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Shot noise and Coulomb effects on non-local electron transport in NSN heterostructures

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Charge and heat dynamics in nano-systems
LPS Orsay, France, October 10-12, 2011

Collaboration:

Dima Golubev

INT, KIT

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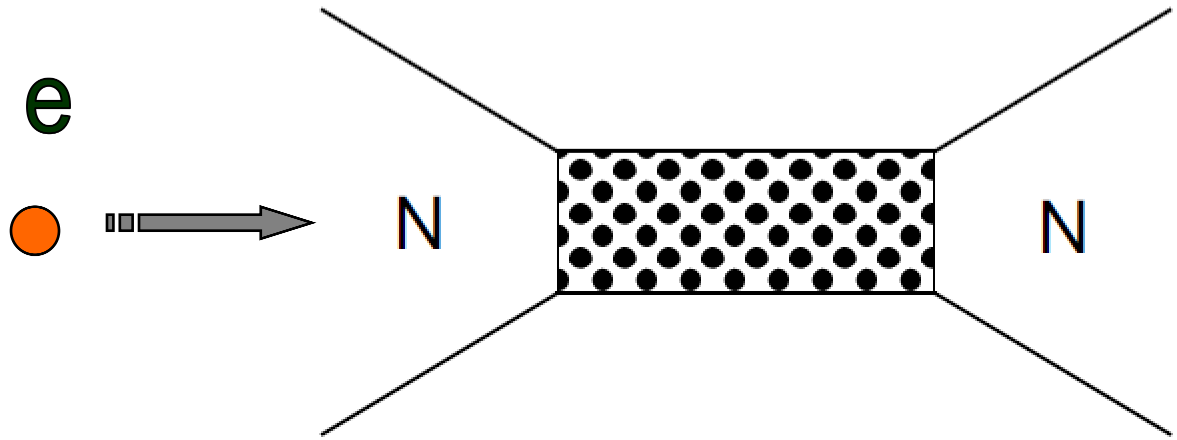
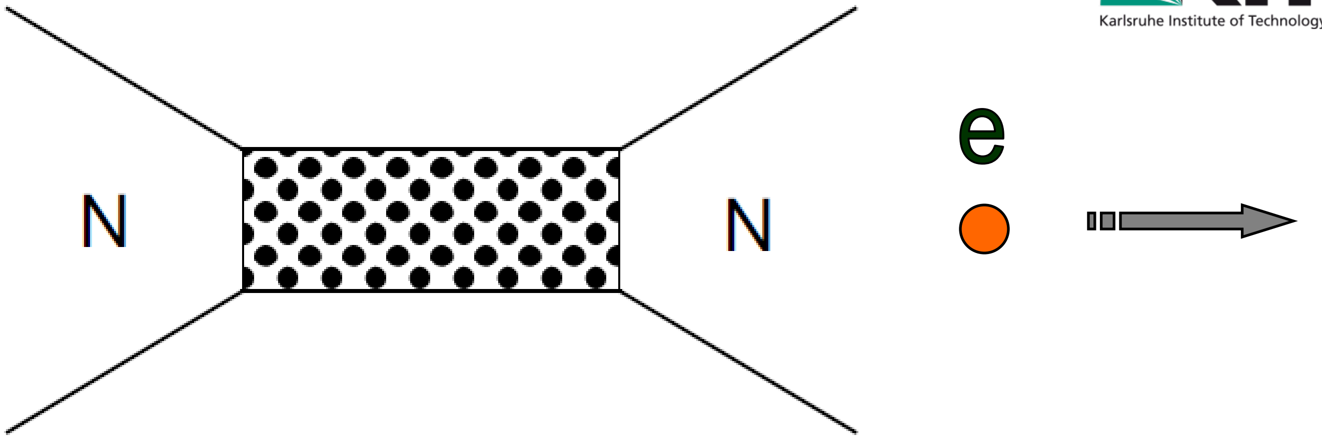
Outline

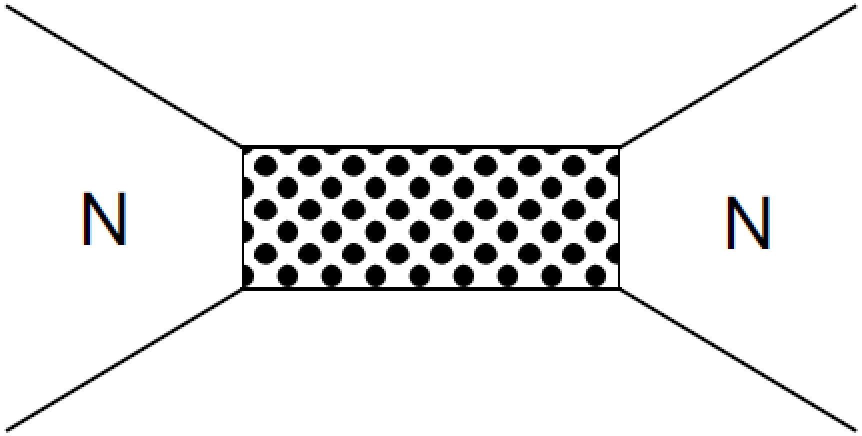
- Introduction: shot noise and e-e interactions in local transport
- Effective action formalism
- NSN systems: non-local shot noise
- NSN systems: non-local transport with e-e interactions
- Summary


e
N
N



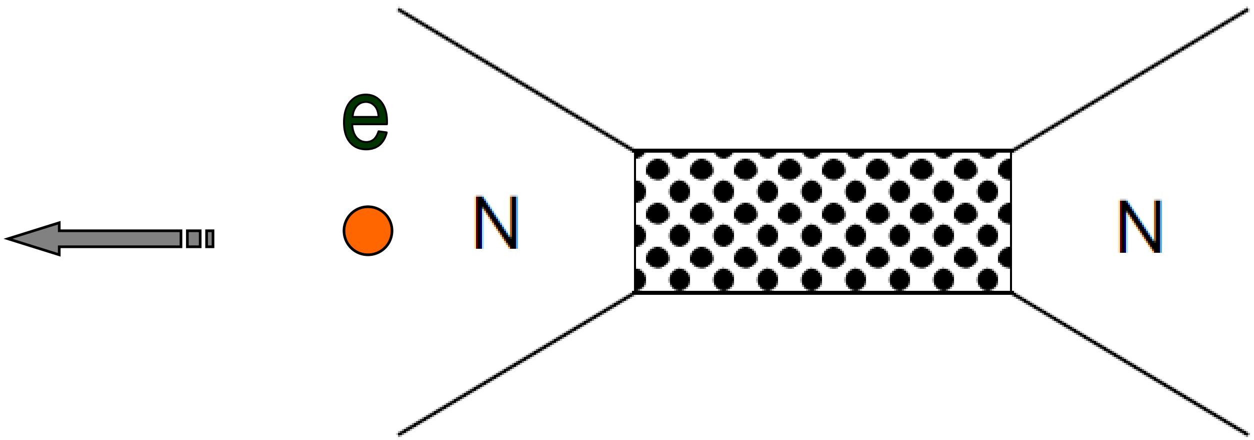
Transmission probability in the n-th channel:

$$T_n$$



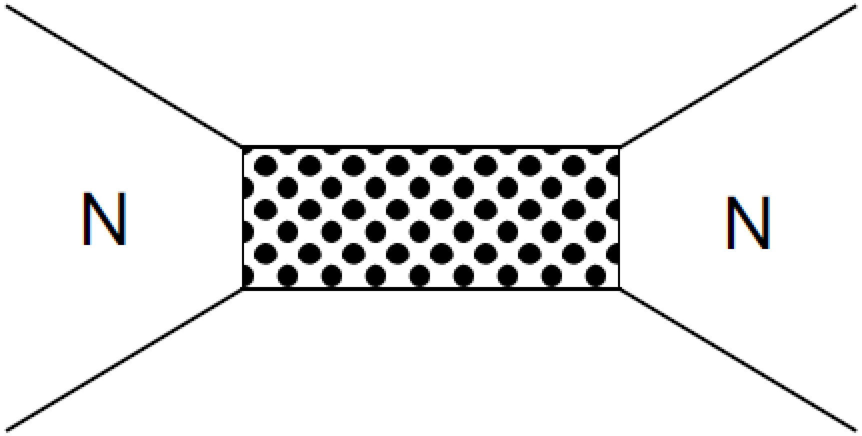
Conductance: Landauer formula

$$G_N = \frac{e^2}{h} 2 \sum_n T_n,$$



Reflection probability in the n-th channel:

$$1 - T_n$$



Shot noise: Khlus et al. formula:

$$\langle |\delta I|^2 \rangle = e|V|G_N \beta_N , \quad \beta_N = \frac{\sum_n T_n(1-T_n)}{\sum_n T_n}$$

Including electron-electron interactions in normal structures...

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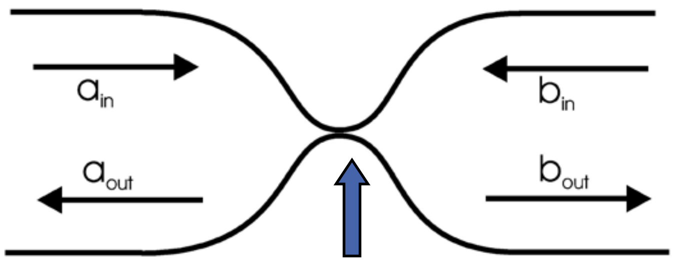
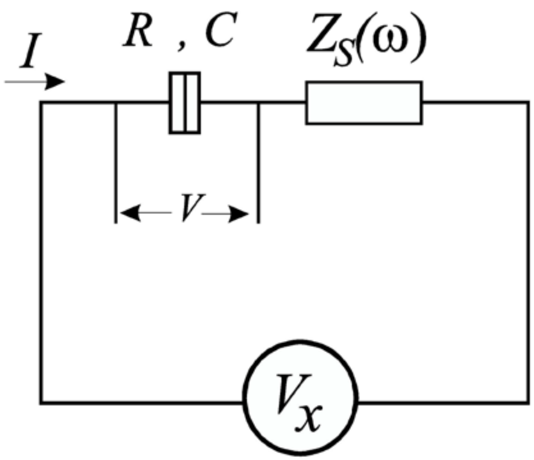


Coherent scatterer



Golubev, A.D.Z. PRL'01:

$$R\frac{dI}{dV} = 1 - \beta f(V,T)$$

→ Universal function

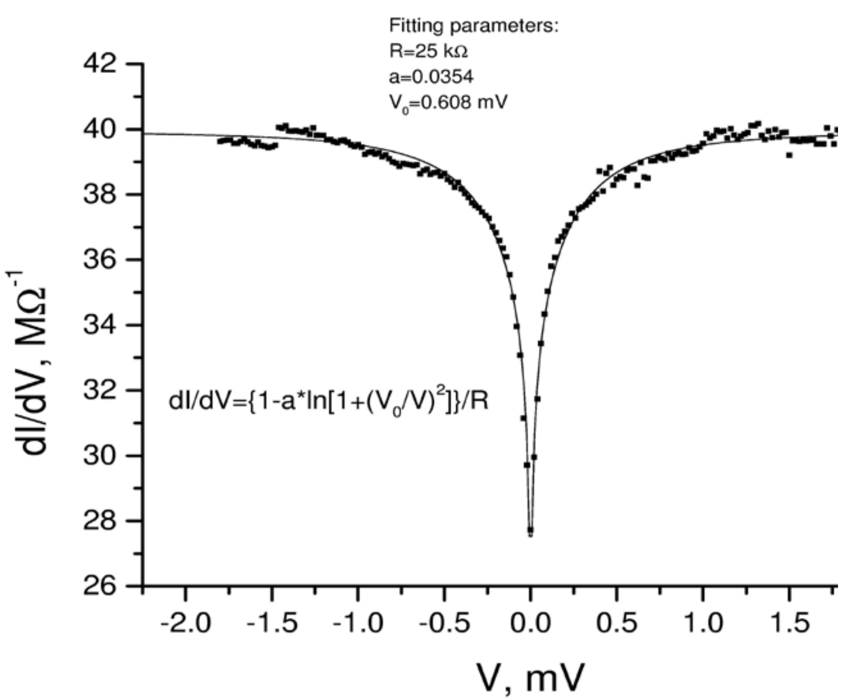
also: Levy Yeati et al. PRL'01

$$\beta = \frac{\Sigma_n T_n(1-T_n)}{\Sigma_n T_n}$$

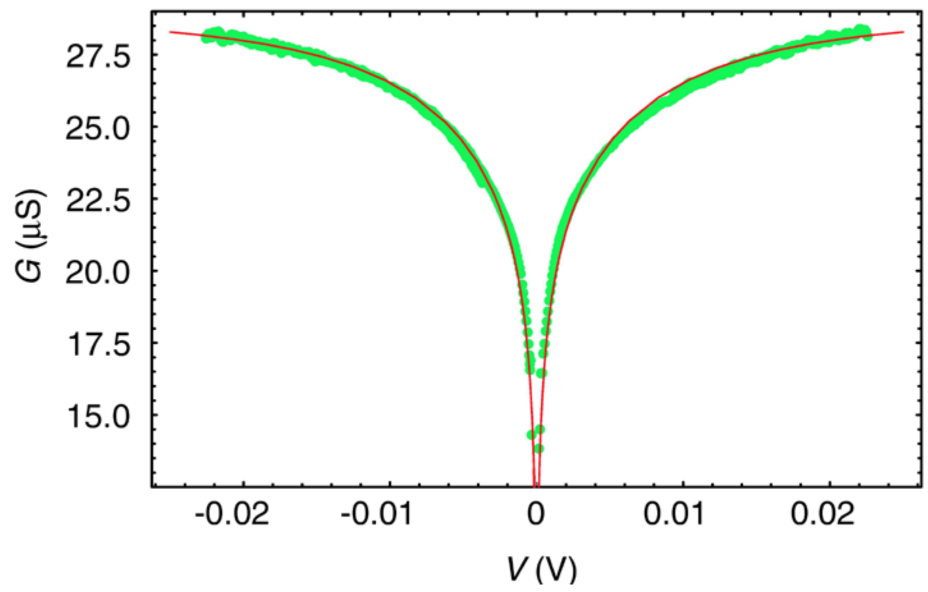
↔ Shot noise

Interaction correction ⟷ Universal function

Granular metals
(Krupenin et al., APL'02)

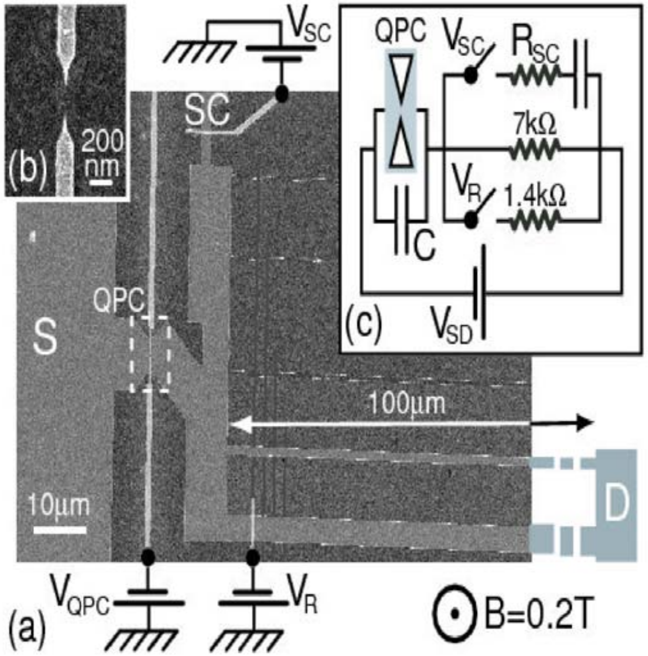


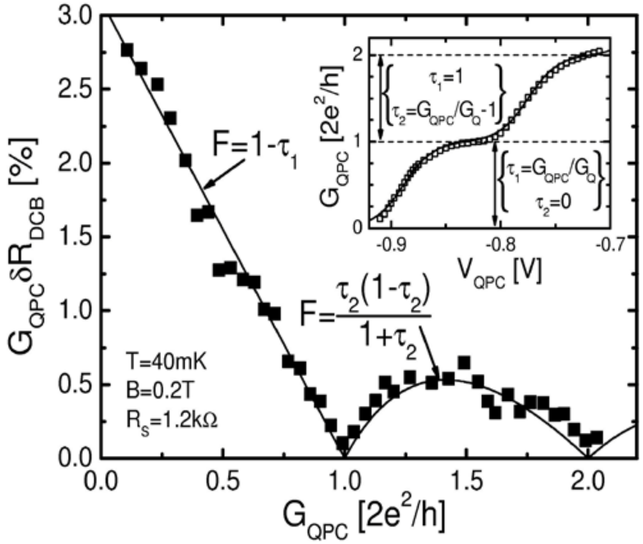
Carbon nanotubes
(Paalanen et al., PRB'04)

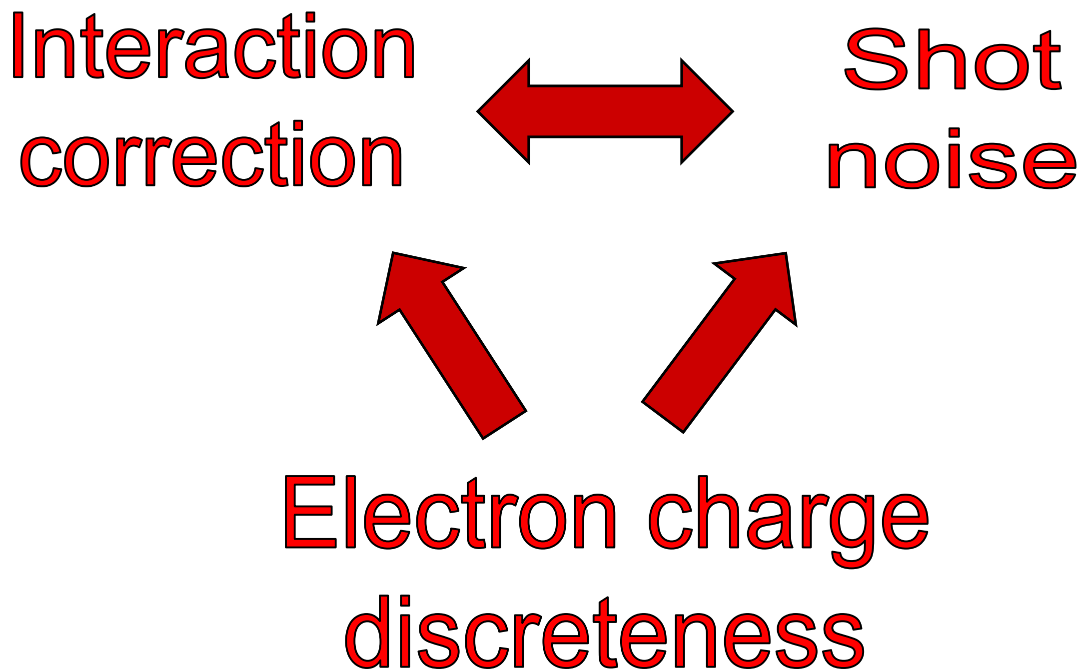


Interaction correction ⟷ Shot noise (Fano factor)

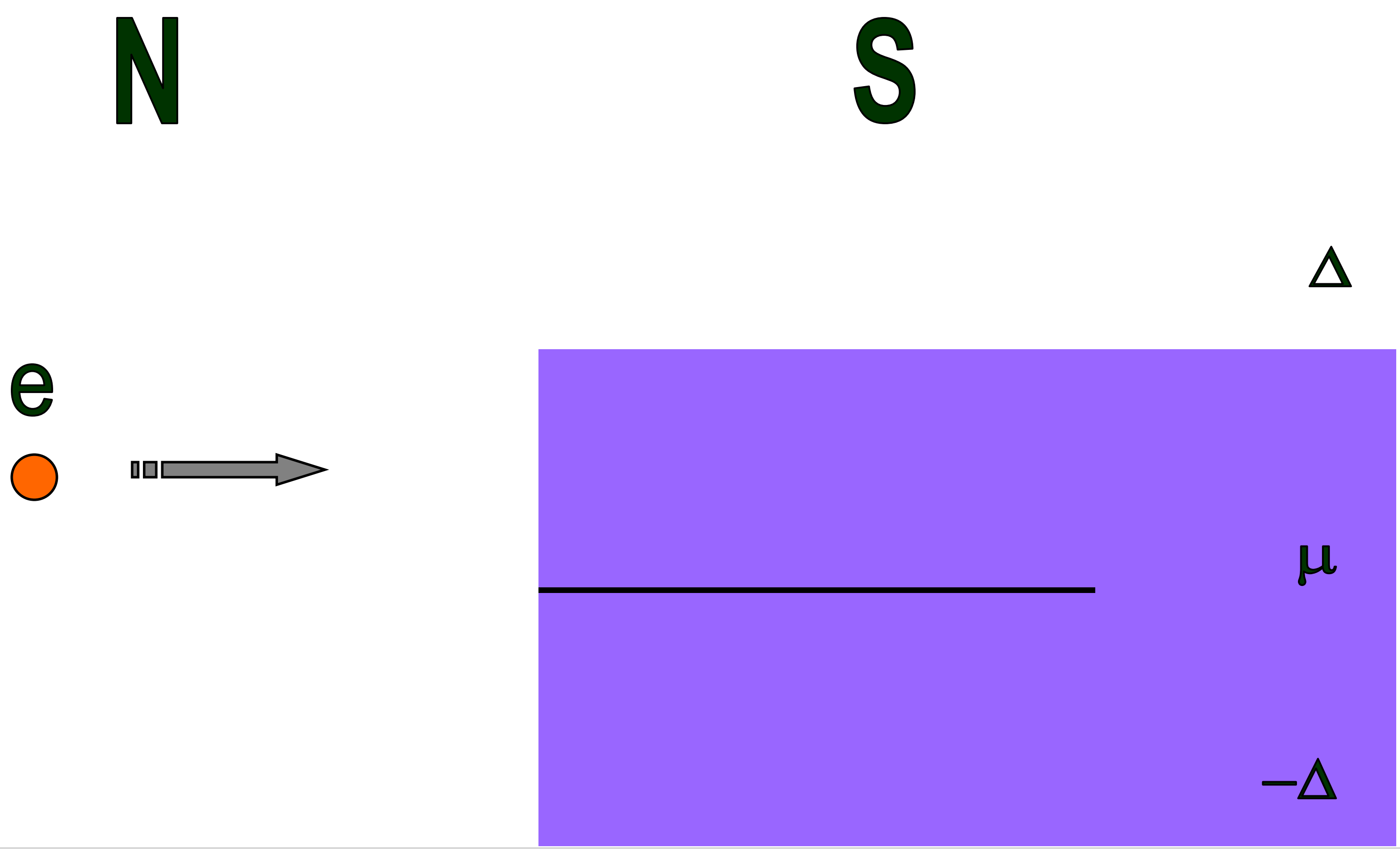
Experiments on break junctions: Altimiras et al. PRL'07



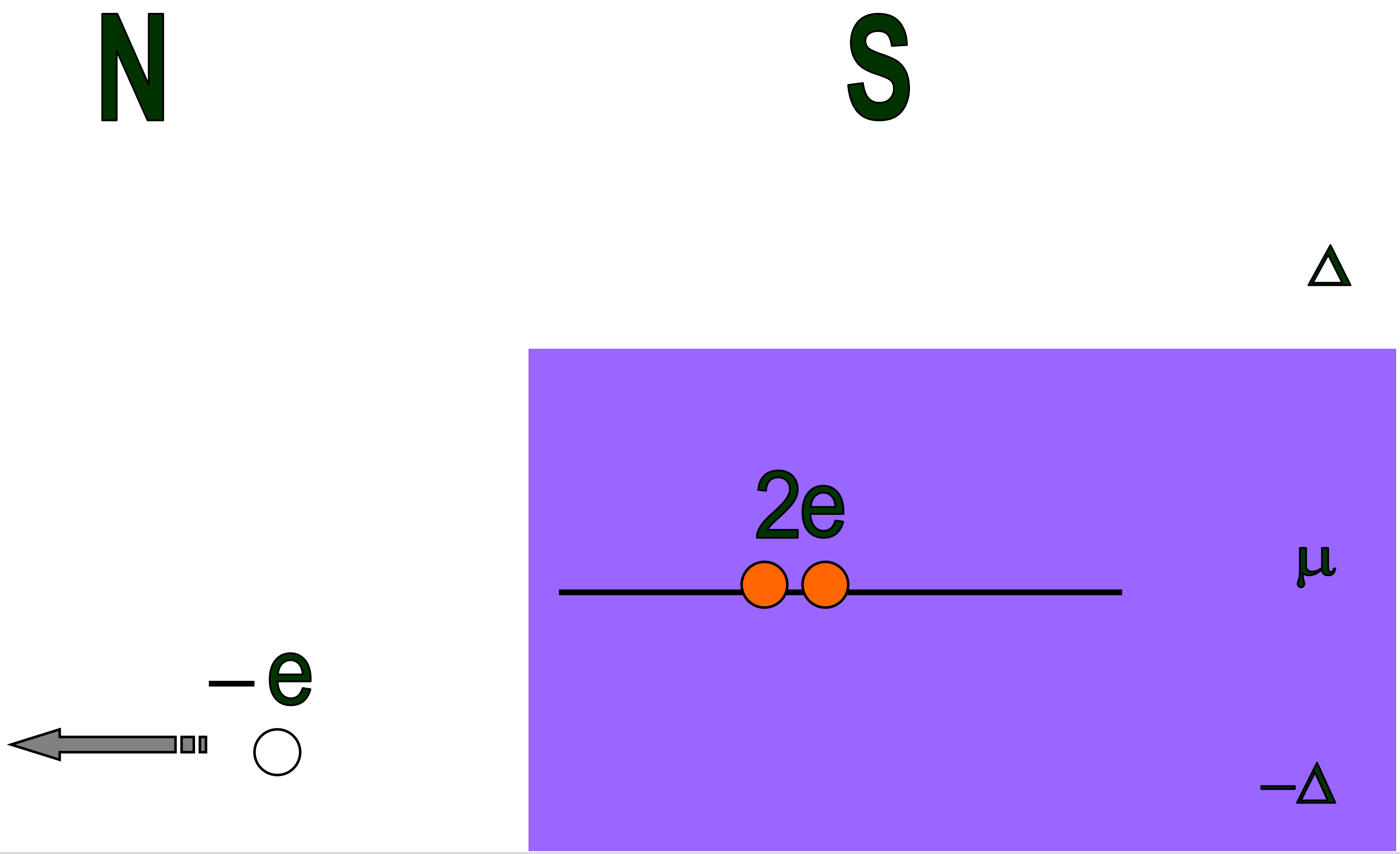


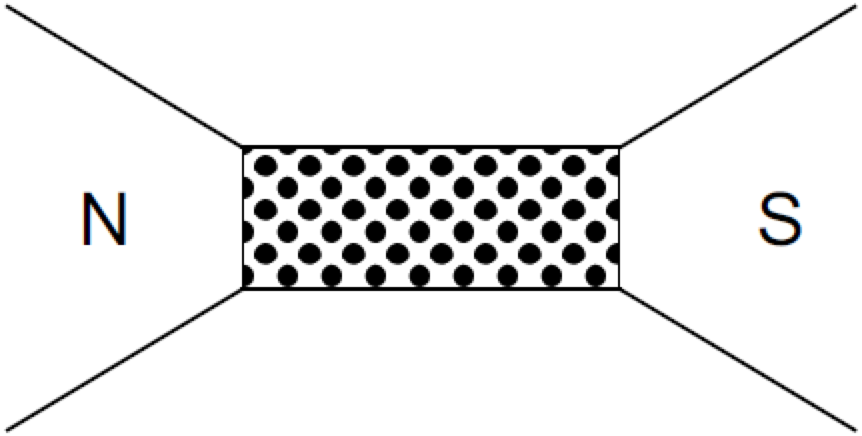

Interaction correction
Shot noise
Electron charge discreteness

Andreev reflection



Andreev reflection



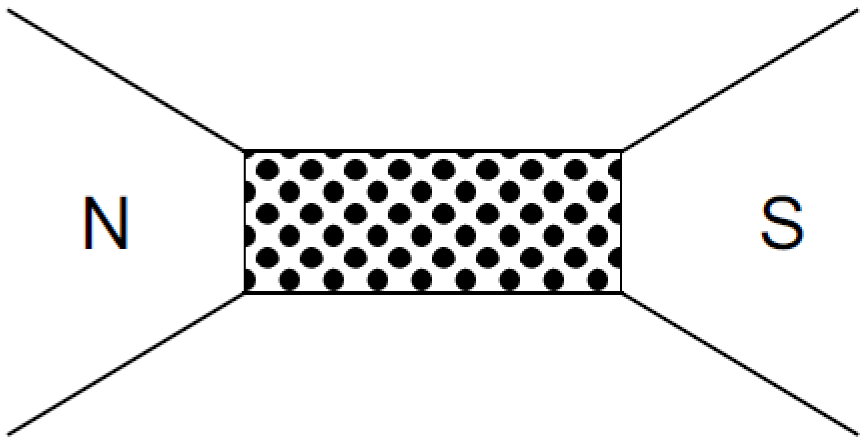


NS conductance: Blonder, Tinkham, Klapwijk

$$G_A = \frac{(2e)^2}{h} \sum_n \mathcal{T}_n,$$

Andreev transmission:

$$\mathcal{T}_n = T_n^2/(2 - T_n)^2.$$



Shot noise: de Jong-Beenakker

$$\langle|\delta I|^2\rangle = 2e|V|G_A\beta_A, \quad \beta_A = \frac{\sum_n T_n(1-T_n)}{\sum_n T_n}$$

$$e^* = 2e$$

A.D.Zaikin - Shot noise and Coulomb effects on non-local electron transport
Charge and heat dynamics in nanosystems Orsay, France, October 10-12, 2011
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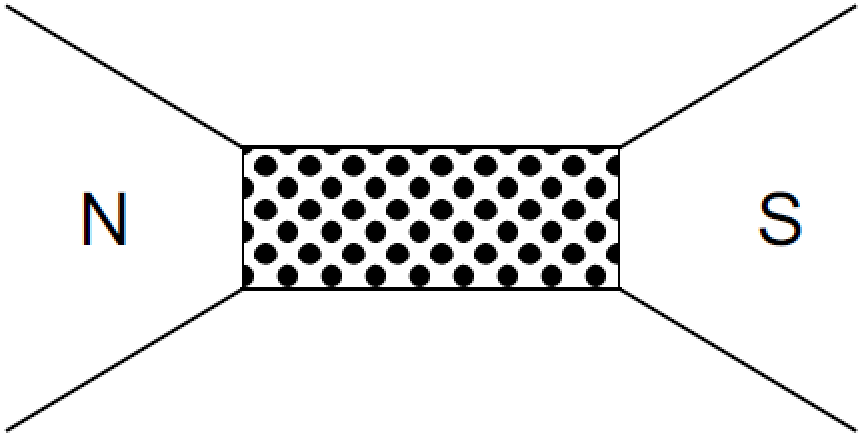
Diffusive conductor



$$G_N = G_A, \quad \beta_N = \beta_A = 1/3$$

$$\langle|\delta I|^2\rangle = 2e|V|G_N/3$$

Shot noise doubling in diffusive NS systems

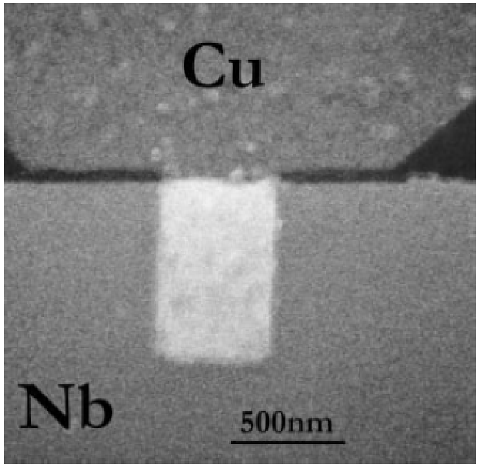
letters to nature (2000)

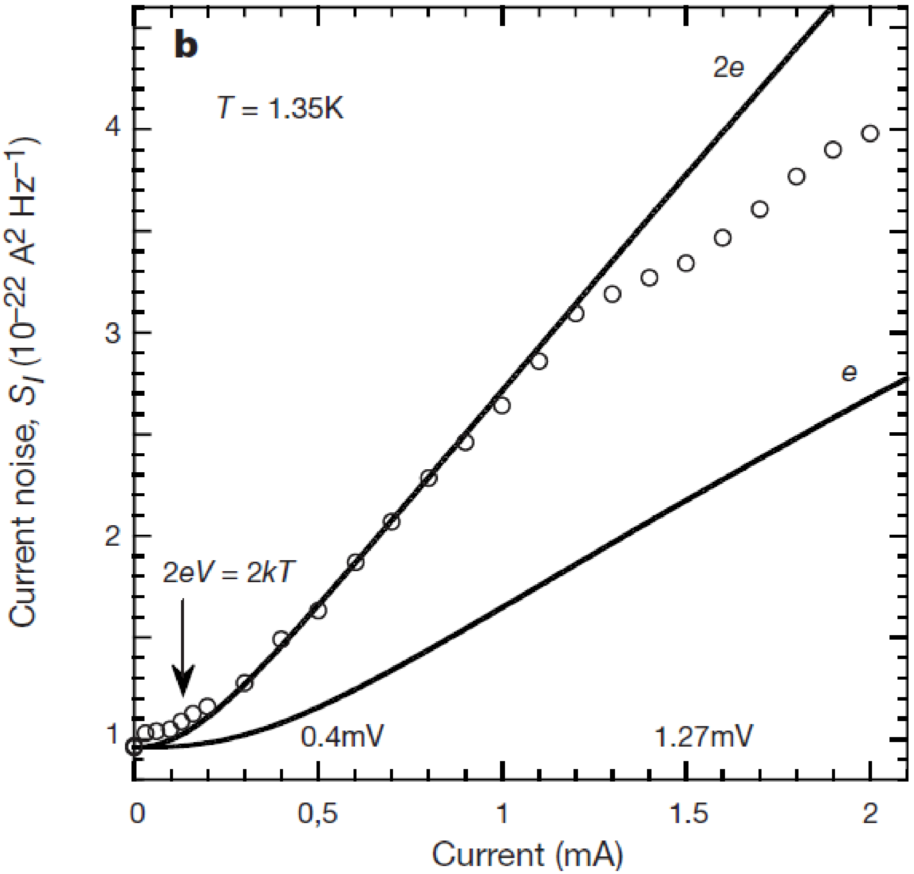
Detection of doubled shot noise in short normal-metal/ superconductor junctions

X. Jehl*, M. Sanquer*, R. Calemczuk* & D. Mailly†

* *DRFMC-SPSMS, CEA-Grenoble, F-38054 Grenoble, France*
† *Laboratoire de Microstructures et de Microélectronique, CNRS-LMM, F-92225 Bagneux, France*

$$S_I = \frac{2}{3}\left[\frac{4k_B T}{R_d} + e^* I \coth\left(\frac{e^* V}{2k_B T}\right)\right]$$





VOLUME 84, NUMBER 15 PHYSICAL REVIEW LETTERS 10 APRIL 2000

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Observation of Photon-Assisted Noise in a Diffusive Normal Metal–Superconductor Junction

A. A. Kozhevnikov, R. J. Schoelkopf, and D. E. Prober
Departments of Physics and Applied Physics, Yale University, New Haven, Connecticut 06520-8284
(Received 12 November 1999)

$$T_N = S_I R_{\mathrm{diff}}/(4k_B)$$

$$T_N = q_{\mathrm{eff}}|V|/(6k_B) = (2e)|V|/(6k_B)$$

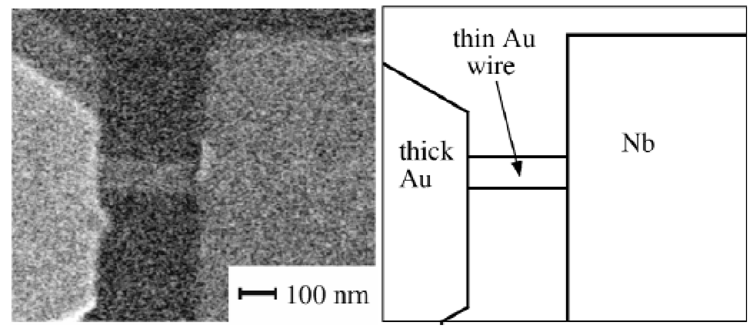
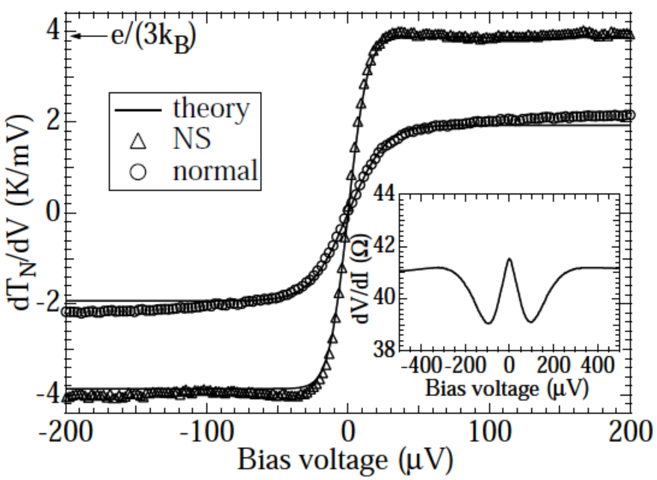
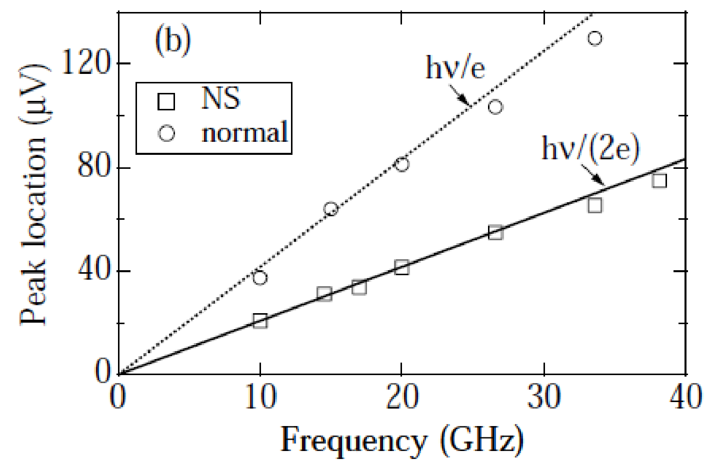
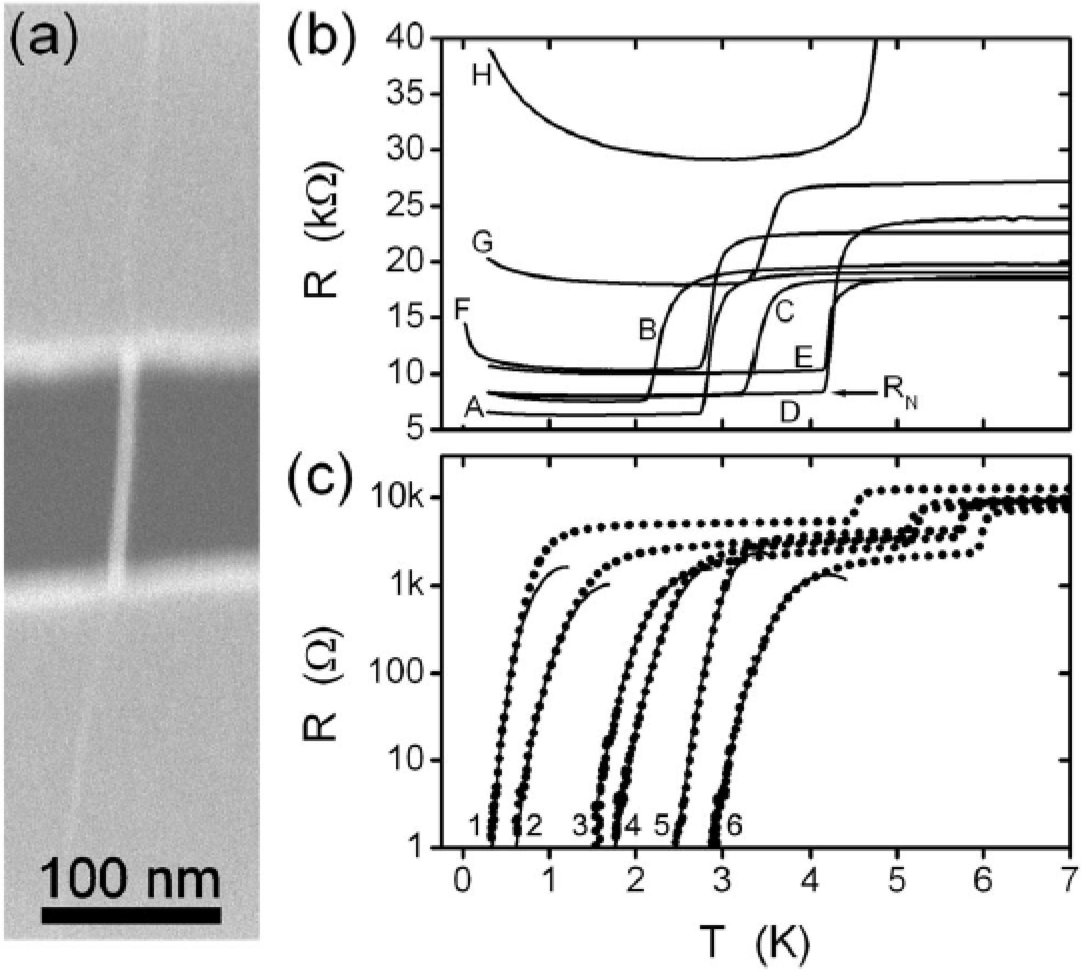


FIG. 1. SEM picture of the device and device schematic.





Bollinger, Rogachev, Bezryadin, EPL (2006)



Bollinger, Rogachev, Bezryadin, EPL (2006)

$$I(V) = VG_0 - \frac{e\beta k_B T}{\hbar}\mathrm{Im}\left[w\Psi\left(1+\frac{w}{2}\right) - iv\Psi\left(1+\frac{iv}{2}\right)\right],$$

where $w = u + iv$, $u = gE_C/\pi^2 k_B T$, $v = eV/\pi k_B T$, and $\Psi(x)$ is the digamma function.

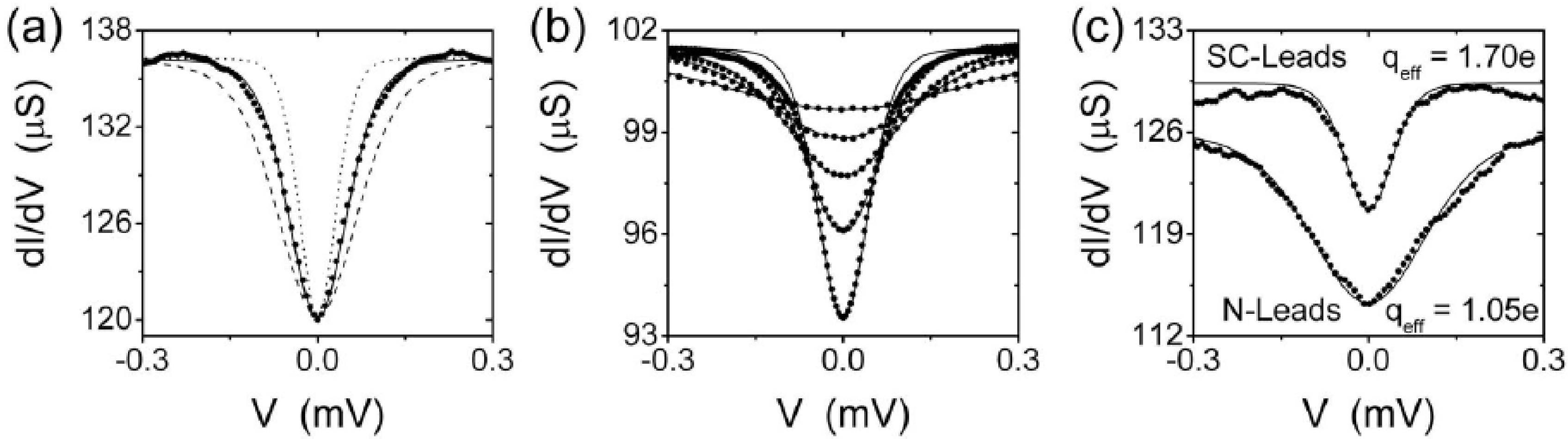


Fig. 3 – (a) The $\mathrm{d}I/\mathrm{d}V$ *vs.* V curves for sample B at $T = 0.28\,\mathrm{K}$. Comparisons to the GZ theory are shown with $q_{eff} = e$ (dashed line), $q_{eff} = 1.29e$ (solid line), and $q_{eff} = 2e$ (dotted line). (b) The $\mathrm{d}I/\mathrm{d}V$ *vs.* V curves for sample E at $T = 0.3$ (deepest dip), 0.5, 0.75, 1.0, and $1.5\,\mathrm{K}$ (shallowest dip). $q_{eff} = 1.53e$. (c) The $\mathrm{d}I/\mathrm{d}V$ *vs.* V curves for sample D at two different magnetic fields. At $B = 0$ the leads are superconducting and $q_{eff} = 1.70e$ whereas at high field ($B = 9\,\mathrm{T}$) the leads are driven normal and q_{eff} drops to $1.05e$. Solid lines are fits to the GZ theory.

Galaktionov,
A.D.Z.'09

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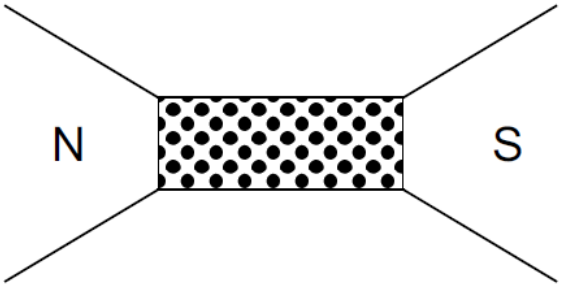
Shot noise:

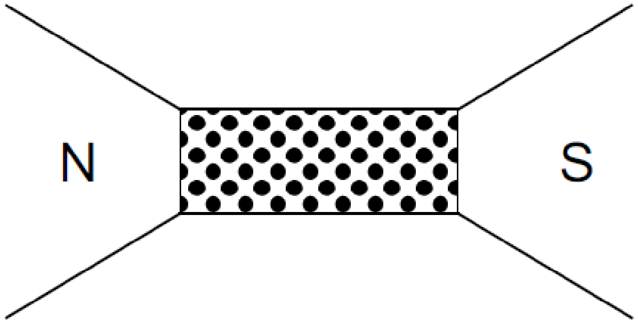


$$\frac{\langle|\delta I|^2_\omega\rangle}{G_A} = (1-\beta_A)\omega\coth\frac{\omega}{2T} + \frac{\beta_A}{2}\sum_\pm(\omega\pm 2eV)\coth\frac{\omega\pm 2eV}{2T}$$

Reduces to:

- De Jong-Beenakker'94 at T=0
- Nagaev-Büttiker'02 in diffusive limit

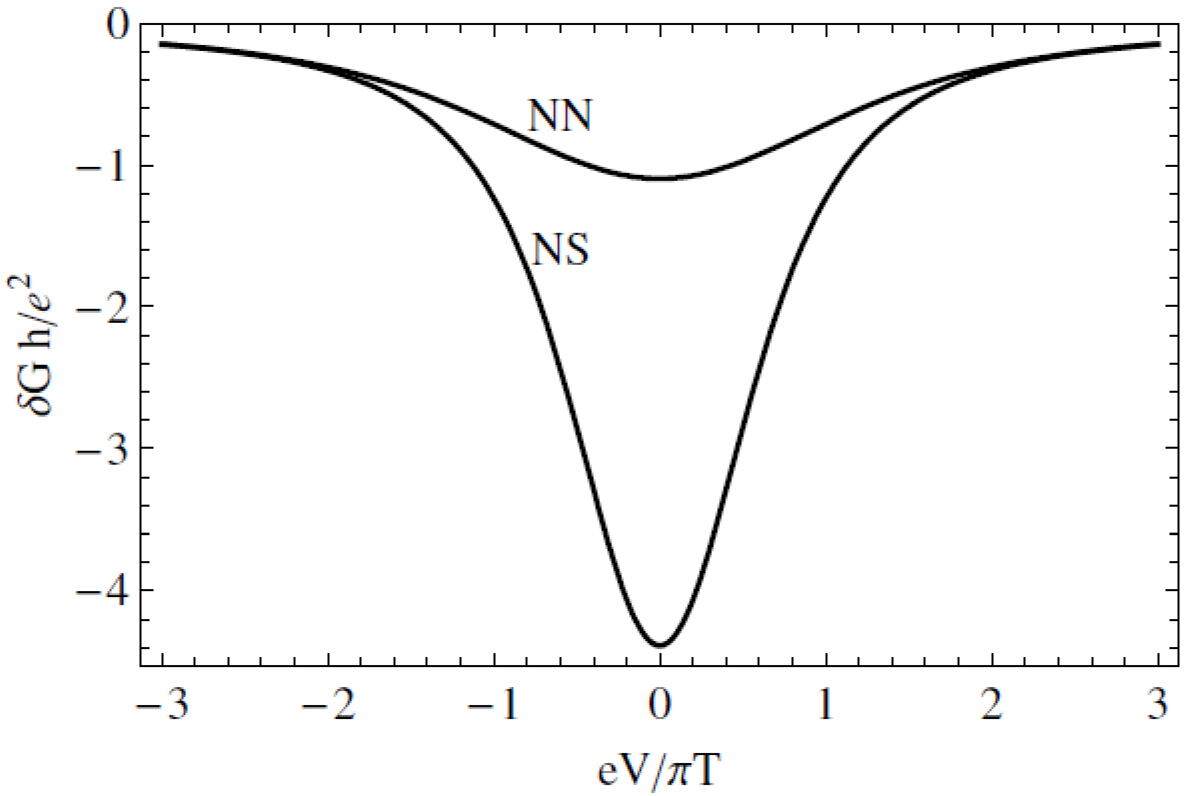
Interaction correction



for $g_A \gg 1$ or $\max(T, eV) \gg E_C = e^2/2C$ we get

$$I = G_A V - 2e\beta_A T \,\mathrm{Im}\left[w\Psi\left(1+\frac{w}{2}\right) - iv\Psi\left(1+\frac{iv}{2}\right)\right]$$

where $\Psi(x)$ is the digamma function, $w = g_A E_C/\pi^2 T + iv$ and $v = 2eV/\pi T$.



The interaction correction $\delta G = dI/dV - G_N$ for short diffusive conductors at $T = G_N/2\pi C$. The upper and lower curves correspond to normal and NS structures respectively.

Diffusive NS structures:

$$e^* = q_{\mathrm{eff}} = 2e$$

↑ Effective charge measured from shot noise

↑ Effective charge measured from e-e interactions

Diffusive NS structures:

e-e interaction correction in NS = 4 X e-e interaction correction in NN

(effective charge)x(shot noise power)

2 2

Crossed Andreev reflection

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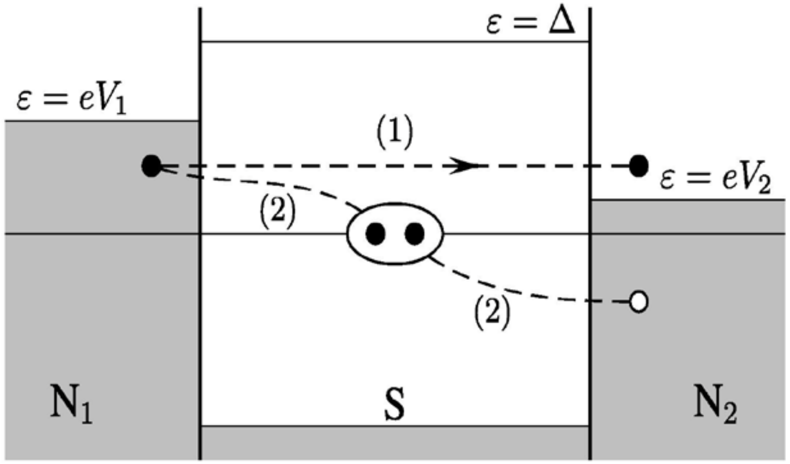
T=0, lowest order in transmission:



EC CAR

$$G_{12} = G_{(1)} - G_{(2)} = 0$$

Falci, Feinberg, Hekking'01

Kalenkov, A.D.Z.'07:

Ballistic NSN structures: NO contribution of CAR to non-local conductance at full transmissions

Arbitrary interface transmissions + disorder

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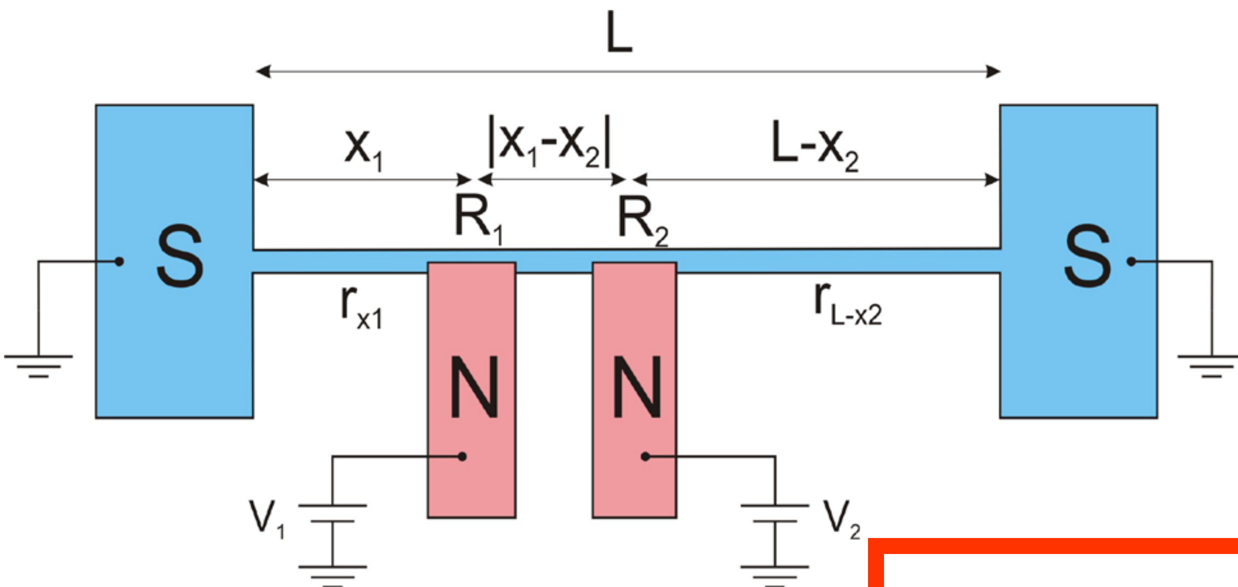
Golubev, Kalenkov, A.D.Z., PRL'09



$$T \ll \Delta$$

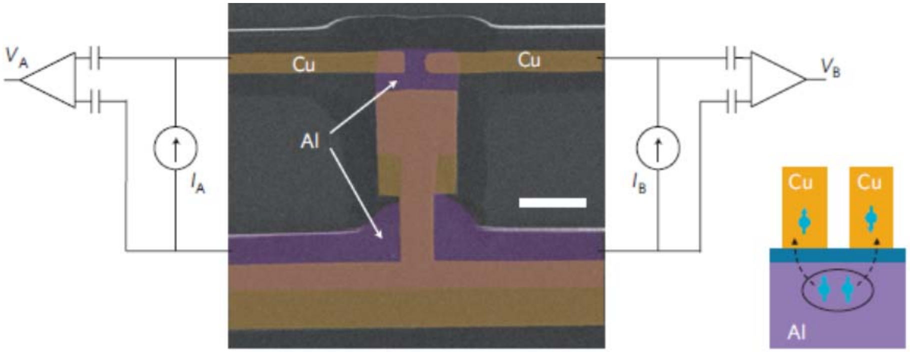
$$R_{12} = \frac{r_{\xi_S}}{2} e^{-|x_2 - x_1|/\xi_S}$$

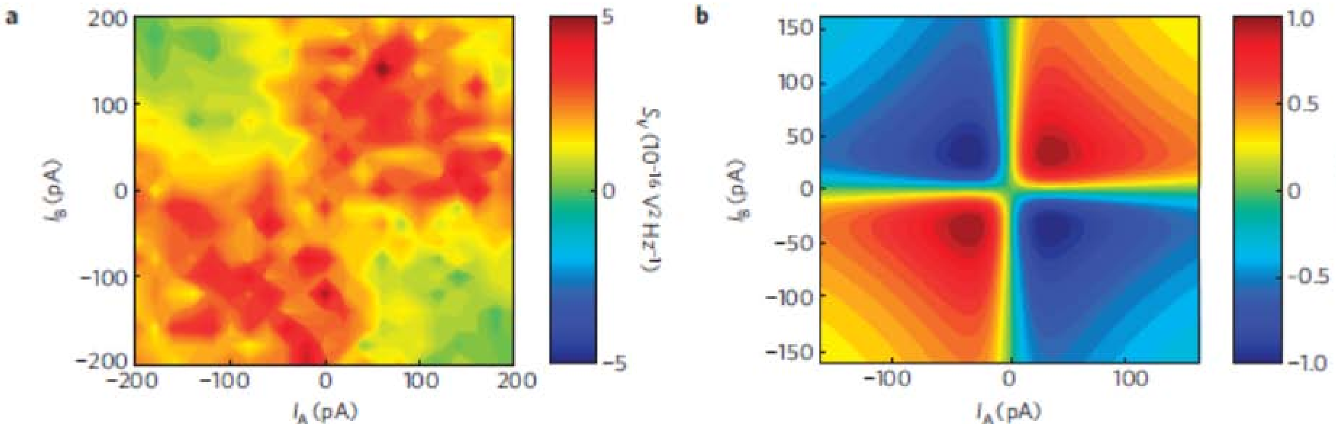
Non-local shot noise in NSN heterostructures

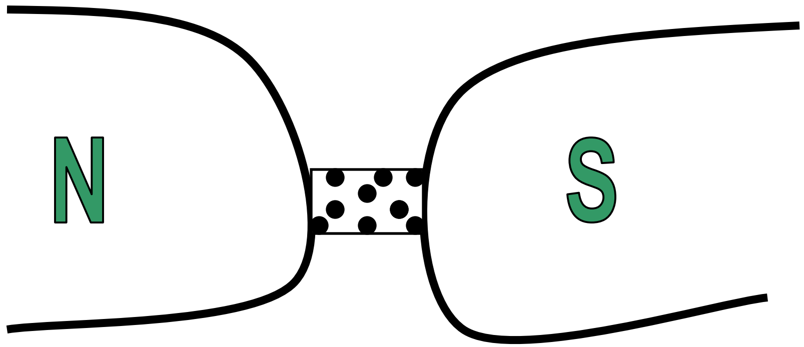
Theory, tunneling limit (Bignon, Houset, Pistolesi, Hekking'04):

$$S_{AB} = S^{CAR} - S^{EC} = 2eG_Q\left[(V_A+V_B)\coth\left(\frac{eV_A+eV_B}{2k_BT}\right)A^{CAR} - (V_A-V_B)\coth\left(\frac{eV_A-eV_B}{2k_BT}\right)A^{EC}\right]$$

Experiment (Wei, Chandrasekhar'10):







$$H = H_0 + H_{\text{int}} + H_{\text{field}} \,,$$

where

$$H_0 = \int \mathrm{d}^3 r\, \psi_\sigma^+(\boldsymbol{r}) \left[-\frac{\hbar^2}{2m} \left(\boldsymbol{\nabla} - \frac{\mathrm{i}e}{\hbar} \boldsymbol{A} \right)^2 - \mu + U(\boldsymbol{r}) \right] \psi_\sigma(\boldsymbol{r}) \,,$$

$$H_{\text{int}} = \int \mathrm{d}^3 r \int \mathrm{d}^3 r'\, \psi_\sigma^+(\boldsymbol{r}) \psi_{\sigma'}^+(\boldsymbol{r}') [-\tfrac{1}{2} g(\boldsymbol{r}) \delta(\boldsymbol{r} - \boldsymbol{r}') \delta_{\sigma,-\sigma'} + e^2 v(\boldsymbol{r} - \boldsymbol{r}')] \psi_{\sigma'}(\boldsymbol{r}') \psi_\sigma(\boldsymbol{r}) \,,$$

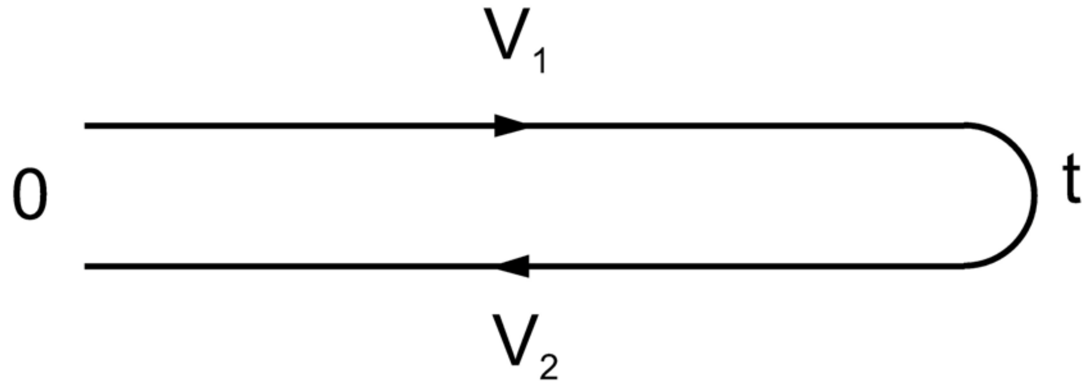
$$H_{\text{field}} = \int \mathrm{d}^3 r\, \frac{1}{8\pi} (\boldsymbol{h} - \boldsymbol{h}_x)^2 \,.$$

BCS

Coulomb

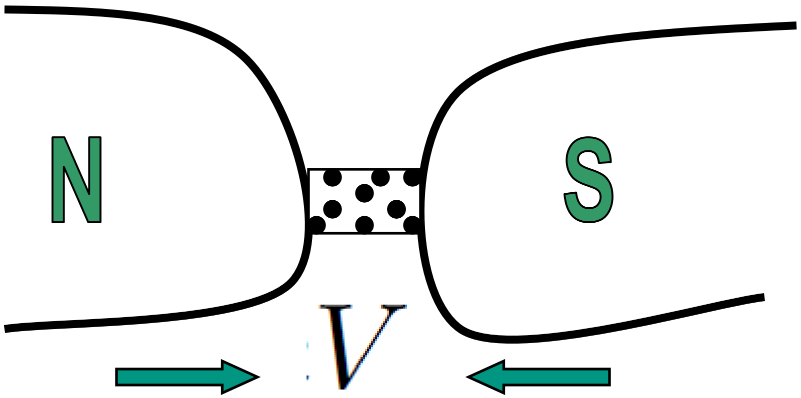
$$v(\boldsymbol{r}) = 1/|\boldsymbol{r}|$$

Real time dynamics: Keldysh contour



Effective action

$$iS = 2\,\mathrm{Tr}\ln \hat{G}_V^{-1} + i\int_0^t dt' \int d\mathbf{r}\,\frac{(\nabla V_1)^2 - (\nabla V_2)^2}{8\pi}.$$



Keldysh phases:

$$\dot{\varphi}_{1,2}(t) \;=\; eV_{1,2}$$

$$J = \int \mathcal{D}\varphi_1 \mathcal{D}\varphi_2 \exp(iS_c[\varphi] + iS_t[\varphi])$$

Charging term:

$$S_c[V] = \frac{C}{2e^2}\int_0^t dt'(\dot{\varphi}_1^2 - \dot{\varphi}_2^2) \equiv \frac{C}{e^2}\int_0^t dt\dot{\varphi}^+\dot{\varphi}^-$$

$$\varphi_+ = (\varphi_1 + \varphi_2)/2 \qquad \varphi_- = \varphi_1 - \varphi_2$$

Electron transfer between N- and S-terminals:

$$S_t[\varphi] = -\frac{i}{2}\sum_n \mathrm{Tr}\ln\left[1 + \frac{T_n}{4}\left(\{\check{G}_N, \check{G}_S\} - 2\right)\right]$$

A.D.Z.'94, Snyman-Nazarov'08

Average current and current noise

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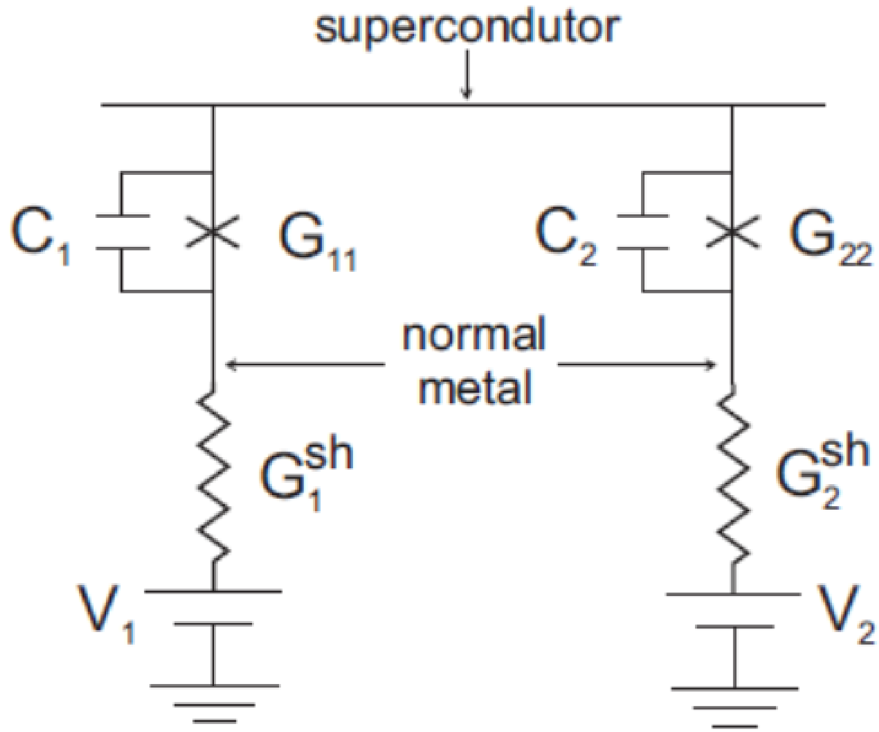
$$\langle \hat{I}(t) \rangle = ie \int \mathcal{D}\varphi_{\pm} \frac{\delta}{\delta\varphi_{-}(t)} e^{iS[\varphi]},$$

$$\frac{1}{2}\langle \hat{I}\hat{I} \rangle_{+} = -e^2 \int \mathcal{D}\varphi_{\pm} \frac{\delta^2}{\delta\varphi_{-}(t)\delta\varphi_{-}(t')} e^{iS[\varphi]},$$

$$\langle \hat{I}\hat{I} \rangle_{+} = \langle \hat{I}(t)\hat{I}(t') + \hat{I}(t')\hat{I}(t) \rangle$$

Non-local shot noise and e-e corrections



Model and key assumptions

- $\tau_r = T_r^2/(2 - T_r)^2$

  i.e. transmissions are the same for all barrier channels

- Large dimensionless conductances

$$g_r = 2\pi(G_r^{\mathrm{sh}} + G_{rr})/e^2 \gg 1$$

- Low energy limit $T, eV_r \ll |\Delta|$

- $e^2 N_r T_r R_\xi/\pi \ll 1$

Final effective action

$$iS_T = iS_{11} + iS_{22} + iS_{12},$$

where

$$iS_{11} = -i\frac{G_{11}}{e^2}\int dt\dot{\varphi}_1\varphi_1^- - \int dtdt'\frac{\varphi_1^-(t)\tilde{S}_{11}^{tt'}\varphi_1^-(t')}{2e^2},$$

$$iS_{12} = i\frac{G_{12}}{e^2}\int dt(\dot{\varphi}_1\varphi_2^- + \dot{\varphi}_2\varphi_1^-) - \int dtdt'\frac{\varphi_1^-(t)\tilde{S}_{12}^{tt'}\varphi_2^-(t')}{e^2},$$

and the term iS_{22} is obtained by interchanging the indices $1 \leftrightarrow 2$

The functions $\tilde{S}_{rl}^{tt'}$ read

$$\tilde{S}_{11}^{tt'} = G_{11}M(t-t')(1-\beta_1+\beta_1\cos[2\varphi_1^{tt'}]) + 2G_{12}M(t-t') \times(\alpha_1-\eta_1\cos[2\varphi_1^{tt'}]) + (G_{12}/2)M(t-t')(\kappa_1^+\cos[\varphi_1^{tt'} + \varphi_2^{tt'}] + \kappa_1^-\cos[\varphi_1^{tt'}-\varphi_2^{tt'}]),$$

$$\tilde{S}_{12}^{tt'} = -G_{12}M(t-t')(1-\beta_1+\beta_1\cos[2\varphi_1^{tt'}]) - G_{12}M(t-t')(1 - \beta_2+\beta_2\cos[2\varphi_2^{tt'}]) + (G_{12}/2)M(t-t')(\gamma_+\cos[\varphi_1^{tt'} + \varphi_2^{tt'}] - \gamma_-\cos[\varphi_1^{tt'}-\varphi_2^{tt'}]).$$

Here we denoted $\varphi_r^{tt'} = \varphi_r(t) - \varphi_r(t')$

$$M(t) = \int \frac{d\omega}{2\pi} e^{i\omega t}\omega \coth\frac{\omega}{2T} = -\frac{\pi T^2}{\sinh^2(\pi T t)}$$

The functions $\tilde{\mathcal{S}}_{rl}^{tt'}$ read

$$\tilde{\mathcal{S}}_{11}^{tt'} = G_{11}M(t-t')(1-\beta_1+\beta_1\cos[2\varphi_1^{tt'}]) + 2G_{12}M(t-t') \times(\alpha_1-\eta_1\cos[2\varphi_1^{tt'}]) + (G_{12}/2)M(t-t')(\kappa_1^{+}\cos[\varphi_1^{tt'} + \varphi_2^{tt'}] + \kappa_1^{-}\cos[\varphi_1^{tt'}-\varphi_2^{tt'}]),$$

$$\tilde{\mathcal{S}}_{12}^{tt'} = -G_{12}M(t-t')(1-\beta_1+\beta_1\cos[2\varphi_1^{tt'}]) - G_{12}M(t-t')(1 -\beta_2+\beta_2\cos[2\varphi_2^{tt'}]) + (G_{12}/2)M(t-t')(\gamma_+\cos[\varphi_1^{tt'} + \varphi_2^{tt'}] - \gamma_-\cos[\varphi_1^{tt'}-\varphi_2^{tt'}]).$$

Here we denoted $\varphi_r^{tt'} = \varphi_r(t) - \varphi_r(t')$ $\quad \beta_r = 1-\tau_r,$

$$\kappa_r^{\pm} = \pm(4\tau_r-3) + 1/\sqrt{\tau_1\tau_2} \quad (r=1,2),$$
$$\gamma_{\pm} = \pm 1 + \left(1-2\tau_1-2\tau_2+4\tau_1\tau_2\right)/\sqrt{\tau_1\tau_2}$$

Our non-local action is equivalent to the following Langevin equations:

$$C_1\dot{v}_1 + (G_1^{\rm sh} + G_{11})v_1 - G_{12}v_2 = G_1^{\rm sh}V_1 + \xi_1^{\rm sh} + \xi_1,$$

$$C_2\dot{v}_2 + (G_2^{\rm sh} + G_{22})v_2 - G_{12}v_1 = G_2^{\rm sh}V_2 + \xi_2^{\rm sh} + \xi_2,$$

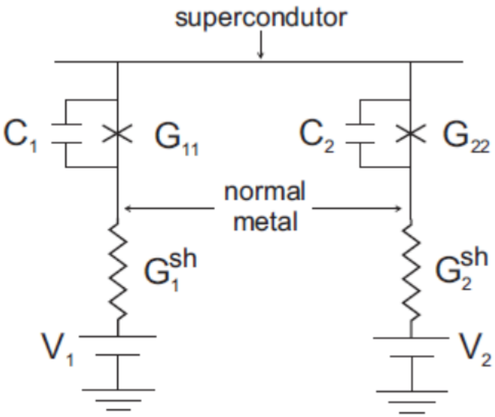
which describe the current balance in our system. Here $\xi_r^{\rm sh}$ are stochastic variables with pair correlators

$$\langle \xi_r^{\rm sh}(t)\xi_r^{\rm sh}(t')\rangle = G_r^{\rm sh}M(t-t'),$$

describing Gaussian current noise in the shunt resistors while the variables ξ_r with the correlators

$$\langle \xi_r(t)\xi_l(t')\rangle = \widetilde{S}_{rl}^{tt'}$$

describe shot noise in NS barriers.



Non-local shot noise:

$$\mathcal{S}_{12}(t,t') = \langle I_1(t)I_2(t') + I_2(t)I_1(t')\rangle$$

$$\mathcal{S}_{12}(\omega) = -2G_{12}(2-\beta_1-\beta_2)W(\omega,0)$$
$$-2G_{12}\beta_1 W(\omega,2V_1) - 2G_{12}\beta_2 W(\omega,2V_2)$$
$$+G_{12}\gamma_+ W(\omega,V_1+V_2) - G_{12}\gamma_- W(\omega,V_1-V_2),$$

Positive cross-correlations due to CAR!

where

$$W(\omega,V) = \frac{1}{2}\sum_{\pm}(\omega \pm eV)\coth\frac{\omega \pm eV}{2T}.$$

$$\beta_r = 1-\tau_r,$$

$$\kappa_r^{\pm} = \pm(4\tau_r - 3) + 1/\sqrt{\tau_1\tau_2} \quad (r=1,2),$$
$$\gamma_{\pm} = \pm 1 + \left(1 - 2\tau_1 - 2\tau_2 + 4\tau_1\tau_2\right)/\sqrt{\tau_1\tau_2}$$

Full transmissions, zero frequency limit:

$$\mathcal{S}_{12}(0) = -8TG_{12} + 2eG_{12}(V_1 + V_2)\coth\frac{e(V_1 + V_2)}{2T}$$

only positive cross-correlations
due to CAR at T=0!

Electron-electron interactions

- $$\partial I_1/\partial V_1 = G_{11} - (4G_{11}\beta_1 - 8G_{12}\eta_1)F(2V_1)/g_1$$
  $$-\delta G_+ F(V_1+V_2) - \delta G_- F(V_1-V_2),$$

$$\eta_r = 2\tau_r(1-\tau_r)/\sqrt{\tau_1\tau_2},$$

- $$\partial I_1/\partial V_2 = -G_{12}[1-(4\beta_2/g_2)F(2V_2)]$$
  $$-\delta G_+ F(V_1+V_2) + \delta G_- F(V_1-V_2),$$

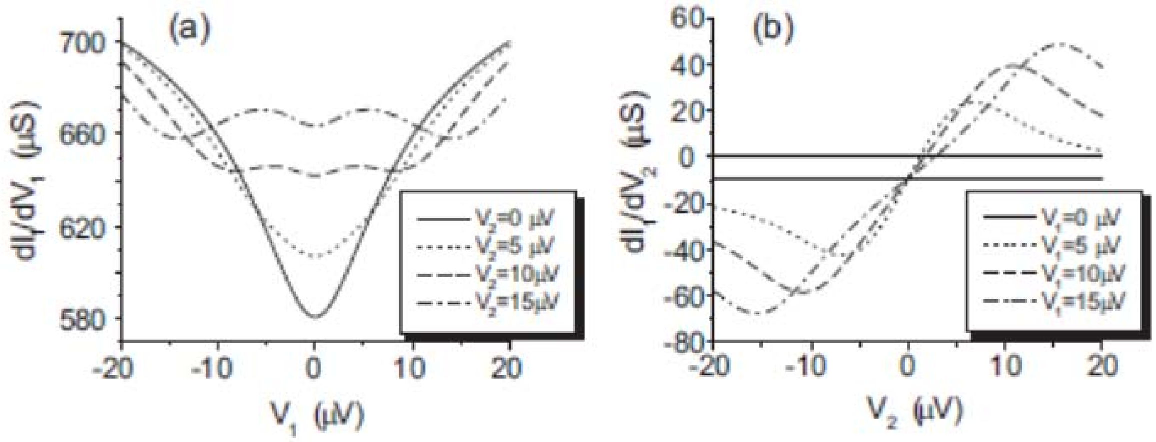
Coulomb anti-blockade due to CAR

where $\delta G_\pm = G_{12}\left(\kappa_1^\pm/g_1 + \gamma_\pm/g_2\right),$

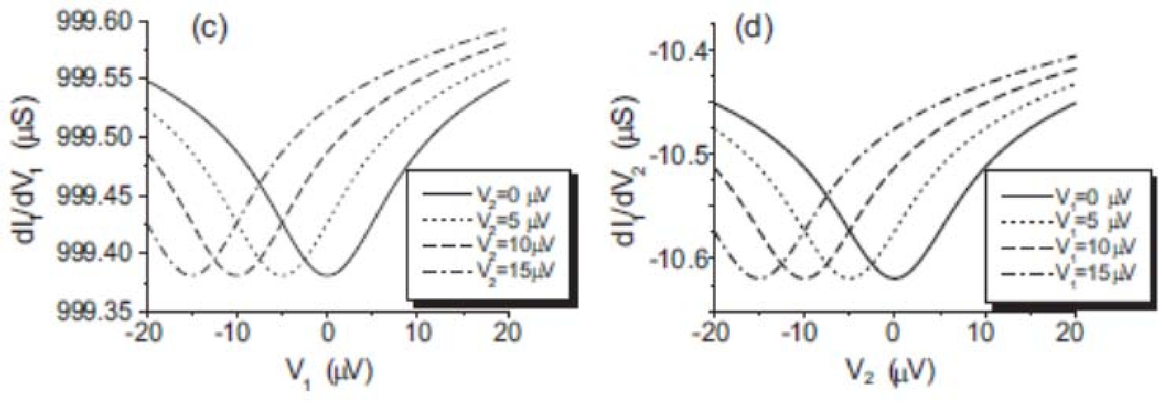
$$F(x) = \mathrm{Re}\,[\Psi(1+k+iax) + (k+iax)\Psi'(1+k+iax) - \Psi(1+iax) - iax\Psi'(1+iax)],$$

$\Psi(x)$ is the digamma function, $a = e/2\pi T$ and $k = 1/2\pi T\tau_{RC}$.

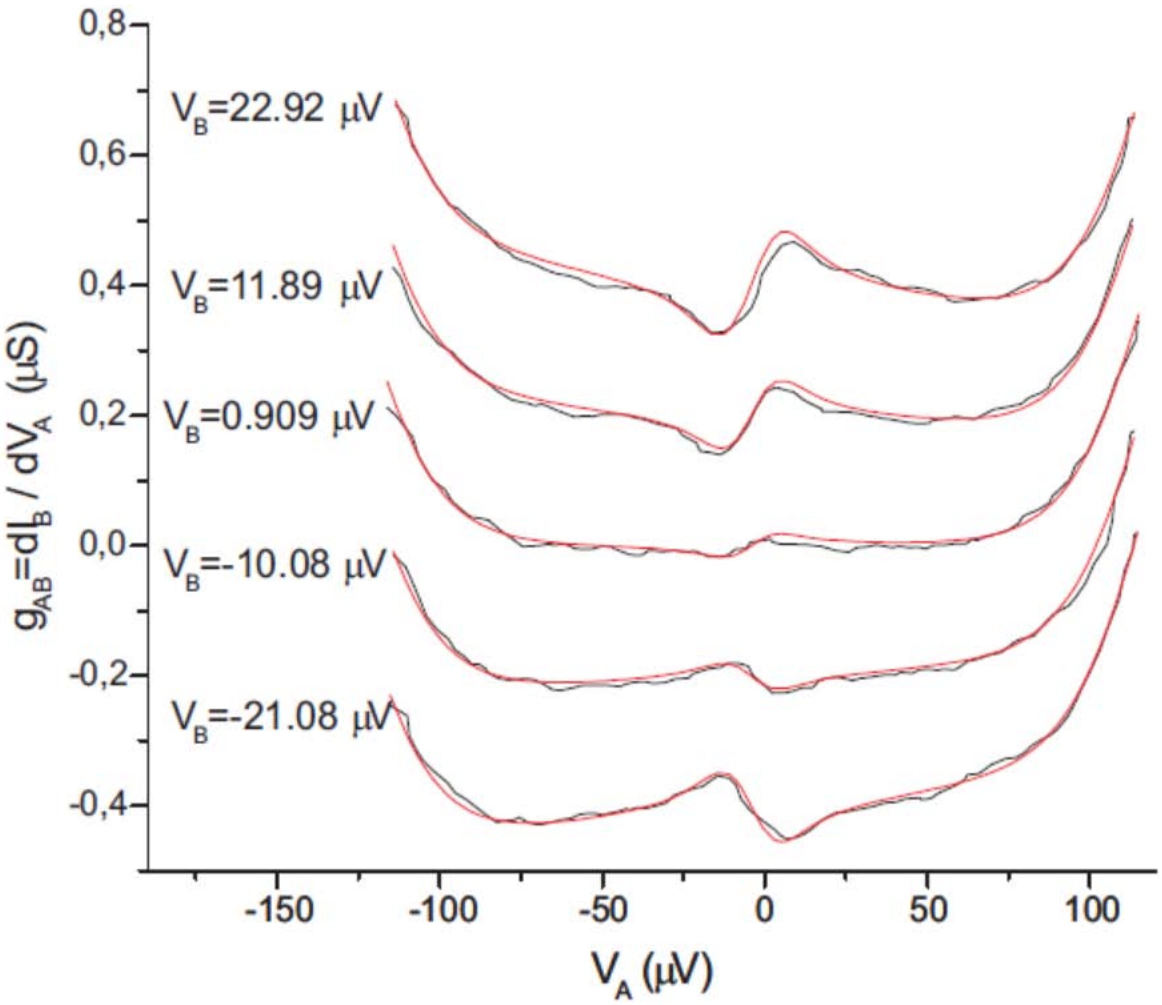
Interaction corrections: tunneling limit



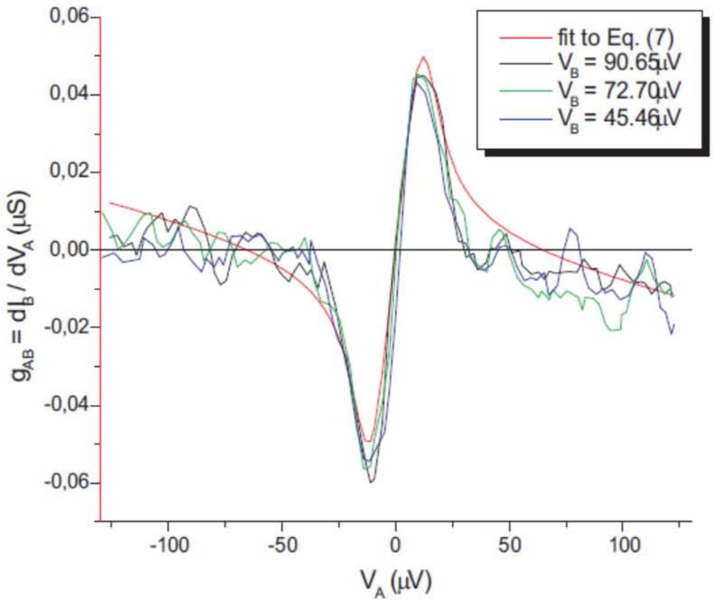
Interaction corrections: full transmissions

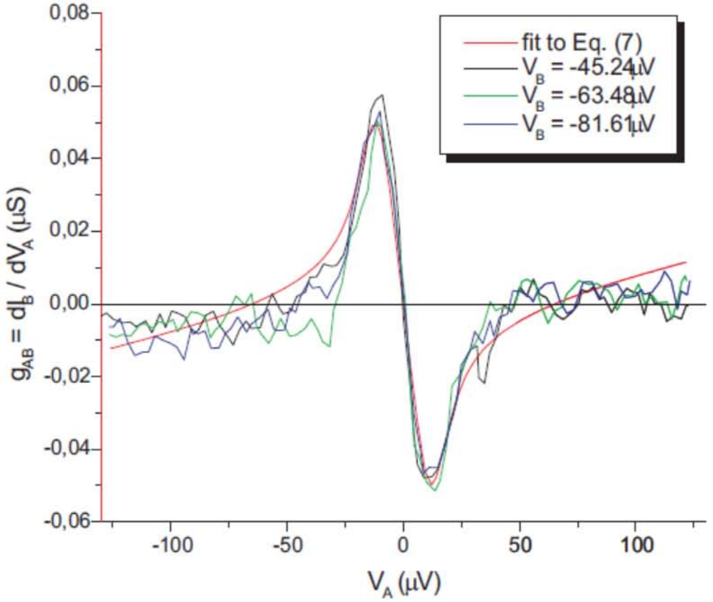


Preliminary comparison with Beckmann's experiments



Preliminary comparison with Beckmann's experiments





Summary

- Fundamental relation between shot noise and Coulomb effects in NS (local transport) and NSN (non-local transport)
- NSN structures: positive cross-correlations of shot noise due to CAR, dominate at large transmissions
- Non-local transport in NSN structures in the tunneling regime: (a) no effect in the linear in voltage regime and (b) S-shaped non-local conductance beyond linear regime
- Non-local transport in NSN structures with high transmissions: Coulomb anti-blockade due to CAR
- Good agreement with recent experiments

Thank you!