Circuit approach to photonic heat transport

applied to superconducting hybrid devices

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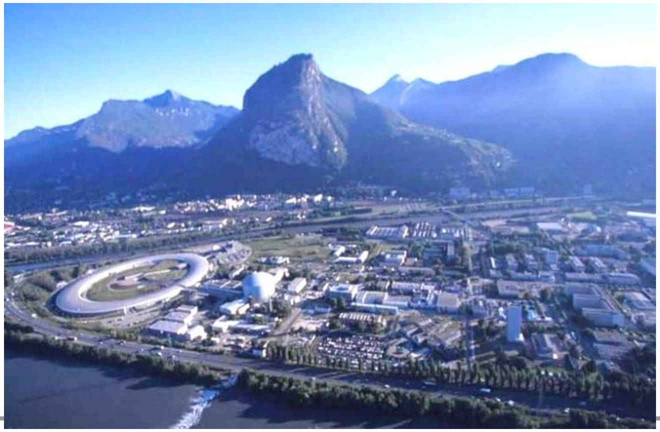
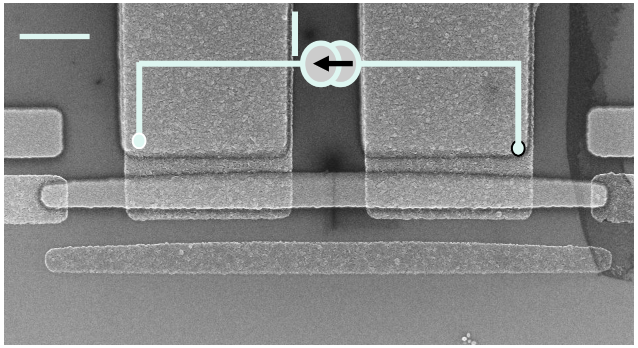
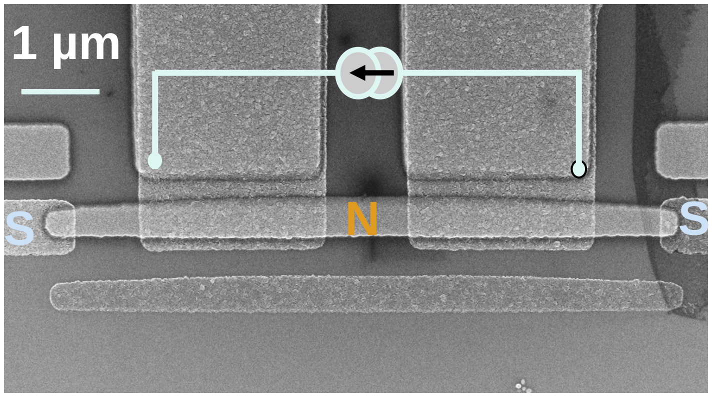
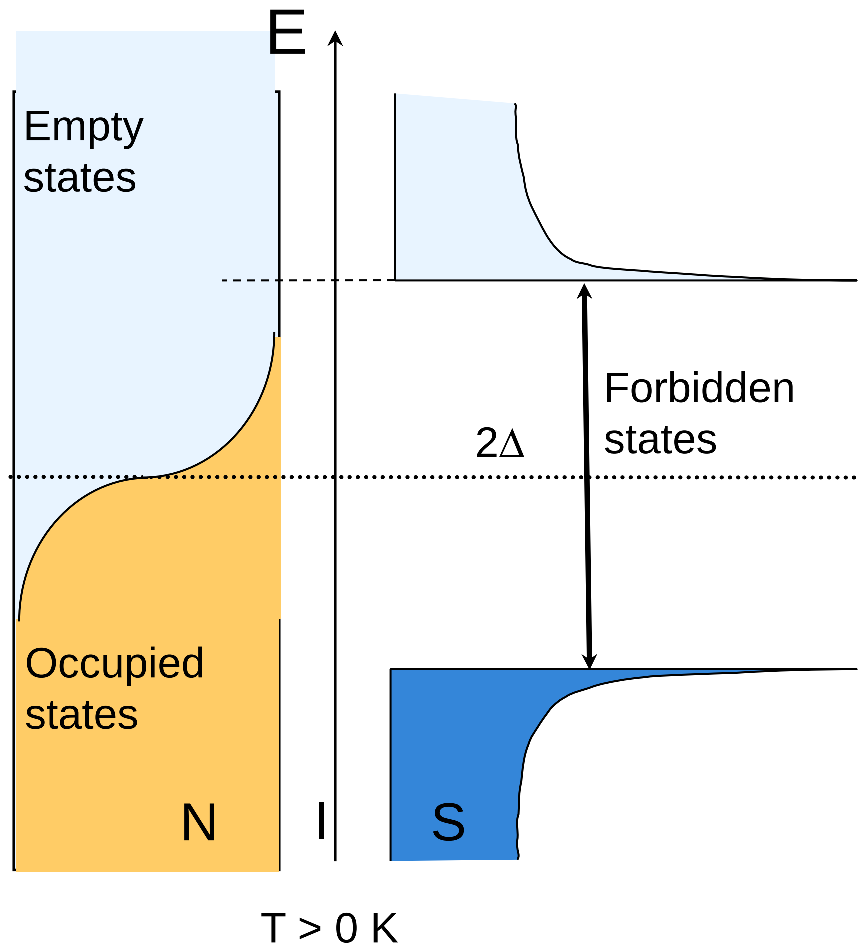




S-I-N-I-S thermometry





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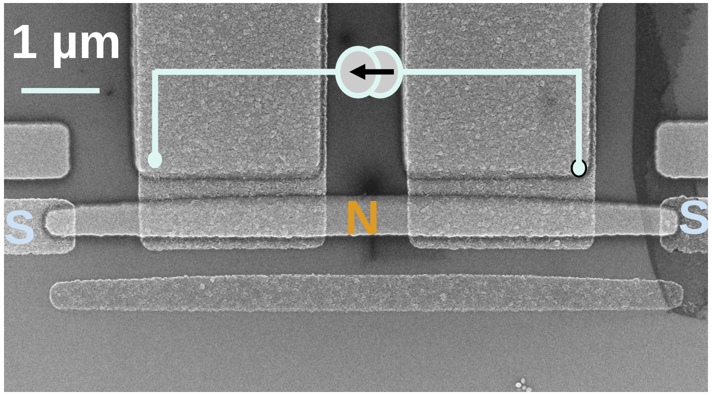

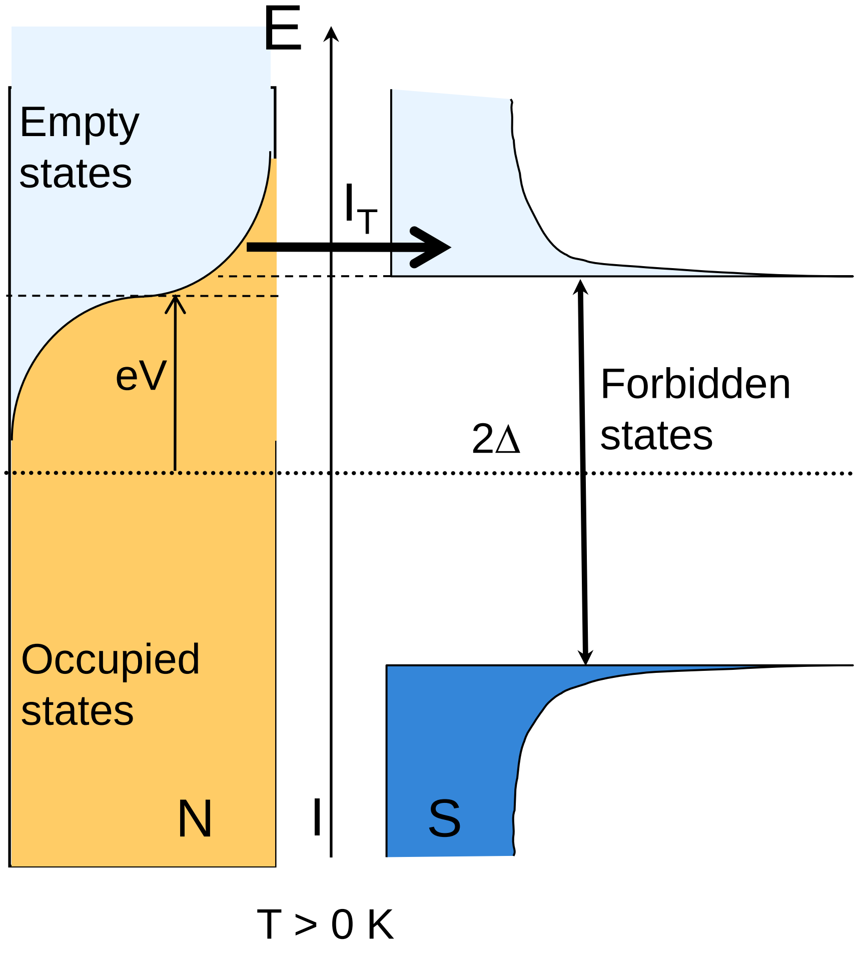

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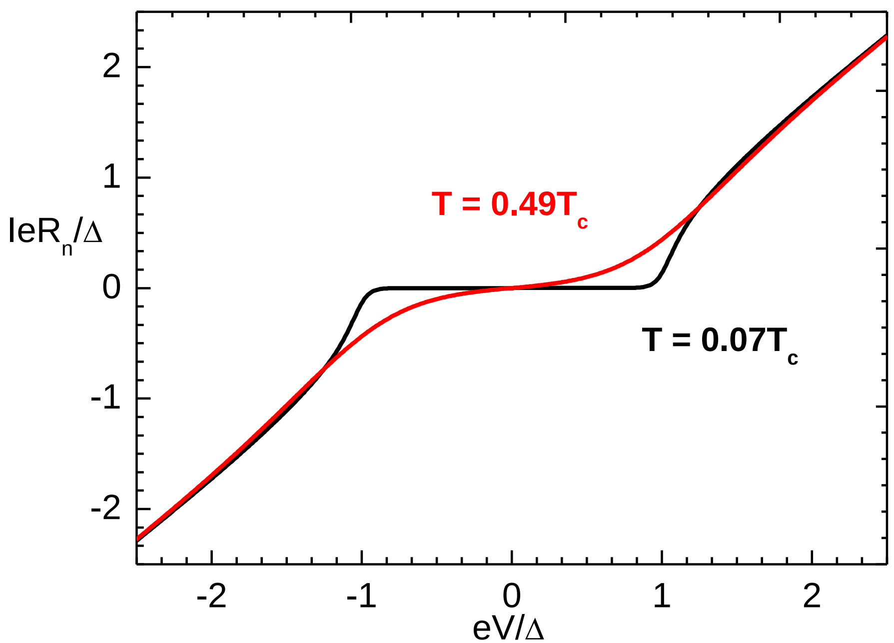
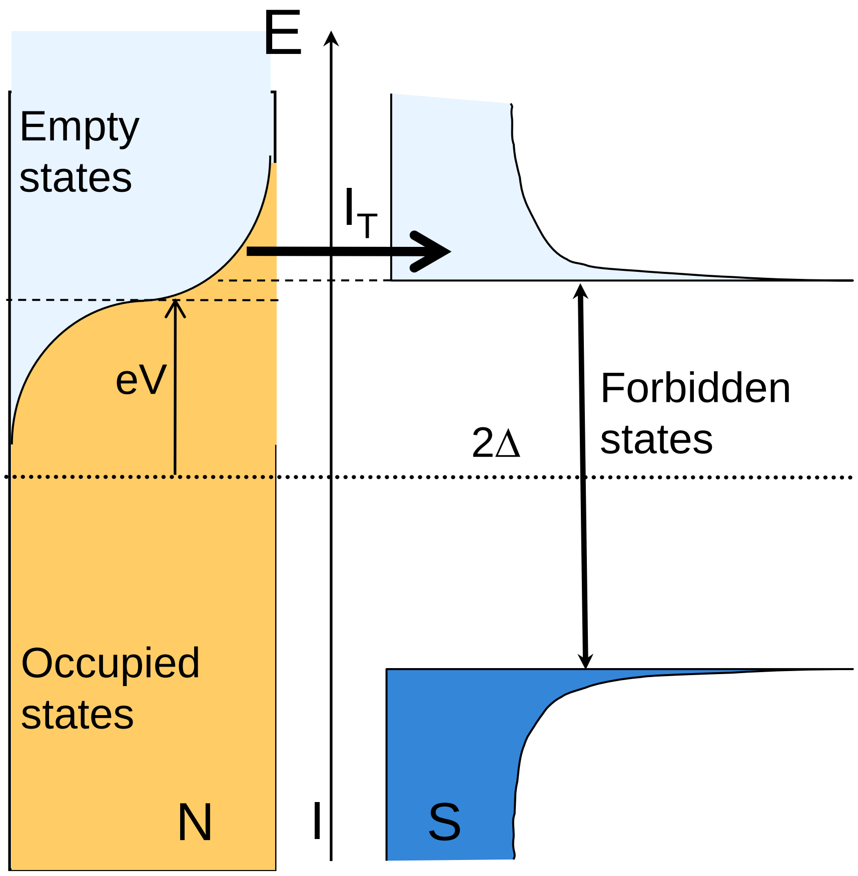


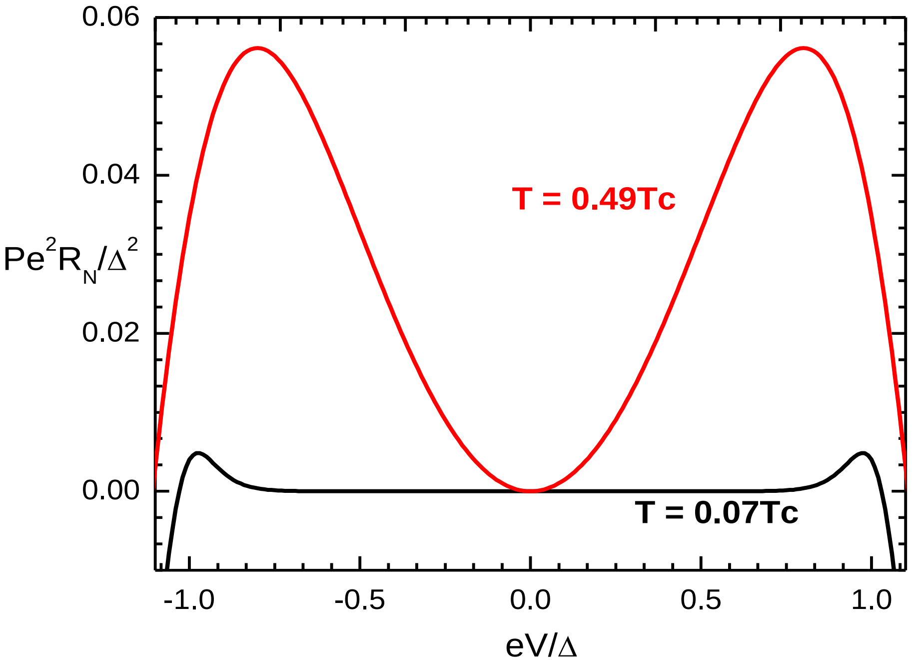


$$I_T = \frac{1}{eR_N}\int_{-\infty}^{\infty} n_S(E)[f_N(E-eV)-f_S(E)]dE$$

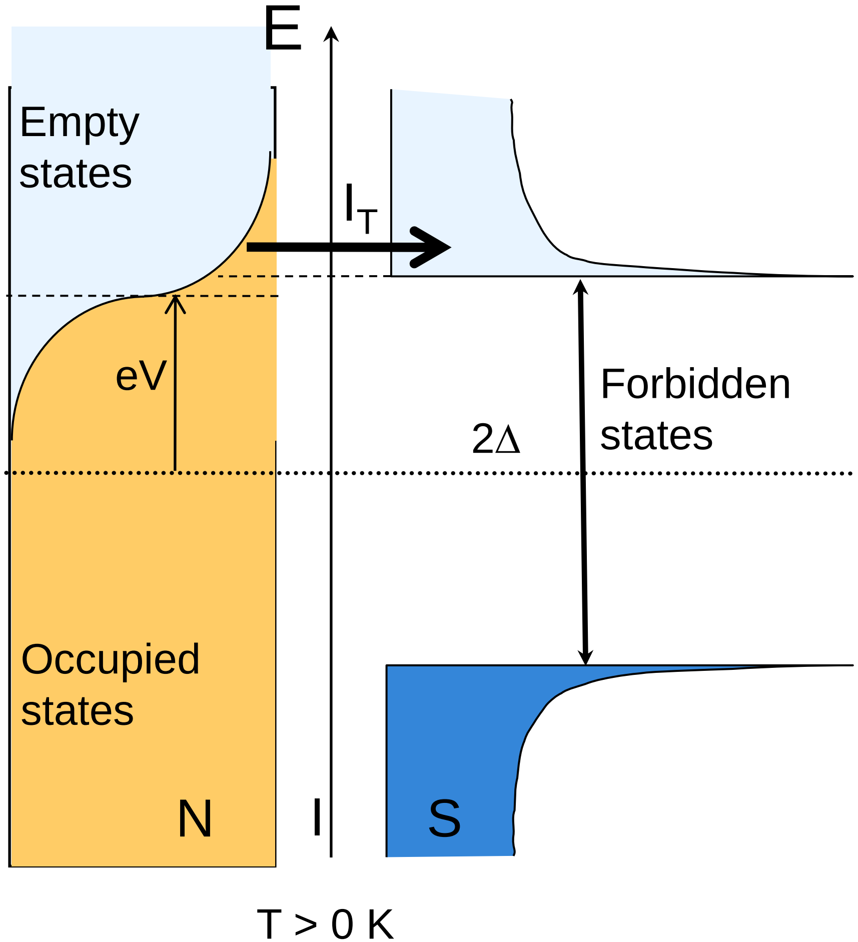
Voltage at fixed current measures **electronic** temperature

T > 0 K

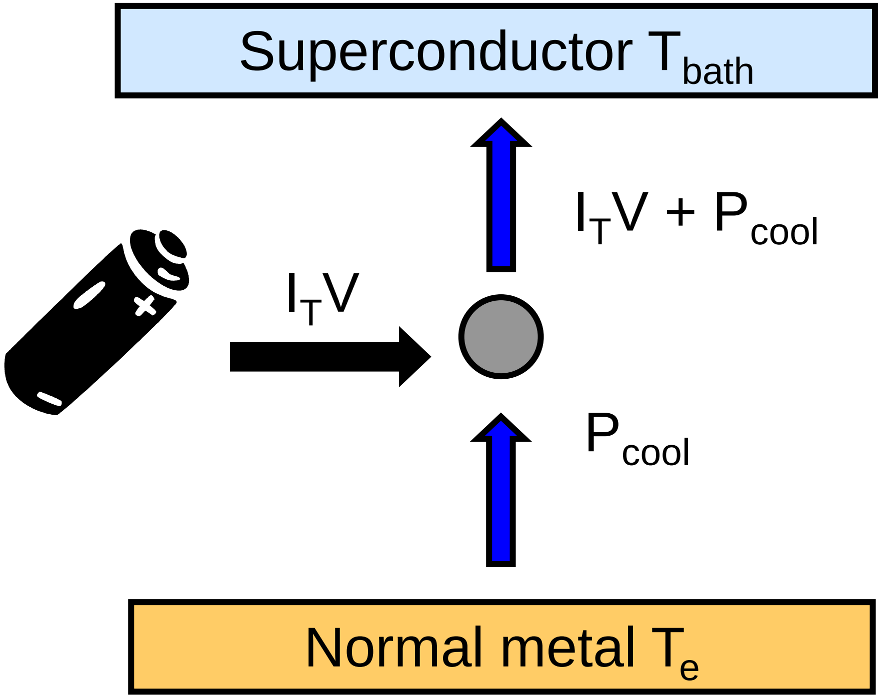
S-I-N-I-S electronic cooling



$$P_{cool} = \frac{1}{e^2 R_N} \int_{-\infty}^{\infty} (E - eV) n_S(E) [f_N(E - eV) - f_S(E)] dE$$

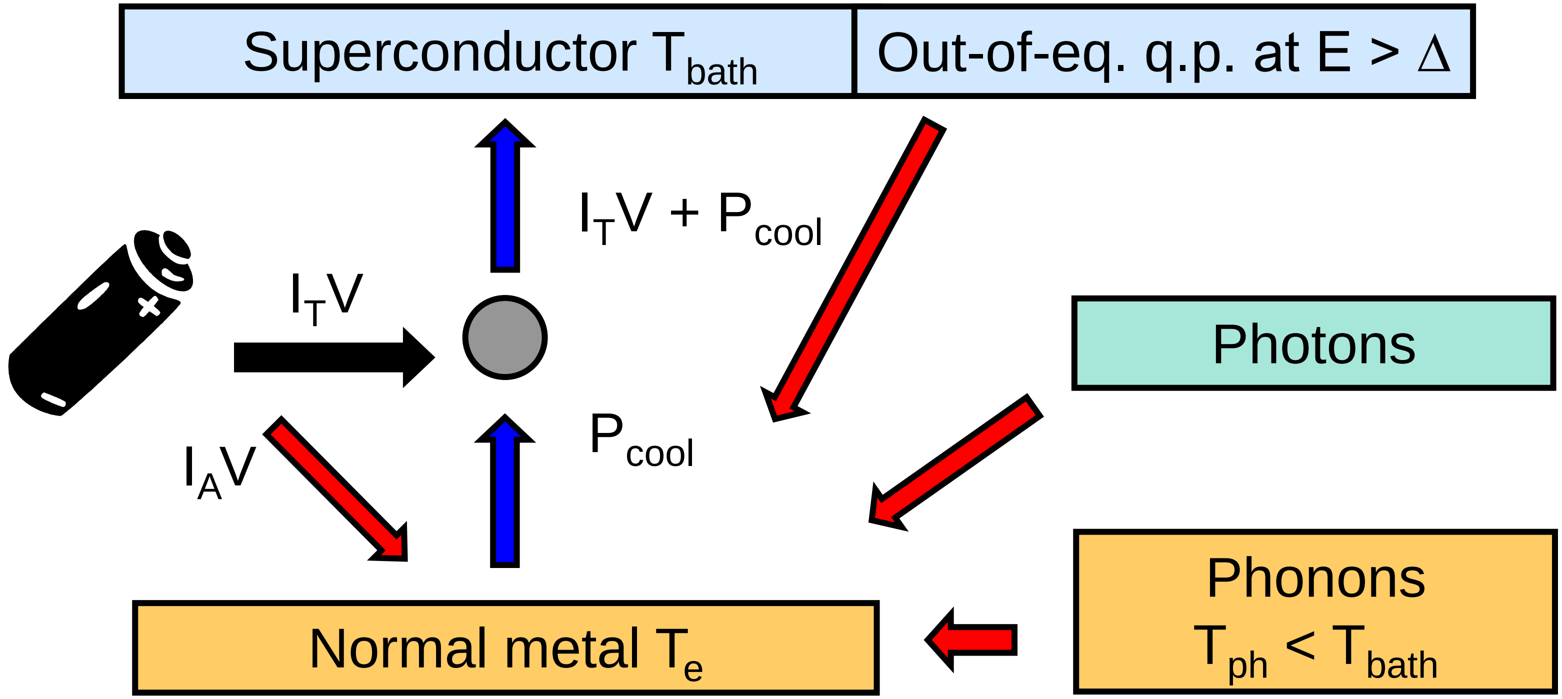


Limitations of electronic refrigeration



F. Giazotto, T. T. Heikkila, A. Luukanen, A. M. Savin and J. P. Pekola, Rev. Mod. Phys. 78, 217 (2006)

Limitations of electronic refrigeration



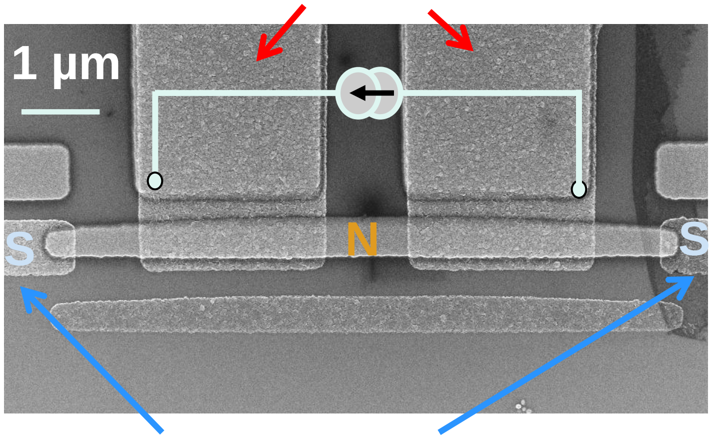
Our objective :

Understand the mechanisms coupling the cooled e- to the thermal baths.

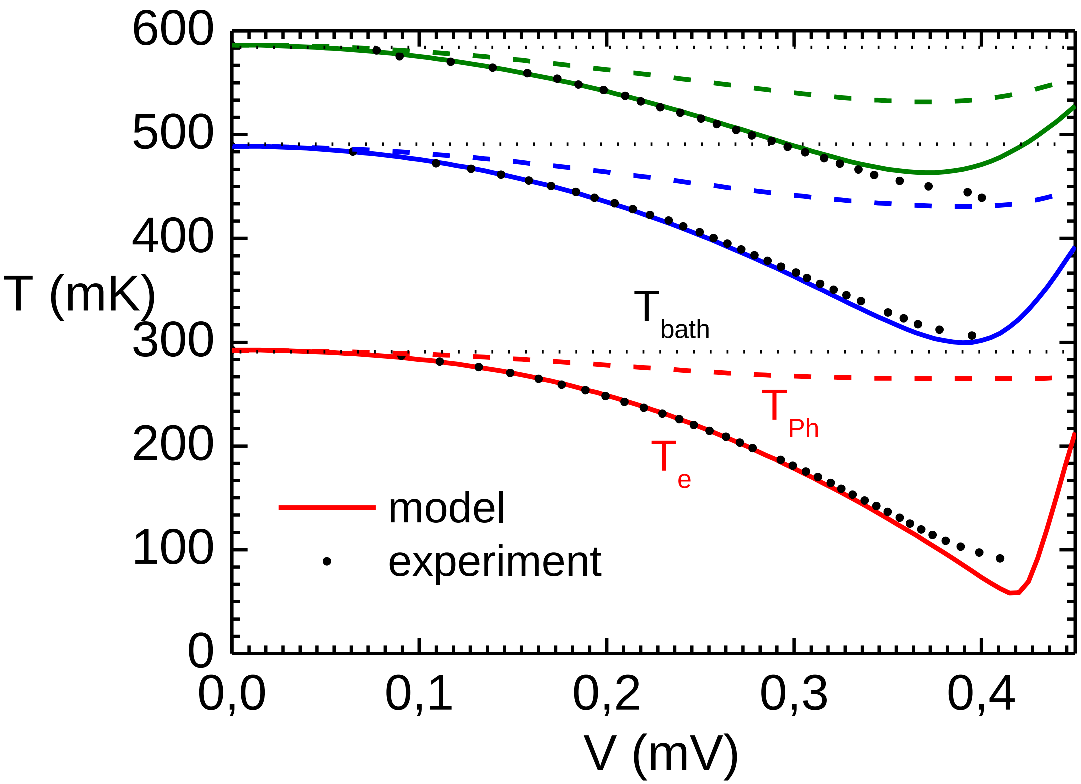
Performance of electronic refrigeration

Al/AlOx/Cu/AlOx/Al device:

Cooler junctions: N island weakly coupled to external world, strong cooling effect expected.



Thermometer junctions



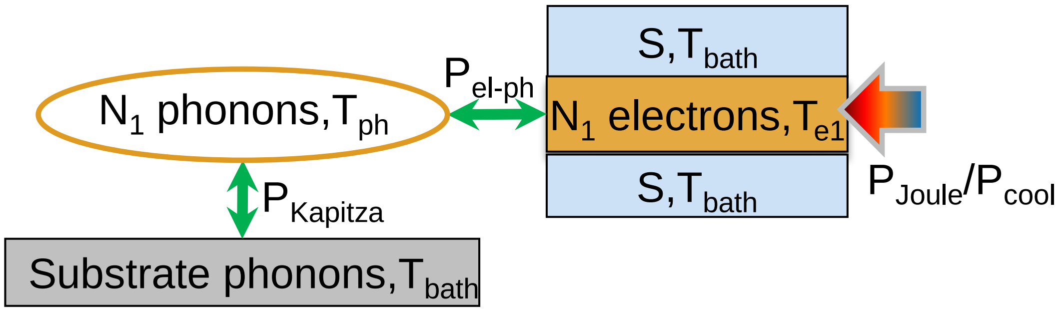
Electrons cool from 0.3 down to < 0.1K

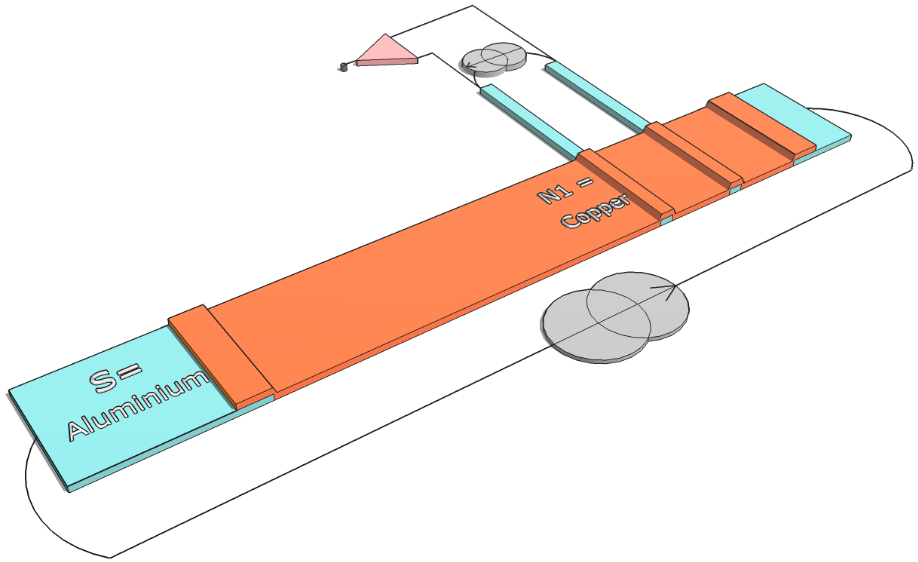
~ 1.5 pW at 0.3 K, area junction 0.45 µm²

S. Rajauria et al., PRL **99**, 047004 (2007)

Probing independently electron & phonon temperature…

Probe phonon temperature with S-I-N-I-S





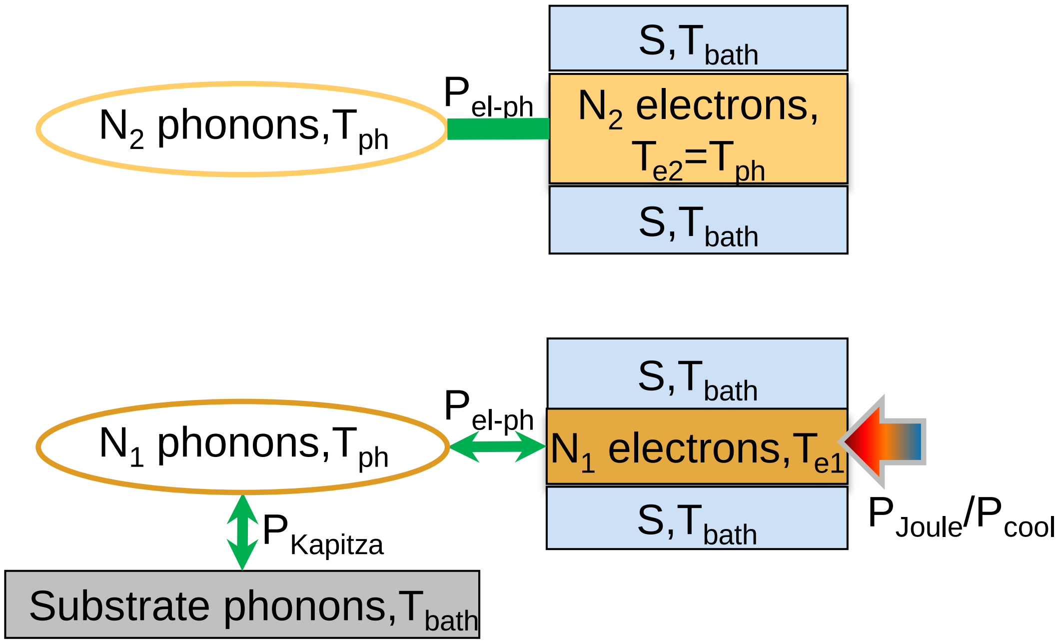
Theoretical expectations :

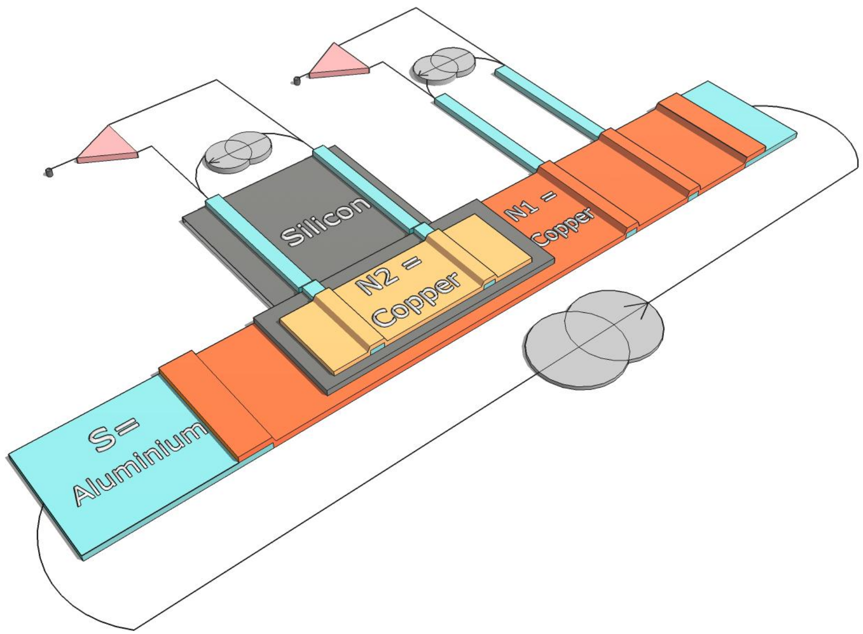
$$P_{el-ph} = \Sigma A d (T_e^5 - T_{ph}^5)$$

$$P_{Kapitza} = K A (T_{ph}^4 - T_{bath}^4)$$

Σ is material dependent prefactor,
K is the Kapitza coefficient,
A area, d thickness.

Probe phonon temperature with S-I-N-I-S





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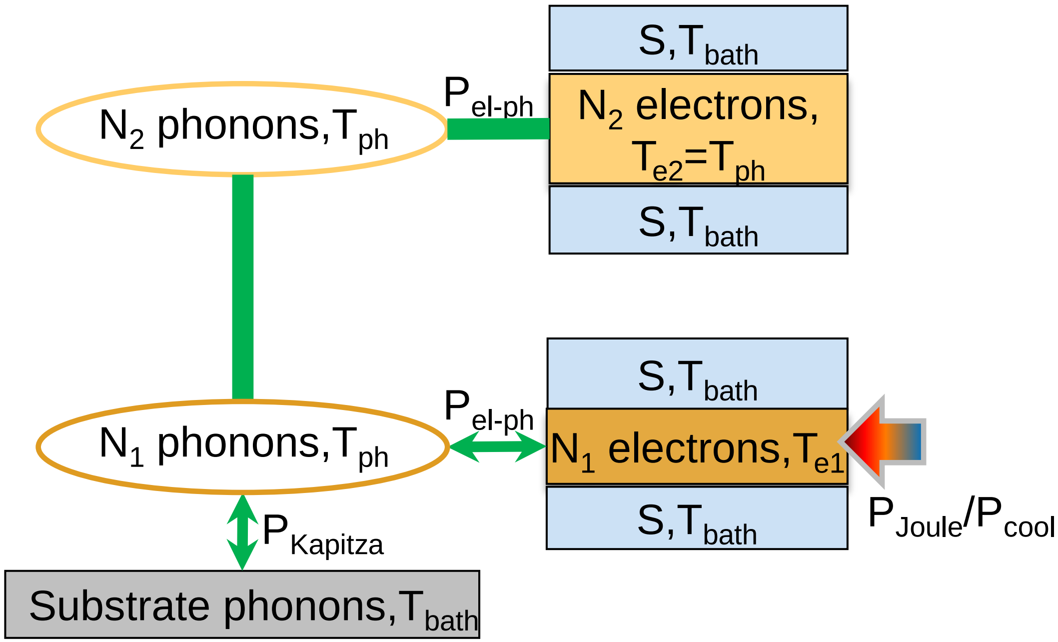

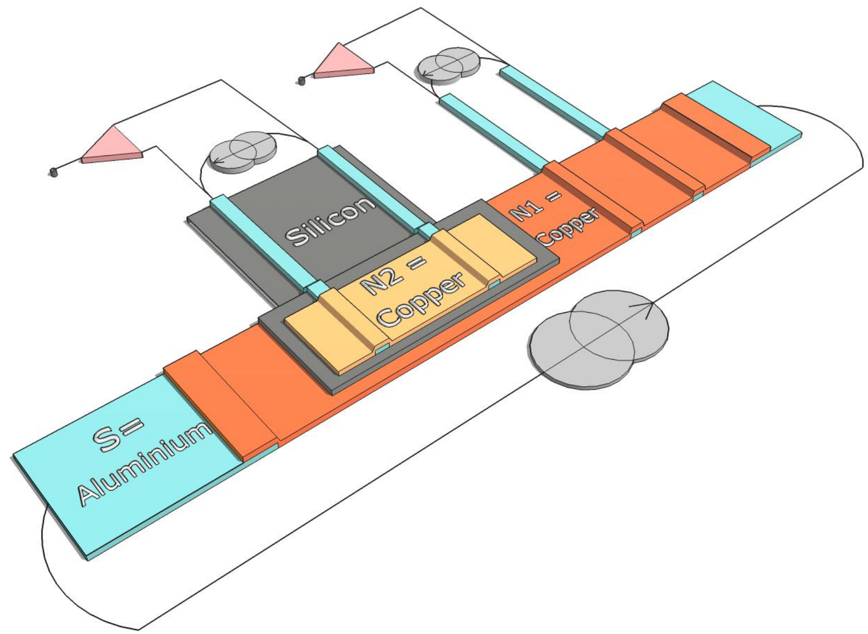

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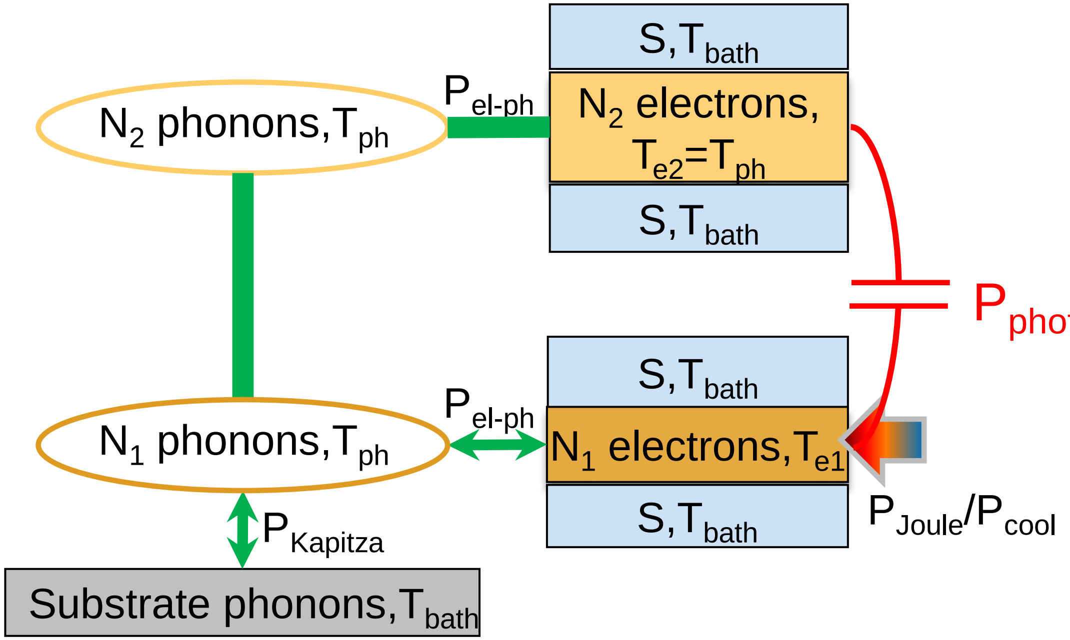
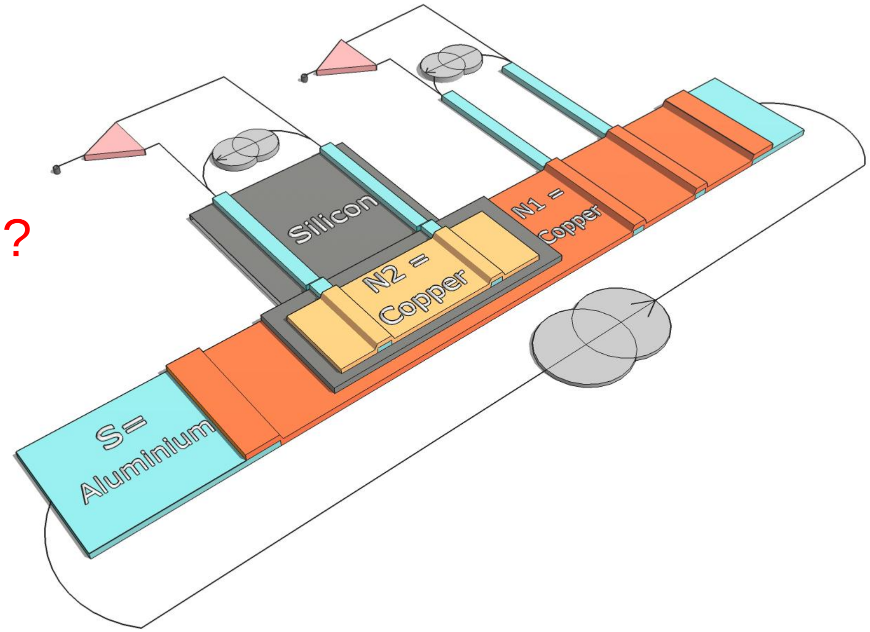
Probe phonon temperature with S-I-N-I-S





Theoretical expectations :

$$P_{el-ph} = \Sigma Ad(T_e^5 - T_{ph}^5)$$

$$P_{Kapitza} = KA(T_{ph}^4 - T_{bath}^4)$$

Σ is material dependent prefactor,
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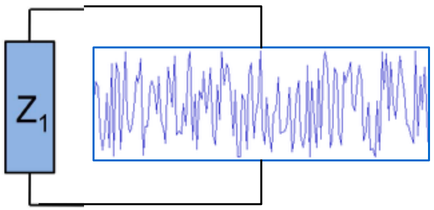
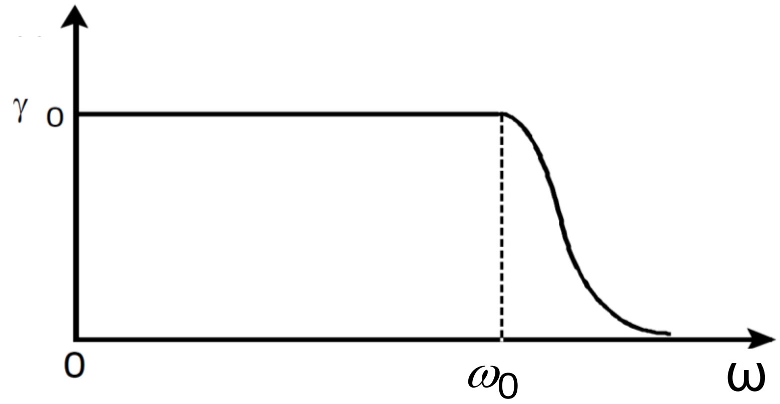
A area, d thickness.

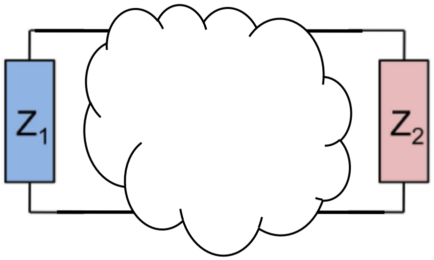
Circuit approach to photonic heat transport

Thermal noise or Johnson-Nyquist noise





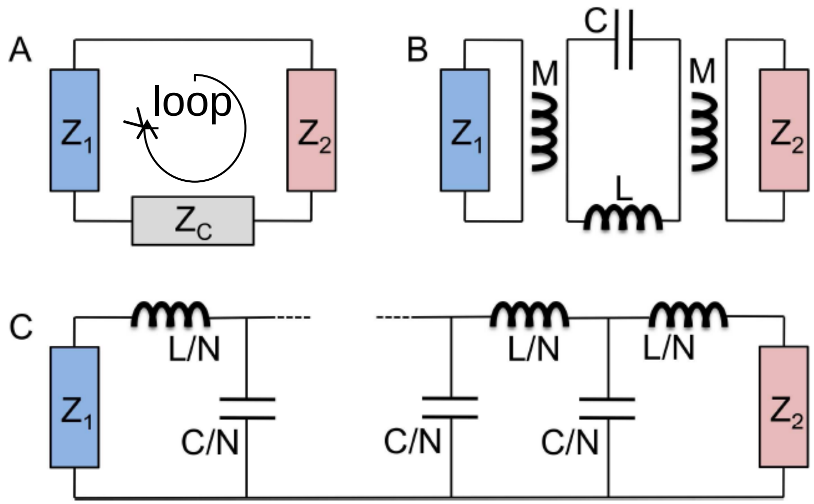
Noise is white below the thermal cut-off.



Photonic power from a hot resistor to a cold one coupled through an impedance?

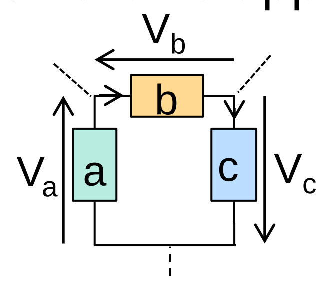
D. Schmidt et al, Phys. Rev. Lett. **93**, 045901 (2004)

Circuit theory introduction

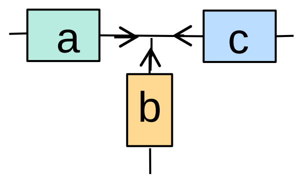


L < λ_ω (~ 1 cm at 1 K): Kirchhoff's laws apply.

voltage: $\sum \Delta V_i = 0$



current: $\sum \Delta I_i = 0$



$$\Delta I_i = \delta I_i + \Delta V_i / Z_i$$

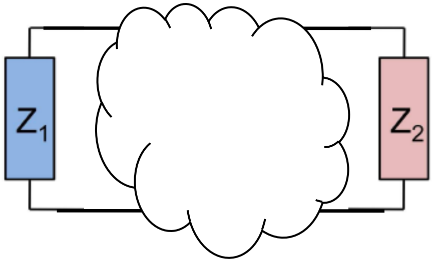
Fluctuation-dissipation theorem ← → *Current through Z_i*

$$\langle \delta I_i(\omega)\delta I_i(\omega')\rangle = 2\pi\delta(\omega+\omega')\hbar\omega\Re e\left[\frac{1}{Z_i(\omega)}\right]\left\{1+2\frac{1}{e^{\hbar\omega/2k_BT}-1}\right\}$$

$n_i(\omega)$

Absorbed power: $P_i(t) = \langle \Delta I_i(t)\Delta V_i(t)\rangle = \int \frac{d\omega}{2\pi}\int \frac{d\omega'}{2\pi} e^{-i\omega t}e^{-i\omega' t}\langle \Delta I_i(\omega)\Delta V_i(\omega')\rangle$

Absorbed power and linear inductance



Landauer-Büttiker – type formula: $P_1 = -P_2 = \int_0^\infty P(\omega) d\omega = \int_0^\infty \frac{d\omega}{2\pi} \hbar\omega \mathrm{T}(\omega)[n_1(\omega) - n_2(\omega)]$

Photon transmission coefficient: $\mathrm{T}(\omega) = \frac{4\Re e[Z_1(\omega)]\Re e[Z_2(\omega)]}{|Z_T(\omega)|^2}$

Linear heat conductance: $K = \frac{Tk_B^2}{\pi\hbar} \int_0^\infty dx \mathrm{T}(x) \frac{x^2}{\sinh^2(x)}$ with $x = \frac{\hbar\omega}{2k_B T}$

Absorbed power and linear inductance

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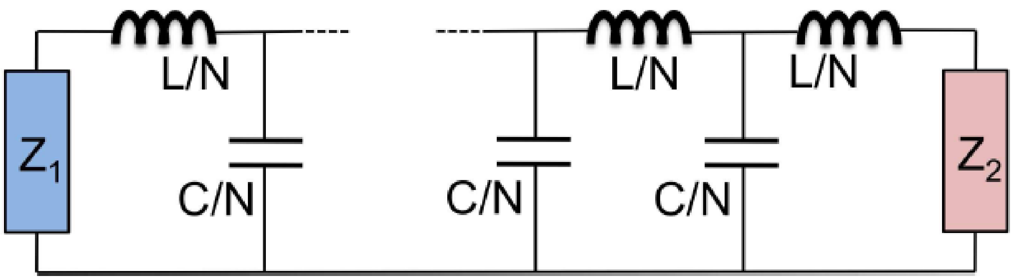
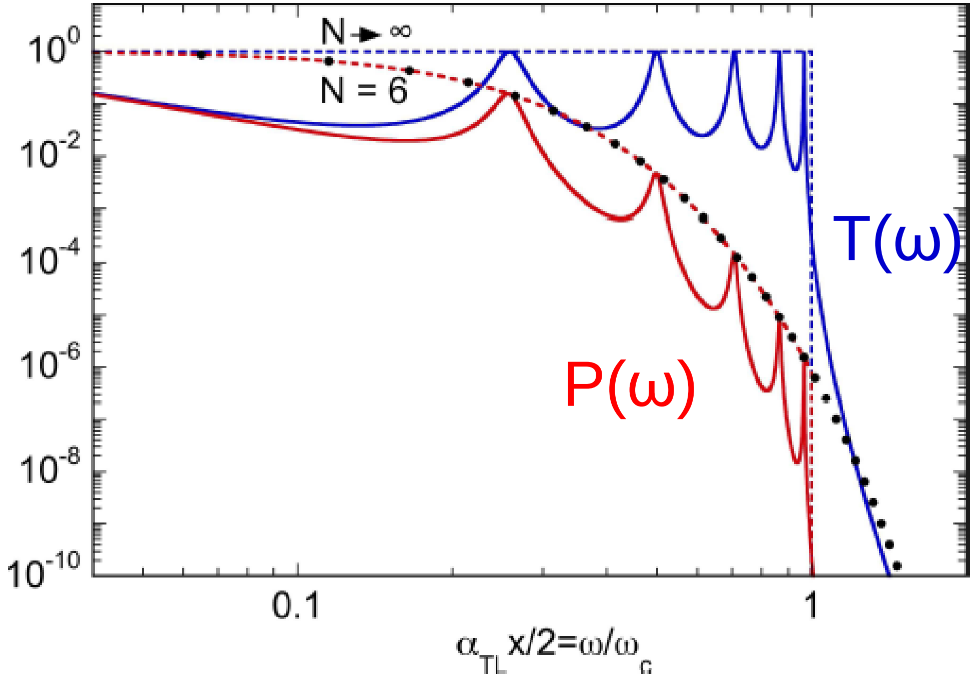
Linear heat conductance: $K = \frac{T k_B^2}{\pi\hbar}\int_0^\infty dx \mathrm{T}(x)\frac{x^2}{\sinh^2(x)}$ with $x = \frac{\hbar\omega}{2k_B T}$

If T(ω) = 1, $K = K_Q = \frac{\pi^2 k_B^2 T}{3h}$ *thermal conductance quanta, ~ 1 pW/K at 1 K,* see M. Meschke et al, Nature (2006).

Photonic heat transport in transmission line



Cut-off frequency $\omega_c = \frac{2}{\sqrt{LC}}$



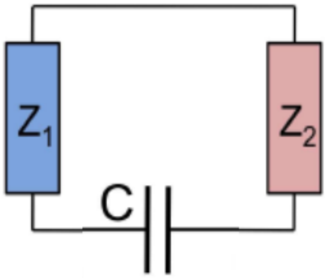
Total impedance Z_T:

$$\omega << \omega_c : \quad Z_T(\omega) = (Z_1 + Z_2)\cos(N\alpha_{TL}) + i\sin(N\alpha_{TL})\sqrt{\frac{L}{C}}[1 + Z_1 Z_2 \frac{C}{L}]$$

$$\omega >> \omega_c : \quad Z_T(\omega) = i\alpha_{TL} x \sqrt{\frac{L}{C}}[-x^2 \alpha_{TL}^2]^{N-1}$$

L. Pascal, H. Courtois, F. W. J. Hekking, Phys. Rev. B **83**, 125113 (2011)

Spectral density with a capacitive coupling



Transmission coefficient

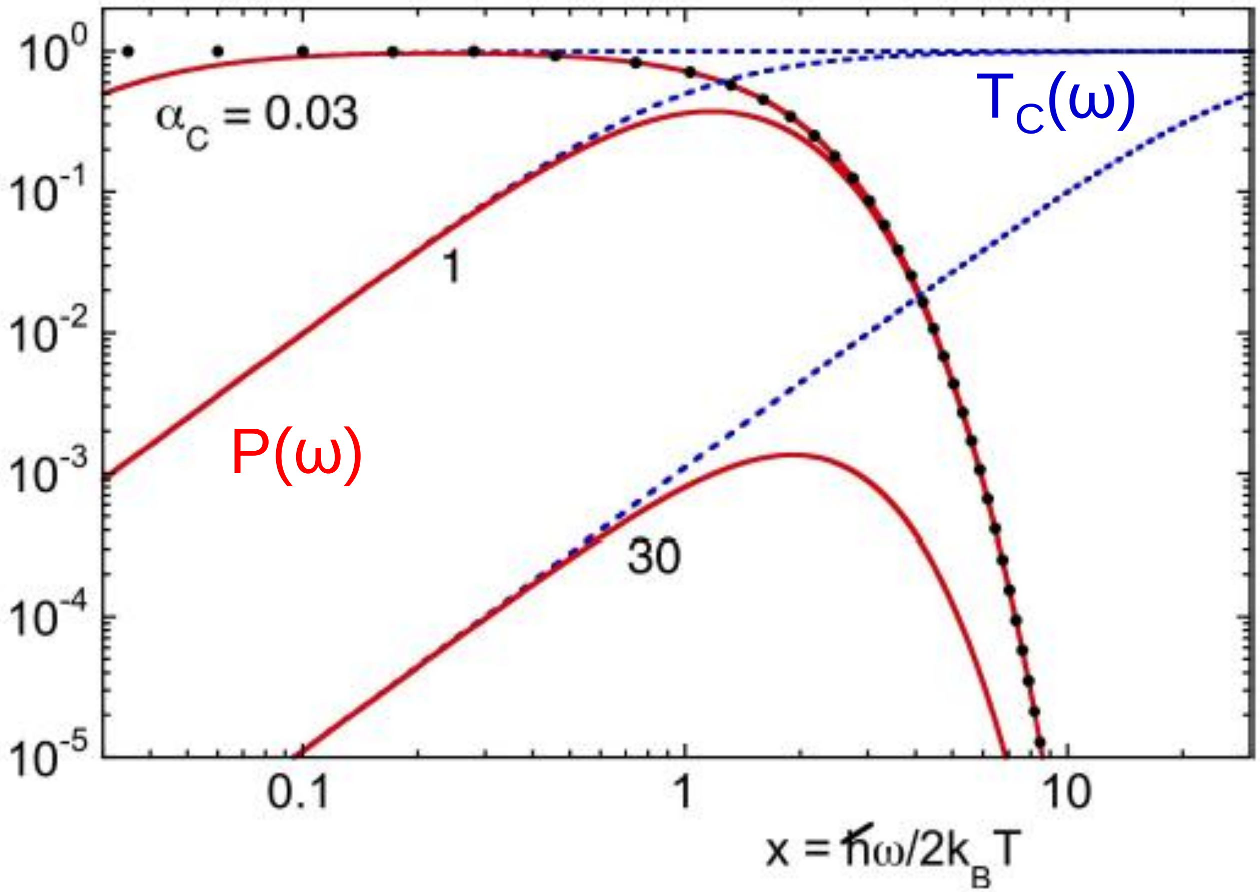
$$T_c(\omega) = \frac{x^2}{\alpha_c^2 + x^2}$$

with

$$\alpha_c = \frac{\hbar}{4RCk_BT}$$

$$x = \frac{\hbar\omega}{2k_BT}$$

Spectral density



Spectral density with a capacitive coupling

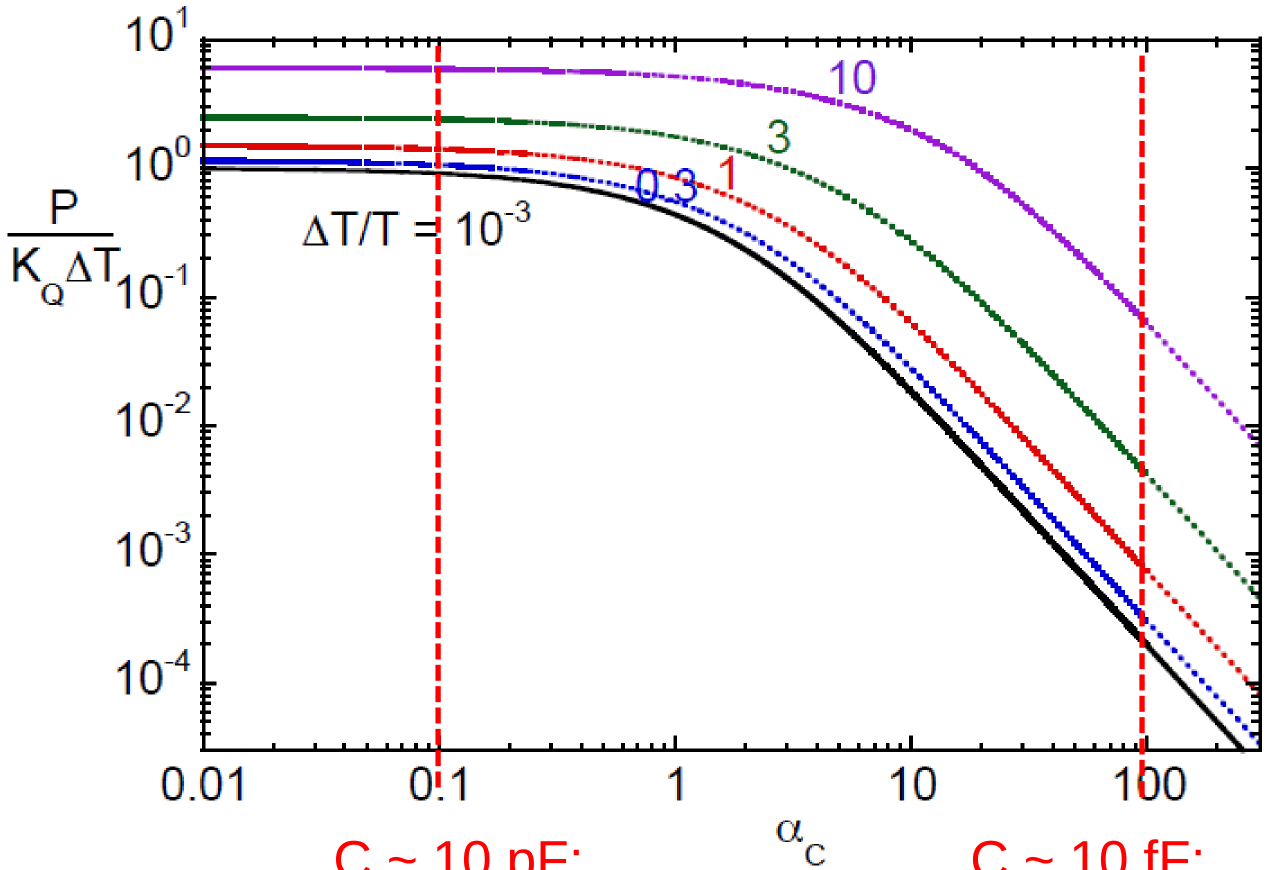
Typical parameters

$R_1 = R_2 = R \sim 50\ \Omega$

$T \sim 40$ mK

$K > K_Q$ for large ΔT.

$$\alpha_c = \frac{\hbar}{4RCk_BT}$$



C ~ 10 pF:
strong coupling

C ~ 10 fF:
small coupling

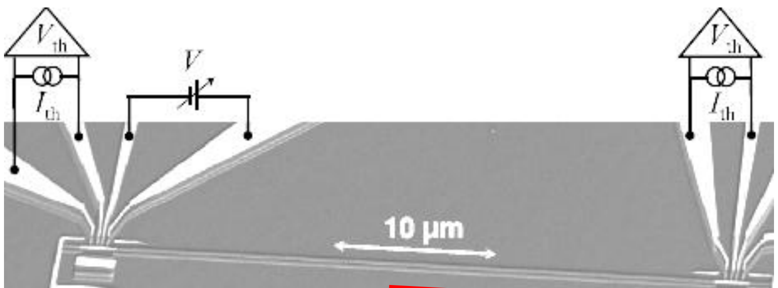
Spectral density with a capacitive coupling

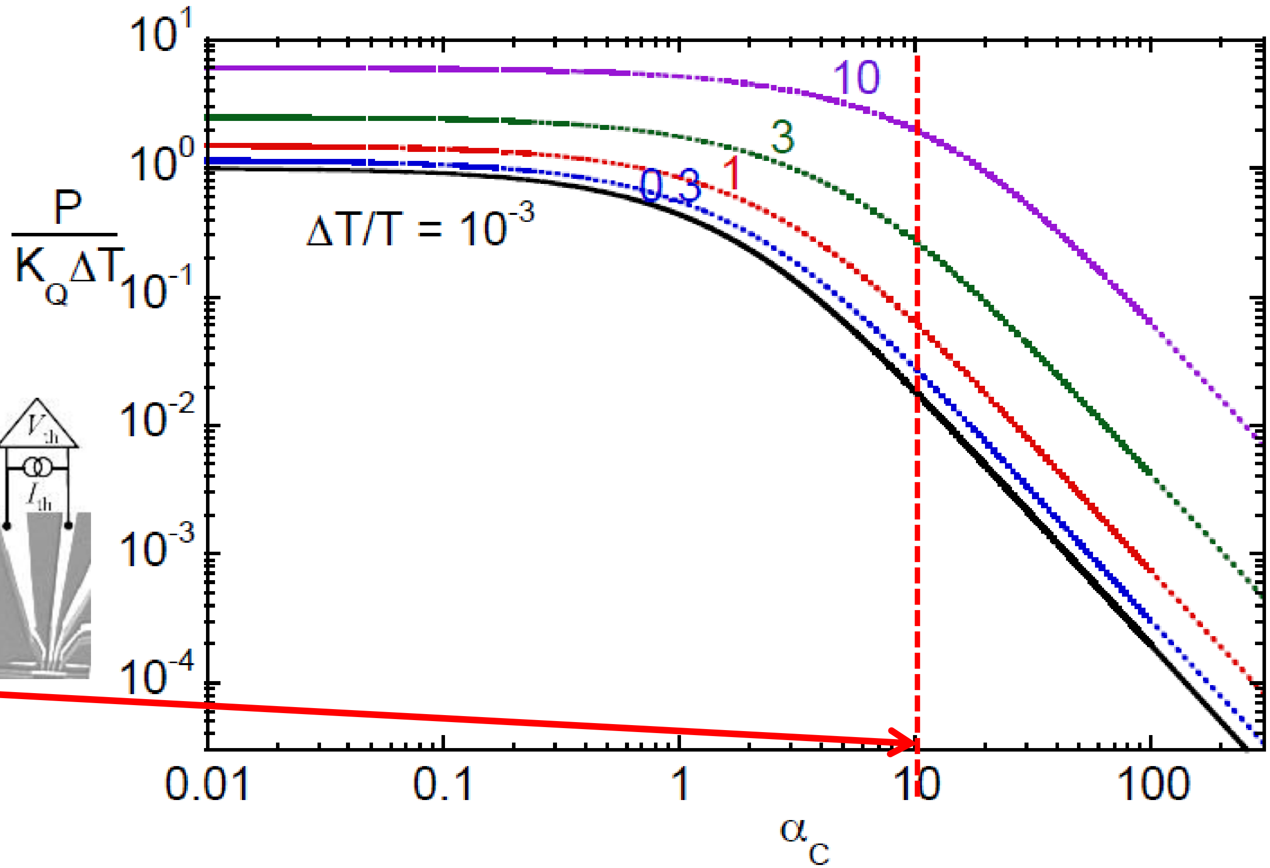
Typical parameters

$R_1 = R_2 = R \sim 230\ \Omega$

T ~ 300 down to 100 mK

C ~ 10 fF:





$$\alpha_c = \frac{\hbar}{4RCk_BT} < 10$$

A. Timofeev et al., PRL **102**, 200801 (2009)

Spectral density with a capacitive coupling

Typical parameters

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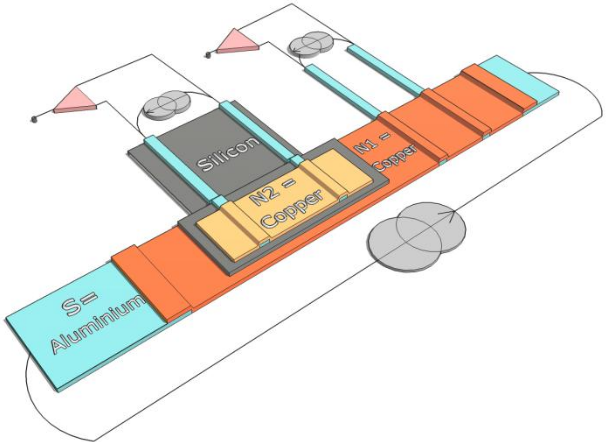
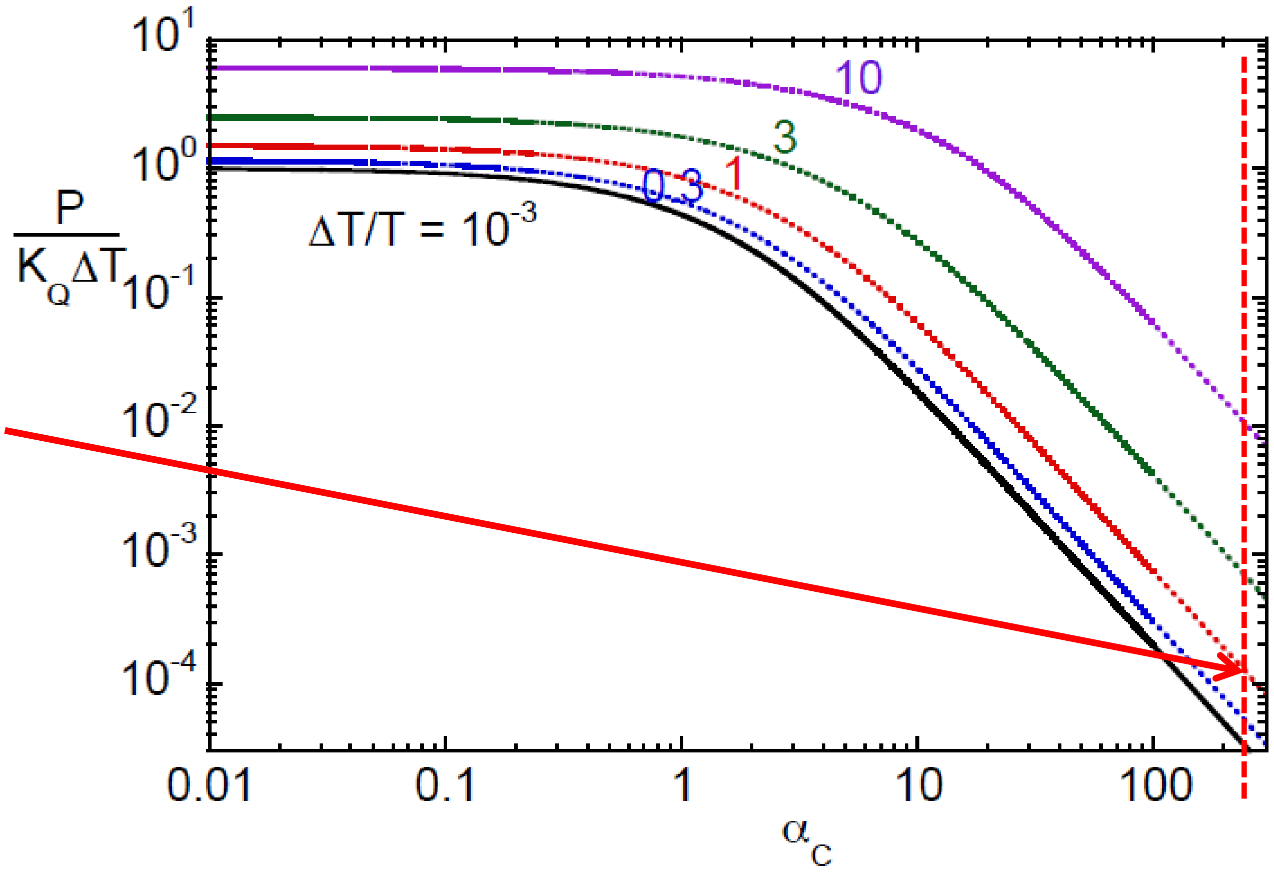
T ~ 300 mK



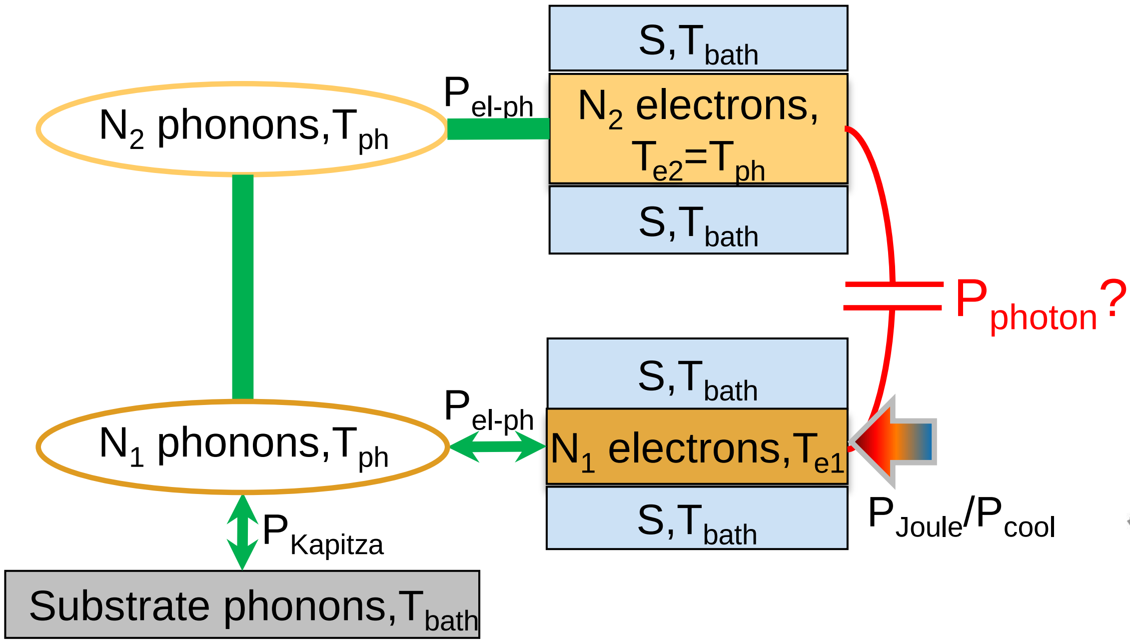


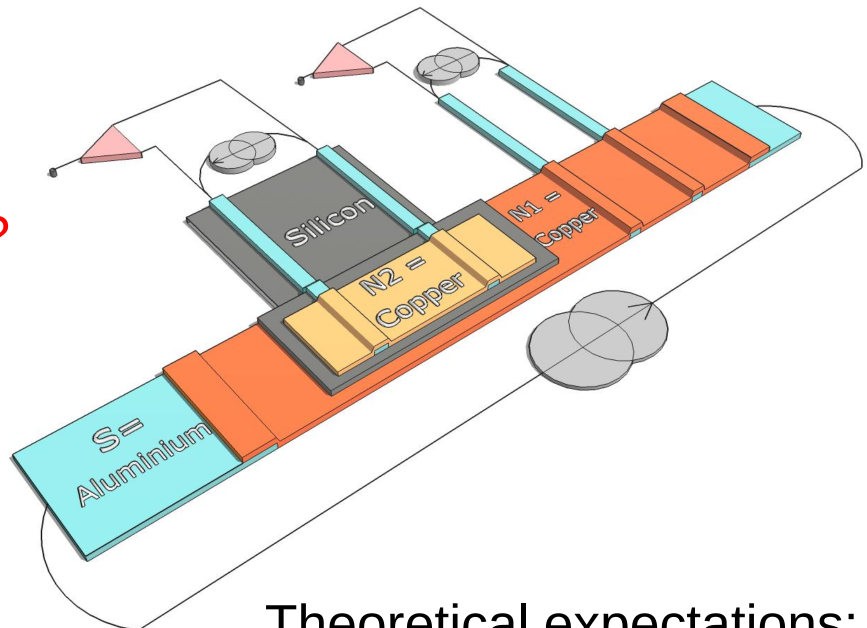
Our work:

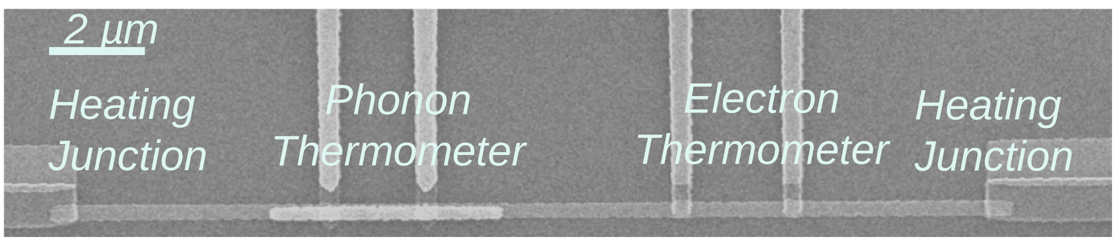
$$\alpha_{c\,\text{exp.}} = \frac{\hbar}{4RCk_BT} >> 10$$

…back to probing independently electron & phonon temperature…

Probing phonon temperature







Theoretical expectations:

$$P_{el-ph} = \Sigma A d (T_e^5 - T_{ph}^5)$$

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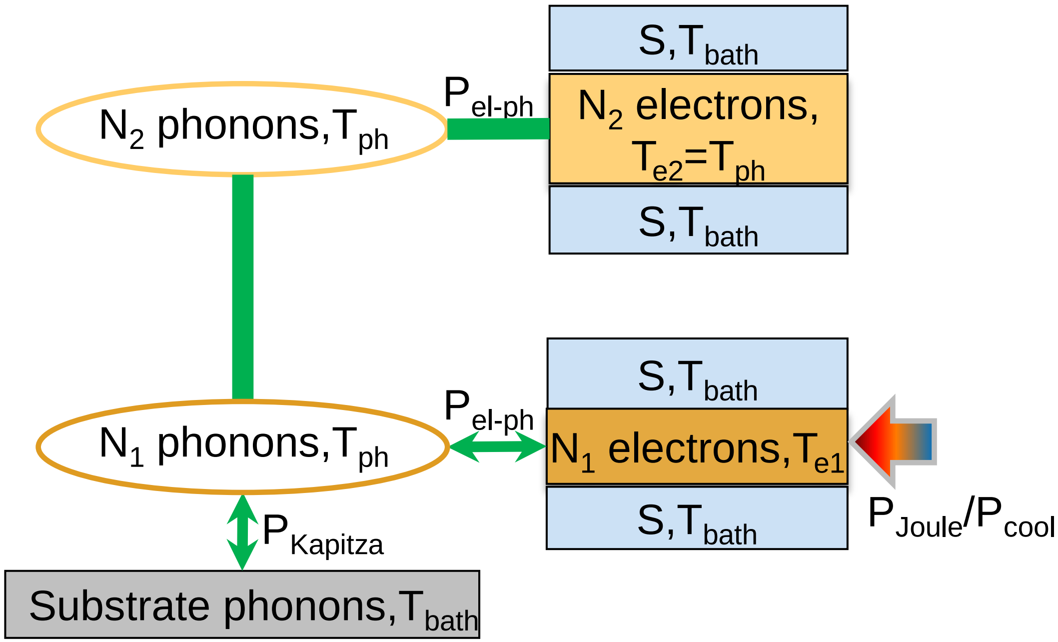
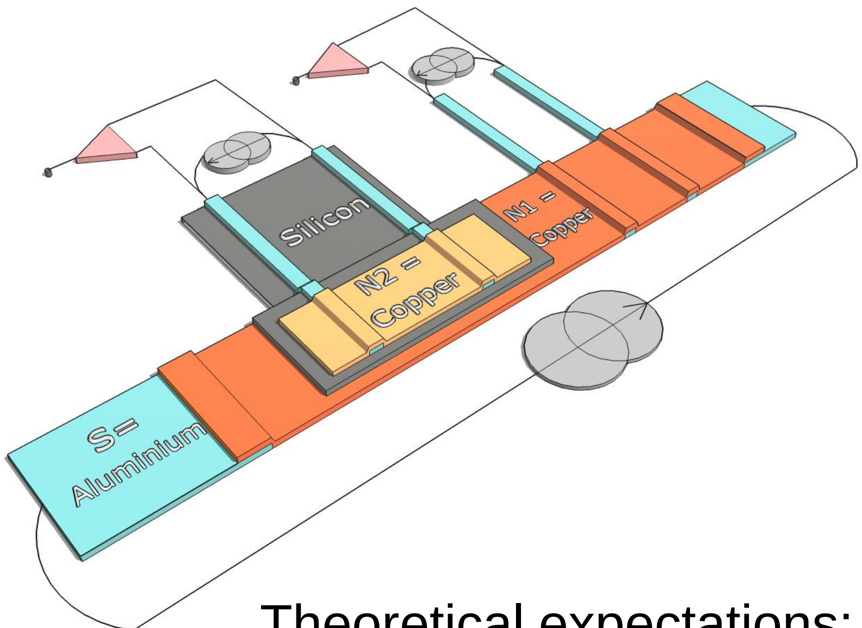
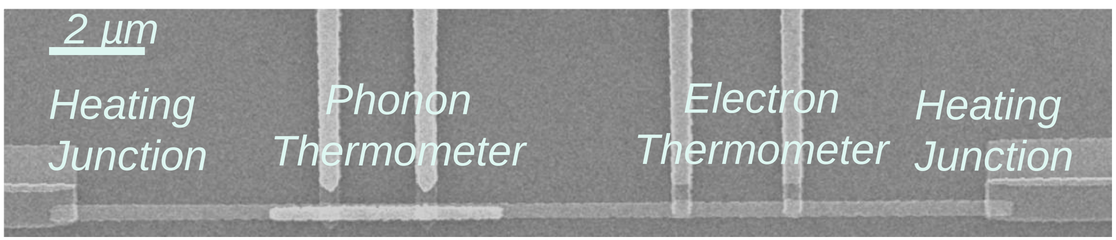
Probing phonon temperature







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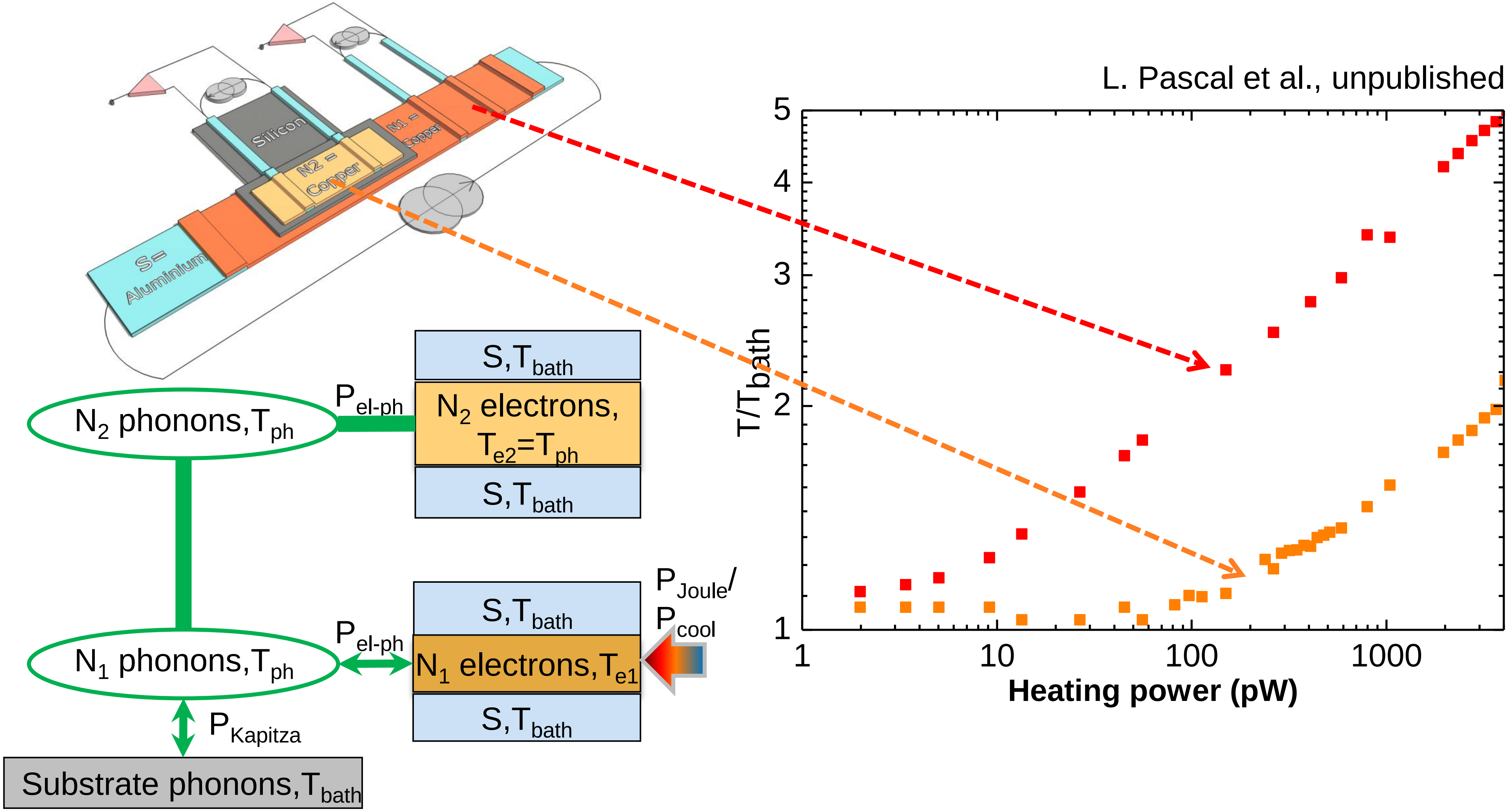

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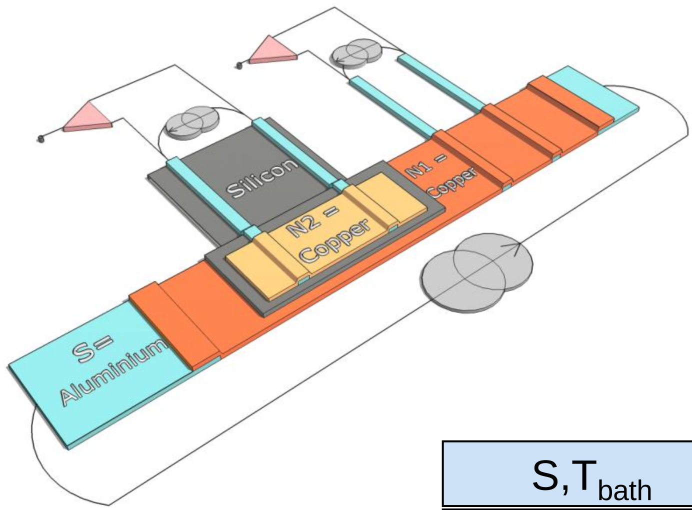
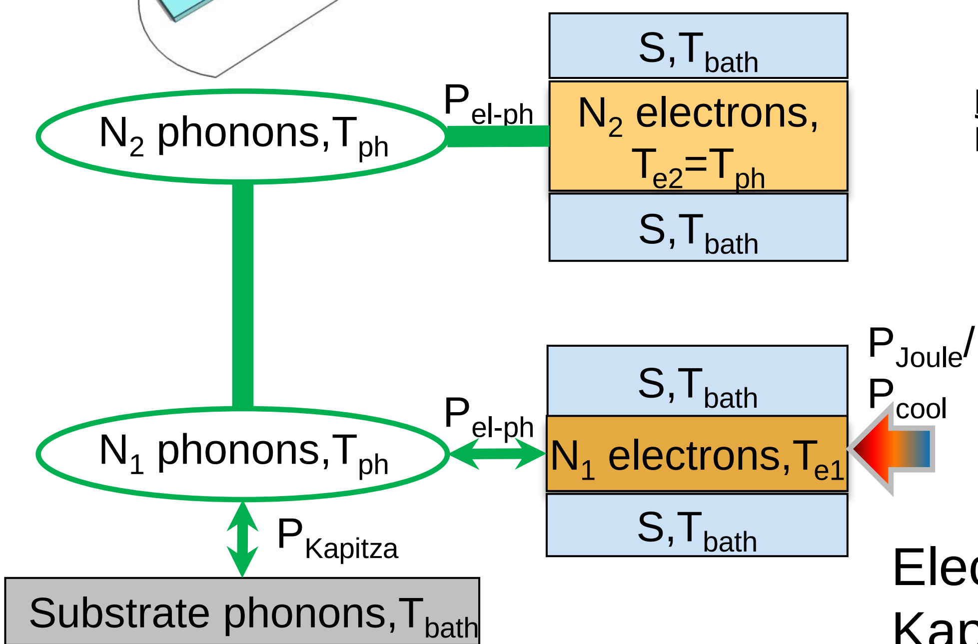
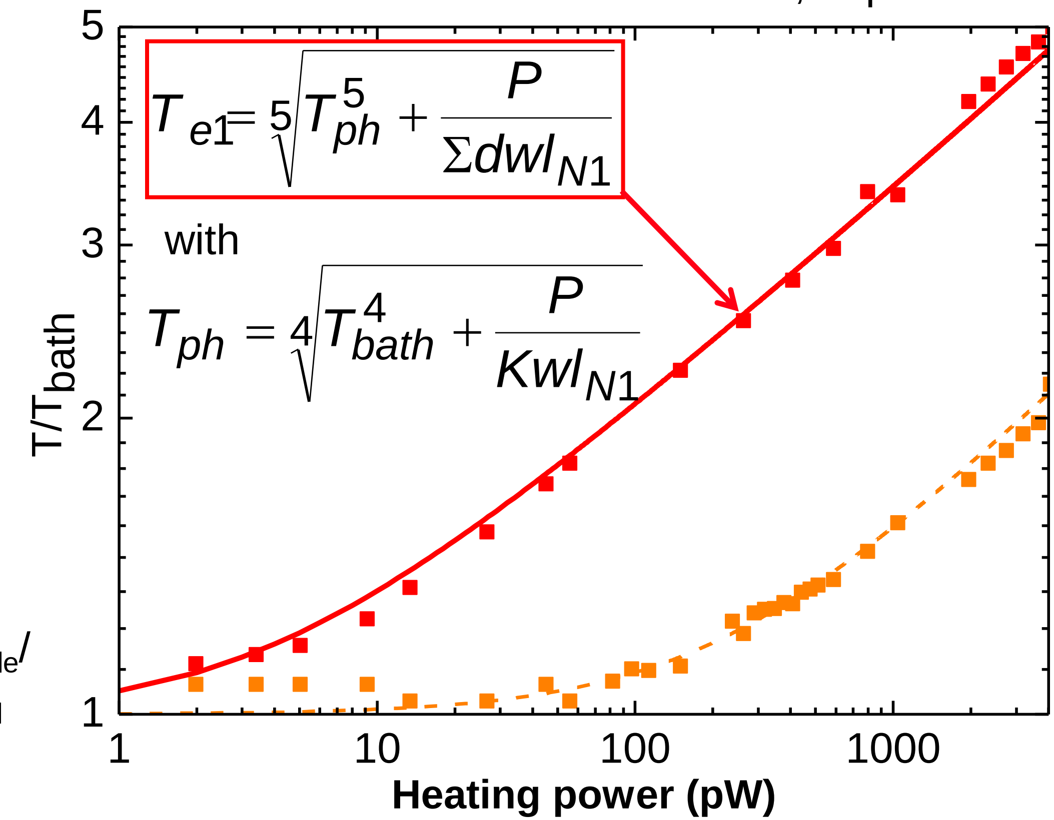




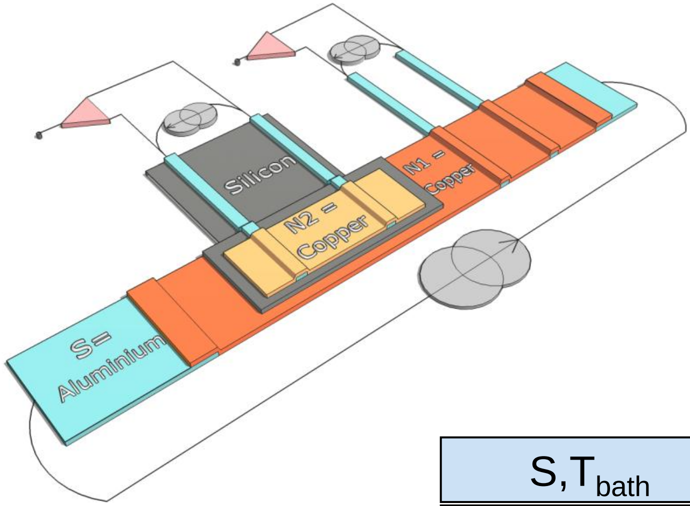
L. Pascal et al., unpublished

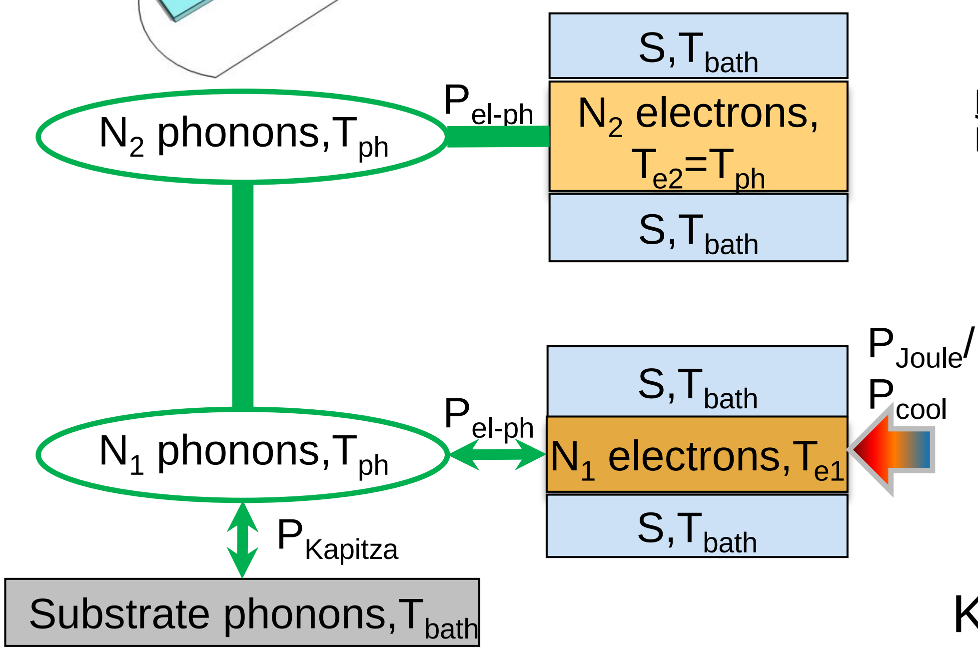


Electron-phonon coupling : $\Sigma = 2\ 10^9$ W.m^{-3}.K^{-5}

Kapitza coupling: $K = 7\ 10^7$ W.m^{-2}.K^{-4}

Probing phonon temperature





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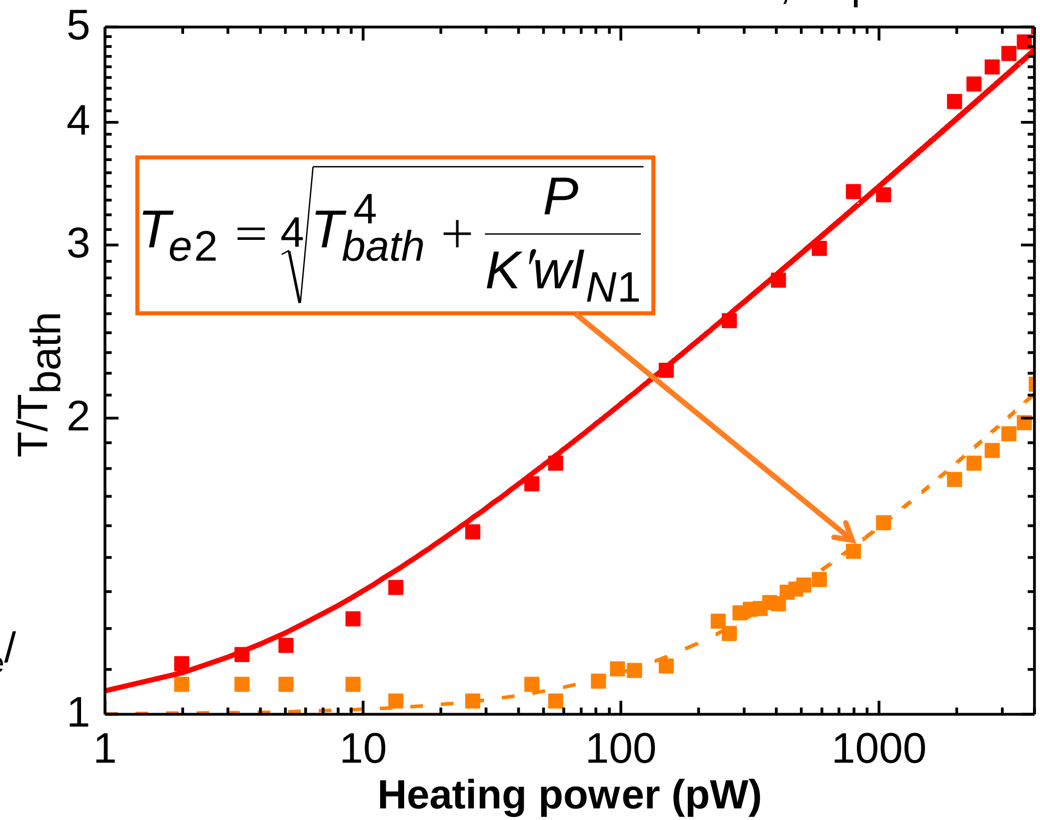
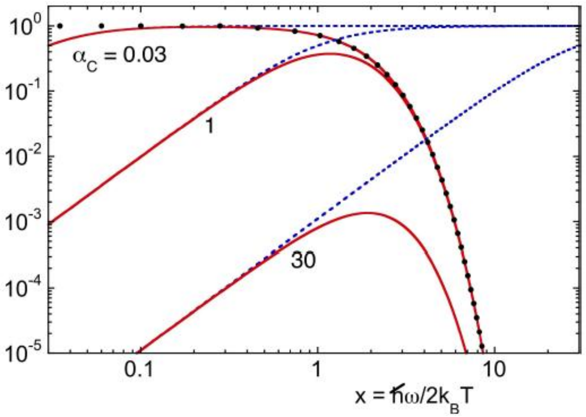
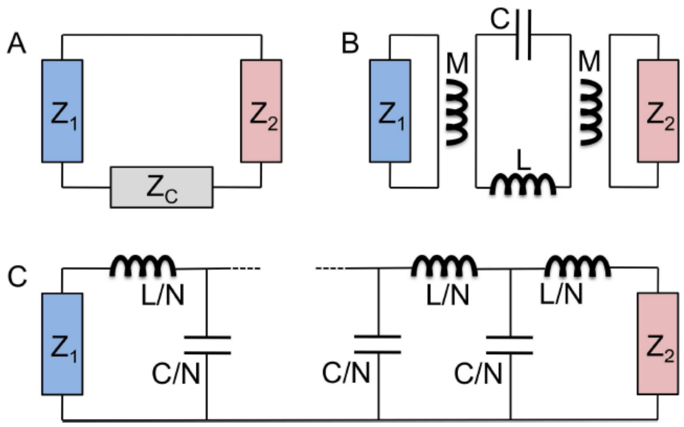
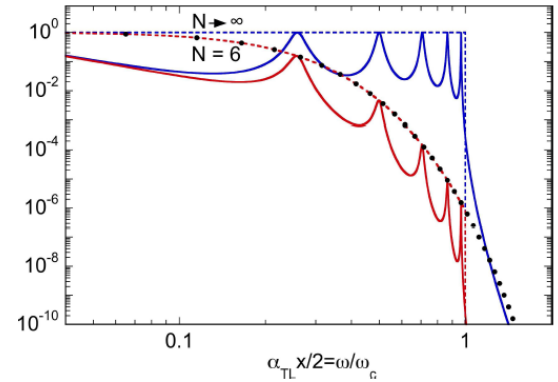


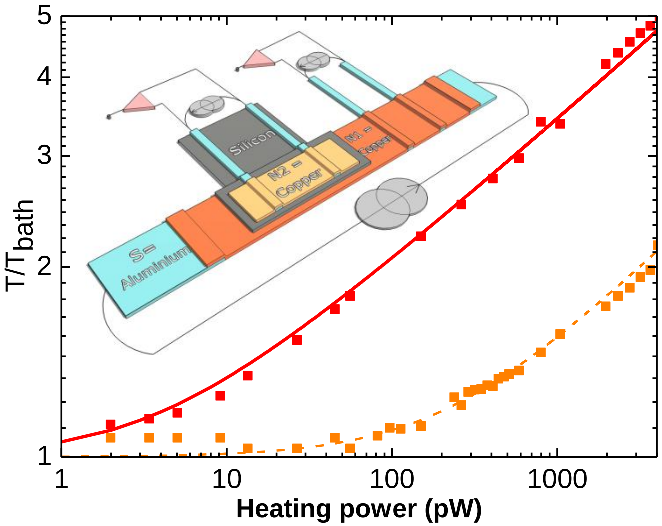
Kapitza coupling : $K'=7\ 10^8\ W.m^{-2}.K^{-4}$

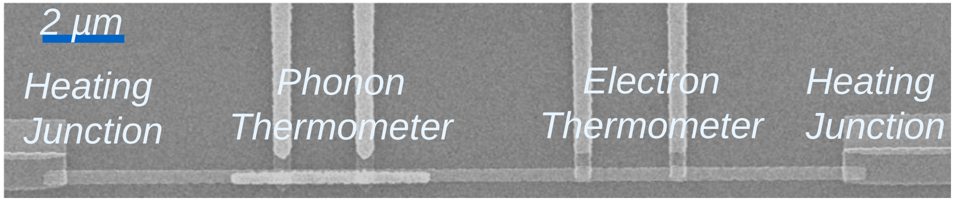
Conclusion & Perspectives

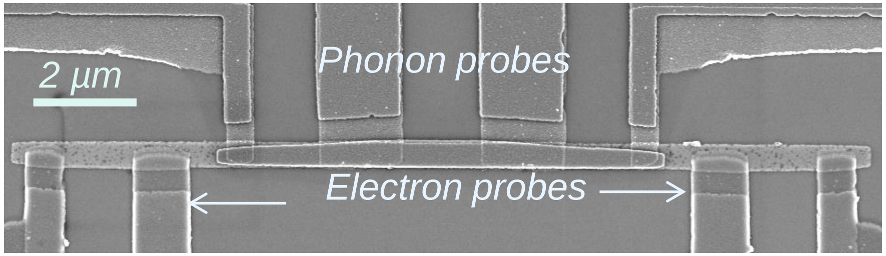












Thanks for your attention !