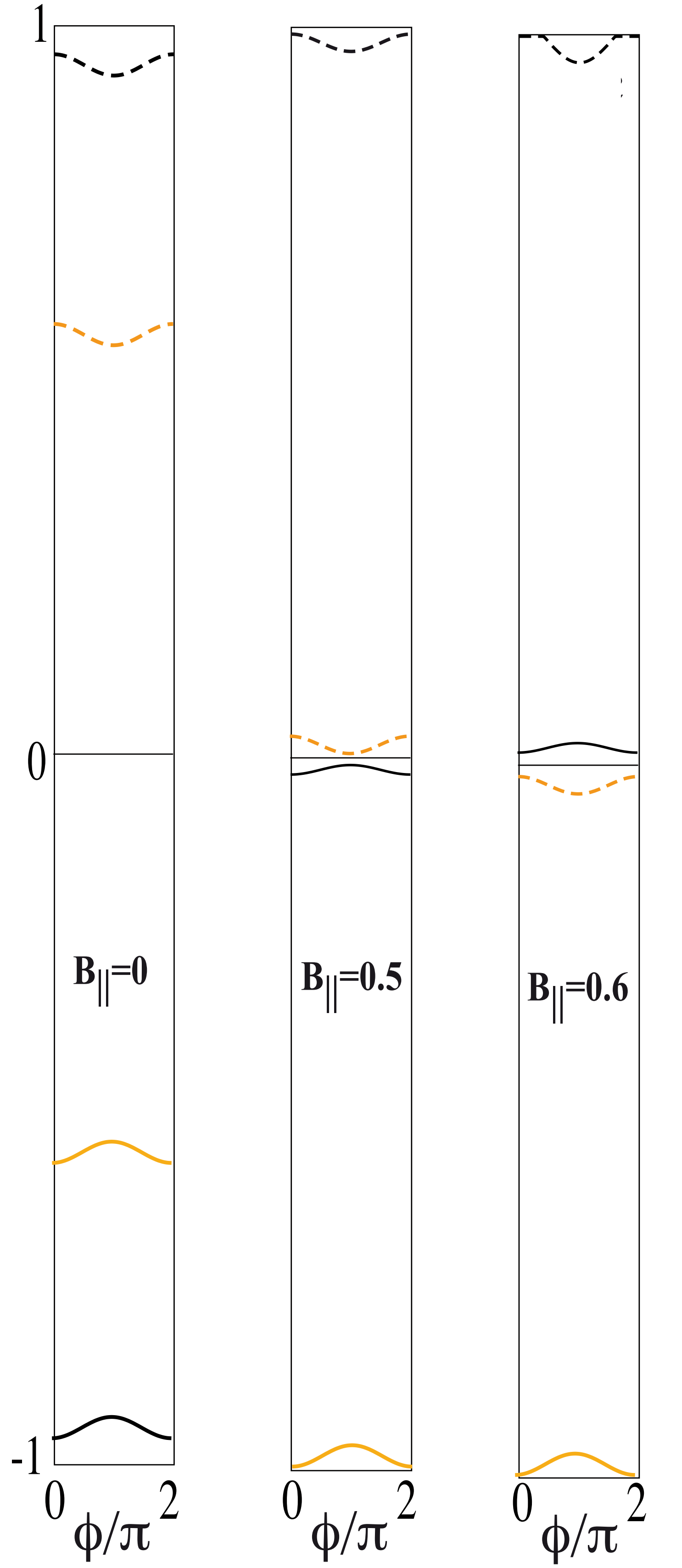
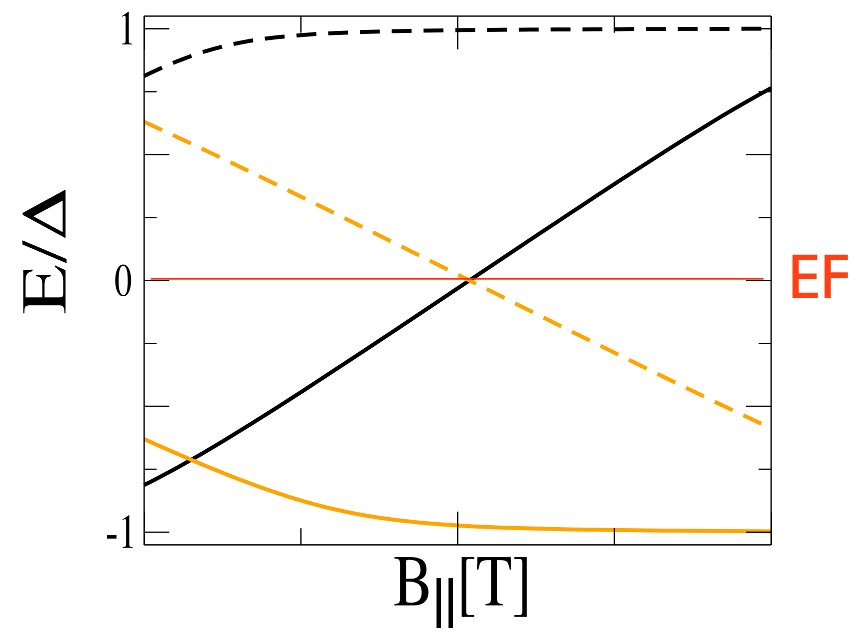
Non-interacting regime II



Magnetic field where spin-polarized orbital states become degenerate





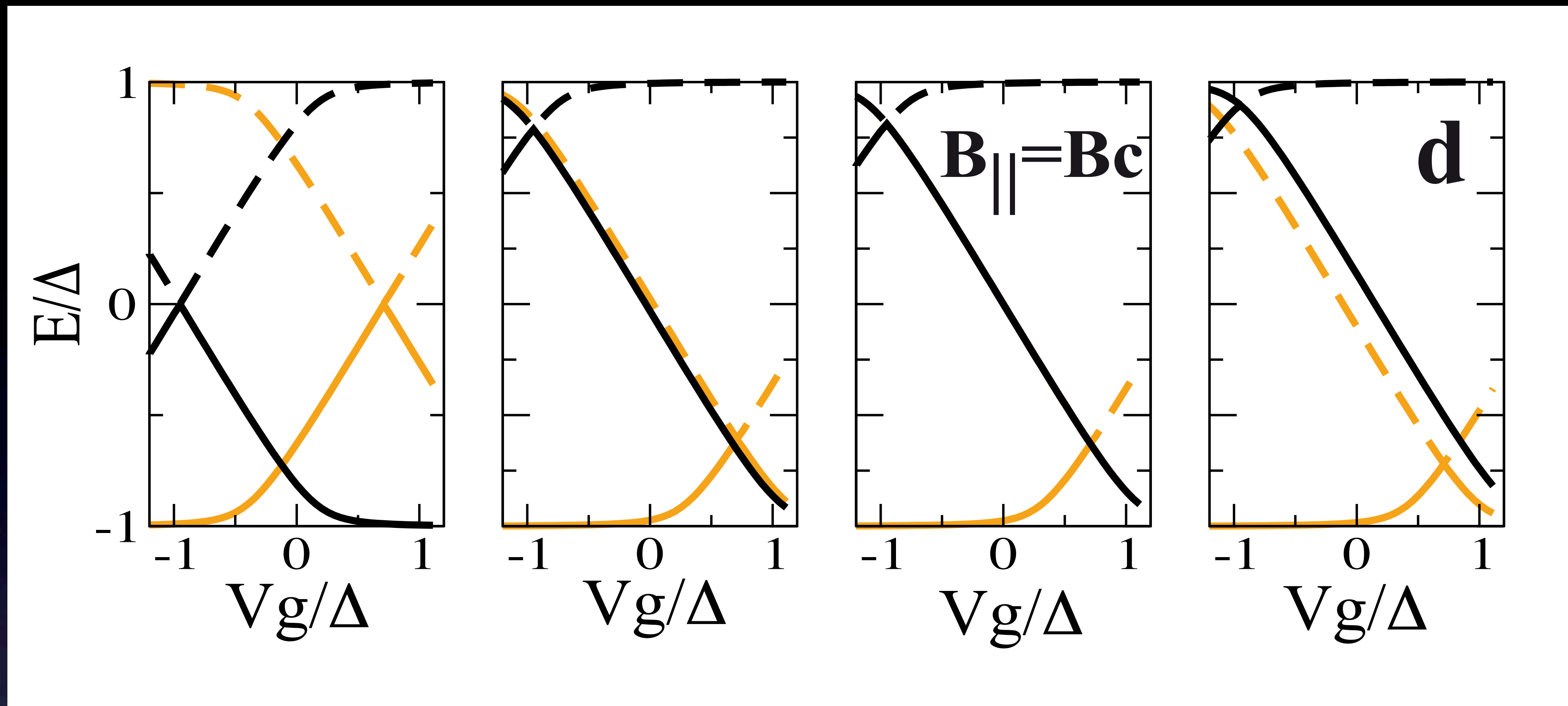


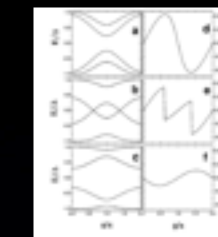
After the crossing, the occupied ABS belong to the <u>same</u> Kramers doublet. Importantly, they have opposite derivative with respect to phase which gives discrete supercurrents of opposite sign.

Only the continuous current (states above the gap) contributes. This reverses the sign of the supercurrent.

In standard QDs this happens in the cotunneling regime only, see *Supercurrent reversal in quantum dots,* J. van Dam et al, Nature 442, 667 (2006).

Gate tunability





At zero magnetic field, the SO splitted ABS show a diamond-like shape, similarly to spin-slit ABS due to Coulomb Blockade

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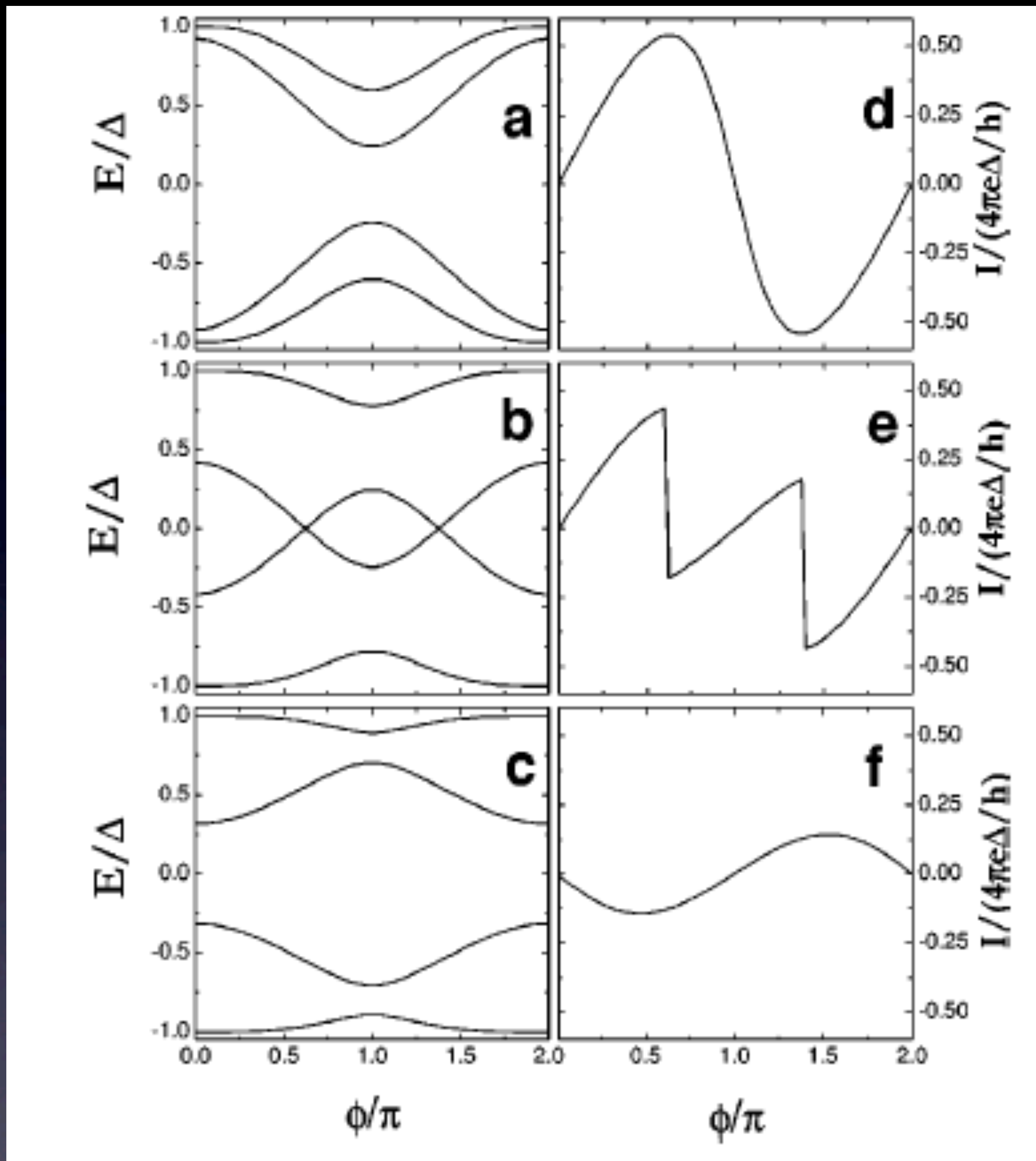
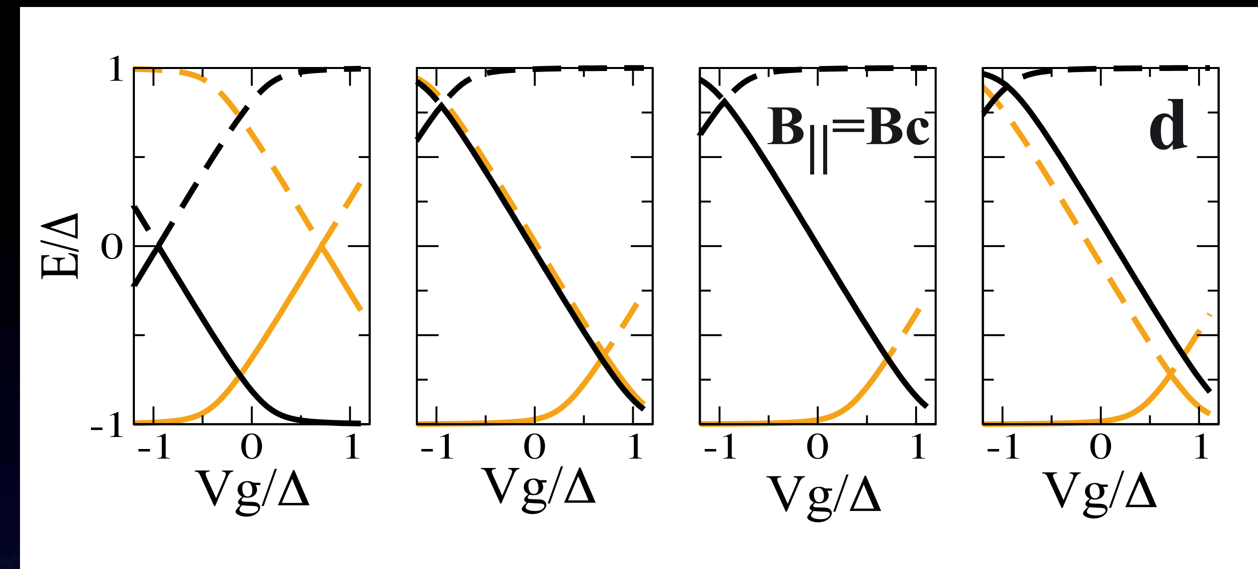
Gate tunability

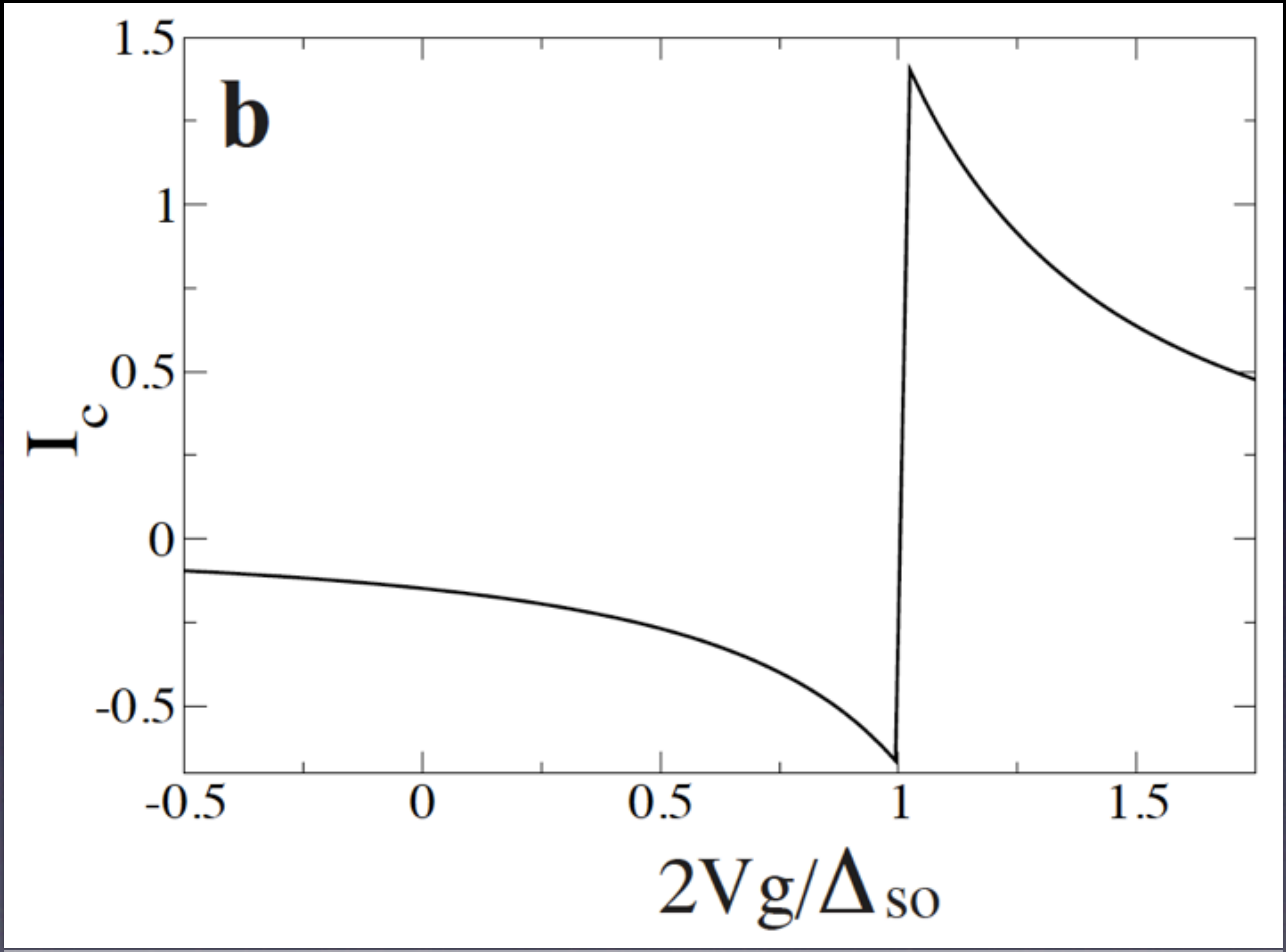




At zero magnetic field, the SO splitted ABS show a diamond-like shape, similarly to spin-slit ABS due to Coulomb Blockade

E. Vecino, A. Martín Rodero and A. L. Yeyati, Phys. Rev. B, 68,035105, 2003

Cotunneling regime (fourth order perturbation theory)



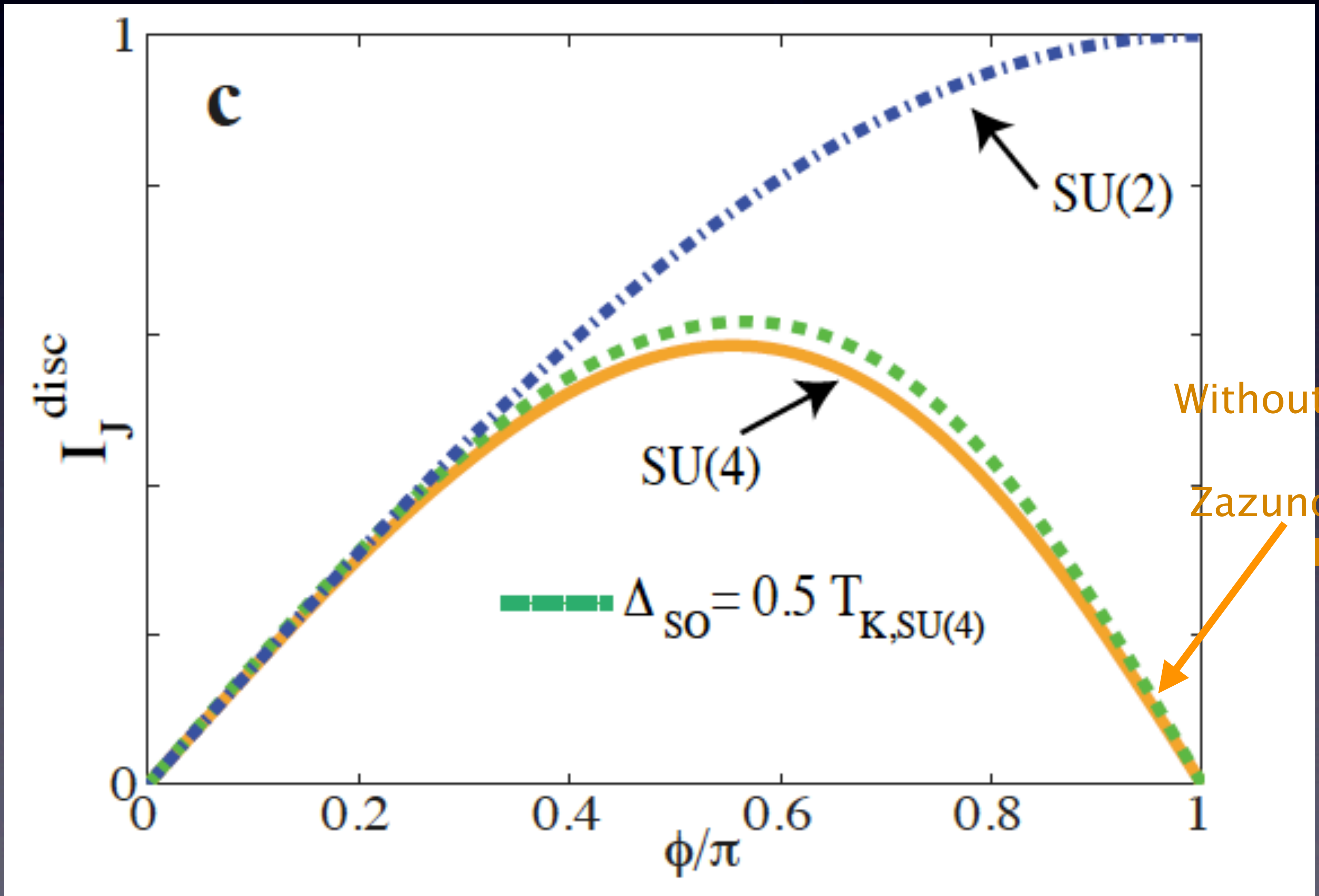
Charge and heat dynamics in nano-systems

IFISC

Kondo regime (slave boson)

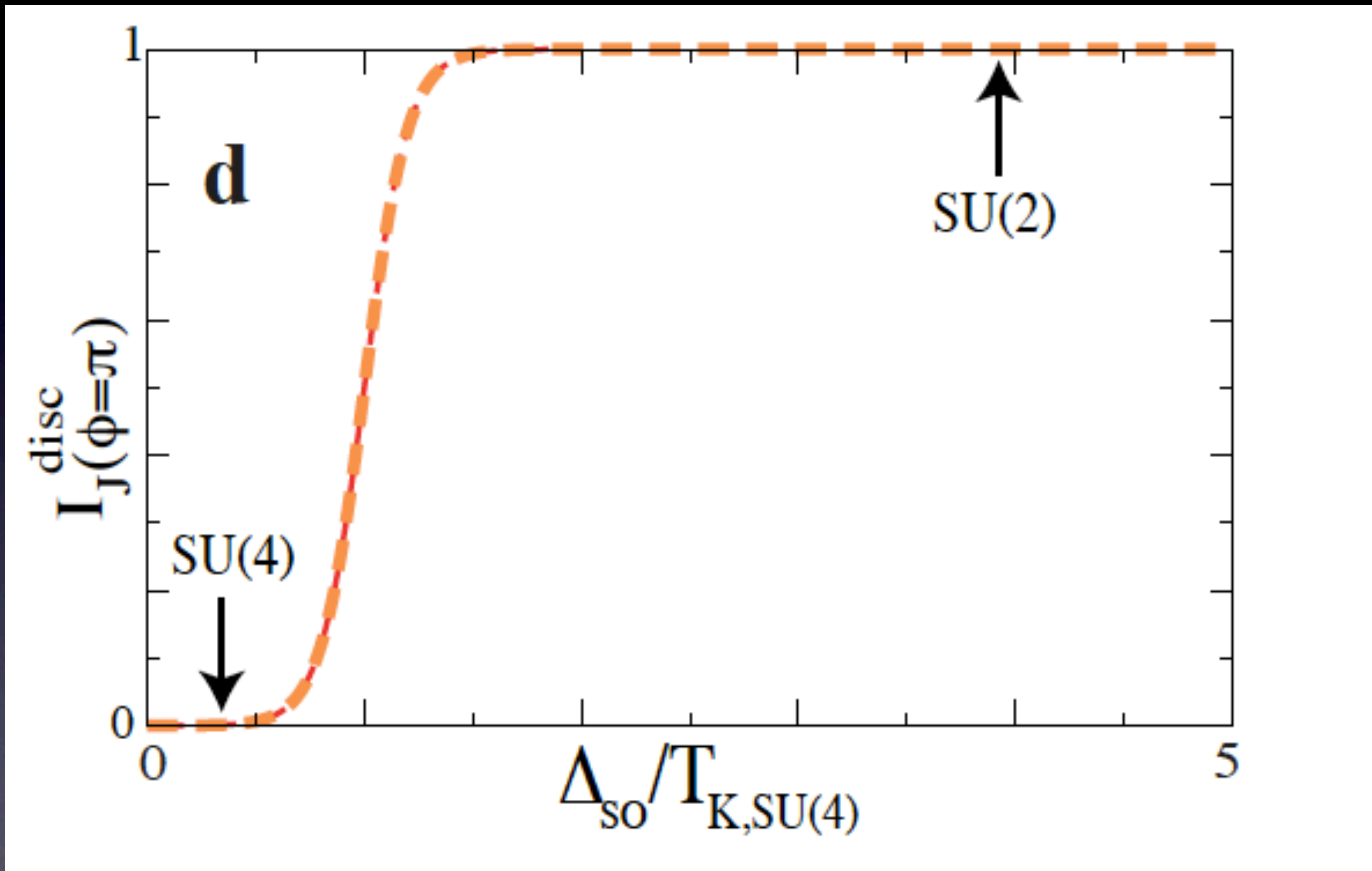
$$I_J^{dis} = \frac{e\Delta}{2\hbar} \sum_{\eta=\pm} \frac{\sin(\phi)}{[(1+\eta\alpha)^2+1][(1+\eta\alpha)^2+cos^2(\frac{\phi}{2})]}$$

$$\alpha = \frac{\Delta_{SO}}{2T_{K,SU(4)}}$$

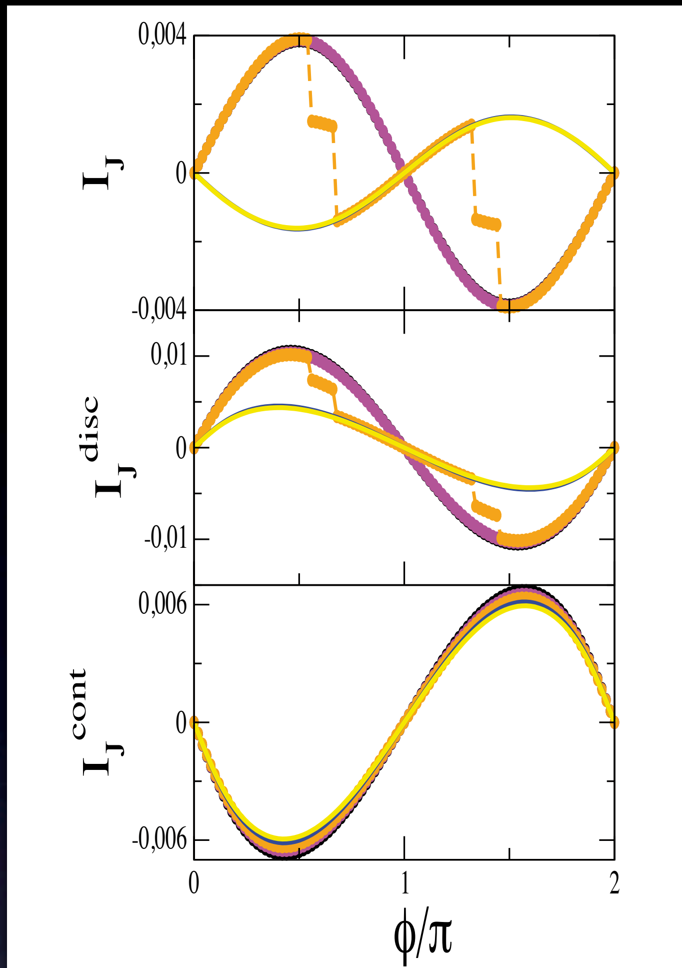


Without SO we recover the results of:
Zazunov, Levy-Yeyati and Egger, PRB **81**, 012502 (2010)

Kondo regime

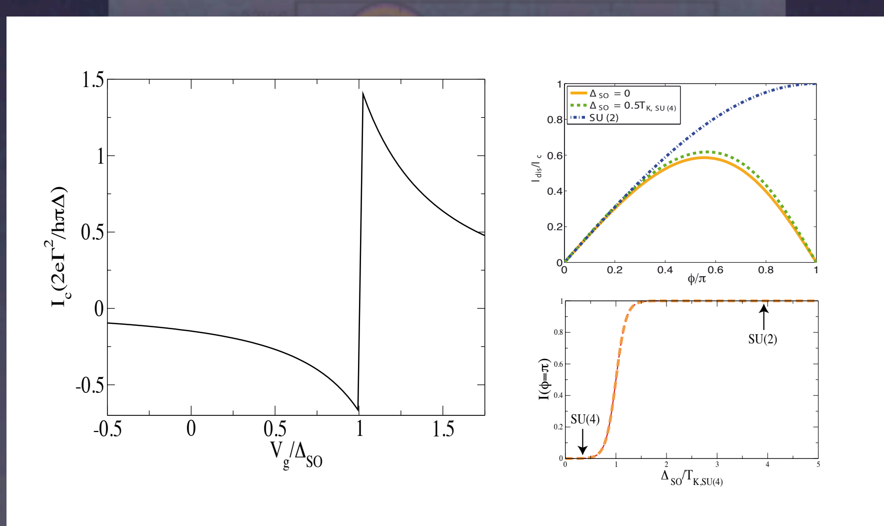


CONCLUSIONS



- The relatively small SO coupling in quantum dot carbon nanotubes induces a 0-pi transition in the Josephson current when an external magnetic field brings spin-polarized orbital levels to degeneracy.

- The transition is also tunable by a gate voltage. This is relevant in view of recent transport experiments in quantum dot carbon nanotubes.



- Cotunneling regime: the transition occurs even at zero magnetic field.

- Kondo regime: the Josephson current is always in the 0 phase for both SU(4) and SU(2) symmetries.

J. S. Lim, R. López, R. Aguado

Phys. Rev. Lett (2011)

arXiv:1104.0513