Finite frequency noise in a quantum dot in the Kondo regime

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Collaborators:

THEORY

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EXPERIMENT

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Hélène Bouchiat

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Motivations

Useful to understand noise

- ``Noise is an enemy of the measurements…”
- However, noise contains a lot of information

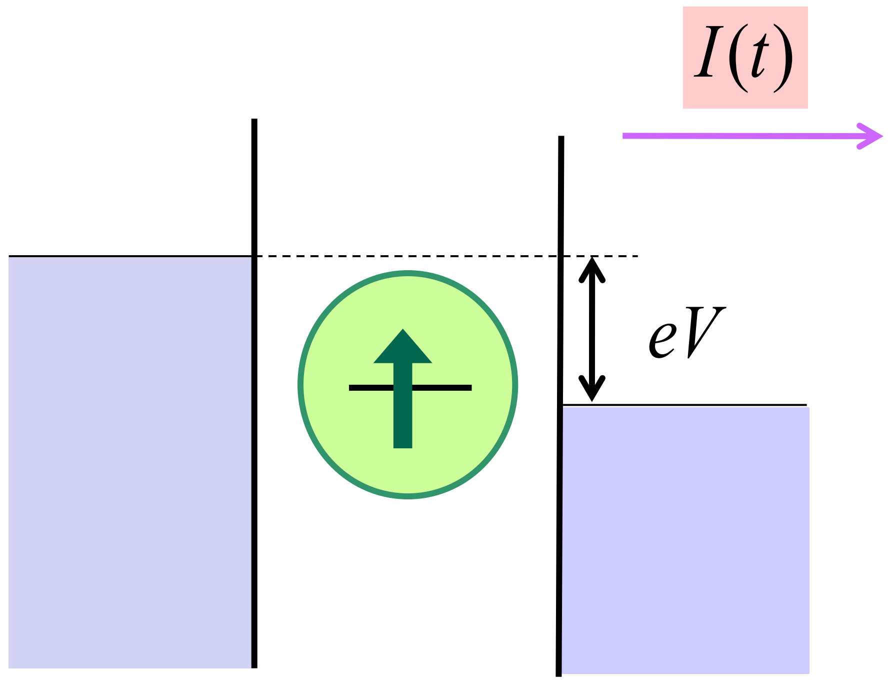
(Spectrum, Fano factor, …)

Non-equilibrium noise spectrum of a quantum dot ????

Relatively little was known…

Experiments have just been starting

Time-dependent noise through a quantum dot?

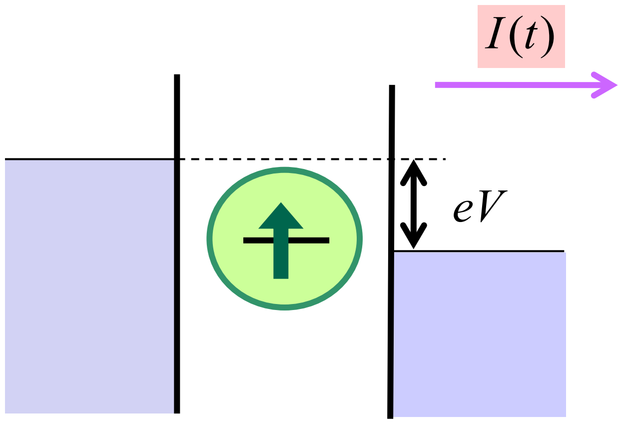


$$S(t)=\left\langle \{I(t),I(0)\}\right\rangle /2=?$$

$$S^{>}(t)=\left\langle I(t)I(0)\right\rangle =?$$

What kind of informations can we grasp in $S(\omega,V)$?

Time-dependent noise through a quantum dot?



Suppose the nano-objet we probe has some intrinsic energy scale associated with correlations.

Call this energy scale T_K the Kondo temperature

How the building of correlations do show up in the noise spectrum ?

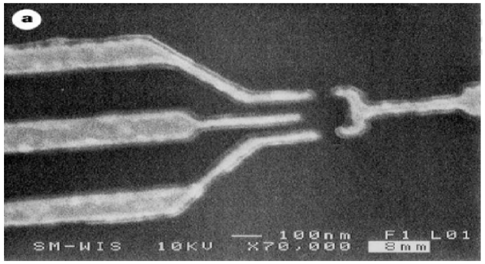
In particular at frequencies larger than T_K

OUTLINE

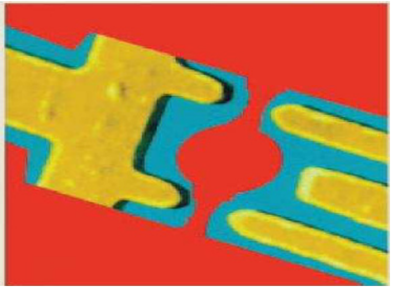
- I. The Kondo effect in artificial magnetic impurities (quantum dots)
- II. The non-equilibrium current noise for the Kondo problem
- III. Conclusions and perspectives

I) The Kondo effect
in
quantum dots

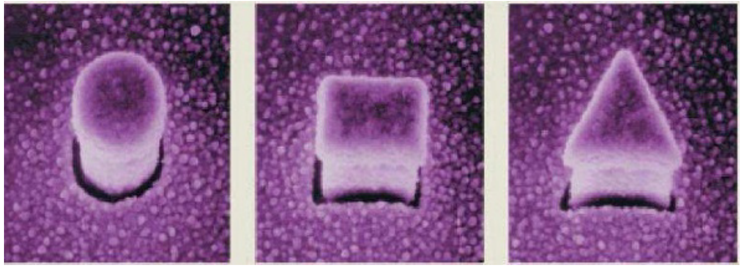
Various quantum dots



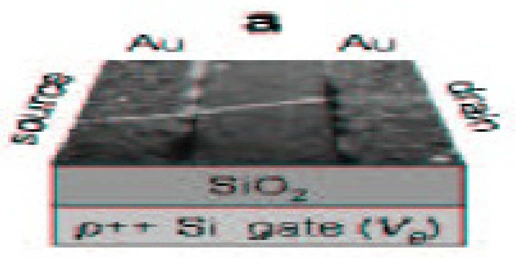
Goldhaber-Gordon et al., Nature (1998)



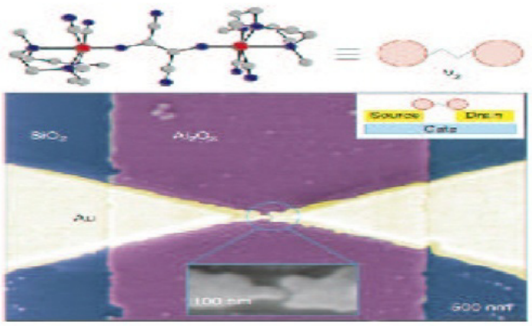
From Kouwenhoven' s group



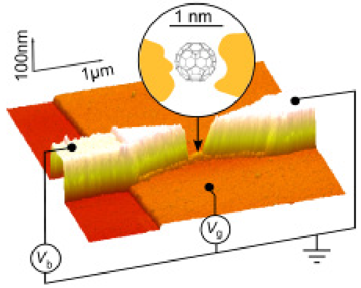
From Tarucha' s group



Cobden et al., Nature (2001)



Bockath et al., Nature (2002)

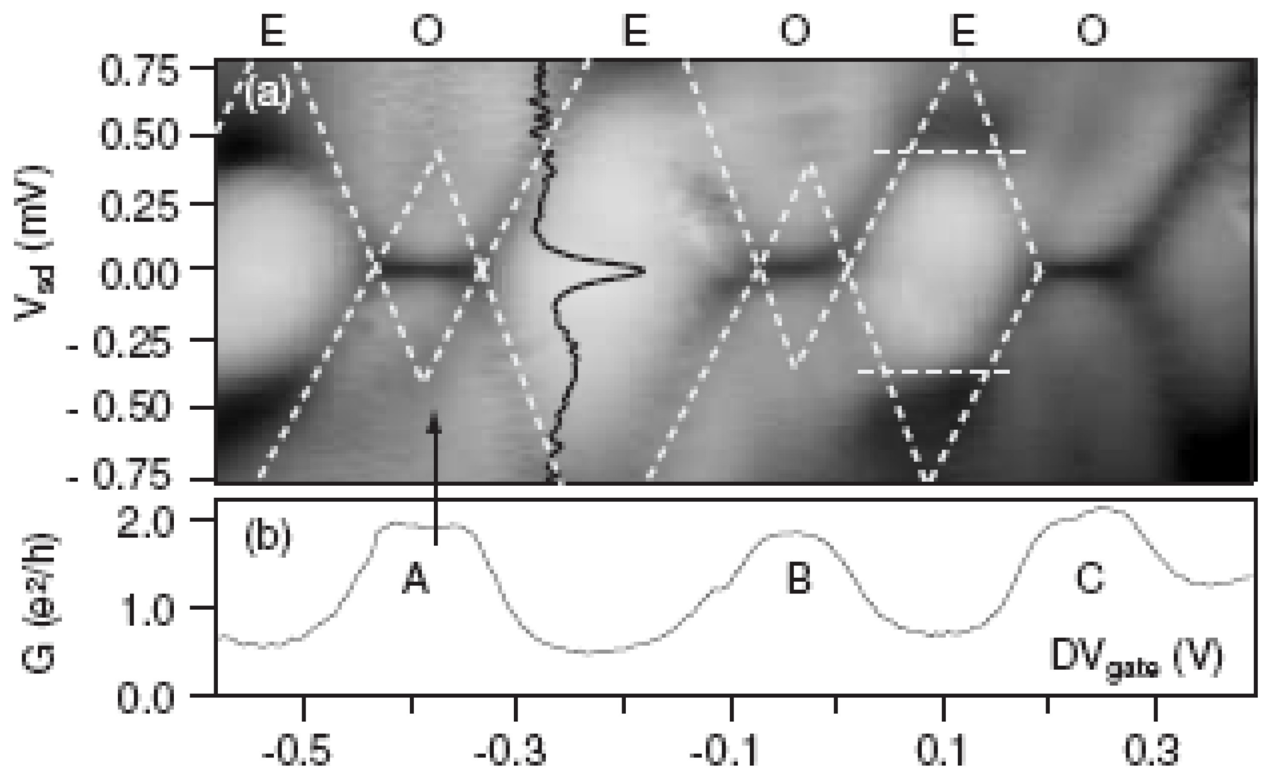


Roch et al., Nature (2008)

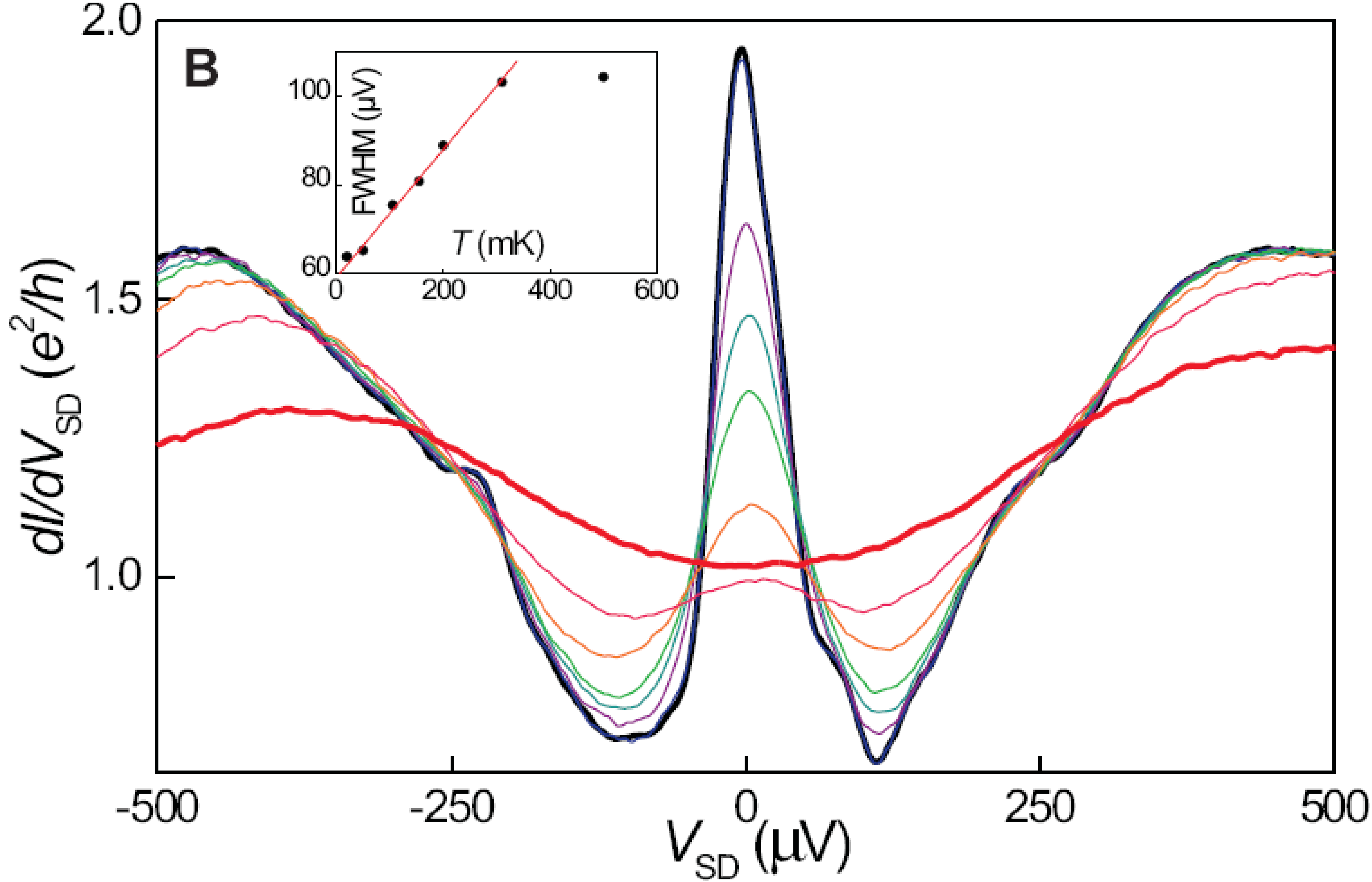
And many others ....

- Quantum dots can hold a few hundreds electrons
- A current can be driven through them

Kondo ridges



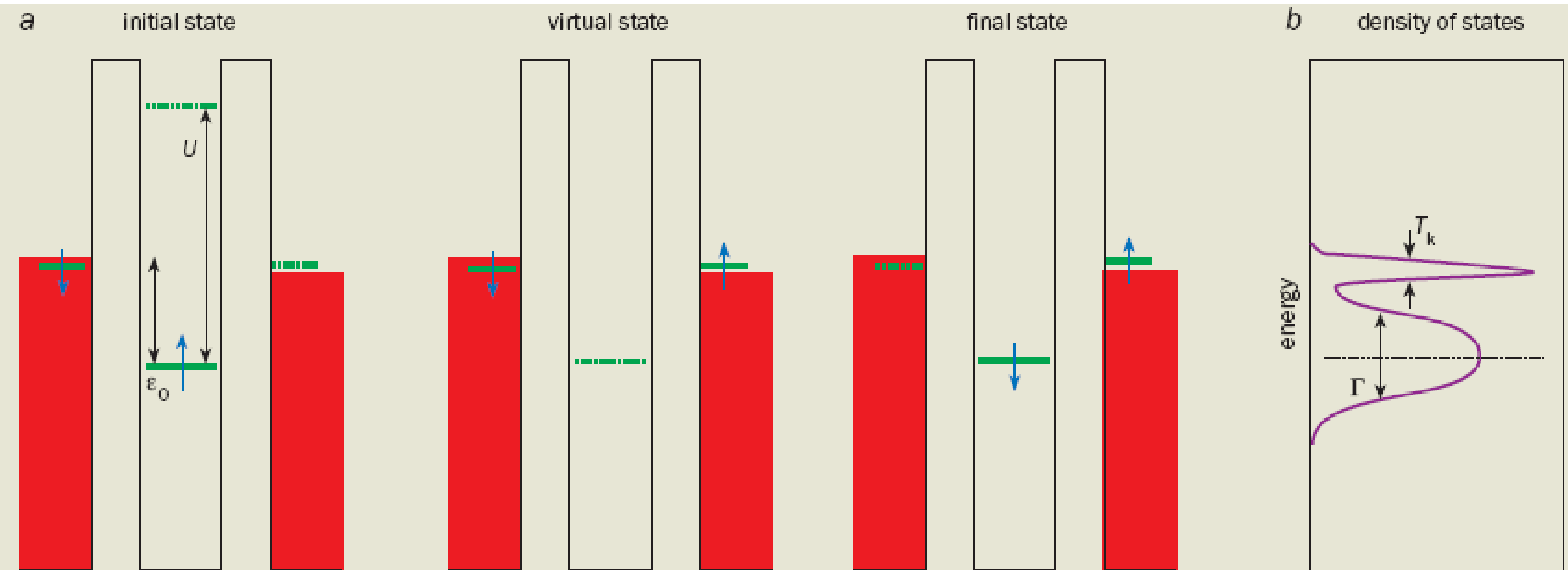
Differential conductance



Van der Wiel et al., Science **289**, 2105 (2000)

Kondo effect in quantum dots

Cotunneling regime



➡ Virtual charge transitions can be accompanied with a spin flip inside the dot !

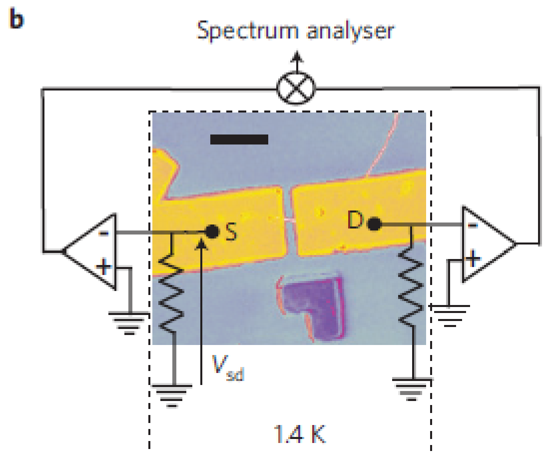
➡ Like an ordinary magnetic impurity in a metal

$$H = \sum_{\vec{k},\sigma} \varepsilon_k \psi^\dagger_{\vec{k}\sigma} \psi_{\vec{k}\sigma} + J\vec{S}_{imp} \cdot \vec{S}_{el}(\vec{r} = 0)$$

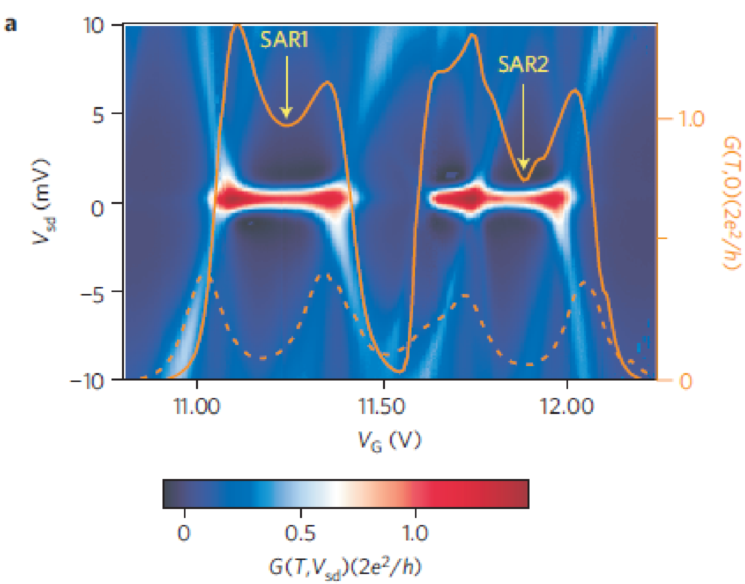
II. The non-equilibrium current noise for the Kondo model

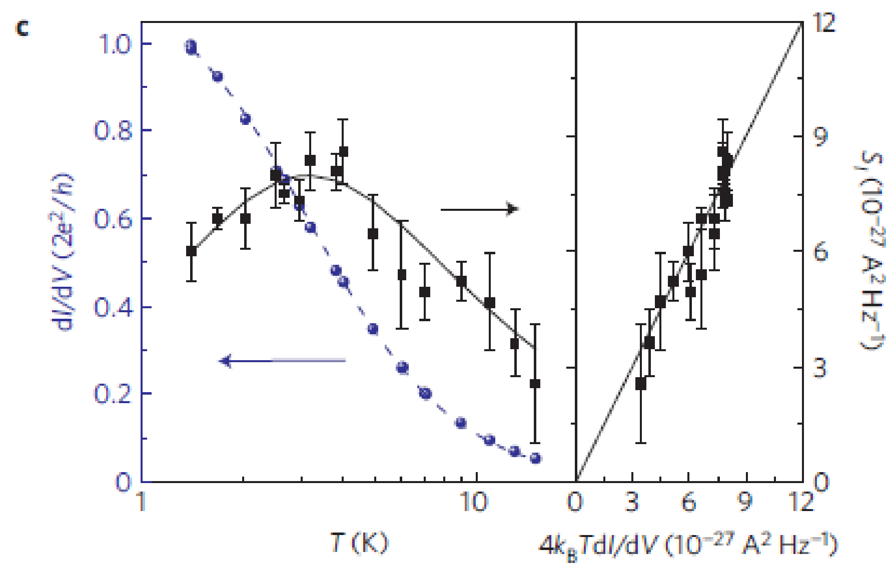
Shot Noise

Delattre et al., Nature Physics (2009)



$$V_{SD} = 0\,!$$

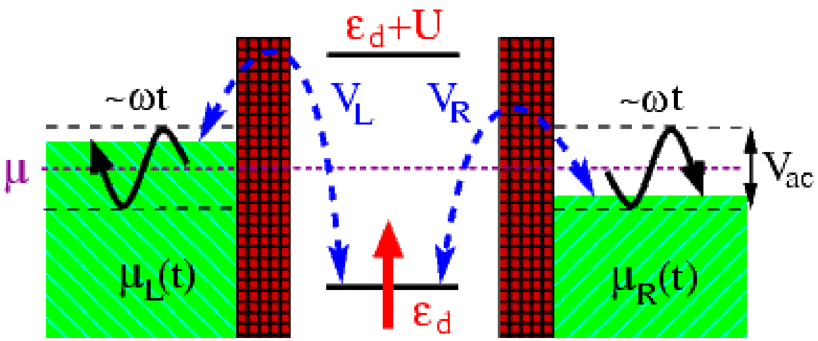


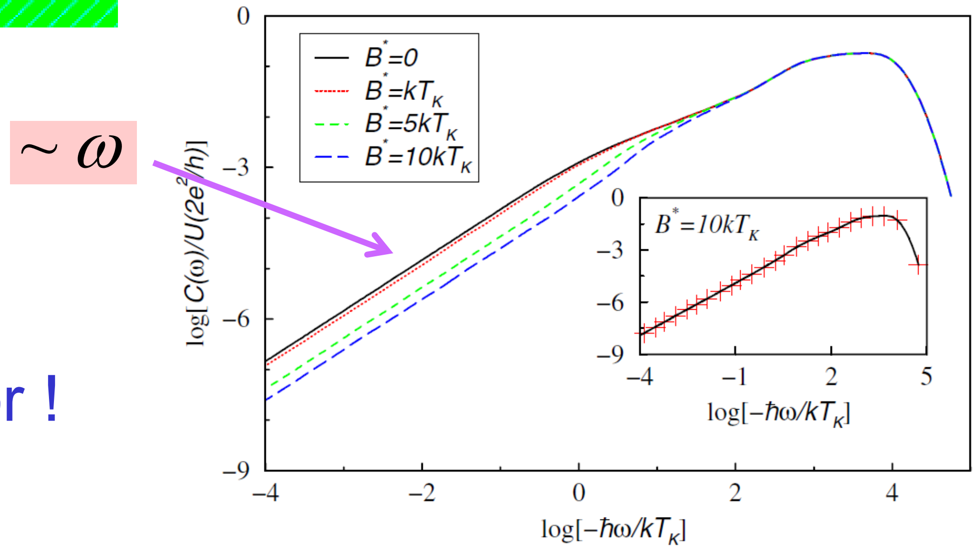


In agreement with FDT…

Equilibrium noise

Study of the ac equilibrium noise

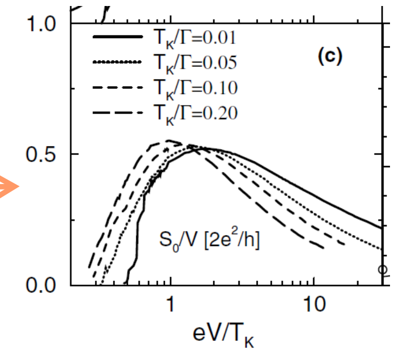




Fermi liquid behavior !

Sindel et al, PRL (2005)

What do we know about current noise of a quantum dot?



- Shot noise $S_{\omega=0}(V,T)$

Theory : Meir and Golub, PRL (2002)
Experiments: Delattre et al., Nature Physics (2009)

- Noise spectrum ?

Toulouse Limit: Schiller and Herschfield (1998)
$T=V=0$: Sindel et al, PRL (2005)
Perturbative calculations: Korb et al, PRB (2007)

Our questions ?

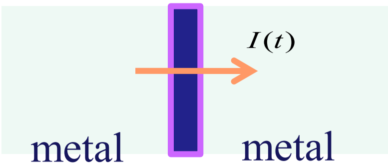
$S(\omega,V,T)=???$

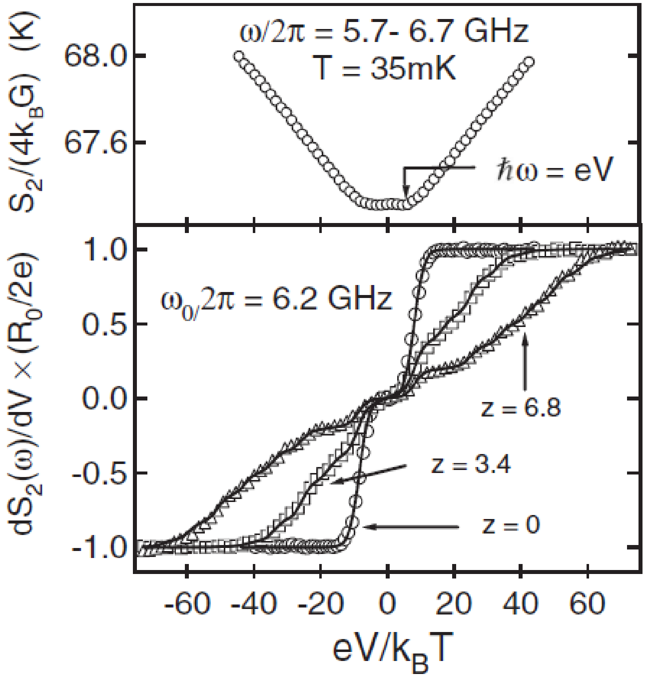
$G(\omega,V,T)=???$
(non-equilibrium linear conducatence)

Are there interesting things in $S(\omega,V)$?

Fermi liquid, no… $(eV << T_K)$ **is not very interesing**

Expect behavior similar to tunnel junction





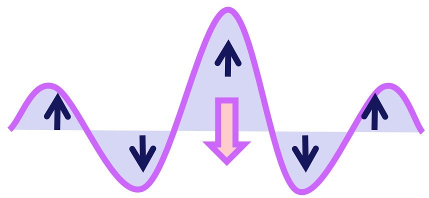
[Gabelli and Reulet, PRL 2008]

Yes, interesting regime: $eV > T_K$

Perturbative, but…

Difficulties

Kondo effect



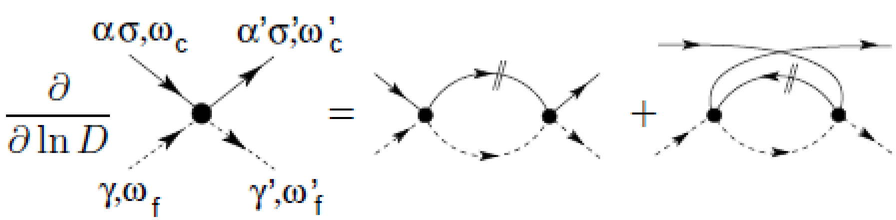
Logarithmic singularities with ``fine structure”

$$\ln(\omega / D), \quad \ln(|\omega - eV| / D)$$

Standard RG is not accurate enough

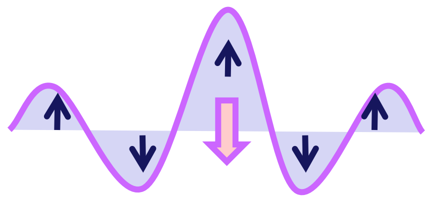
Paaske, Rosch, Kroha, Wölfle:

Scaling equations (heuristic) for the vertex function



Difficulties

Kondo effect



⇨ Logarithmic singularities with „fine structure"

$$\ln(\omega / D), \quad \ln(|\omega - eV| / D)$$

⇨ Standard RG is not accurate enough

Paaske, Rosch, Kroha, Wölfle:

Scaling equations (heuristic) for the vertex function:

⇨ **Retarded interactions !**

Current conservation???

What is $I(t)$???

Other non-equilibrium RG schemes

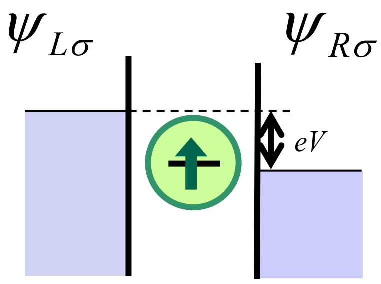
- Volker Meden
  (functional RG)
- Stefan Kehrein
  (flow equation)
- Herbert Schöller, Sabine Andergassen, Misha Pletnikov, …
  (reduced density matrix formalism)

Dynamical correlations ?? Current conservation ?

- Frithjof Anders and Avi Schiller
  (NRG)

Simple model

Kondo Hamiltonian:



$$H_{\rm int} = \frac{1}{2} \sum_{\alpha,\beta=L,R} \sum_{\sigma,\sigma'} j_{\alpha,\beta}\, \mathbf{S}\, \psi^\dagger_{\alpha\sigma} \sigma_{\sigma\sigma'} \psi_{\beta\sigma'} \ .$$

$$\alpha \in \{L, R\}$$

$$\psi_{\alpha\sigma} = \int c_{\alpha\sigma}(\xi) e^{-|\xi| a} \ d\xi$$

$$H_0 = \sum_{\alpha,\sigma} \int \mathrm{d}\xi \ \ (\xi + \mu_\alpha) \ \ c^\dagger_{\alpha\sigma}(\xi) c_{\alpha\sigma}(\xi),$$

$$\mu_\alpha = \pm eV/2$$

Use pseudofermions (Abrikosov)

$$\hat{S}^i \ \rightarrow \ \sum_{s,s'} \frac{1}{2} \ \ f^\dagger_s \sigma^i_{s,s'} f_{s'} \qquad\qquad \sum_s f^\dagger_s f_s \ = \ 1$$

Path integral on the Keldysh contour

Action: $\mathcal{S} = \mathcal{S}_{\text{lead}} + \mathcal{S}_{\text{spin}} + \mathcal{S}_{\text{int}}$

Interaction part:

Keldysh label

$$\mathcal{S}_{\text{int}} = \sum_{\kappa} \sum_{\alpha\beta} s_{\kappa} \frac{1}{4} \int \mathrm{d}t_1\, \mathrm{d}t_2\, g_{\alpha\beta}(t_1 - t_2)\;\; \bar{f}^{\kappa}(T_{12})\vec{\sigma} f^{\kappa}(T_{12}) \cdot \bar{\psi}^{\kappa}_{\alpha}(t_1)\vec{\sigma}\psi^{\kappa}_{\beta}(t_2)$$

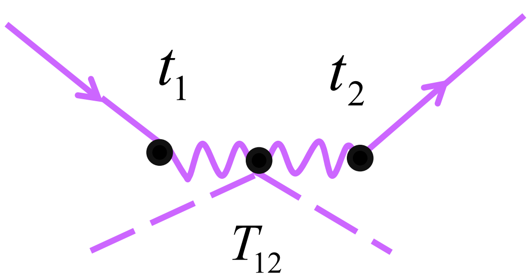
$s_{\kappa} = \pm 1$

$T_{12} = (t_1 + t_2)/2$

Keldysh sign:

Interaction initially local:

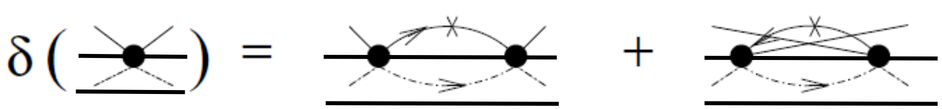
$$g^{(0)}_{\alpha\beta}(t) = j_{\alpha\beta}\;\; \delta(t)$$



Generating the RG equations

1. Expand $\langle U(-\infty,-\infty)\rangle$ in S_{int}

2. Rescale $a \to a'$, $g_{\alpha\beta}(t,a) \to g'_{\alpha\beta}(t,a')$



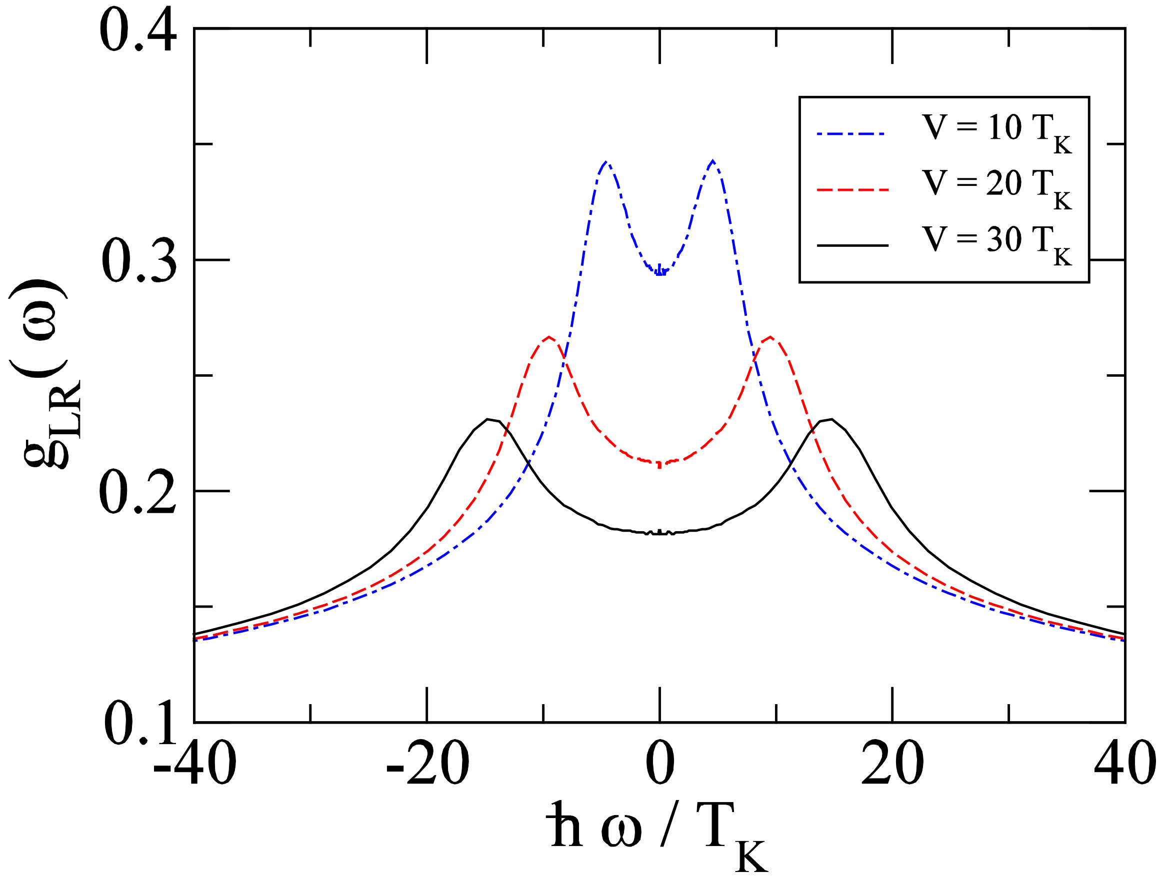
$$\frac{\mathrm{d}\mathbf{g}(\omega)}{\mathrm{d}l} = \mathbf{g}(\omega)\ \mathbf{q}(\omega,a)\ \mathbf{g}(\omega)$$

$l = \ln(a/a_0)$ $\quad$ $g_{\alpha\beta} \to \mathbf{g}$ $\quad$ Cut-off function

``Standard" RG:

$$g_{\alpha\beta}(t) \to \delta(t)\ \int \mathrm{d}t\ g_{\alpha\beta}(t)$$

Renormalized vertex function



Current vertex

Current operator from equation of motion:

$$\hat{I}_L(t) = -\hat{I}_R(t) = \sum_{\alpha\beta} \frac{1}{2} v^L_{\alpha\beta} \hat{\mathbf{S}}(t) \cdot \hat{\psi}^\dagger_\alpha(t) \sigma \hat{\psi}_\beta(t)$$

$$\mathbf{v}^L = -\mathbf{v}^R = \begin{pmatrix} 0 & -i\, j_{LR} \\ i\, j_{LR} & 0 \end{pmatrix}$$

Generating functional:

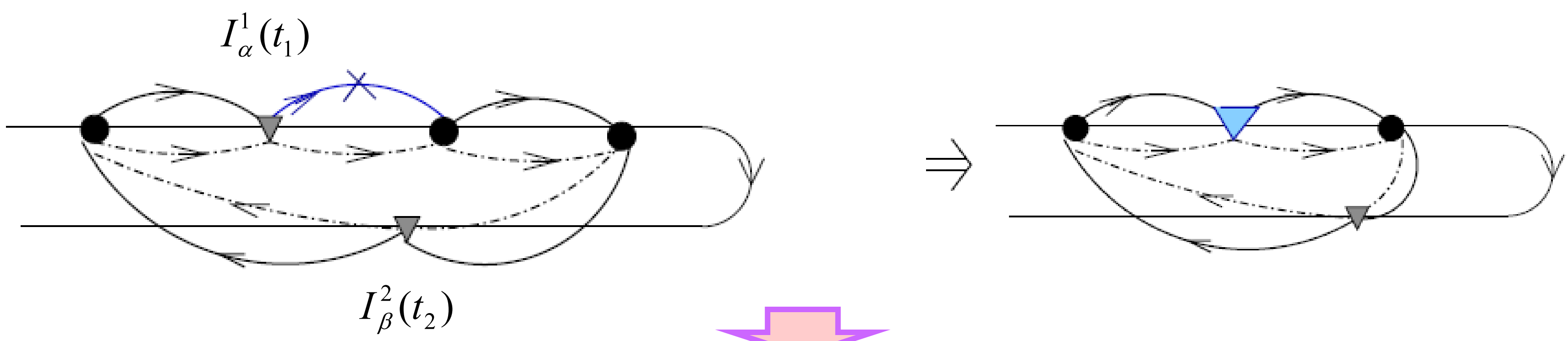
$$Z[h^\kappa_\alpha(t)] \equiv \langle e^{-i \sum_{\kappa,\alpha} \int \mathrm{d}t\, h^\kappa_\alpha(t) I^\kappa_\alpha(t)} \rangle_S$$

RG: 1. Expand $Z[h^\kappa_\alpha(t)]$ in $h^\kappa_\alpha(t)$ and in S_{int}

2. Rescale $a \to a'$, $g_{\alpha\beta}(t,a) \to g'_{\alpha\beta}(t,a')$

and $I^\kappa_\alpha(t)$

Current vertex renormalization



Current necessarily becomes non-local in time !

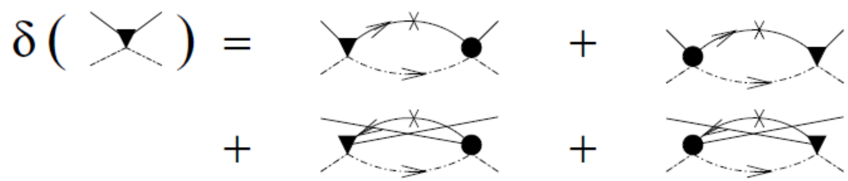
time of the measurement!

$$I_L^\kappa(t) = \frac{e^2}{4}\sum_\kappa\sum_{\alpha\beta}\int \mathrm{d}t_1\mathrm{d}\,t_2\, V_{\alpha\beta}(t_1-t, t-t_2, a)$$
$$\bar{f}^\kappa(t)\vec{\sigma}f^\kappa(t)\cdot\bar{\psi}^\kappa_\alpha(t_1)\vec{\sigma}\psi^\kappa_\beta(t_2)\ ,$$

C. P. Moca, PS, C.-H. Chung, G. Zarand, PRB 83, 201303(R) (2011)

RG equations

Current vertex scaling:



$$\frac{\mathrm{d}\mathbf{V}^L(\omega_1,\omega_2)}{\mathrm{d}l} = \mathbf{V}^L(\omega_1,\omega_2)\,\mathbf{q}(\omega_2,a)\,\mathbf{g}(\omega_2) + \mathbf{g}(\omega_1)\,\mathbf{q}(\omega_1,a)\,\mathbf{V}^L(\omega_1,\omega_2)$$

Initial condition: $\mathbf{V}^{L/R}(\tau_1,\tau_2,a_0) = \delta(\tau_1)\,\delta(\tau_2)\,\mathbf{v}^{L/R}$

Current conservation:

$$\mathbf{v}^L = -\mathbf{v}^R$$

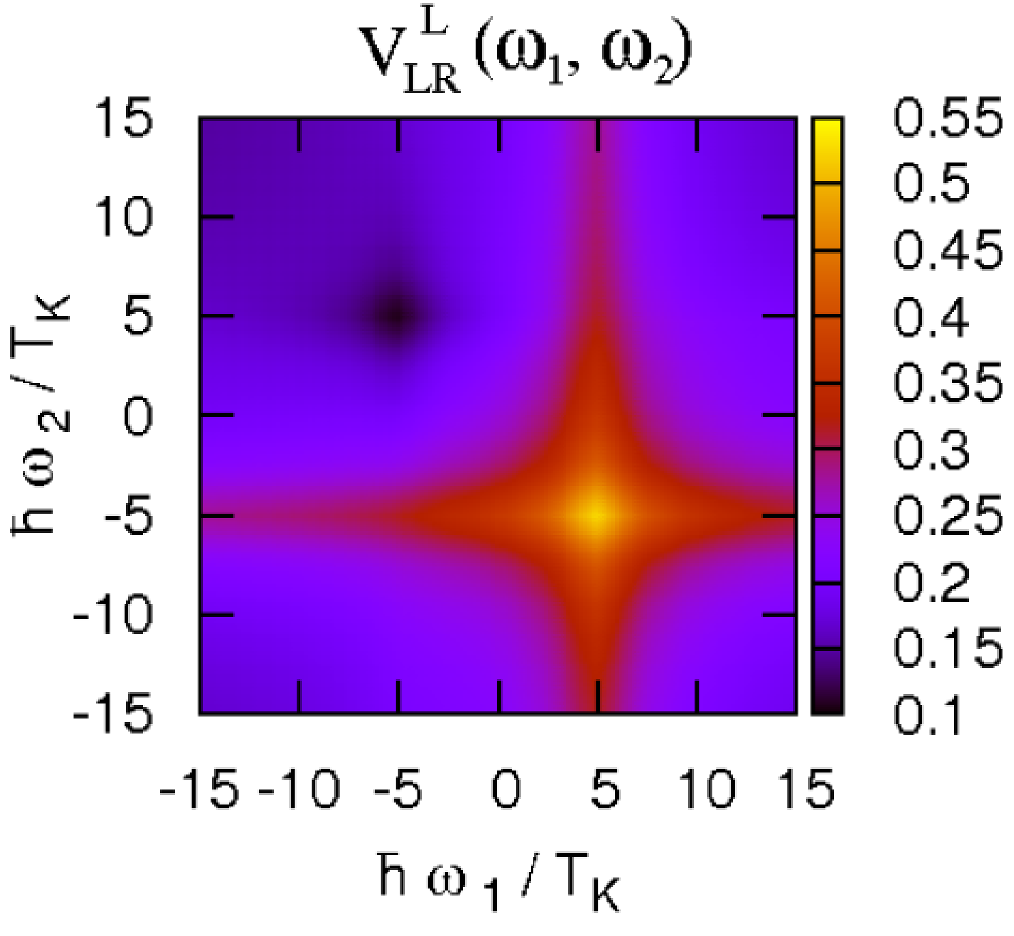
$$V^L_{\alpha\beta}(\omega_1,\omega_2) = -V^R_{\alpha\beta}(\omega_1,\omega_2)$$

$$I_L^\kappa(t) + I_R^\kappa(t) \equiv 0$$

Automatically satisfied at the functional level !

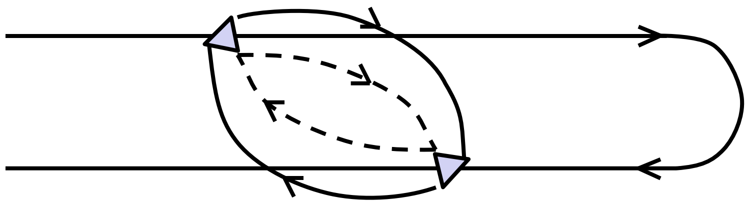
Current vertex

$eV / T_K = 10$



Computing the noise

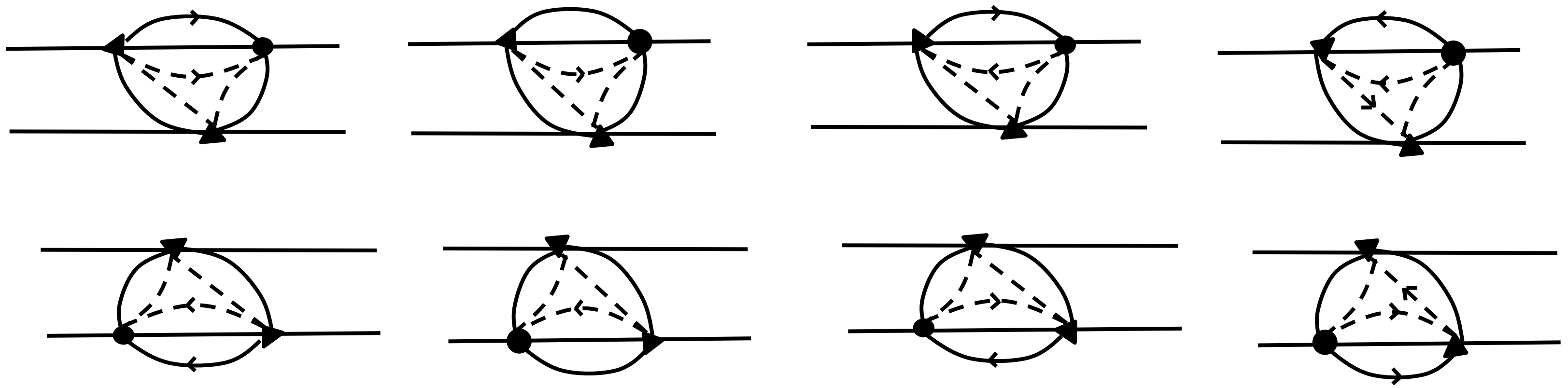
Functional RG noise formula:



$$S^{>}_{LL}(\omega) = \frac{e^2}{2} S(S+1) \int \frac{\mathrm{d}\tilde{\omega}}{2\pi} \mathrm{Tr}\{\mathbf{V}^L(\tilde{\omega}_-, \tilde{\omega}_+) \mathbf{G}^{>}(\tilde{\omega}_+) \mathbf{V}^L(\tilde{\omega}_+, \tilde{\omega}_-) \mathbf{G}^{<}(\tilde{\omega}_-)\} \, .$$

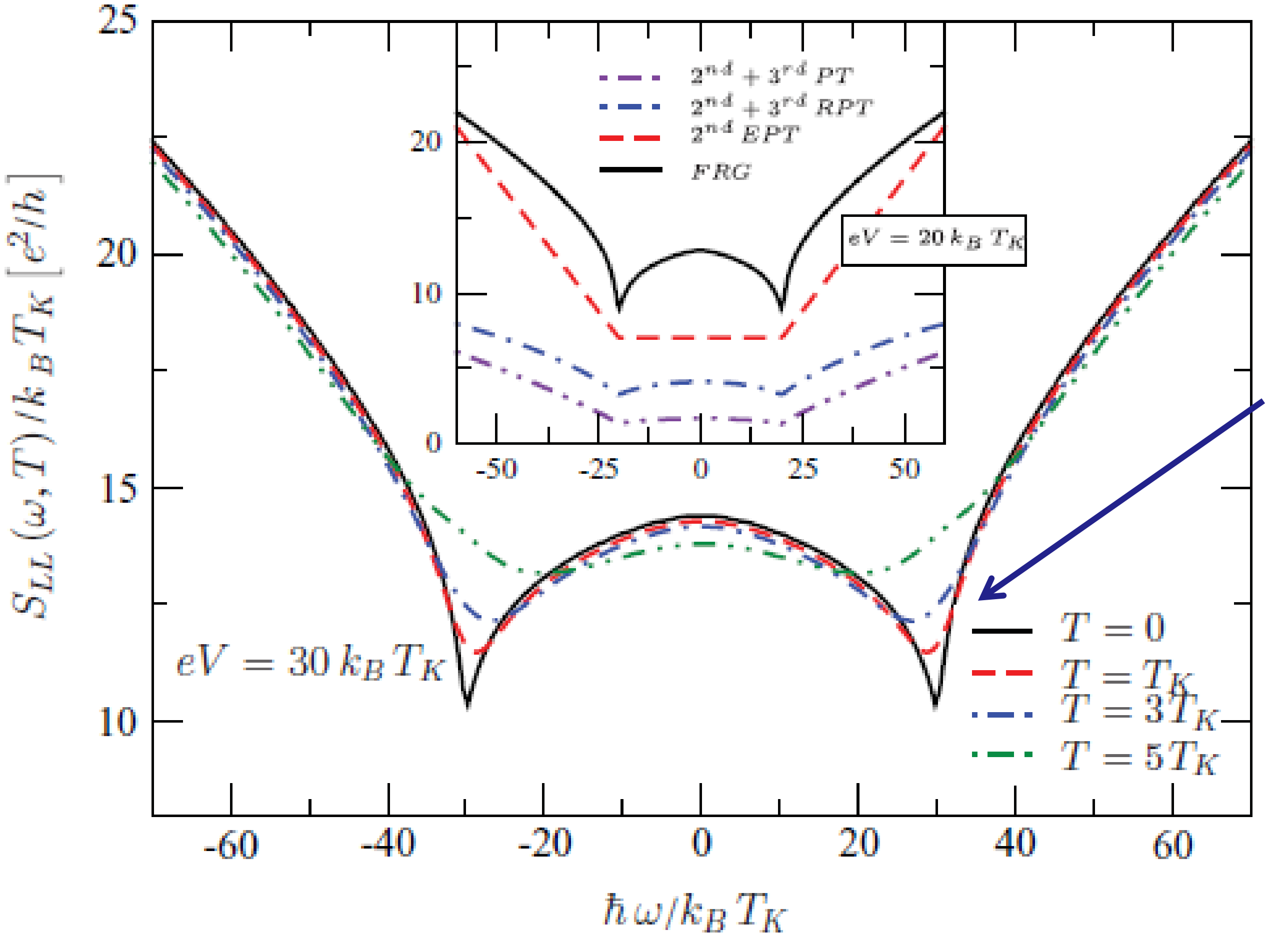
$$\tilde{\omega}_\pm = \tilde{\omega} \pm \frac{\omega}{2} \qquad G^{>/<}_{\alpha\beta}(\omega) = \pm i \; 2\pi \; \delta_{\alpha\beta} \; f(\pm(\omega - \mu_\alpha))$$

Perturbation theory



$$S^{>}_{LL}(t) = -e^2\,\frac{3}{4}|j_{LR}|^2\cos(eVt)\left\{\frac{1}{(t-i\,a)^2} + 2(j_{LL}+j_{RR})\frac{\ln(1+i\,t/a)}{t(t-2\,i\,a)}+\ldots\right\}$$

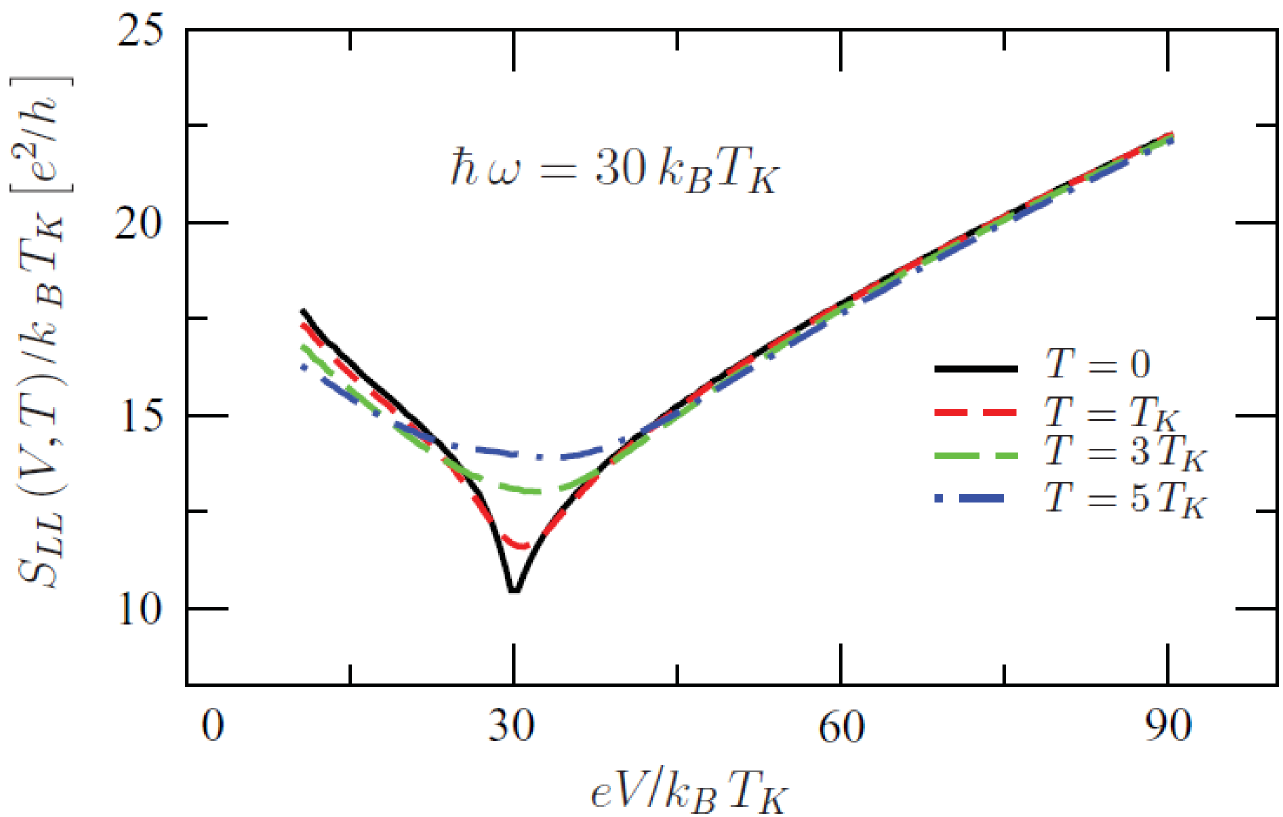
Symmetrized noise spectra



Non-equilibrium Kondo effect

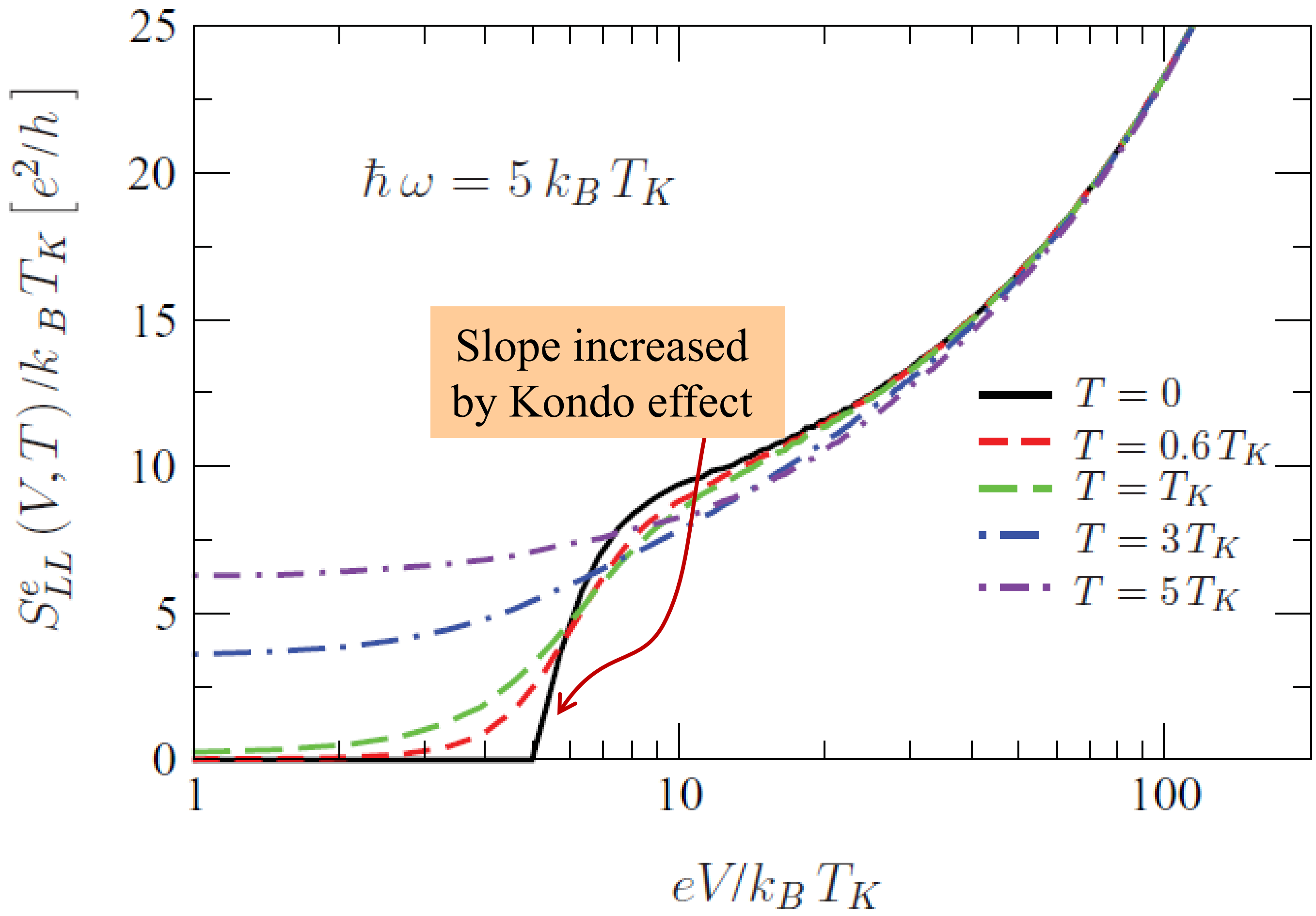
C. P. Moca, PS, C.-H. Chung, G. Zarand, PRB 83, 201303(R) (2011)

Noise spectra at fixed frequency



C. P. Moca, PS, C.-H. Chung, G. Zarand, PRB 83, 201303(R) (2011)

Emission noise



C. P. Moca, PS, C.-H. Chung, G. Zarand, PRB 83, 201303(R) (2011)

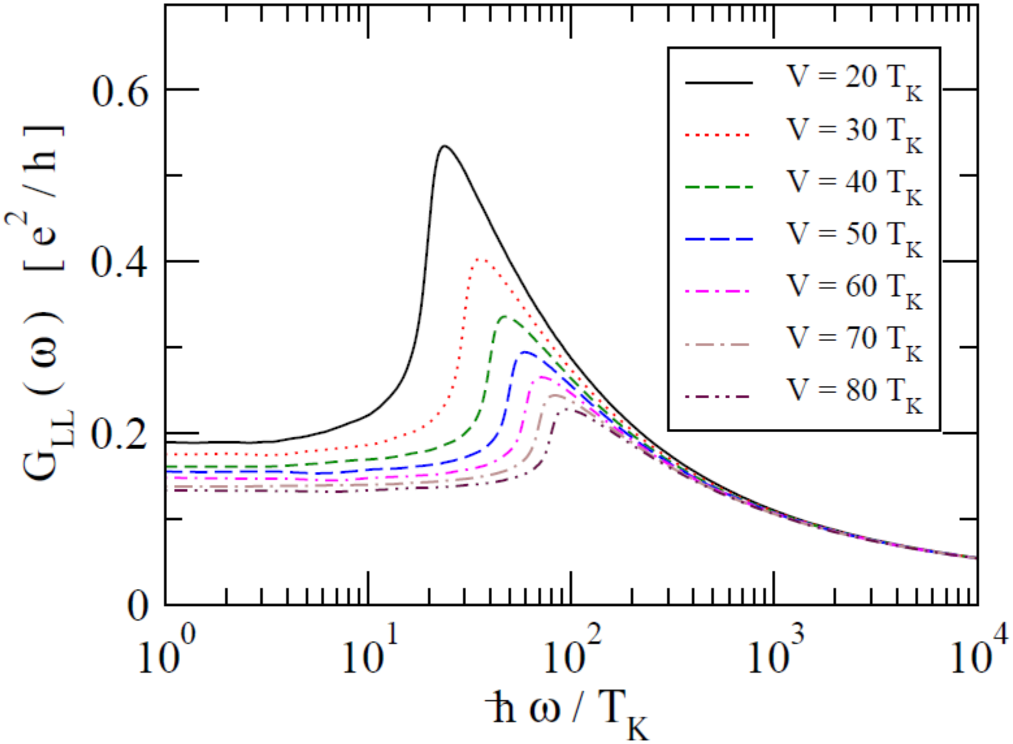
Non-equilibrium ac linear conductivity

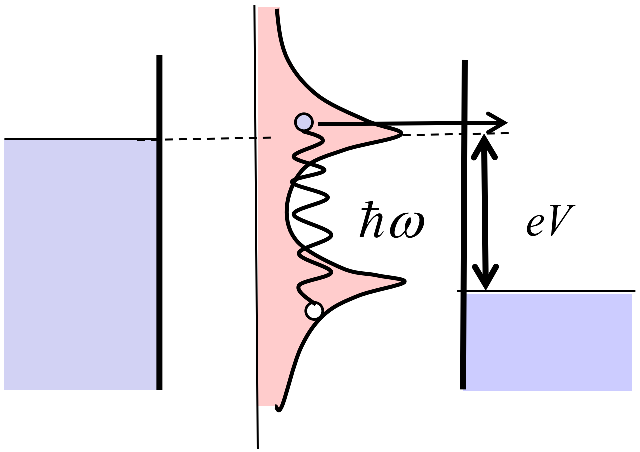
$$V_{SD} = V + \delta V_\omega \cos(\omega t) \quad \Rightarrow \quad I(t) = \mathrm{Re}\, G(\omega, V)\, \delta V_\omega \cos(\omega t) + \ldots$$

Non-equlibrium ac-conductance

$$G_{LL}(\omega, V) = \frac{1}{\hbar\omega}\left(S^{>}_{LL}(\omega) - S^{<}_{LL}(\omega)\right)$$

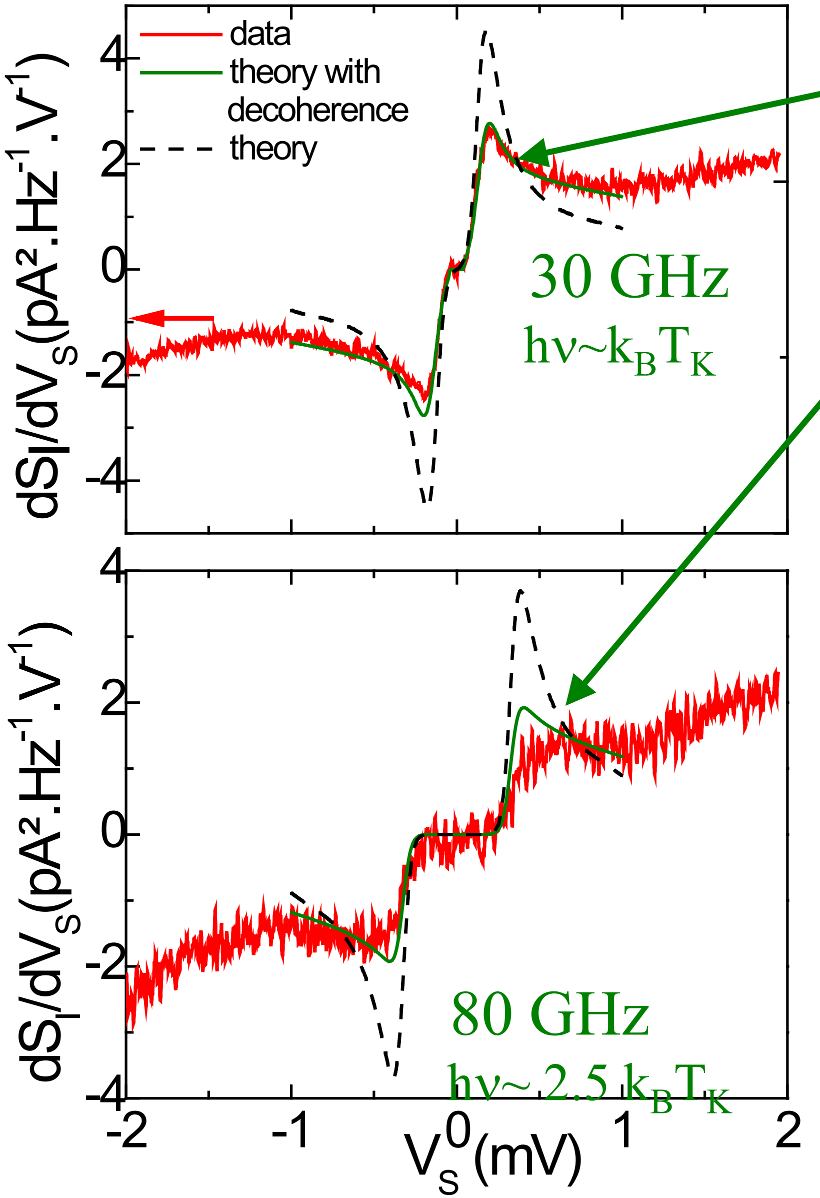
Safi 09'





C. P. Moca, PS, C.-H. Chung, G. Zarand, PRB 83, 201303(R) (2011)

Comparison with the experimental data



Fits OK using a single bias dependent spin decoherence rate function

J. Basset, A. Kasumov, C.P. Moca, G. Zarand, PS, H. Bouchiat, R. Deblock, arXiv:1110.570

Conclusions

- Real time functional RG formalism

  ⇨ simple and transparent derivation of equations of Rosch et al.

  ⇨ Current-conserving scheme
- RG equations for current vertex
- $S(\omega,V)$ shows strong anomalies at $\omega=\pm eV$
- simple perturbation theory or RG is insufficient
- $G(\omega,V)$ shows split Kondo resonance
- Good agreement with experimental data

In progress

- magnetic fields…
- More complicated systems

Path integral on the Keldysh contour II

Leads:

$\mathcal{S}_{\text{lead}}$ Quadratic,

⇨ correlation function is enough to know

$$\langle \psi^{\kappa}_{\alpha\sigma}(t)\bar{\psi}^{\kappa'}_{\alpha'\sigma'}(0)\rangle_{\mathcal{S}_{\text{lead}}} = i\, e^{-i\mu_\alpha t}\delta_{\alpha\alpha'}\delta_{\sigma\sigma'} \begin{pmatrix} \frac{-1}{t-\ i\ a\ \text{sgn}\ t} & \frac{-1}{t+\ i\ a} \\ \frac{-1}{t-\ i\ a} & \frac{-1}{t+\ i\ a\ \text{sgn}\ t} \end{pmatrix}$$

Same holds for pseudofermions….