Phase-charge duality in Josephson junction circuits: effect of microwave irradiation*

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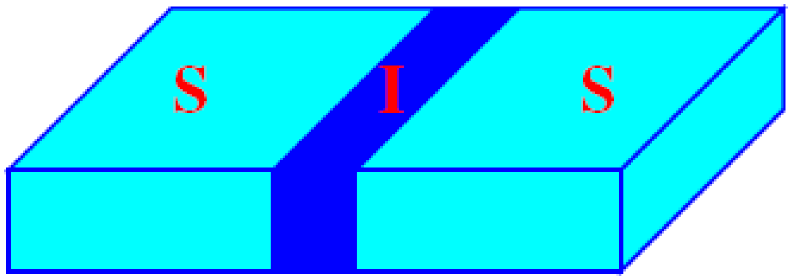
I. Ultrasmall Josephson junctions

Some basic notions

Small Josephson junction



Josephson relations:

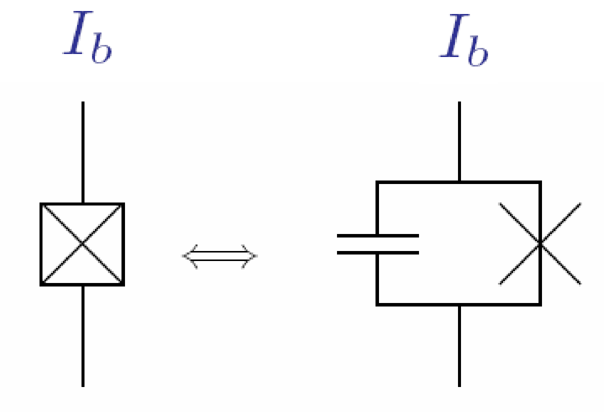
$$I = I_c \sin\phi$$

$$\dot{\phi} = 2eV/\hbar$$

Small Josephson junction: two energy scales

$$E_C = (2e)^2/2C$$

$$E_J = \hbar I_c/(2e)$$



Hamiltonian

$$H = E_c(Q/2e)^2 - E_J\cos\phi - I_b\phi$$

Commutator

$$[\phi, Q] = 2ie$$

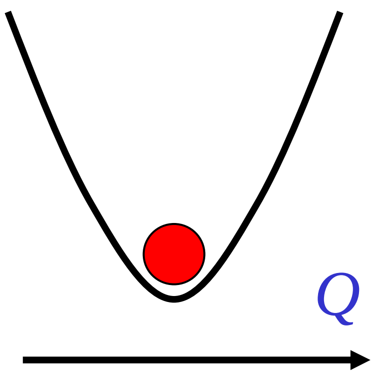
Superconductor vs. insulator behavior: consequence of phase-charge duality

$$H = E_c(Q/2e)^2 - E_J \cos\phi$$

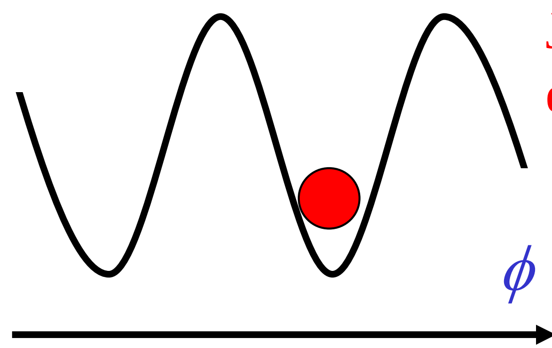
Localized charge

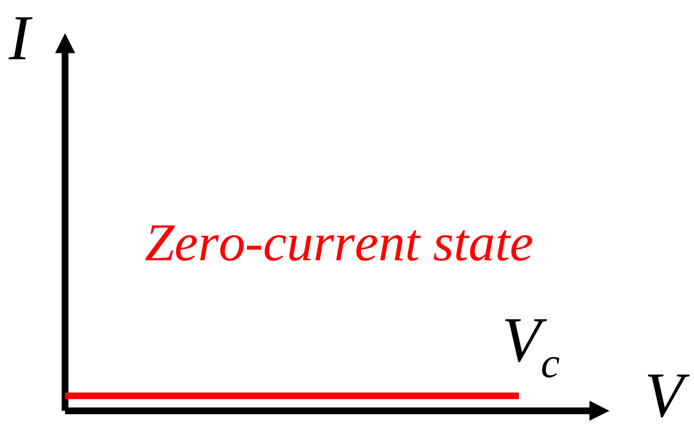
Localized phase

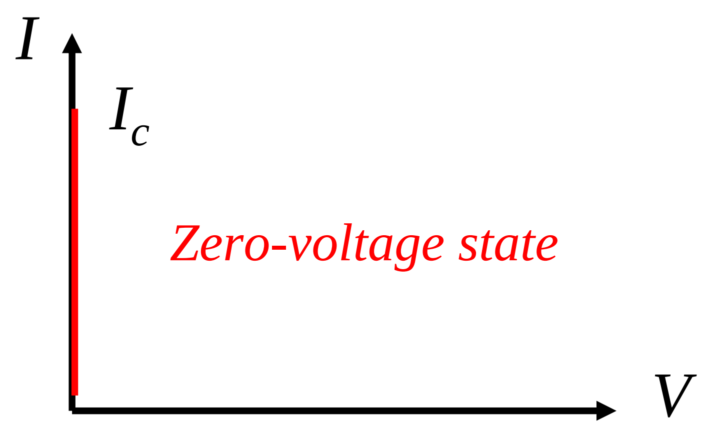
Coulomb blockade



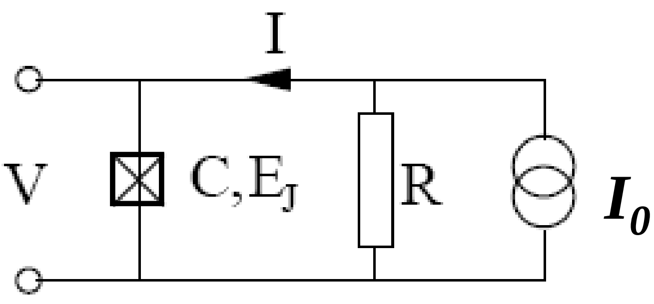
Josephson effect







Effect of environment: RCSJ model

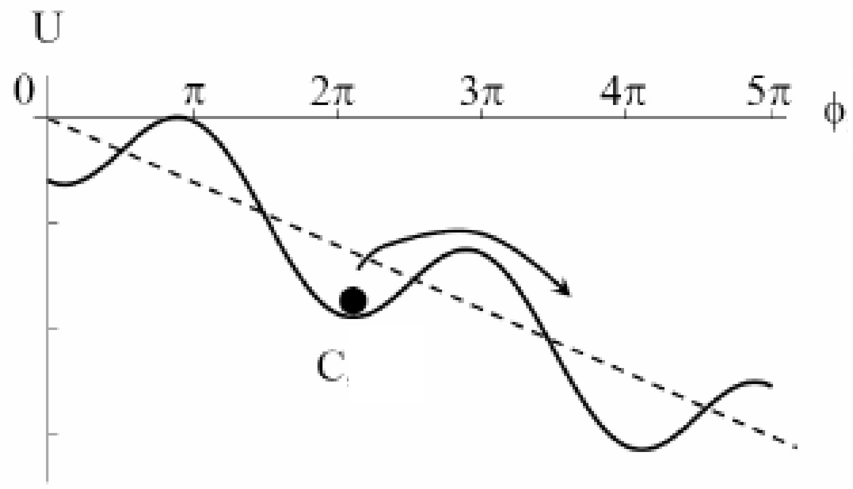


$$\frac{d^2\phi}{dt^2} + \sigma\frac{d\phi}{dt} + \sin\phi = i_0$$

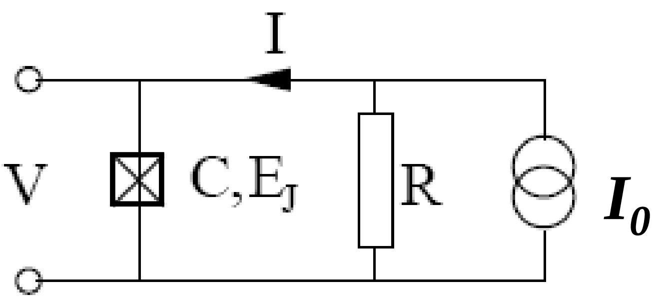
displacement current — *current trhu resistor* — *supercurrent*

$$\omega_p = (2eI_c/\hbar C)^{1/2} \qquad \sigma = G(\hbar/2eI_cC)^{1/2} \qquad i_0 = I_0/I_c$$

Dynamics of fictitious phase particle



Effect of environment: RCSJ model

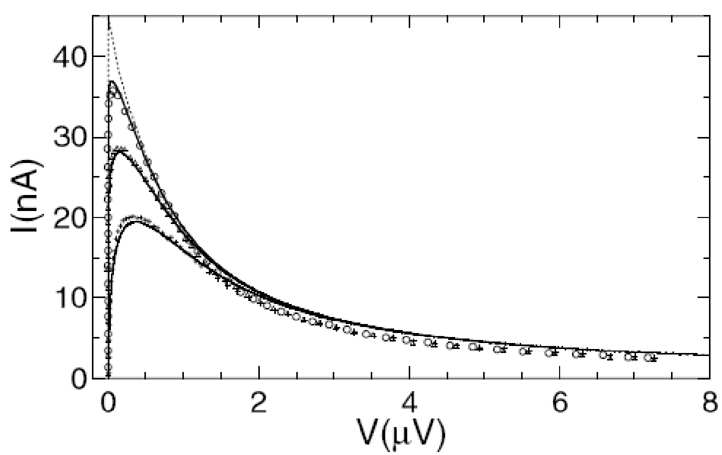


$$\cancel{\frac{d^2\phi}{dt^2}} + \sigma\frac{d\phi}{dt} + \sin\phi = i_0 + \delta i_{ther}$$

displacement current — *current trhu resistor* — *supercurrent*

$$\omega_p = (2eI_c/\hbar C)^{1/2} \qquad \sigma = G(\hbar/2eI_cC)^{1/2} \qquad i_0 = I_0/I_c$$

Overdamped limit: effect of thermal noise



$$I(V_B) = I_0\,\mathrm{Im}\left[\frac{I_{1-2i\beta eV_B/\hbar R_B}(\beta E_J)}{I_{-2i\beta eV_B/\hbar R_B}(\beta E_J)}\right]$$

Steinbach et al., '01

Ivanchenko & Zilbermann '69

Effect of microwaves: Shapiro voltage steps

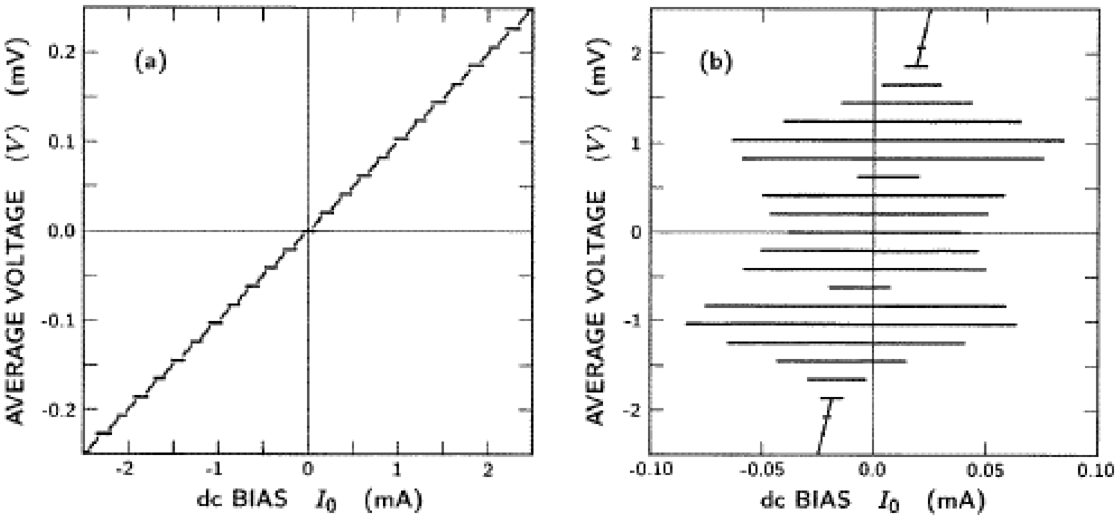
Dynamics of phase

$$\frac{d^2\phi}{dt^2} + \sigma\frac{d\phi}{dt} + \sin\phi = i_0 + i_1 \sin\Omega_1 t.$$

Finite voltage: Josephson oscillations $\langle V\rangle = \left\langle \frac{d\phi}{dt} \right\rangle = f_J$

Phase-locking: voltage-Shapiro steps at $V_n = nhf_1 / 2e$

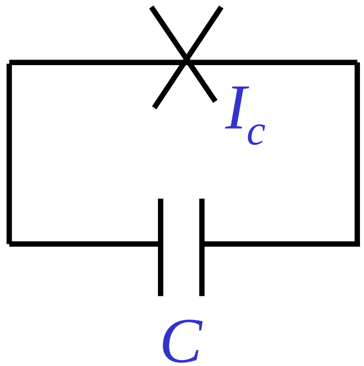
$\sigma > 1$ $\sigma < 1$



Kautz '96

Duality: Josephson junction versus phase-slip junction

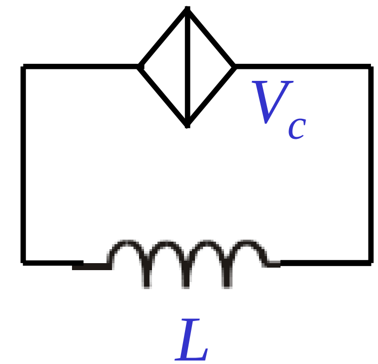
Cooper pair tunneling



Duality

$$[\theta, Q] = -2ie$$

Phase-slip



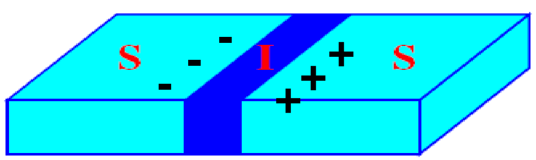
$$\hat{H} = \frac{\hat{Q}^2}{2C} - E_J \cos\hat{\theta}$$

$$\hat{\theta} \leftrightarrow \hat{q}$$

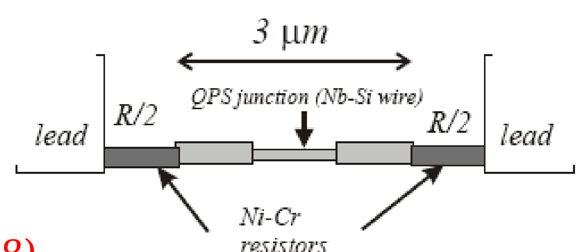
$$\hat{H} = \frac{\hat{\theta}^2}{2L} - \Delta_0 \cos\hat{q}$$

$$E_J = \frac{\hbar}{2e} I_c$$

$$\Delta_0 = eV_c$$

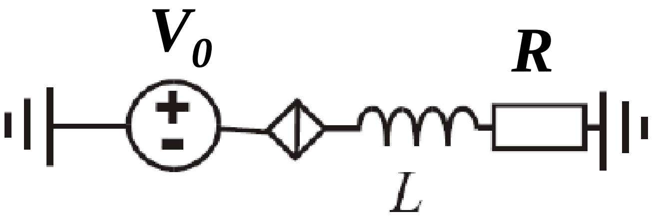


Schmid (1983)
Likharev, Averin, Zorin (1985)
Schön, Zaikin (1990)
Ingold and Nazarov (1992)
Weiss (1999)
Beloborodov et al. (2002)
Zazunov et al. (2008)
Arutyonov, Golubev & Zaikin (2008)



J.E. Mooij and Yu.V. Nazarov (2006)

Consequences of duality: IV characteristics & Shapiro steps

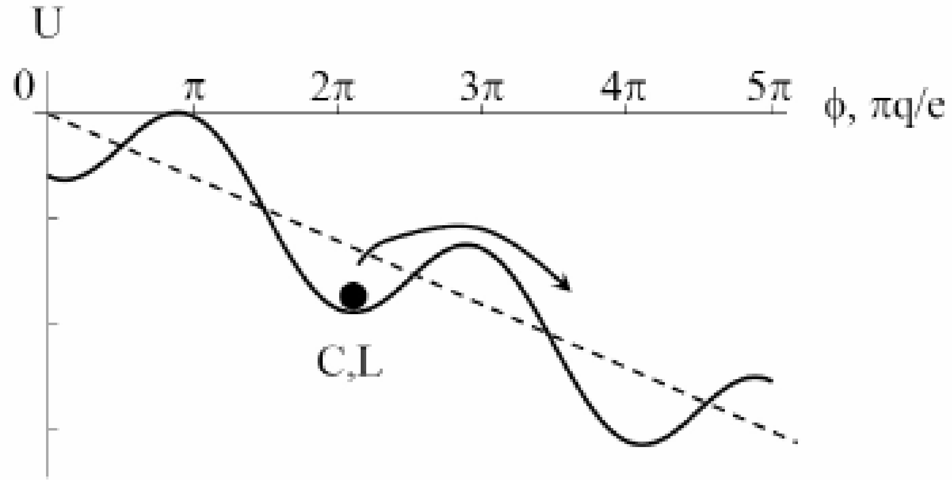


$$\frac{d^2q}{dt^2} + \rho\frac{dq}{dt} + \sin q = v_0$$

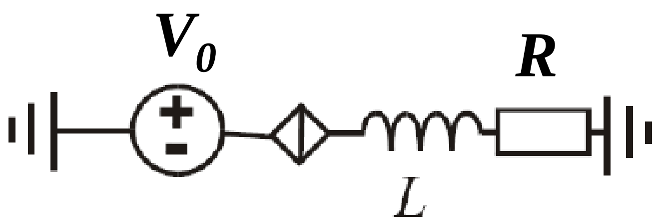
inductance *resistor* *phase slip element*

$$\omega_c = (\pi V_c/eL)^{1/2} \qquad v_0 = V_0/V_c \qquad \rho = R(e/\pi V_c L)^{1/2}$$

Phase vs. charge particle



Consequences of duality: IV characteristics & Shapiro steps

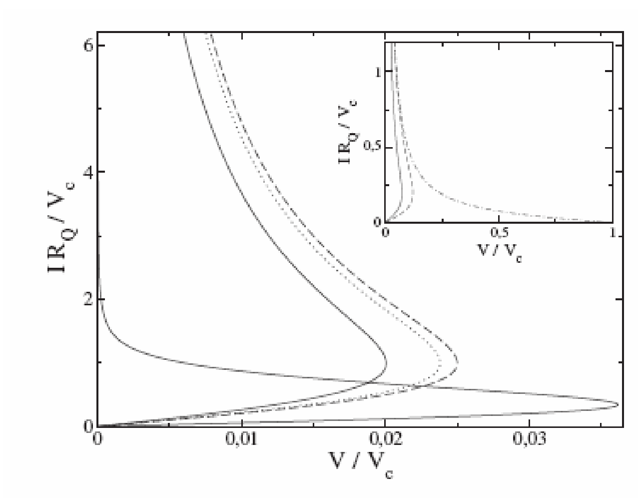


$$\frac{d^2q}{dt^2} + \rho\frac{dq}{dt} + \sin q = v_0 + \delta v_{MW}(t)$$

inductance *resistor* *phase slip element*

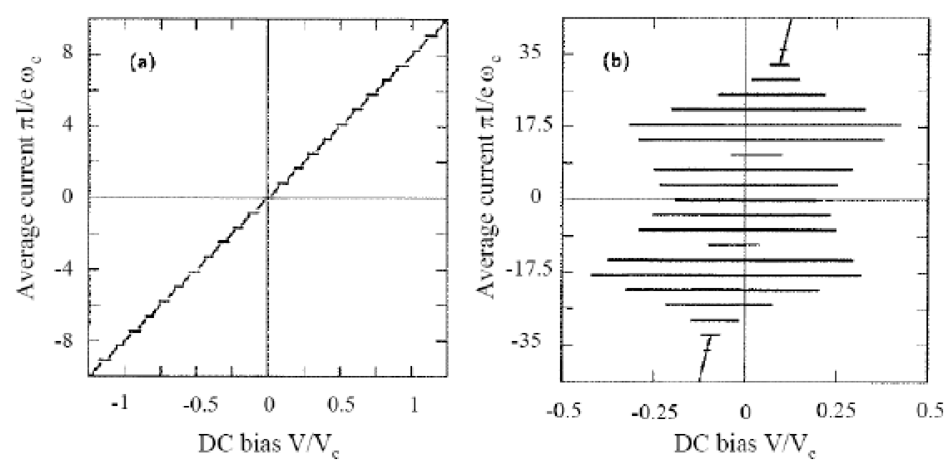
$$\omega_c = (\pi V_c/eL)^{1/2} \qquad v_0 = V_0/V_c \qquad \rho = R(e/\pi V_c L)^{1/2}$$

Expected I-V characteristics



Effect of microwaves: current-Shapiro steps

$\rho > 1$ $\rho < 1$

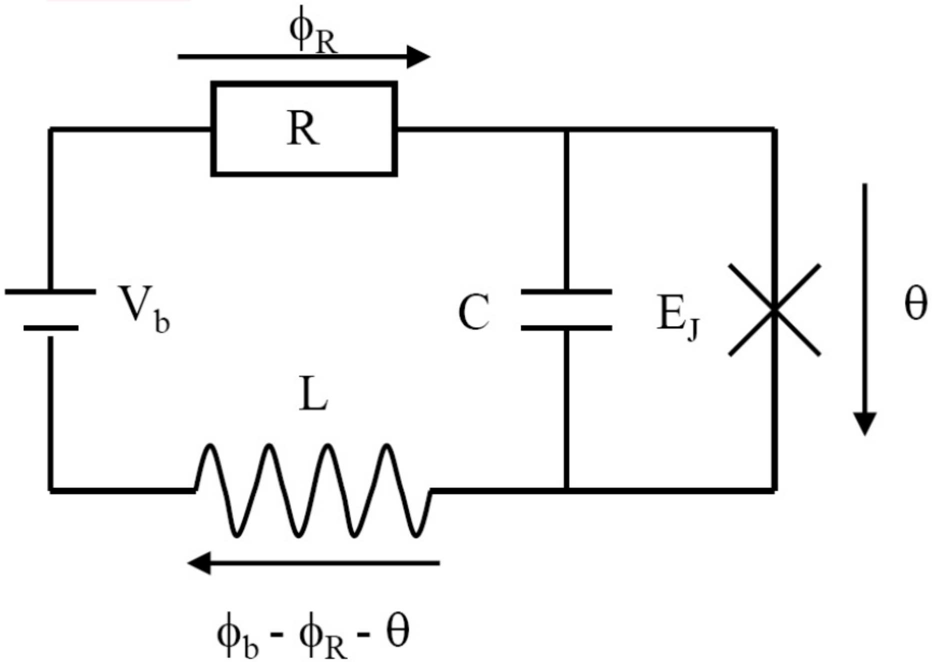


II. Voltage-biased junction in an inductive environment

Single Josephson junction as a phase-slip element

CJRL-model: Circuit & Hamiltonian

Apenko (1989), Schön & Zaikin (1990), Zazunov, Didier & FH (2008), Guichard & FH (2009)



$$\hat{H} = \hat{H}_J + \hat{H}_L + \hat{H}_B'$$

Josephson junction

$$\hat{H}_J = \frac{\hat{Q}^2}{2C} - E_J \cos\hat{\theta}$$

Inductance

$$\hat{H}_L = \left(\frac{\hbar}{2e}\right)^2 \frac{(\phi_b - \hat{\phi}_R - \hat{\theta})^2}{2L}$$

Bath Hamiltonian

$$\hat{H}_B' = \sum_{i=1}^{\infty} \frac{\hat{\Pi}_i^2}{2} + \frac{1}{2}\tilde{\omega}_i^2 \hat{\Xi}_i^2$$

Coupling

$$\hat{\phi}_R = \sum_i \lambda_i \hat{\Xi}_i$$

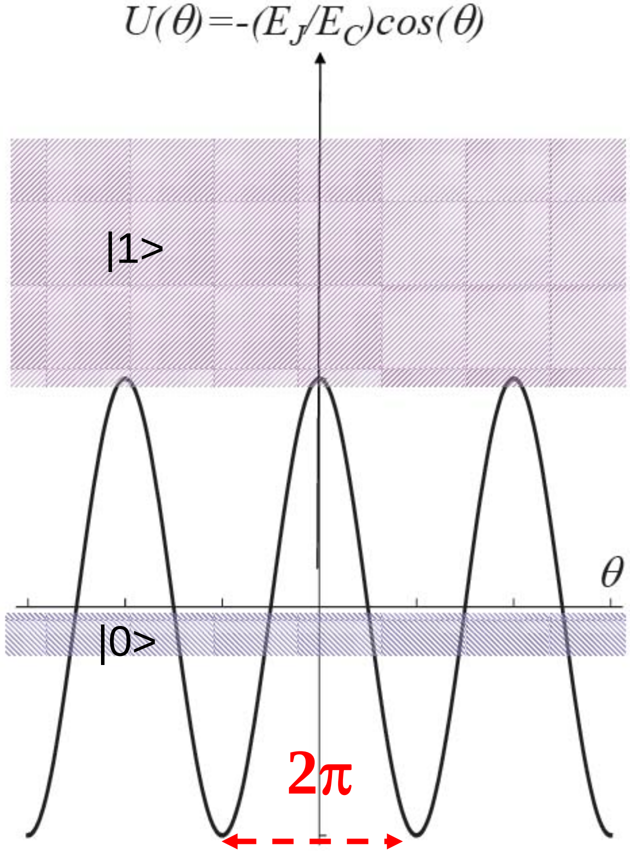
Bias phase

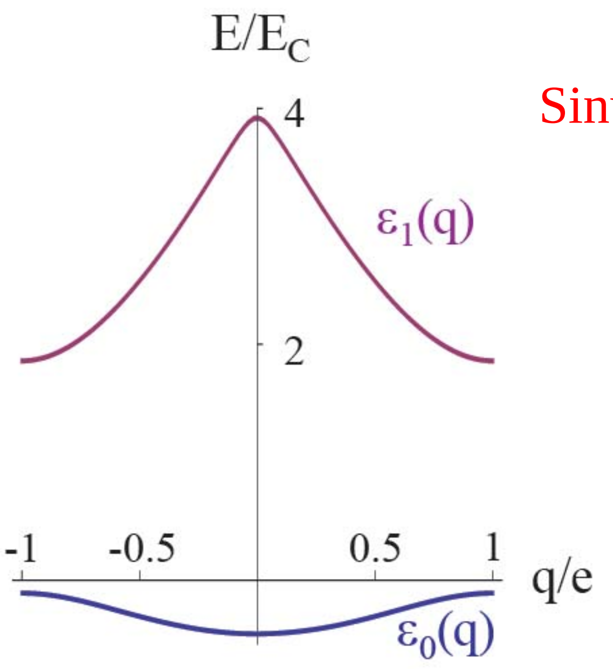
$$\frac{\hbar}{2e}\dot{\varphi}_b = V_b$$

Single junction: Bloch Bands

Likharev,Averin,Zorin (1985)

Periodic Hamiltonian: $\hat{H}_J = \frac{\hat{Q}^2}{2C} - E_J \cos\hat{\theta}$ $\quad$ *$E_J >> E_C$* → *Tight-Binding Model*





Sinusoïdal lowest band:

$$\epsilon_0(q) = -\Delta_0 \cos \pi q/e$$

Phase-slip amplitude:

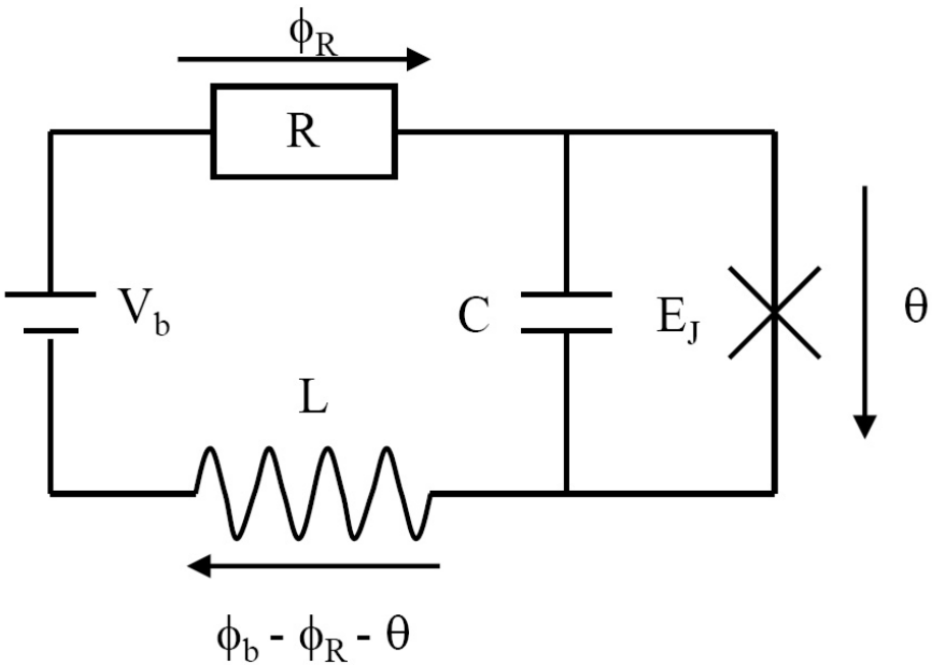
$$\Delta_0 \approx \left(E_J^3 E_C\right)^{1/4} \exp\left(-\sqrt{8E_J / E_C}\right)$$

Quasicharge

$$\hat{H} = \frac{\hat{Q}^2}{2C} - E_J \cos\hat{\theta} + \frac{1}{2L}\left(\varphi_b - \hat{\varphi}_R - \hat{\theta}\right)^2 + H_{bath}$$

Dynamics in lowest Bloch band

$$\hat{H} = \varepsilon_0(\hat{q}) + \frac{1}{2L}\left(\varphi_b - \hat{\varphi}_R - \hat{\theta}\right)^2 + H_{bath}$$



Equation of motion for quasicharge

$$\frac{d\hat{q}}{dt} = i[\hat{H}, \hat{q}] = d\hat{H} / d\varphi_b = \hat{I}$$

Quasicharge = total momentum of JJ that is conserved without external current

Equation of motion for quasicharge

Guichard & FH (2009)

$$L\ddot{\hat{q}} + \partial\epsilon_0(q)/\partial q = V_b + \delta\hat{V}$$

$$\delta\hat{V} = \hat{v}(t) - \int_0^t Z(t-t')\dot{\hat{q}}(t')$$

$$Z(\omega) = \left(\frac{\hbar}{2e}\right)^2 \sum_{i=1}^{\infty} \lambda_i^2 \frac{i\omega}{(\omega+i\eta)^2 - \tilde{\omega}_i^2}$$

$$\int dt e^{i\omega t}\langle\{\hat{v}(t),\hat{v}(0)\}\rangle/2$$

$$= \hbar\omega\Re e[Z(\omega)]\coth(\hbar\omega/2k_BT)$$

$$E_J >> E_C$$

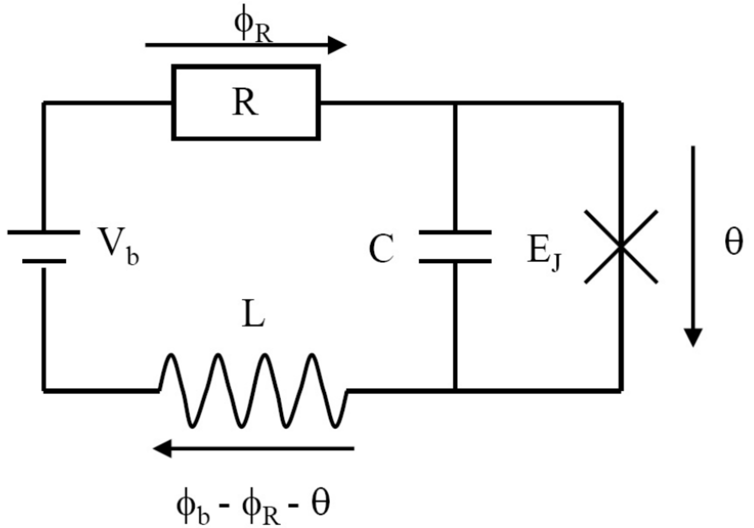
$$Z(\omega) \to R$$

$$L\ddot{\hat{q}} + R\dot{\hat{q}} + V_c \sin\pi q/e = V_b + \hat{v}$$

$$\omega_c = (\pi V_c/eL)^{1/2}$$

$$\rho = R(e/\pi V_c L)^{1/2}$$

$$d^2\hat{\bar{q}}/d\tau'^2 + \rho d\hat{\bar{q}}/d\tau' + \sin\hat{\bar{q}} = \bar{V}_b + \hat{\bar{v}}$$



Exactly dual to usual RCSJ model!

Equation of motion for quasicharge: quasiclassical approach

Guichard & FH (2009)

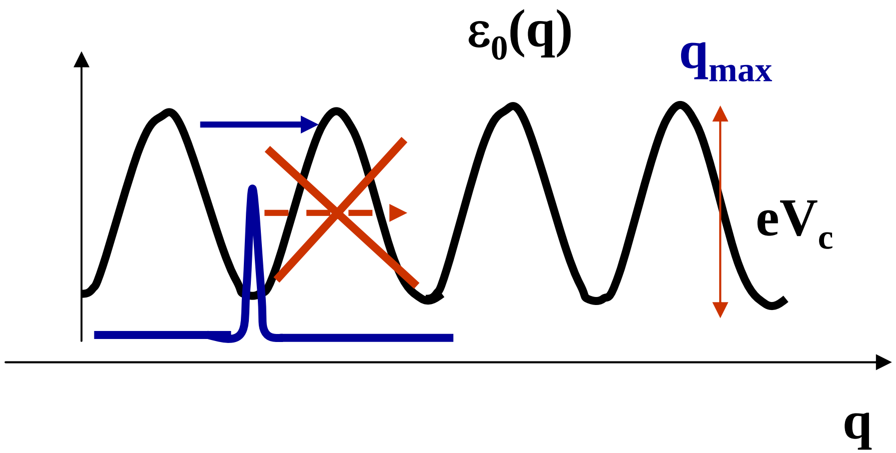
$$L\ddot{\hat{q}} + R\dot{\hat{q}} + V_c \sin \pi q/e = V_b + \hat{v}$$

Quasiclassical dynamics

Narrow wave packets, thermal noise

$$\boxed{L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + V_c \sin(\pi q/e) = V_b + v}$$

Ignore wave-packet spreading on scale $1/\omega_c$, L/R



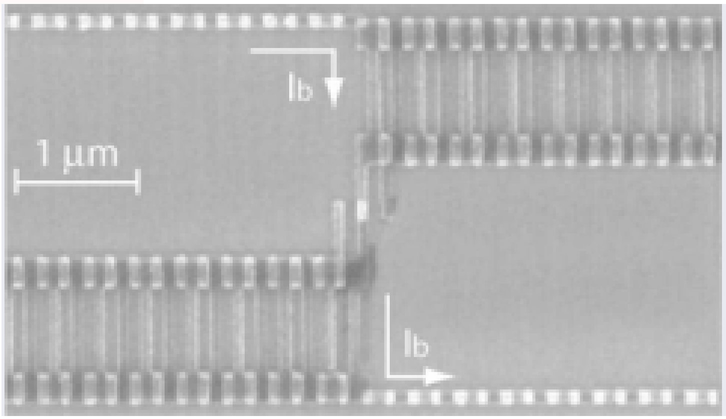
Classical noise:

$$\langle v(t)v(0)\rangle = 2k_BTR\delta(t)$$

ω_c, $R/L < T < \Delta_0$

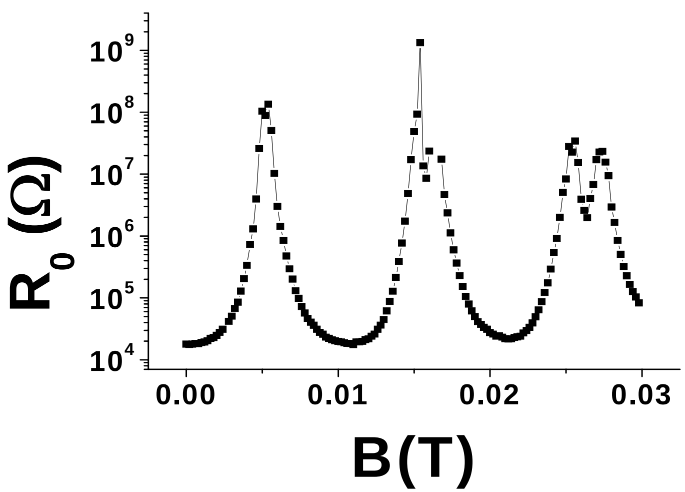
Overdamped quasicharge dynamics in a phase-slip junction: experiment

Corlevi, Guichard, FH & Haviland (2006); Beloborodov, FH & Pistolesi (2002)

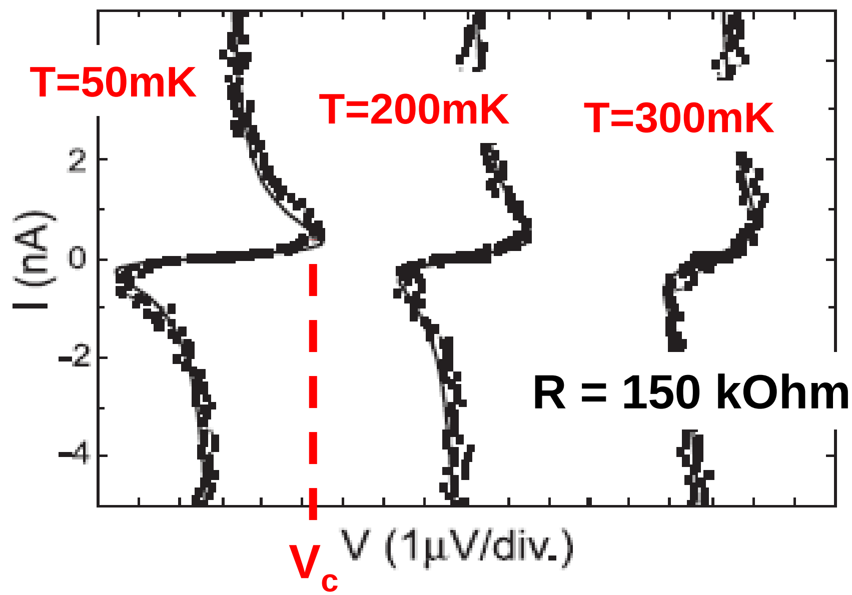


$$\boxed{L\cancel{\frac{d^2q}{dt^2}} + R\frac{dq}{dt} + V_c \sin(\pi q/e) = V_b + v_{ther}}$$

$$\langle V\rangle = V_C \Im\mathrm{m}\left[\frac{I_{1-i\beta e I_b R/\pi}(\beta e V_C/\pi)}{I_{-i\beta e I_b R/\pi}(\beta e V_C/\pi)}\right]$$



Dual to Steinbach et al.!

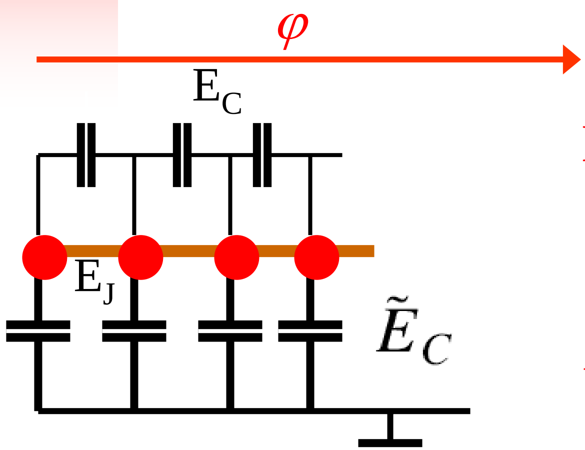


III. One-dimensional junction arrays

Josephson junction chain as a phase-slip element
(providing its own inductance)

Phase-biased Josephson junction array

Matveev, Larkin, Glazman (2002)



Ignore self-capacitance: $\tilde{E}_C \gg E_J, E_C.$

Action: $$S = \int_0^\beta dt \sum_{n=1}^{N} \left\{ \frac{\dot{\theta}_n^2}{2E_C} + E_J[1 - \cos\theta_n(t)] \right\}.$$

Phase bias leads to constraint: $$\sum_{n=1}^{N} \theta_n(t) = \varphi$$

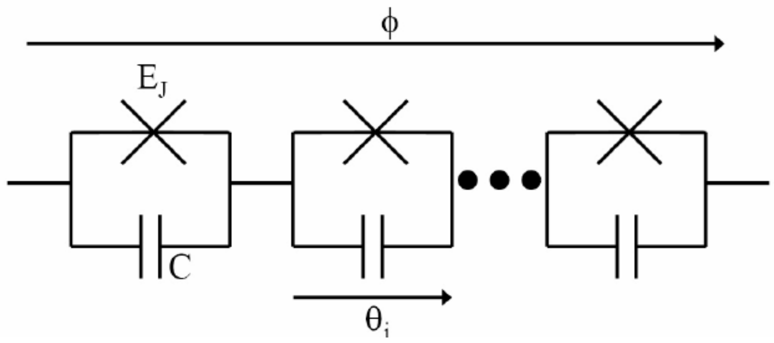
Strong Josephson coupling & long array: $E_J \gg E_C$ $\quad N \gg 1$

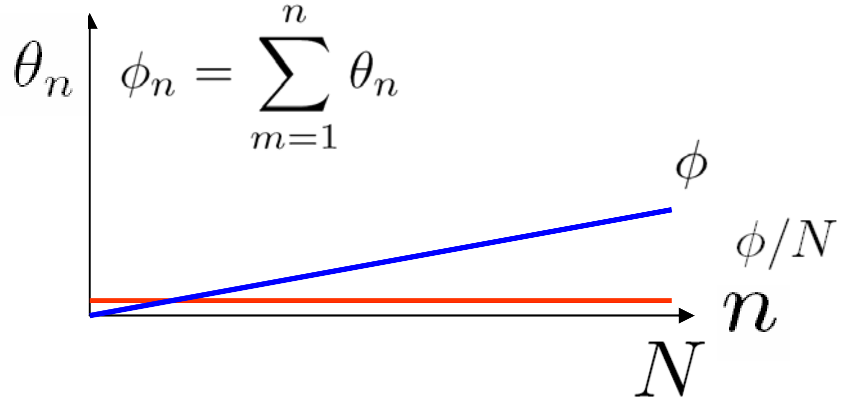
Classical energy of chain: minimize potential energy for large N, imposing the constraint

$$\theta_n = \phi/N \ll 1 \longrightarrow E_{class} = \frac{E_J}{2N}\phi^2$$ *(inductor)*

Phase-slips in phase-biased arrays

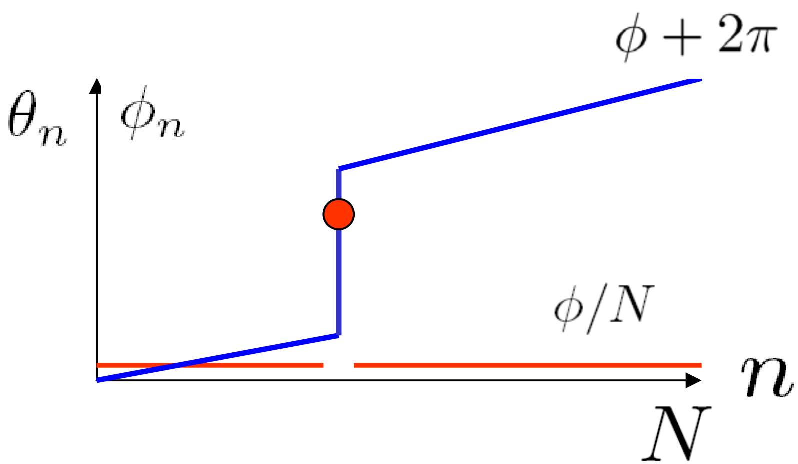
Matveev, Larkin, Glazman (2002)

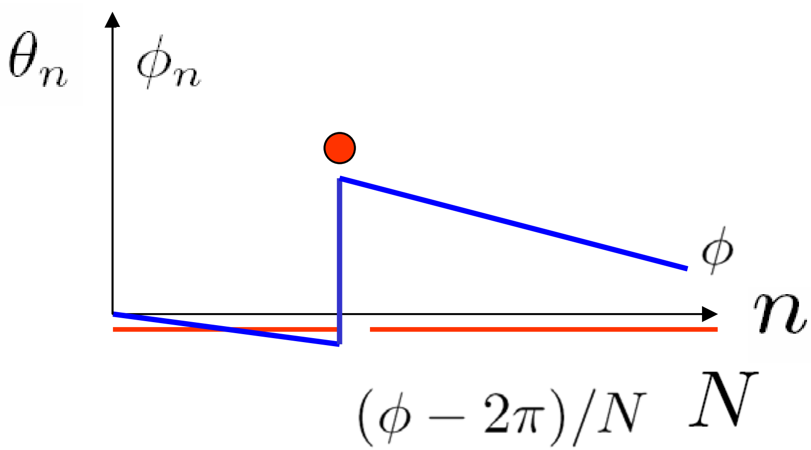




Phase slip: energy unchanged,
but constraint violated

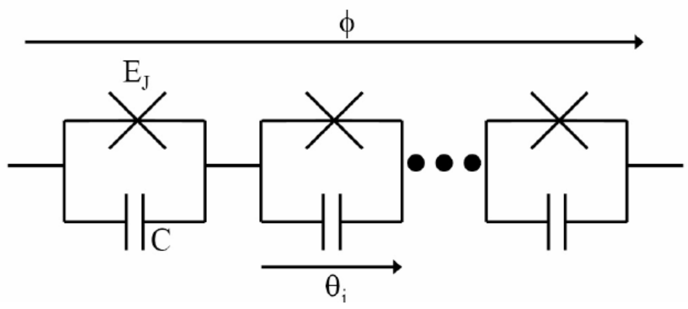
Phase slip combined with small adjustment:
constraint satisfied





Hamiltonian for quantum phase-slip array

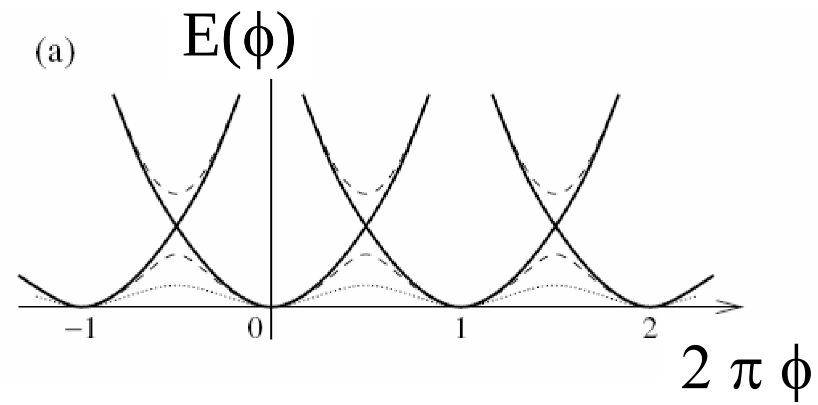
Matveev, Larkin, Glazman (2002)



Classical energy for *m* phase slips:

$$E_J(\phi - 2\pi m)^2/2N$$

Effect of capacitance: quantum fluctuations



Phase slip at junction k:

$$\theta_k(t) = 4 \arctan \exp[\sqrt{E_J E_C}\,(t - t')],$$

Phase slip amplitude:

$$\Delta_0 \approx \left(E_J^3 E_C\right)^{1/4} \times \exp\left(-\sqrt{8E_J / E_C}\right)$$

Hamiltonian:

Phase-slip can occur on any junction

$$\hat{H}_{\mathrm{ar}} = \frac{E_J}{2N}(2\pi\hat{m} - \phi)^2 - \frac{N\Delta_0}{2}\sum_m [|m+1\rangle\langle m| + h.c.]$$

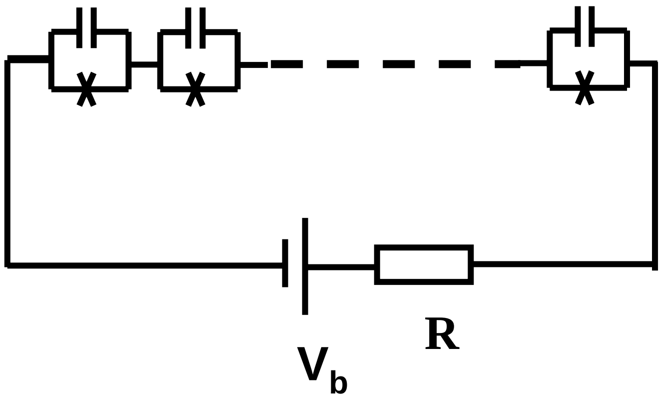
Phase-slip array in quasi-charge representation

Guichard & FH (2009)

Introduce « quasicharge representation » $[\hat{q},\hat{m}] = -ie/\pi$

Rewrite Hamiltonian $\hat{H}_{\mathrm{ar}} = (E_J/2N)(2\pi\hat{m}-\phi)^2 - N\Delta_0 \cos\pi\hat{q}/e$

Quasicharge dynamics related to current $\dot{\hat{q}} = (2e/\hbar)d\hat{H}/d\breve{\phi}$



$$L_{\mathrm{ar}}\ddot{\hat{q}} + R\dot{\hat{q}} + V_{c,\mathrm{ar}}\sin\pi\hat{q}/e = V_b + \hat{v}$$

$$L_{\mathrm{ar}} = \hbar N/2eI_c \quad V_{c,\mathrm{ar}} = NV_c$$

Tunable parameters!

Again exactly dual to usual RCSJ model!

IV. Phase-slip junction under microwave irradiation

Current Shapiro steps: towards a current standard?

Microwaves & Shapiro steps

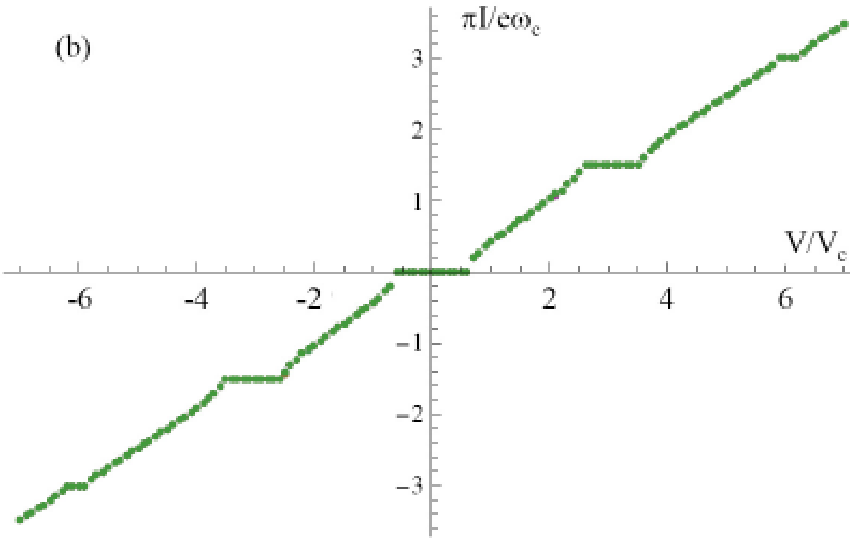
Guichard & FH (2009)

$$\frac{d^2q}{dt^2} + \rho\frac{dq}{dt} + \sin q = v_0 \qquad v_0 = V_b/V_c$$

$$V_b(t) = V + V_{\mathrm{MW}}\sin(\omega_{\mathrm{MW}}t)$$

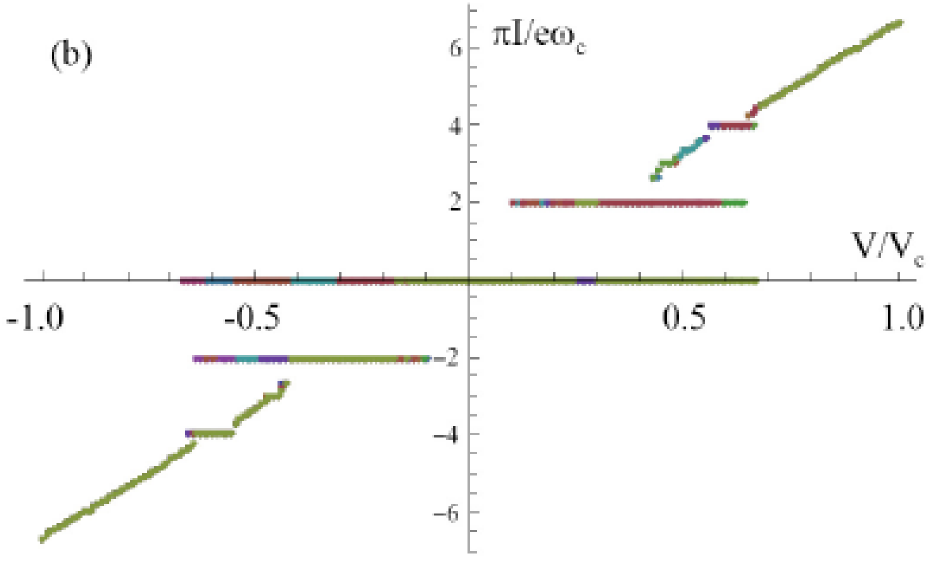
Overdamped case

$\rho=2,\ V_{MW}=5V_c,\ \omega_{MW}=1.5\omega_c$

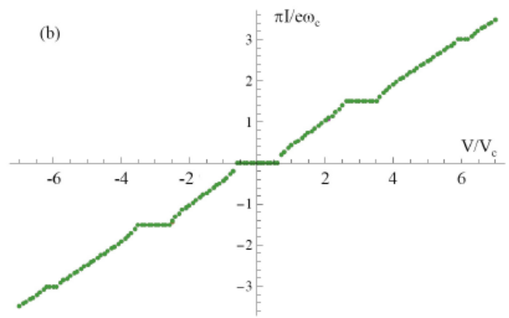


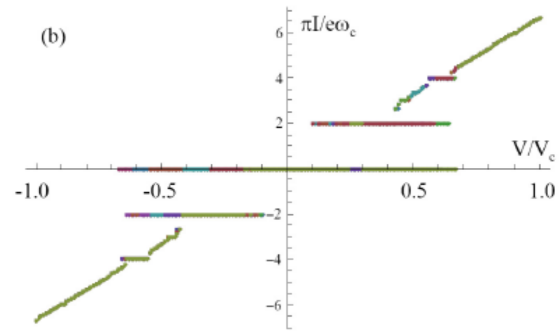
Underdamped case

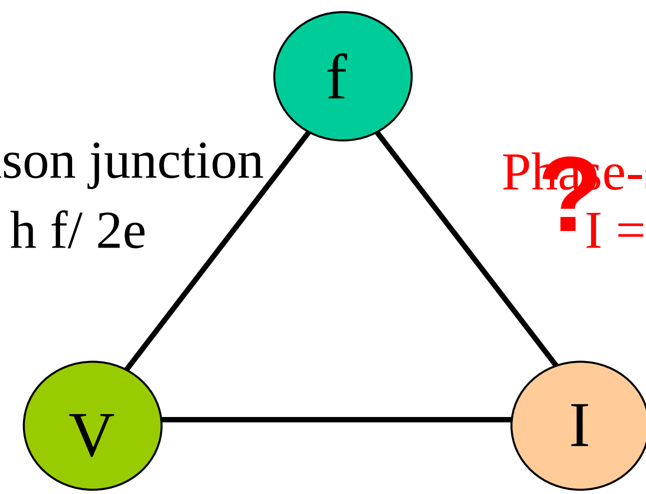
$\rho=0.15\ V_{MW}=3V_c,\ \omega_{MW}=2\omega_c$



Phase-slip junctions, Shapiro steps & the metrological triangle







Josephson junction
$V = n\, h\, f / 2e$

Phase-slip junction ?
$I = m\, e\, f$

Quantum Hall effect
$I = p\, e^2\, V/h$

Conclusions

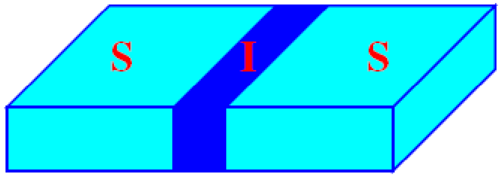
I. Ultrasmall Josephson junctions

Some basic notions:

-Junction in electromagnetic environment

-Voltage-Shapiro steps

-Duality

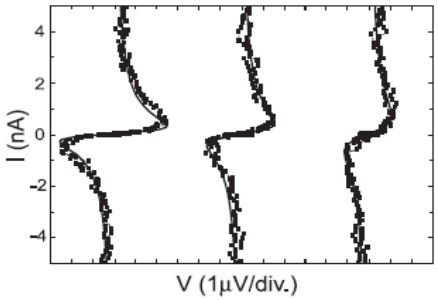


II. Voltage-biased junction in an inductive environment

Single Josephson junction as a phase-slip element:

- Hamiltonian

- Duality with an external inductance

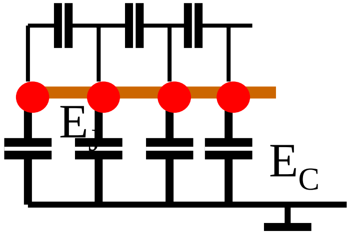


III. One-dimensional junction arrays

Josephson junction chain as a phase-slip element:

- Quantum phase-slips in a phase-biased array

- Duality with intrinsic inductance



IV. Effect of microwaves

- Current-Shapiro steps: closing the metrological triangle?