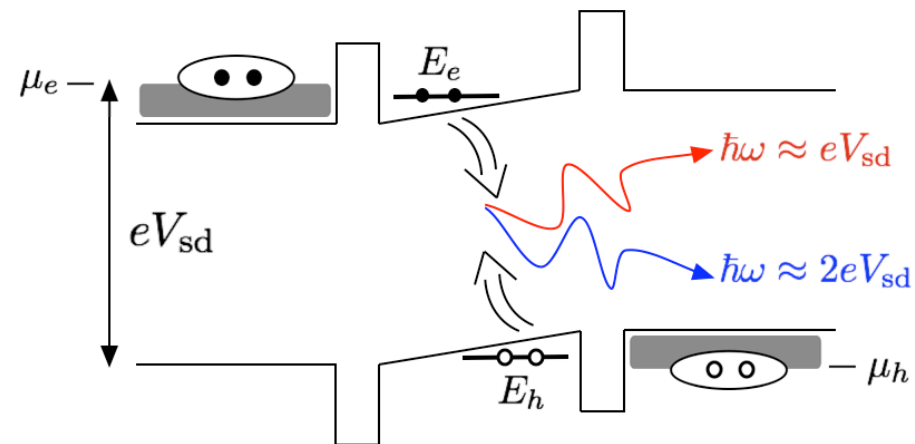


The Josephson light-emitting diode

P. Recher, Yu.V. Nazarov, and L.P. Kouwenhoven, arXiv:0902.4468



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Motivation

- QD as single photon sources:
 - tunable light
 - photo emission at optical frequencies by demand

P. Michler et al., Science 00; C. Santori et al., PRL 01

- QD as source for entangled two photon cascade

O. Benson et al., PRL 00; O. Gywat et al., PRB 02; N. Akopian et al., PRL 06

- Semiconductor-superconductor junctions:

(supercurrent through nanowires, creation of spin-entangled electrons)

Doh et al., Science 05

P. Recher et al., PRB 01

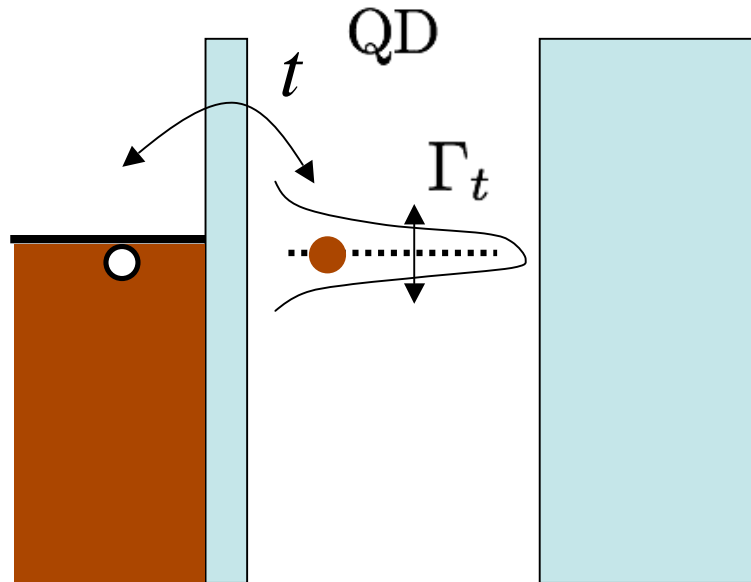
J.A. van Dam et al., Nature 06

G.B. Lesovik et al., EPJB 01

- Combination of superconductivity and optics ?

Preliminaries

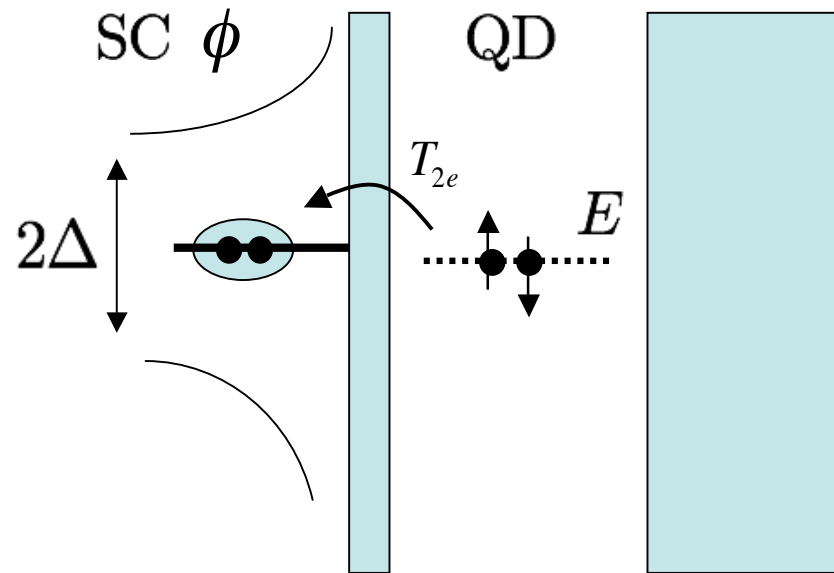
- Level coupled to **normal** leads: broadening Γ_t



$$\Gamma_t = 2\pi\nu|t|^2 : \text{tunneling rate}$$

ν is DOS per spin in the lead

Level coupled to **superconducting** leads: coherent transfer of Cooper pairs



Amplitude: $T_{2e} \approx (1/2)\Gamma_t e^{-i\phi}$ in 2nd-order tunneling with $|T_{2e}| \ll |\Delta|$

The **phase factor** $\exp(-i\phi)$ is associated with **creation of a Cooper pair** in SC

- Note that SC stays in the ground state by transferring charge $2e$, DOS of Cooper pairs is “discrete” => **no broadening of level on QD**

Effective model for QD level coupled to SC lead

$$H_e = E \sum_{\sigma} c_{\sigma}^{\dagger} c_{\sigma} + \tilde{\Delta} c_{\uparrow}^{\dagger} c_{\downarrow}^{\dagger} + \tilde{\Delta}^* c_{\downarrow} c_{\uparrow} + U \hat{n}_{\uparrow} \hat{n}_{\downarrow}$$

$\tilde{\Delta}^*$: induced pair-potential on QD = T_{2e}

$$E, |\tilde{\Delta}|, U \ll |\Delta|$$

Diagonalization gives rise to **4 levels**:

doublet: $c_{\uparrow}^{\dagger}|0\rangle$ and $c_{\downarrow}^{\dagger}|0\rangle$ with energy E

2 singlets:

$$|g\rangle = -e^{-i\phi}|u\rangle|0\rangle + |v\rangle|2\rangle, \quad \text{with } |2\rangle = c_{\uparrow}^{\dagger}c_{\downarrow}^{\dagger}|0\rangle$$

with energy:

$$\varepsilon_g = \tilde{E} - (\tilde{E}^2 + |\tilde{\Delta}|^2)^{1/2}, \quad (\tilde{E} = E + (U/2))$$

and

$$|e\rangle = e^{-i\phi}|v\rangle|0\rangle + |u\rangle|2\rangle$$

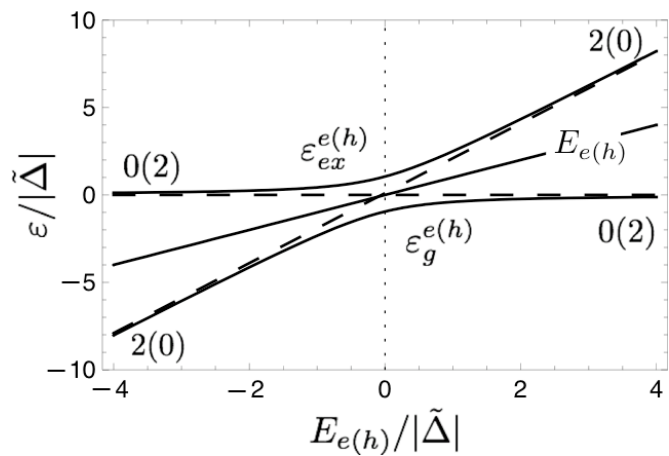
with energy:

$$\varepsilon_{ex} = \tilde{E} + (\tilde{E}^2 + |\tilde{\Delta}|^2)^{1/2}$$

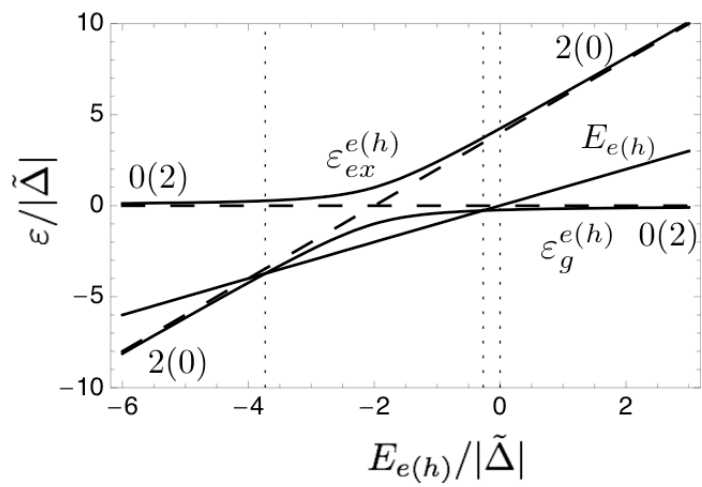
We have introduced standard BCS coherence factors:

$$|u|, |v| = (1/\sqrt{2})[1 \pm \tilde{E}/(\tilde{E}^2 + |\tilde{\Delta}|^2)^{1/2}]^{1/2}$$

tunable by QD level energies (e.g. by gate voltage)

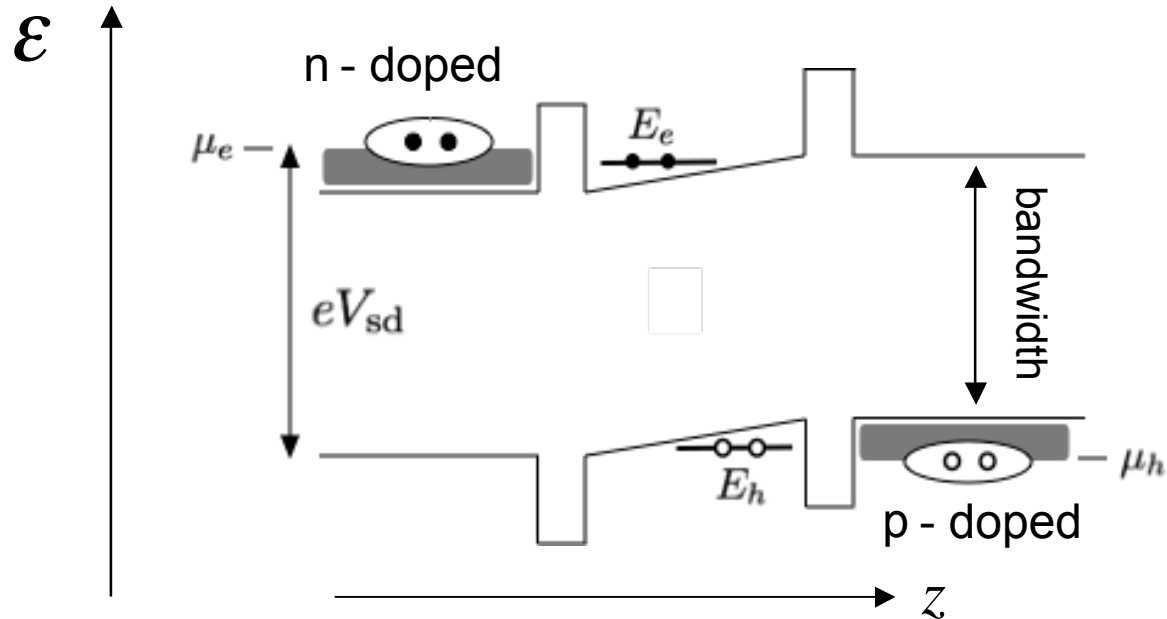


$$U = 0.1||\tilde{\Delta}|$$



$$U = 4||\tilde{\Delta}|$$

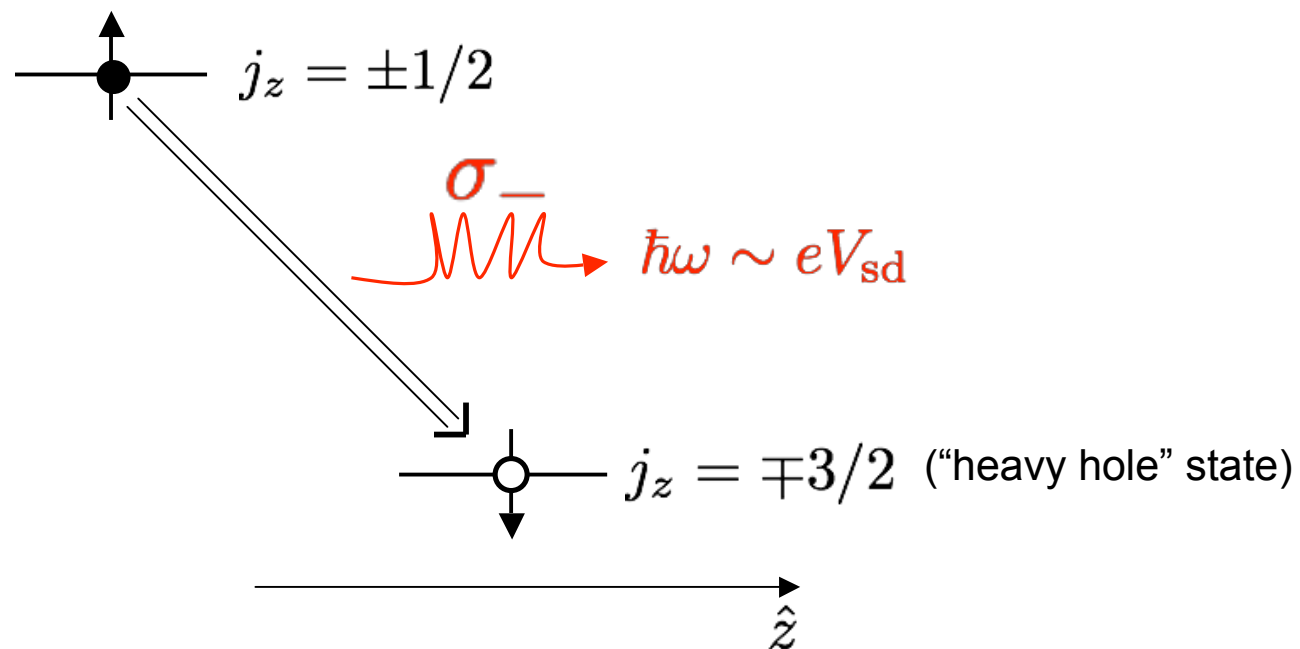
p-n junction with SC leads



- **electron level** (energy E_e) and **hole level** (energy E_h) coupled to respective SC leads.

Although bias voltage V_{sd} is applied, **no charge transfer** from n to p possible **without annihilation of electrons and holes**.

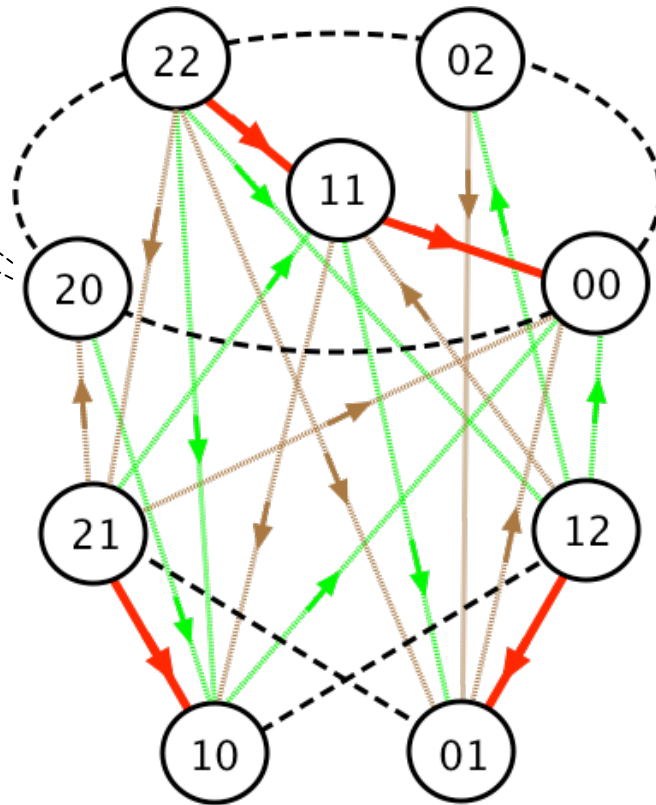
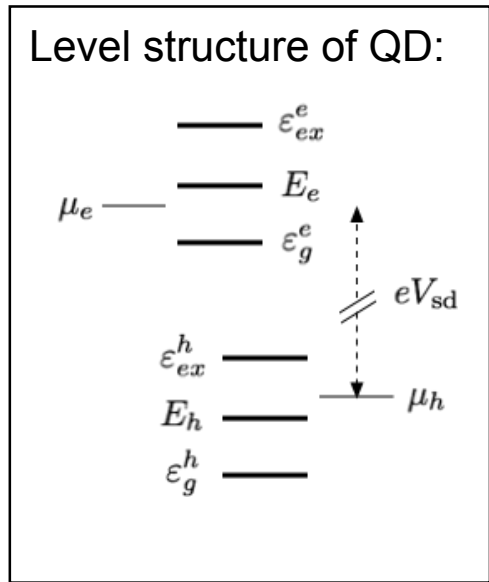
Electron-hole recombination via emission of “red” photons



$$H_{\text{int},1} = G \sum_q \left(a_{q,-}^\dagger h_\downarrow c_\uparrow + a_{q,+}^\dagger h_\uparrow c_\downarrow \right) e^{-ieV_{sd}t} + \text{H.c.}$$

Setup dynamics

$\#e\#h$



Cycle 1:
 $\#e + \#h = \text{even}$

Cycle 2:
 $\#e + \#h = \text{odd}$

----- : mixed by superconductivity

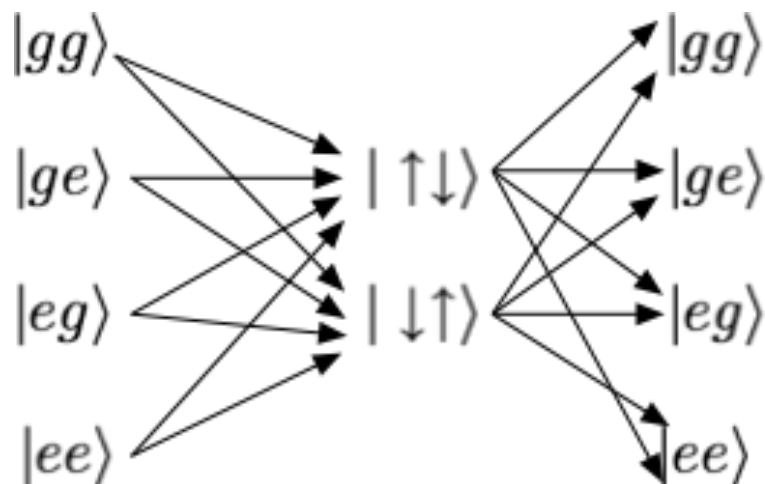
- “red” photon emission proceeds along arrows (bold) leaving parity unchanged
- cycles connected by **single-particle tunneling** to/from SC leads (n-side brown, p-side green) + photon at $\hbar\omega \sim eV_{sd} - \Delta$
 emission rates smaller by factor $|\tilde{\Delta}|/|\Delta|$

“Biexciton-exciton” cascade as an example

normal state picture:



superconducting leads:



transition rates:

$$W_{|g\rangle_e|g\rangle_h \rightarrow |1\rangle_e|1\rangle_h} = \Gamma_{ph} |v_e u_h|^2$$

$$W_{|1\rangle_e|1\rangle_h \rightarrow |g\rangle_e|g\rangle_h} = \Gamma_{ph} |u_e v_h|^2$$

depend on *coherence factors* !!
=> tunable by gate voltage

Note: emission peak widths limited by emission $\hbar/\Gamma_{ph} \propto \hbar/G^2$ time only !

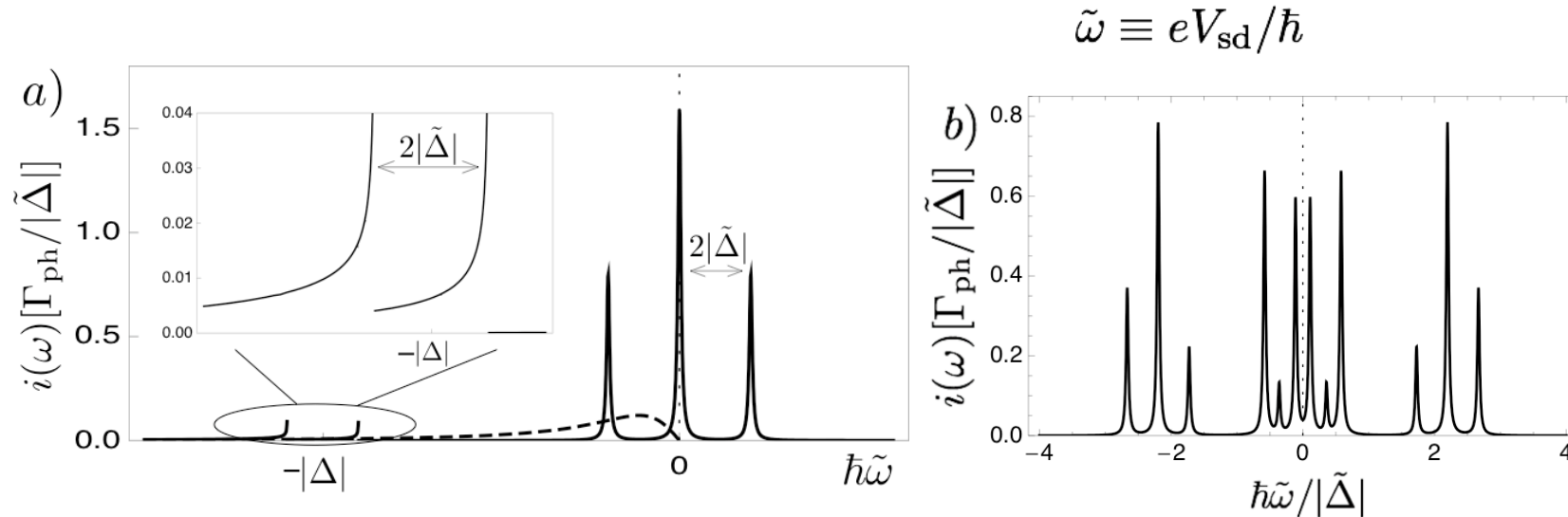
8 distinct emission peaks in total

QD state : $|\alpha\beta\rangle \equiv |\alpha\rangle_e \otimes |\beta\rangle_h$ has energy $\epsilon_\alpha^e + \epsilon_\beta^h$

We thereby neglect possible interactions V_{eh} between electrons and holes which would not qualitatively change our results as long $V_{eh} \ll |\Delta|$

Emission intensity of “red” photons

- Solving Master equation for QD states we can compute emission intensity :

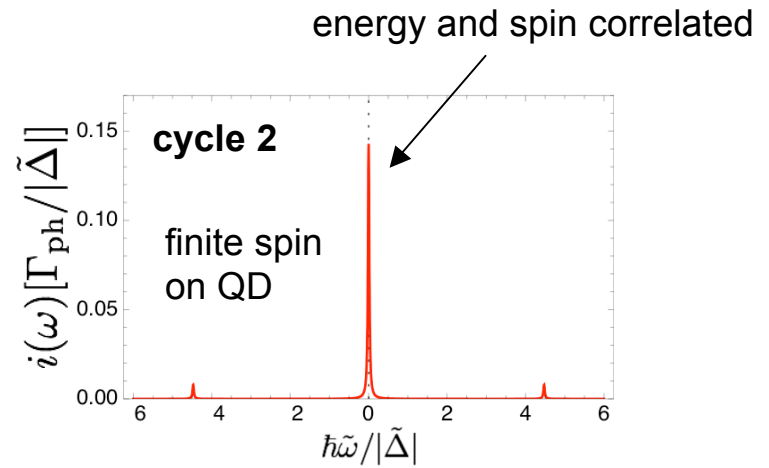
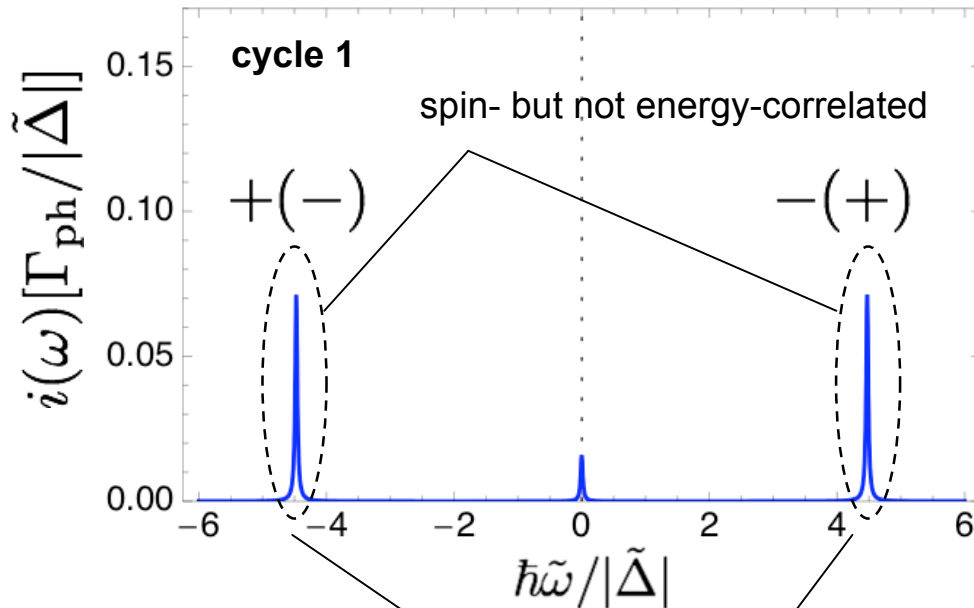


a) On resonance: $E_e = E_h = U = 0$

We find **three discrete peaks** at $\hbar\tilde{\omega} \sim 0$ shifted by induced gap and a **continuous part** (inset) due to quasiparticle creation in leads at $\hbar\tilde{\omega} \sim -|\Delta|$
 Dashed curve is **emission for normal leads** in same regime

b) General asymmetric case: $E_e = -0.81$, $E_h = -0.53$, $U = 0.47$ (units of $|\tilde{\Delta}|$)
 => **12 discrete peaks** due to emission from both cycles

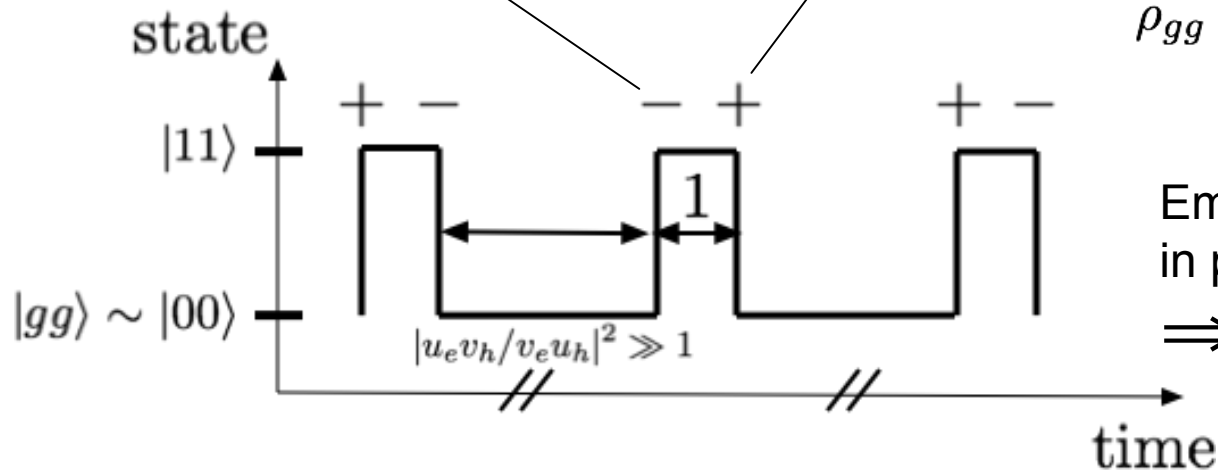
Correlated emission of photon pairs



$$E_e = -E_h = 2|\tilde{\Delta}|$$

$$\Rightarrow |u_e| = |v_h| \sim 0.97 \quad (U=0)$$

QD mostly in $|gg\rangle \sim |00\rangle$:
 $\rho_{gg} \sim 0.8$

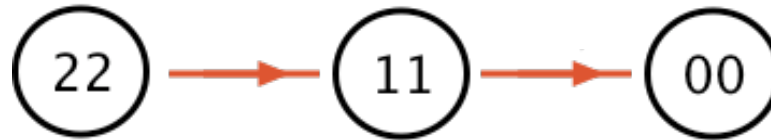


Emission is correlated
in polarization pairs (+,-)

\Rightarrow polarization-entangled pairs

Josephson radiation at optical frequencies

----- emission of a single “blue” photon: -----



+1 photon ! (not 2)

How possible ? At which frequency ?

Energy conservation requires $\hbar\omega \sim 2eV_{sd}$

Hamiltonian $H_{\text{int},1}$ destroys one electron-hole pair with each emitted photon.

=> need annihilation without emission => static electric field \mathbf{E}_0

$$H_{\text{int},0} = (V_0^+ h_\downarrow c_\uparrow + V_0^- h_\uparrow c_\downarrow) e^{-ieV_{sd}t} + \text{H.c.},$$

$$\text{with } V_0^\pm \propto E_{0,x} \mp iE_{0,y}$$

- **rate** for emission of single blue photon (polarization $p = \pm$):

$$W_{i \rightarrow f}^p = (2\pi/\hbar) |\mathcal{A}_{i \rightarrow f}^p|^2 \delta(\varepsilon_{QD}^f - \varepsilon_{QD}^i + \hbar\omega - 2eV_{sd})$$

To second order in $H_{\text{int}} = H_{\text{int},1} + H_{\text{int},0}$

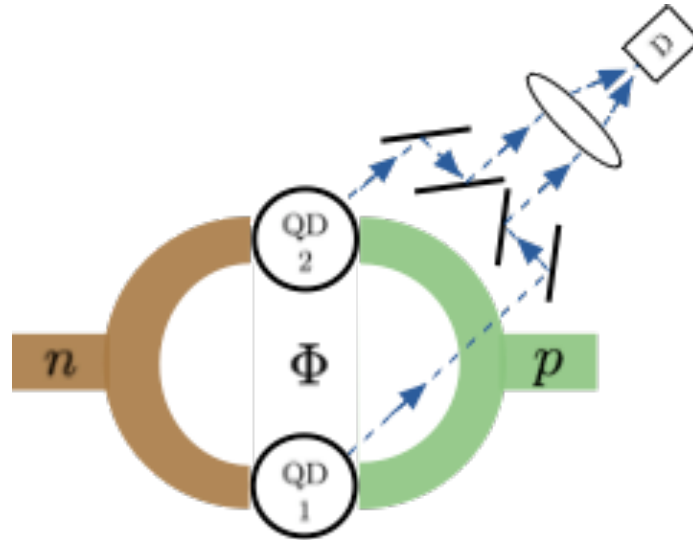
$$\mathcal{A}_{i \rightarrow f}^p = GV_0^p \langle f|00\rangle \langle 22|i\rangle \frac{2(E_e + E_h) - \varepsilon_{QD}^i - \varepsilon_{QD}^f}{(eV_{sd})^2}$$

Note that at $\hbar\omega = 2eV_{sd}$, emission is coherent, i.e. $|i\rangle = |f\rangle$

$$\text{Then: } \langle f|00\rangle \langle 22|i\rangle = \pm e^{i(\phi_e - \phi_h)} |u_e u_h v_e v_h|$$

↑
phase difference between SCs

Probe coherence with SQUID

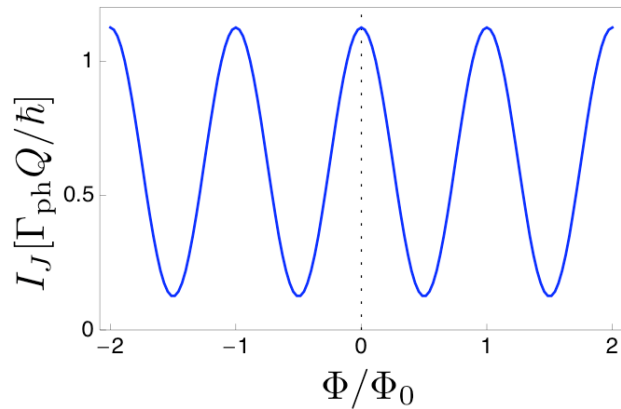


$$I_J = \sum_{\alpha, \beta} \rho_{\alpha} \rho_{\beta} |\mathcal{A}_{1, \alpha}^p e^{i2\pi l_1 / \lambda_J} + \mathcal{A}_{2, \beta}^p e^{i2\pi l_2 / \lambda_J}|^2 = \sum_{\alpha} \rho_{\alpha} |\mathcal{A}_{1, \alpha}^p|^2 + \sum_{\beta} \rho_{\beta} |\mathcal{A}_{2, \beta}^p|^2 + 2\text{Re} \sum_{\alpha, \beta} \rho_{\alpha} \rho_{\beta} [\mathcal{A}_{1, \alpha}^p (\mathcal{A}_{2, \beta}^p)^*]$$

with **interference** contribution $2\text{Re}[\mathcal{A}_{1, \alpha}^p (\mathcal{A}_{2, \beta}^p)^*] \propto \cos[2\pi((l_1 - l_2)/\lambda_J + \Phi/\Phi_0)]$

periodic in flux Φ through SQUID with period $\Phi_0 = hc/2e$

Intensity at Josephson frequency

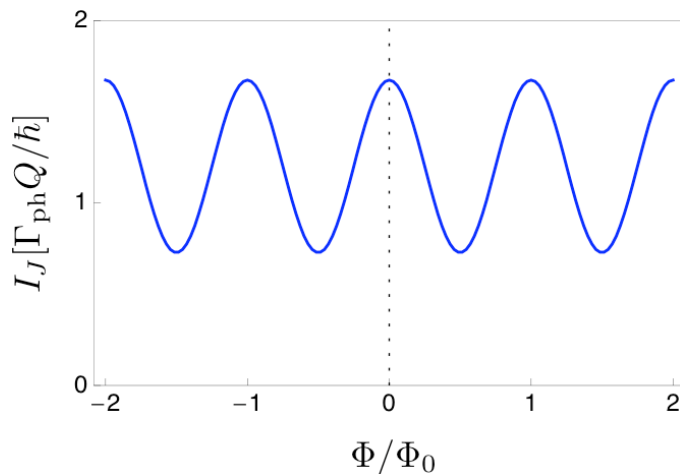


$$E_e = -E_h = 1, U = 0 \quad (\text{for both QDs})$$

$$Q = 2|\mathbf{d} \cdot \mathbf{E}_0|^2 |\tilde{\Delta}|^2 / (eV_{sd})^4$$

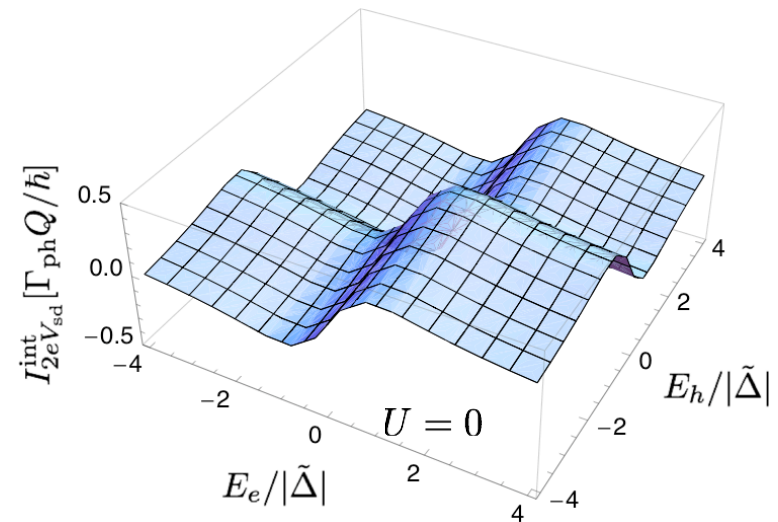
d : Dipole moment of QDs

E_0 : Static electric field (in-plane)



$$E_{e(h)}^{(1,2)} = 1 - U_{(1,2)}/2$$

$$U_1 = 3.6 \quad U_2 = 4.2$$

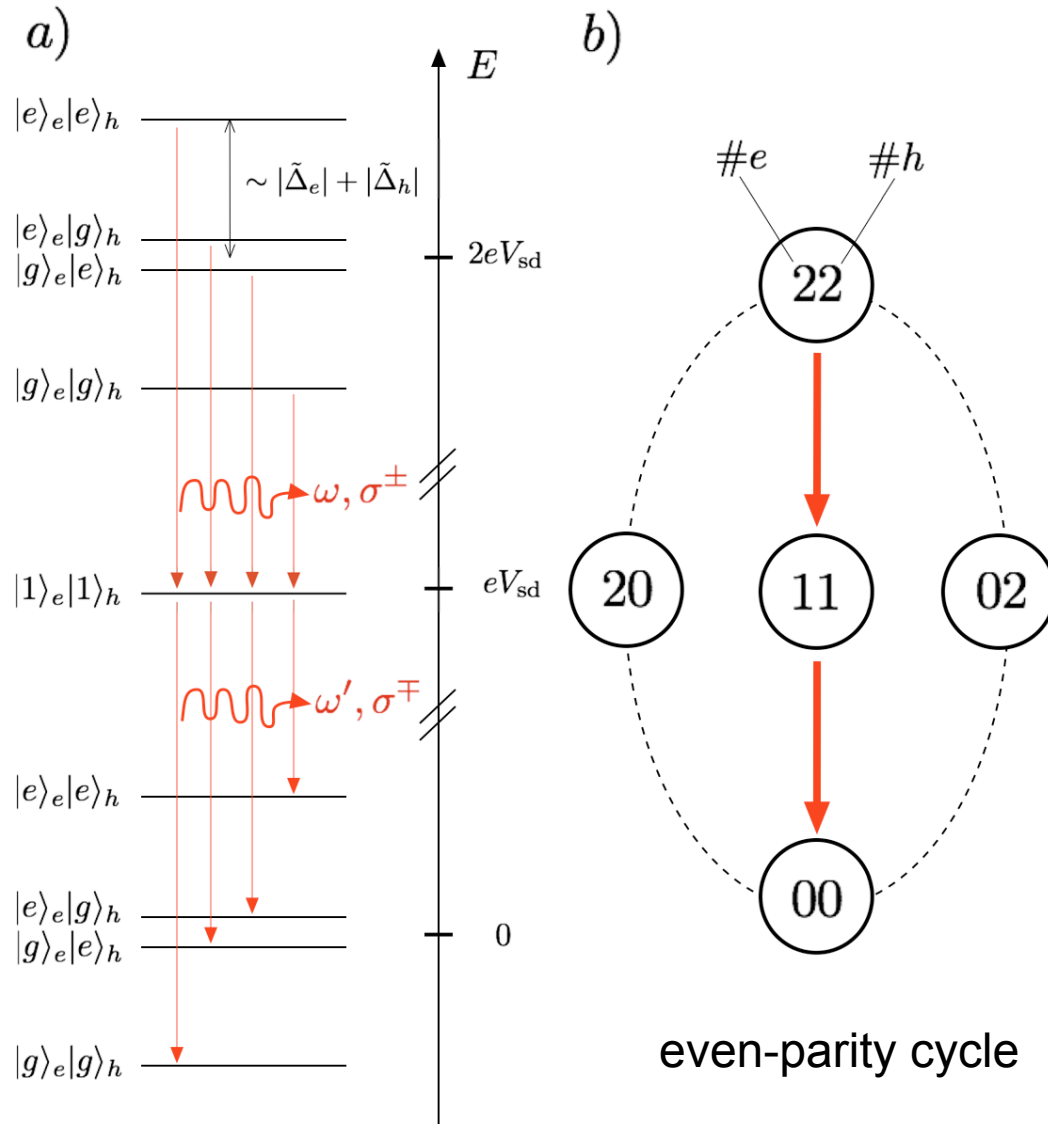


$$E_e = 1, E_h = -1 \quad \text{QD2}$$

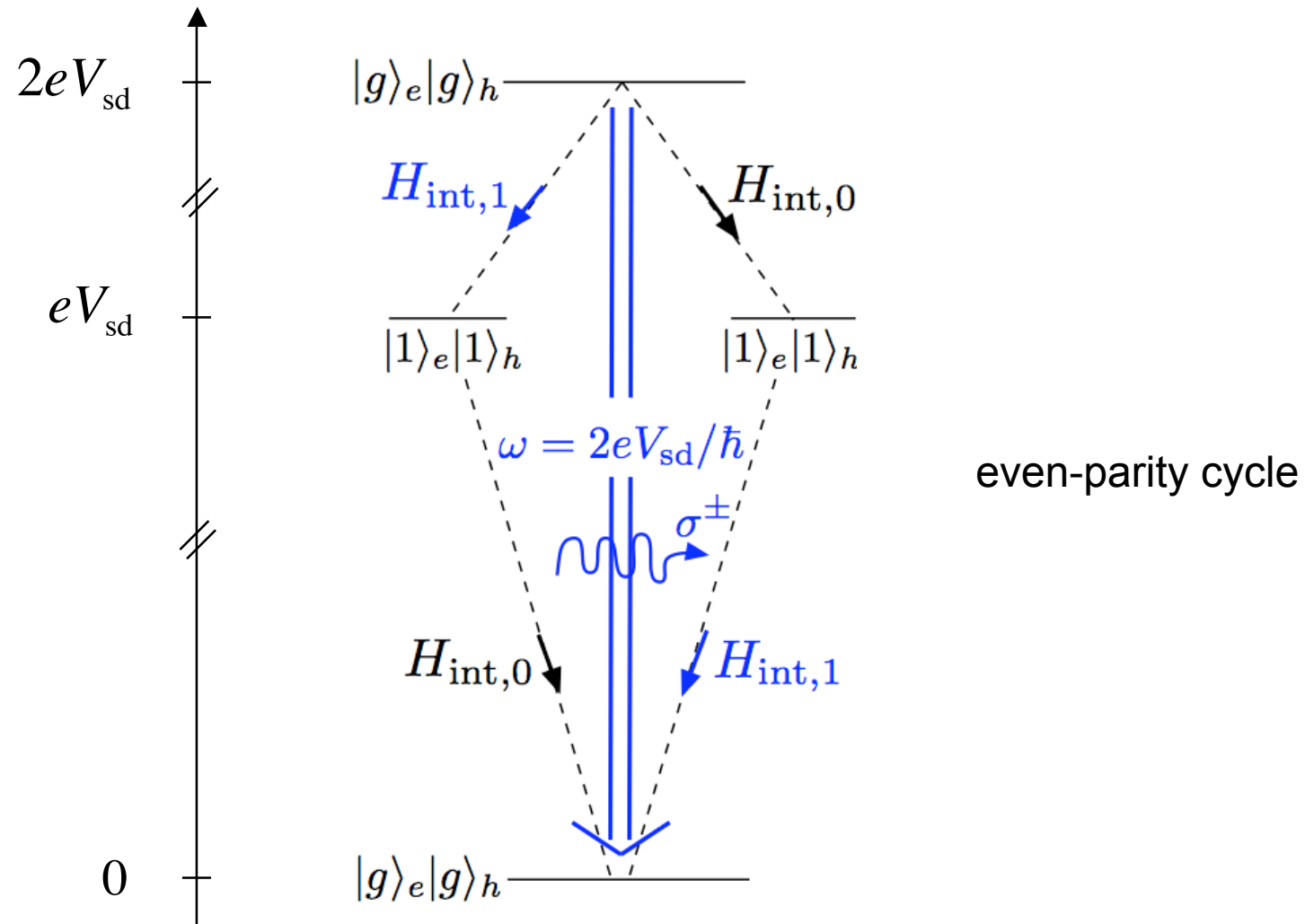
Conclusions

- Setup where electron level and hole level in QD coupled to SC leads
- Emission proceeds in several discrete peaks at frequencies $\hbar\omega = eV_{sd}$ as well as a continuous tail at $\hbar\omega \sim eV_{sd} - |\Delta|$ characteristic of induced superconductivity in QD
- Coherent coupling to SCs allows to tune the correlated 2 photon emission cascade
- photons at $\hbar\omega = 2eV_{sd}$ are coherent and reminiscent of Josephson radiation

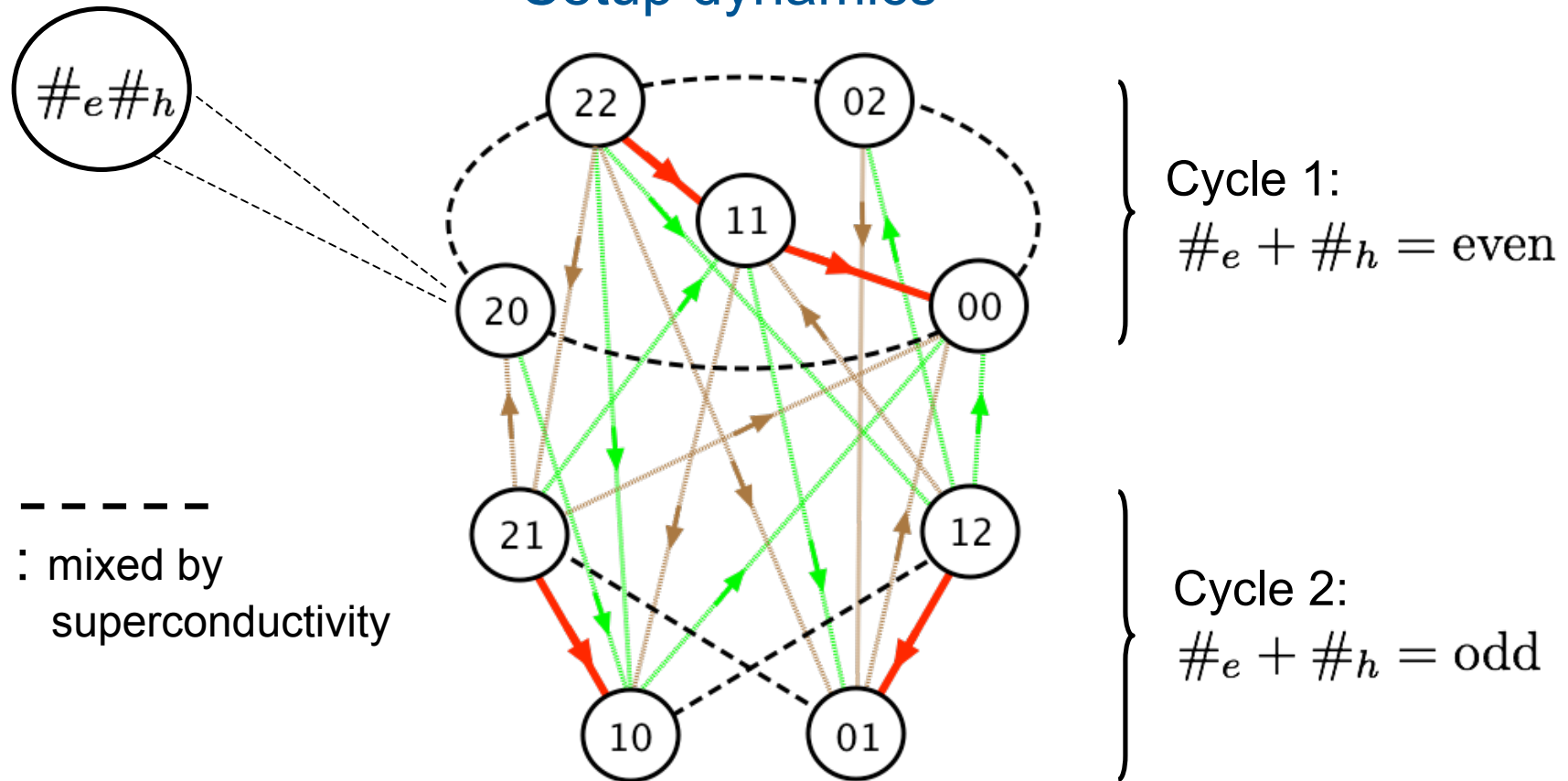
Energetics



Process of “blue”-photon emission



Setup dynamics



- “red” photon emission proceeds along arrows (bold) **leaving parity unchanged**
- cycles connected by **single-particle tunneling to/from SC leads** (n-side brown, p-side green) + photon at $\hbar\omega \sim eV_{sd} - \Delta$
 emission rates smaller by factor $|\tilde{\Delta}|/|\Delta|$