MIXED, CHARGE AND HEAT NOISES IN THERMOELECTRIC NANOSYSTEMS

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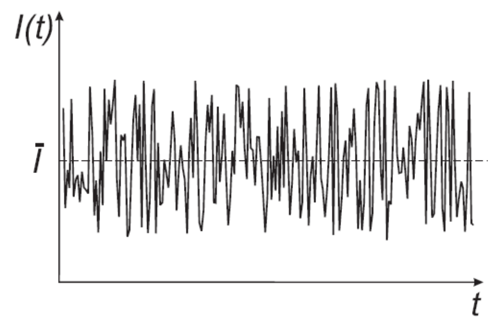
INTRODUCTION

CURRENT FLUCTUATIONS



"The noise is the signal"
Rolf LANDAUER

NOISE

$$\mathcal{S}(\omega) = \int_{-\infty}^{+\infty} dt\, e^{i\omega t} \langle \delta \hat{I}(0)\ \delta \hat{I}(t) \rangle$$

$$\delta \hat{I}(t) = \hat{I}(t) - \langle \hat{I} \rangle$$

AT EQUILIBRIUM

$$\mathcal{S}(\omega = 0) = 2k_B T_0 G$$

$\Rightarrow$ Gives the linear conductance through the fluctuation-dissipation theorem

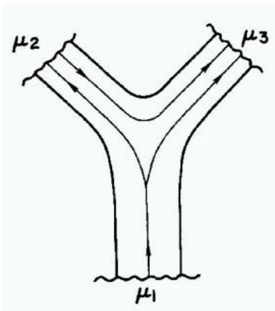
SCHOTTKY RELATION IN THE POISSIONIAN LIMIT (T=0)

$$\mathcal{S}(\omega = 0) = e^* I$$

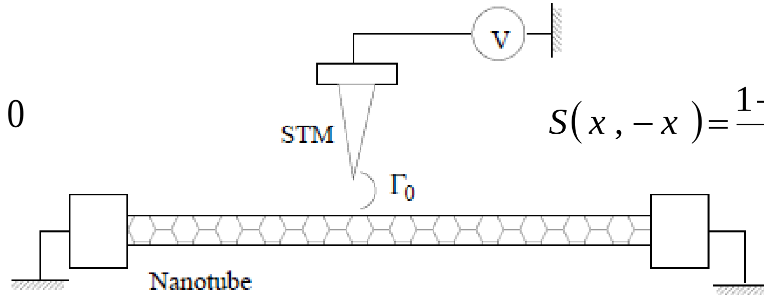
e^*/e = Fano factor (1/3, 1, 5/3, 2, ...)

STATISTICS AND DYNAMICS

CROSS-CORRELATOR



$$S_{23} = -\frac{4e^2}{h}\mathrm{Tr}\left[s_{21}s_{21}^{*}s_{31}s_{31}^{*}\right] < 0$$



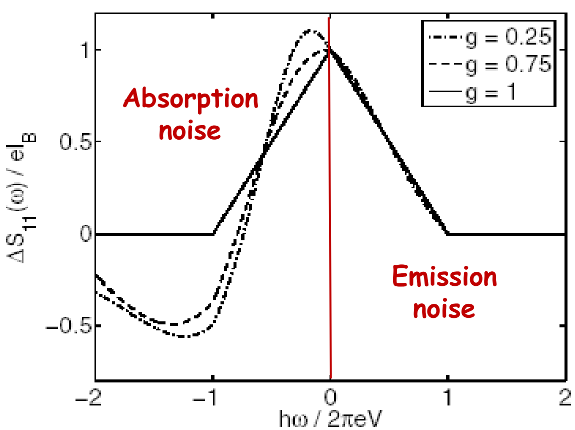
$$S(x\,,-x\,) = \frac{1-K_{c+}^2}{2}e|I| \; > 0$$

MARTIN/LANDAUER, PRB 45, 1742 (1992)

CREPIEUX et al., PRB 67, 205408 (2003)

$\Rightarrow$ Its sign gives the statistics of the excitations

FINITE FREQUENCY NOISE



$$G(\omega) = \mathrm{Re}\left[Y(\omega)\right] = \frac{S(-\omega) - S(\omega)}{2\hbar\omega}$$

$\Rightarrow$ The asymmetry of the finite-frequency noise is related to the ac conductance

CREPIEUX et al., PRB 78, 205422 (2008)

HEAT CURRENT NOISE

$$\mathcal{S}^{JJ}(\omega)=\int_{-\infty}^{+\infty} dt\, e^{i\omega t}\left\langle \delta\hat{J}(0)\,\delta\hat{J}(t)\right\rangle$$

$$\delta\hat{J}(t)=\hat{J}(t)-\left\langle\hat{J}\right\rangle \qquad \hat{J}(t)=\hat{I}^{E}(t)-\frac{\mu}{e}\hat{I}(t)$$

Heat fluctuations give information on higher-order cumulant of charge counting statistics

KINDERMANN/PILGRAM, Phys. Rev. B 69, 155334 (2004)

Finite-frequency symmetrized heat noise

SERGI, Phys. Rev. B 83, 033401 (2011)

Proposal for the detection of single-electron heat transfer statistics

SANCHEZ/BUTTIKER, Eur. Phys. Lett. 100, 47008 (2012)

Fluctuations of heat current emitted from a single-particle source

BATTISTA et al., Phys. Rev. Lett. 110, 126602 (2013)

Heat fluctuations in driven quantum conductor

MOSKALETS, Phys. Rev. Lett. 112, 206801 (2014)

Energy and power fluctuations in ac-driven coherent conductor

BATTISTA et al., Phys. Rev. B 90, 085418 (2014)

PURPOSE OF THIS WORK

⇒ Find the information contained in the correlator mixing charge and heat currents, and its link with thermoelectric conversion

CHARGE NOISE

$$\mathcal{S}_{\mathrm{pq}}^{\mathrm{II}} = \int_{-\infty}^{\infty} \left\langle \delta \hat{I}_p(0) \delta \hat{I}_q(t) \right\rangle dt$$

HEAT NOISE

$$\mathcal{S}_{\mathrm{pq}}^{\mathrm{JJ}} = \int_{-\infty}^{\infty} \left\langle \delta \hat{J}_p(0) \delta \hat{J}_q(t) \right\rangle dt$$

MIXED NOISES

$$\mathcal{S}_{\mathrm{pq}}^{\mathrm{IJ}} = \int_{-\infty}^{\infty} \left\langle \delta \hat{I}_p(0) \delta \hat{J}_q(t) \right\rangle dt$$

$$\mathcal{S}_{\mathrm{pq}}^{\mathrm{JI}} = \int_{-\infty}^{\infty} \left\langle \delta \hat{J}_p(0) \delta \hat{I}_q(t) \right\rangle dt$$

p, q = reservoirs
p=q ⇒ auto-correlation
p≠q ⇒ cross-correlation

CHARGE CURRENT

$$\hat{I}_p(t) = -e\dot{N}_p$$

$$\delta \hat{I}_p(t) = \hat{I}_p(t) - \left\langle \hat{I}_p \right\rangle$$

HEAT CURRENT

$$dQ = dE - \mu dN$$

$$\hat{J}_p(t) = \hat{I}_p^E(t) - \frac{\mu_p}{e} \hat{I}_p(t)$$

$$\delta \hat{J}_p(t) = \hat{J}_p(t) - \left\langle \hat{J}_p \right\rangle$$

GIAZOTTO et al, Rev. Mod. Phys. 78, 217 (2006)
SANCHEZ et al., New J. Phys. 15, 125001 (2013)

LINEAR RESPONSE

RELATIONS BETWEEN NOISES AND CONDUCTANCES

$$\mathcal{S}_{\mathrm{pp}}^{\mathrm{II}} = 2k_B T_0 G$$

$$\mathcal{S}_{\mathrm{pp}}^{\mathrm{JJ}} = 2k_B T_0^2 \tilde{\kappa}$$

$$\mathcal{S}_{\mathrm{pp}}^{\mathrm{IJ}} = \mathcal{S}_{\mathrm{pp}}^{\mathrm{JI}} = -2k_B T_0^2 S G$$

G = electrical conductance
S = Seebeck coefficient
κ = thermal conductance
T_0 = average temperature

$$S = -\frac{V}{T}\bigg|_{I=0}$$

$$\kappa = \tilde{\kappa} - S^2 T_0 G$$

⇒ Fluctuation-dissipation theorem applies for each kind of noises

KUBO et al., J. Phys. Soc. Jpn. 12, 1203 (1957)

FIGURE OF MERIT

$$ZT_0 = \frac{S^2 T_0 G}{\kappa} = \frac{\left(\mathcal{S}_{\mathrm{pq}}^{\mathrm{IJ}}\right)^2}{\mathcal{S}_{\mathrm{pq}}^{\mathrm{II}} \mathcal{S}_{\mathrm{pq}}^{\mathrm{JJ}} - \left(\mathcal{S}_{\mathrm{pq}}^{\mathrm{IJ}}\right)^2}$$

Independent of p and q

CREPIEUX / MICHELINI, arXiv:1403:8035 (2014)

⇒ ZT_0 is not upper bounded

LITTMANN/DAVIDSON, J. App. Phys. 32, 217 (1961)

Argument of entropy production

$$\frac{\left(\mathcal{S}_{\mathrm{pq}}^{\mathrm{IJ}}\right)^2}{\mathcal{S}_{\mathrm{pq}}^{\mathrm{II}} \mathcal{S}_{\mathrm{pq}}^{\mathrm{JJ}}} \leq 1$$

Cauchy-Swartz inequality

WHAT ABOUT THE NON-LINEAR REGIME ?

$$\begin{pmatrix} I \\ J \end{pmatrix} \neq \begin{pmatrix} G & SG \\ \Pi G & \tilde{\kappa} \end{pmatrix} \begin{pmatrix} V \\ T \end{pmatrix}$$

$\Rightarrow$ Optimization of the figure of merit does not guarantee a maximum of thermoelectric efficiency

$$\eta_{\max} \neq \eta_C \frac{\sqrt{1+ZT_0}-1}{\sqrt{1+ZT_0}+1} \qquad \eta_C = \text{Carnot efficiency}$$

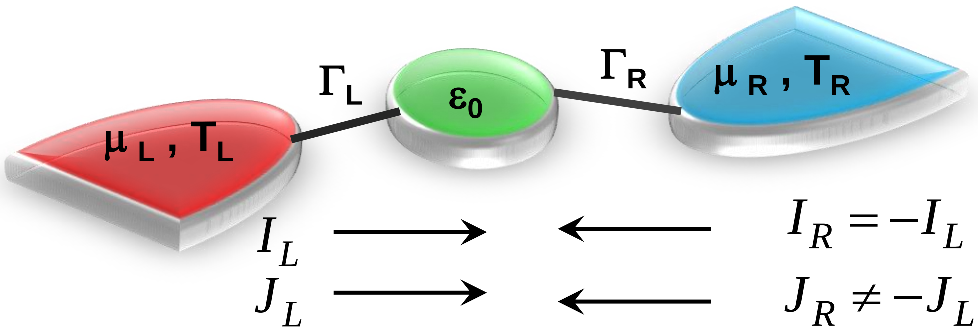
$\Rightarrow$ One has rather to consider the ratio between electric and thermal powers

WHITNEY's talk

$$\eta = \frac{P_{out}^{th}}{P_{in}^{el}}$$ Voltage to heat conversion

$$\eta = \frac{P_{out}^{el}}{P_{in}^{th}}$$ Heat to voltage conversion

SINGLE LEVEL QUANTUM DOT



$$\mu_{L,R} = \varepsilon_F \pm eV/2$$

$$T_{L,R} = T_0 \pm T/2$$

$$\Gamma = \Gamma_L + \Gamma_R$$

LANDAUER-LIKE EXPRESSION

$$\mathcal{S}_{\mathrm{pq}}^{\alpha\beta} = \pm \frac{1}{h} \int_{-\infty}^{\infty} \left(\varepsilon - \mu_p\right)^{n_\alpha} \left(\varepsilon - \mu_q\right)^{n_\beta} F(\varepsilon) d\varepsilon$$

$$\alpha, \beta \in \{I, J\} \qquad n_I = 0 \qquad n_J = 1$$

$$F(\varepsilon) = \mathcal{T}(\varepsilon)\left[f_L(\varepsilon)(1 - f_L(\varepsilon)) + f_R(\varepsilon)(1 - f_R(\varepsilon))\right] + \mathcal{T}(\varepsilon)\left[1 - \mathcal{T}(\varepsilon)\right]\left[f_L(\varepsilon) - f_R(\varepsilon)\right]^2$$

$$\mathcal{T}(\varepsilon) = \frac{\Gamma^2}{(\varepsilon - \varepsilon_0)^2 + \Gamma^2} \qquad f_{L,R}(\varepsilon) = \left[1 + \exp\left(\frac{\varepsilon - \mu_{L,R}}{k_B T_{L,R}}\right)\right]^{-1}$$

$\Rightarrow$ This approach allows to study the noises varying V, T, T_0, ε_0 and Γ

NOISES

GENERAL RELATION

$$\sum_{p,q}\mathcal{S}^{\mathrm{I\,I}}_{\mathrm{pq}}=\sum_{p,q}\mathcal{S}^{\mathrm{I\,J}}_{\mathrm{pq}}=\sum_{p,q}\mathcal{S}^{\mathrm{J\,I}}_{\mathrm{pq}}=0$$

⇒ Charge conservation

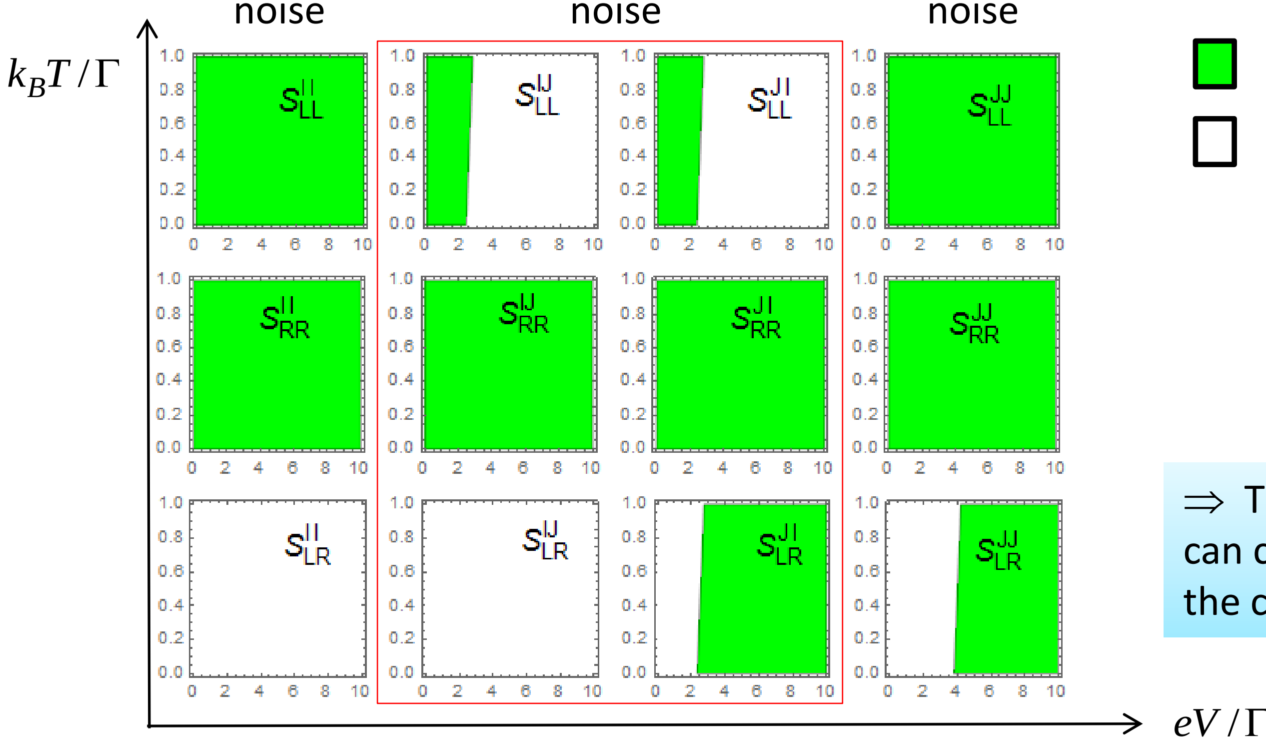
$$\sum_{p,q}\mathcal{S}^{\mathrm{J\,J}}_{\mathrm{pq}}=V^2\,\mathcal{S}^{\mathrm{I\,I}}_{\mathrm{LL}}$$

⇒ Power conservation

$$\hat{J}_L+\hat{J}_R=V\,\hat{I}_L\Leftrightarrow P^{th}=P^{el}$$

CHANGE OF SIGN

Charge noise | Mixed noise | Heat noise



■ region of positive sign

□ region of negative sign

$$\varepsilon_0/\Gamma=2$$

$$k_BT_0/\Gamma=1$$

$$\mathcal{S}^{\alpha\beta}_{\mathrm{RL}}=\mathcal{S}^{\beta\alpha}_{\mathrm{LR}}$$

⇒ The heat cross-correlator can change its sign contrary to the charge cross-correlator

SCHOTTKY REGIME

WEAK TRANSMISSION $\mathcal{T}(\varepsilon) << 1$

Tight energy-charge coupling: $J_R = (\varepsilon_0 - \mu_R)\, I_R$

ESPOSITO et al., Eur. Phys. Lett. 85, 60010 (2009)

NOISES

$$\mathcal{S}_{LR}^{\rm II} = C\, e\, I_R$$
$$\mathcal{S}_{LR}^{\rm JJ} = C(\varepsilon_0 - \mu_L)\, J_R$$
$$\mathcal{S}_{LR}^{\rm IJ} = C\, e\, J_R = C(\varepsilon_0 - \mu_R)\, I_R$$

$$C = \coth\left(\frac{\varepsilon_0 - \mu_R}{2k_B T_R} - \frac{\varepsilon_0 - \mu_L}{2k_B T_L}\right) = 1 \text{ when } T_{L,R} = 0$$

$\Rightarrow$ Noises are proportional to currents

EFFICIENCY

$$\eta = \frac{P^{th}}{P^{el}} = \left|\frac{J_R}{I_R V}\right|$$

$$J_R = \frac{\mathcal{S}_{LR}^{\rm IJ}}{Ce} \qquad I_R = \frac{\mathcal{S}_{LR}^{\rm II}}{Ce} \qquad eV = \frac{\mathcal{S}_{LR}^{\rm IJ}}{C\, I_L} - \frac{\mathcal{S}_{LR}^{\rm JJ}}{C\, J_R}$$

EQUIVALENTLY

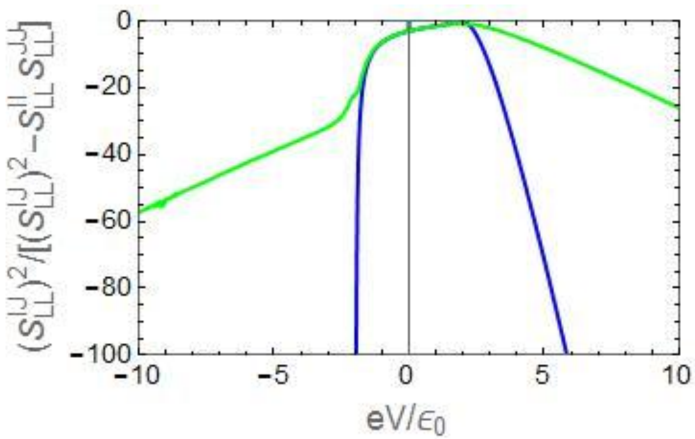
$$\eta = \frac{\left(\mathcal{S}_{\rm LR}^{\rm IJ}\right)^2}{\left(\mathcal{S}_{\rm LR}^{\rm IJ}\right)^2 - \mathcal{S}_{\rm LR}^{\rm II}\mathcal{S}_{\rm LR}^{\rm JJ}}$$

Independent on C

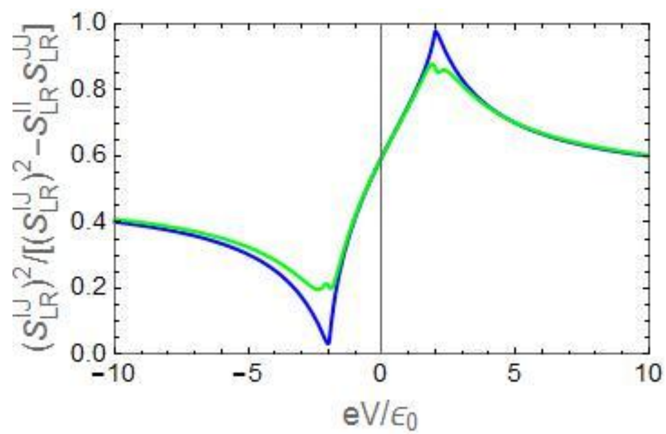
$\Rightarrow$ The efficiency can be written as a ratio of noises

NUMERICAL VERIFICATION

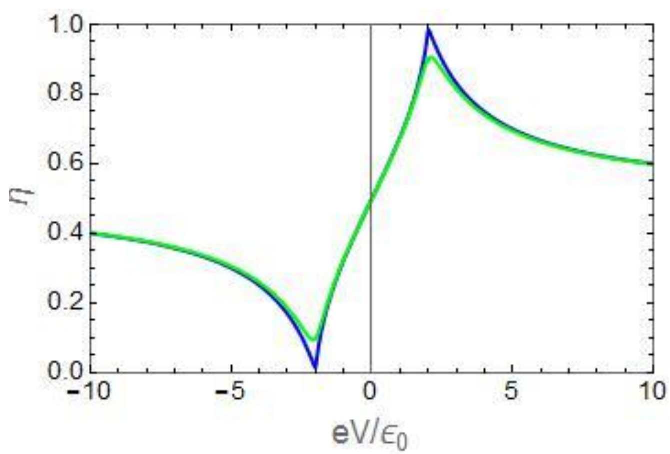
AUTO-RATIO



CROSS-RATIO



EFFICIENCY



$\Gamma/\epsilon_0=0.01$

$\Gamma/\epsilon_0=0.1$

$k_B T/\varepsilon_0 = 0$

$k_B T_0/\varepsilon_0 = 0.001$

$\Rightarrow$ The efficiency fits with the cross-ratio !
But it has no relation with the auto-ratio

CONCLUSION

MIXED NOISE IN DISTINCT RESERVOIRS

$$\mathcal{S}_{\mathrm{LR}}^{\mathrm{IJ}} = \int_{-\infty}^{\infty} \left\langle \delta \hat{I}_L(0) \delta \hat{J}_R(t) \right\rangle dt$$

= a measure of thermoelectric conversion

LINEAR RESPONSE REGIME

$$ZT_0 = \frac{\left(\mathcal{S}_{\mathrm{pq}}^{\mathrm{IJ}}\right)^2}{\mathcal{S}_{\mathrm{pq}}^{\mathrm{II}} \mathcal{S}_{\mathrm{pq}}^{\mathrm{JJ}} - \left(\mathcal{S}_{\mathrm{pq}}^{\mathrm{IJ}}\right)^2}$$

SCHOTTKY REGIME

$$\eta = \frac{\left(\mathcal{S}_{\mathrm{LR}}^{\mathrm{IJ}}\right)^2}{\left(\mathcal{S}_{\mathrm{LR}}^{\mathrm{IJ}}\right)^2 - \mathcal{S}_{\mathrm{LR}}^{\mathrm{II}} \mathcal{S}_{\mathrm{LR}}^{\mathrm{JJ}}}$$

CREPIEUX / MICHELINI, arXiv:1403:8035 (2014)

PERSPECTIVES

- ❑ Mixed noise in a three terminals systems
- ❑ More realistic transmission coefficient $\mathcal{T}(\varepsilon, V)$
- ❑ Effect of coulomb interactions
- ❑ Mixed noise at finite frequency and/or with ac-driven

Thanks to P. Eyméoud, M. Guigou, and R. Whitney