High Frequency Third Cumulant of Quantum Noise

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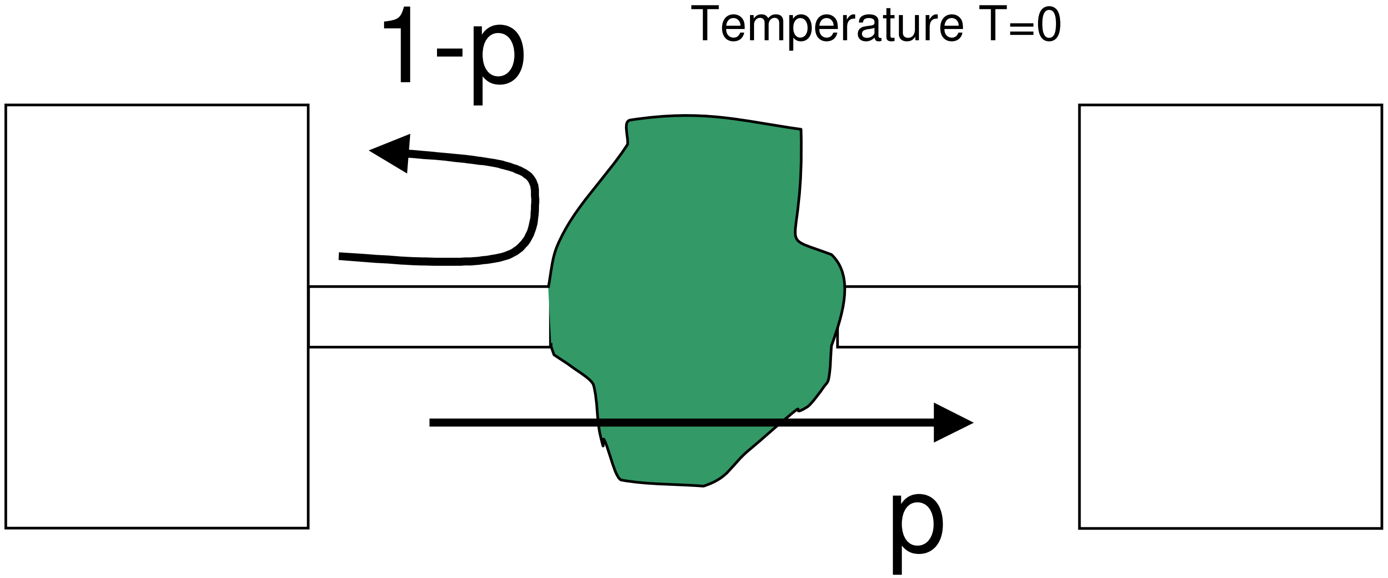
DC Transport in Disordered Systems



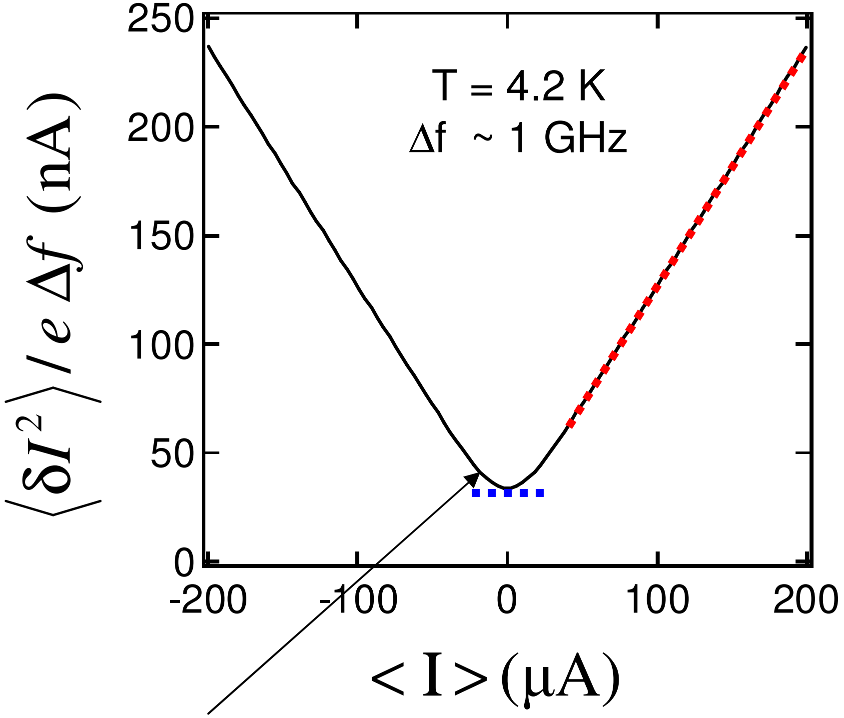
Despite their quantum nature, electrons behave as classical particules…

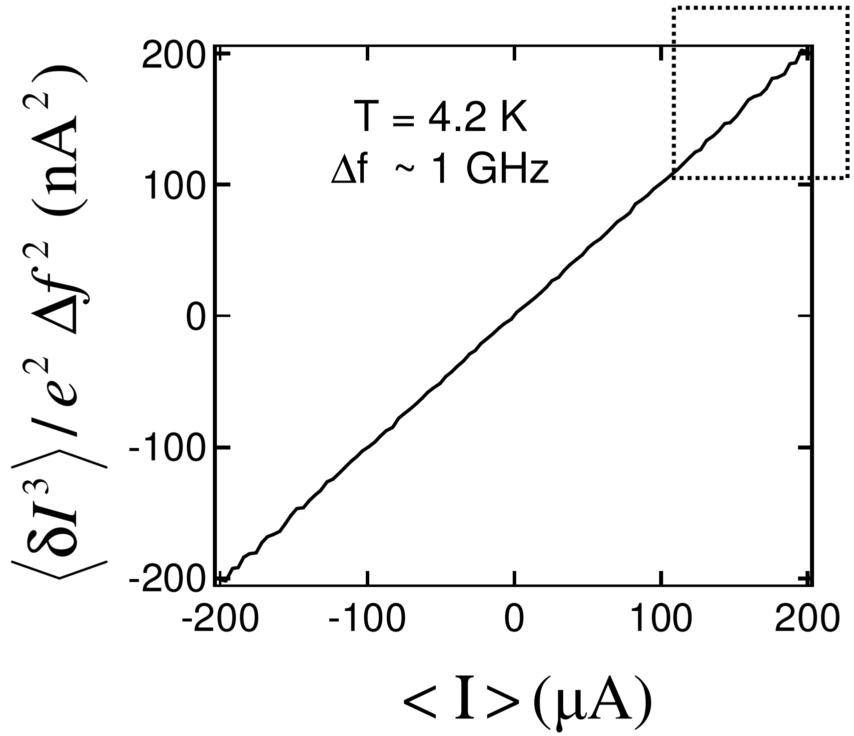
Levitov, Lee & Lesovik, '96

What about finite frequencies ?

Experiment (ω=0,T=4.2K)

Tunnel junction made by L. Spietz at Yale





crossover

$$eV = k_B T$$

Equilibrium noise

$$\langle \delta I^2 \rangle = 4 k_B T G$$

Temperature independent !

Signal to noise: $\dfrac{S_3}{(S_2)^{3/2}} \approx \dfrac{1}{\sqrt{I}}$

Quantum mechanics: ordering of operators?

Average current: $I_{DC} = \langle \hat{I} \rangle$

Noise S_2:

$$S_2(\omega) = \int dt e^{i\omega t} \begin{cases} \langle \hat{I}(0)\hat{I}(t) \rangle & \text{Absorption} \\ \langle \hat{I}(t)\hat{I}(0) \rangle & \text{Emission} \\ \frac{1}{2}\left(\langle \hat{I}(0)\hat{I}(t) \rangle + \langle \hat{I}(t)\hat{I}(0) \rangle \right) & \text{Classical} \end{cases}$$

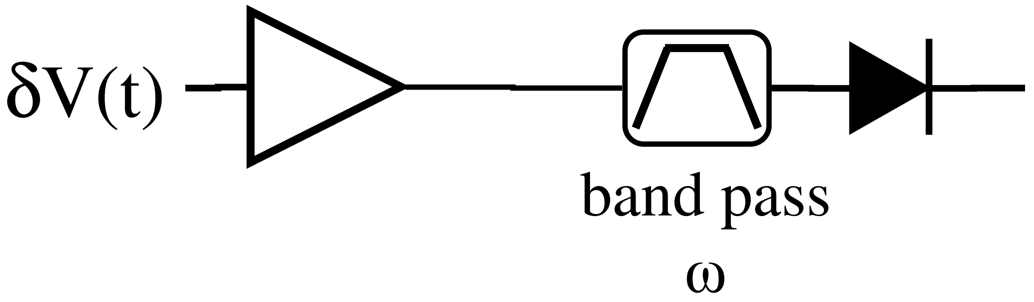
But for simple systems:

$$S_2^{abs}(\omega) = S_2^{em}(-\omega)$$

$$S_2^{sym}(\omega) = S_2^{em}(\omega) + \frac{1}{2} G\hbar\omega$$

How to measure $S_2(\omega)$?

1) « Classical » detection with a linear amplifier



2) « Quantum » detection with a photo-detector

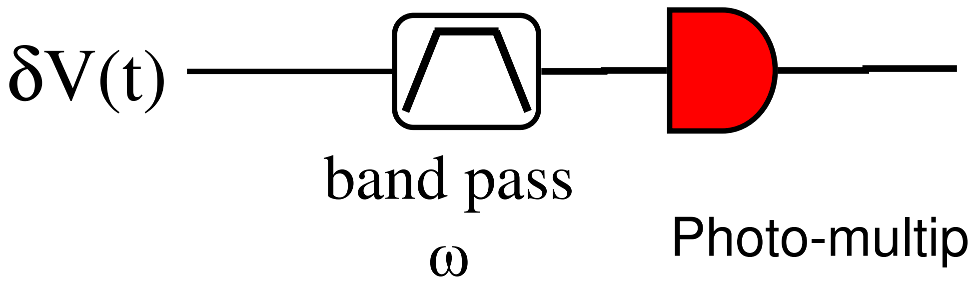


Photo-multiplier: absorbs photons

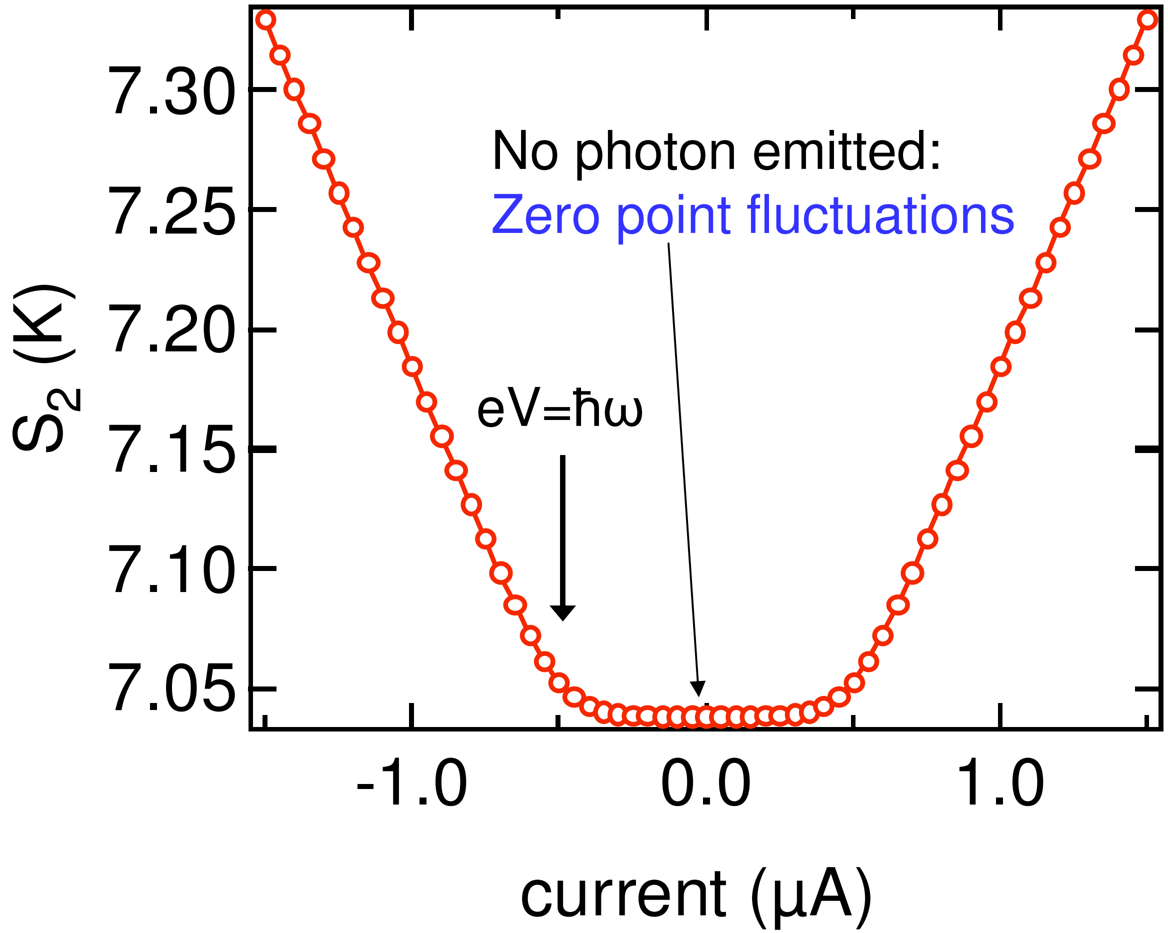
S_2 in the quantum regime $\hbar\omega > k_B T, eV$

$T_{phonons}$= 22 mK
$T_{electrons}$= 27 mK

f = 5.5 - 6.5 GHz
hf/k_B = 290 mK
Ghf/e = 0.50µA

It is not possible to separate the noise of the amplifier from the ZPF !

Tunnel junction R=50Ω



The third cumulant S_3 ?

$$S_3(\omega_1,\omega_2) = \langle I(\omega_1) I(\omega_2-\omega_1) I(-\omega_2)\rangle$$

Measures phase correlations at 3 different frequencies !

* Classical result: in a Dirac peak, all the Fourier components are IN PHASE

* Quantum regime: correlations involving zero point fluctuations ?

We have measured: $S_3(0,\omega) = \langle I(0) I(\omega) I(-\omega)\rangle$

low freq. current fluctuations → $I(0)$

ZPF: $I(\omega) I(-\omega)$

How to calculate S_3 ?

$$I(\omega) = \frac{h}{e}\int dE \quad [(1-r^*(E)r(E+\hbar\omega))a_L^+(E)a_L(E+\hbar\omega) - r^*(E)t(E+\hbar\omega)a_L^+(E)a_R(E+\hbar\omega)$$
$$-t^*(E)r(E+\hbar\omega)a_R^+(E)a_L(E+\hbar\omega) - t^*(E)t(E+\hbar\omega)a_R^+(E)a_R(E+\hbar\omega)]$$

For ω=0, *r* and *t* energy independent:

$$\hat{I} = p(a_L^+ a_L - a_R^+ a_R) - \sqrt{p(1-p)}(a_L^+ a_R + a_R^+ a_L)$$

To leading order in *p*:

$$\left\langle \hat{I}^3 \right\rangle \propto p^2(1-p)$$

WRONG !! Reason: causality is not respected …

Beenakker & Schomerus, ‘01
Salo, Hekking & Pekola, ‘06

S_3 and Q mechanics: ordering ???

$$S_3(\omega,\omega') = \int dt\,dt' e^{i(\omega t+\omega' t')} \left\langle \hat{I}(0,t,t'?)\hat{I}(0,t,t'?)\hat{I}(0,t,t'?) \right\rangle$$

The result depends on ORDERING:

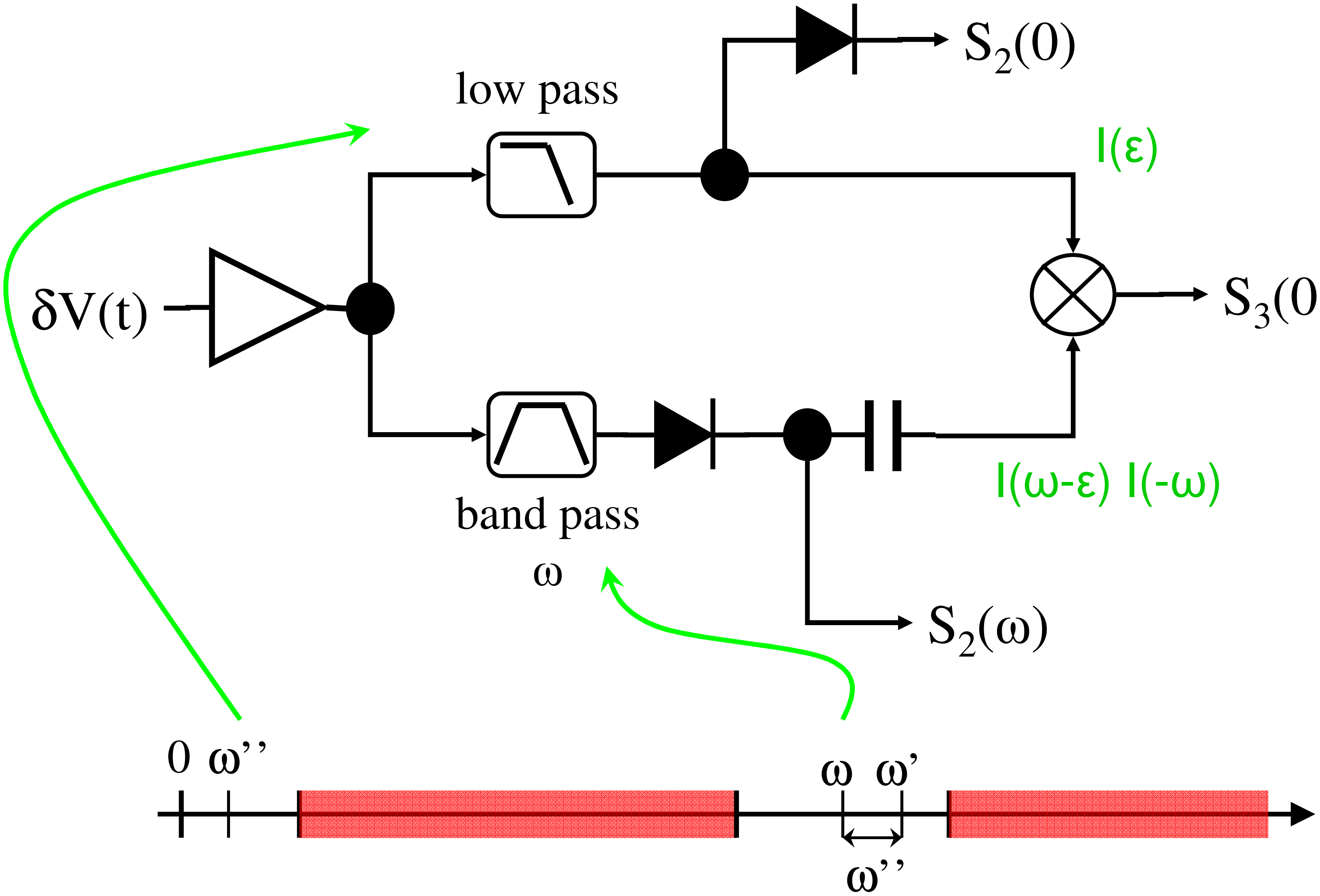
$$S_3(0,0) = \frac{e^2}{h} V \cdot \begin{cases} p(1-p)(1-2p) & \text{Keldysh ordering} \\ p^2(1-p) & \text{Fully symmetrized} \end{cases}$$

At finite frequency, Keldysh ordering, for a tunnel junction:

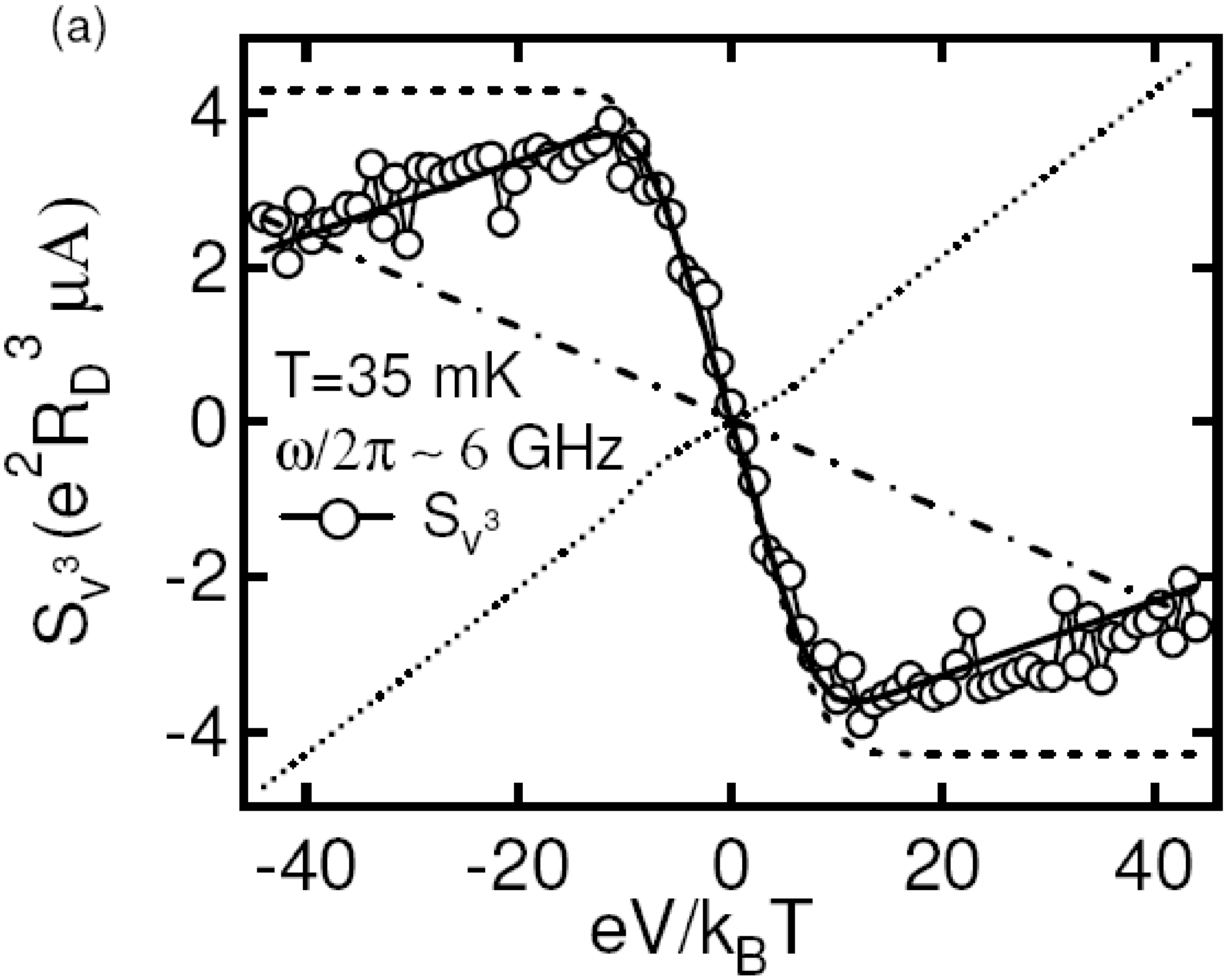
$$S_3(\omega_1,\omega_2) = e^2 I$$ Independent of frequency !!

Galaktionov, Golubev & Zaikin, '08

How to measure $S_3(0,\omega)$?

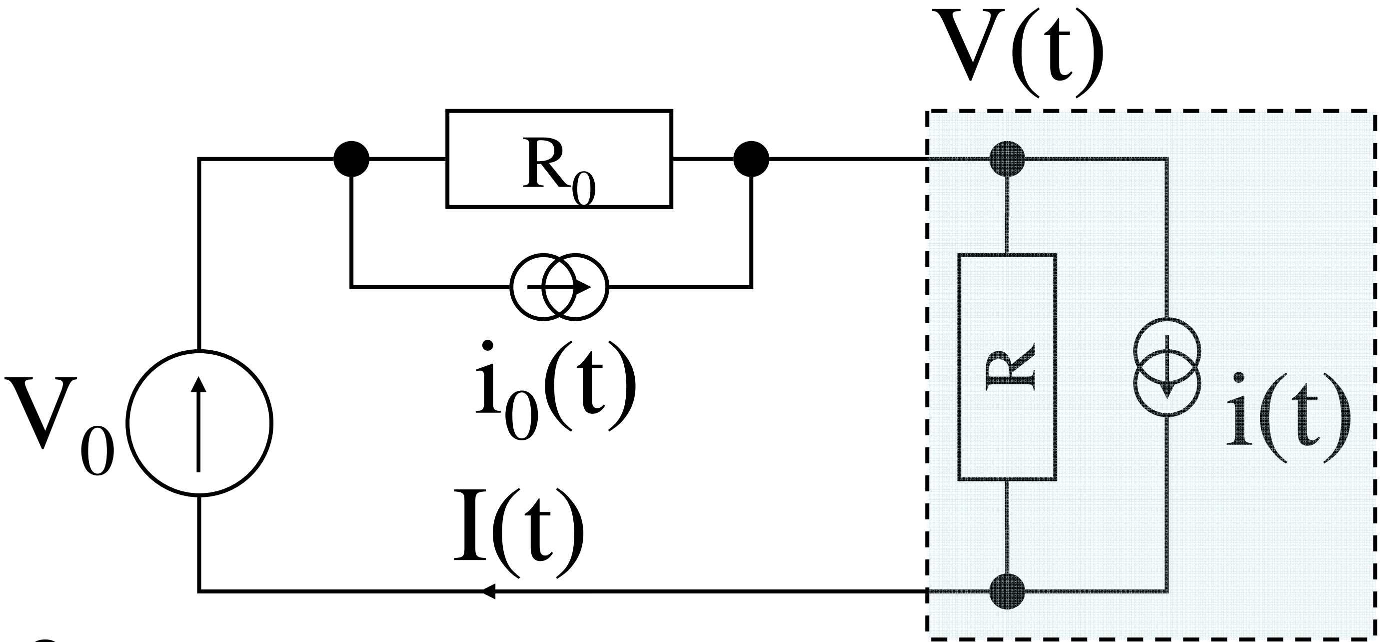


$S_3(0,\omega)$: a first experiment in the Quantum regime



R=44 Ω

Environmental effects



$$\delta V(t) = (R // R_0)(i_0 - i)$$

$$\left\langle \delta V^2 \right\rangle = \left(R // R_0\right)^2 \left(\left\langle i_0^2 \right\rangle + \left\langle i^2 \right\rangle - 2\left\langle i_0 i \right\rangle \right)$$

$$\left\langle \delta V^3 \right\rangle = \left(R // R_0\right)^3 \left(\left\langle i_0^3 \right\rangle - \left\langle i^3 \right\rangle + 3\left\langle i_0 i^2 \right\rangle - 3\left\langle i_0^2 i \right\rangle \right)$$

The probability distribution P(i) depends on V(t)

Feedback and noise of the environment

Kindermann
Nazarov
Beenakker

* The noise of the sample is modulated by external voltage fluctuations:

$$\left\langle i_0 i^2 \right\rangle = \left\langle i_0 S_2(V(t)) \right\rangle \cong \left\langle i_0 \frac{dS_2}{dV} \delta V(t) \right\rangle = \left\langle i_0^2 \right\rangle (R // R_0) \frac{dS_2}{dV}$$

Noise of the environment: T_{env}

Noise susceptibility

* The noise of the sample is modulated by its own current fluctuations through the external impedance:

$$\left\langle i^3 \right\rangle = \left\langle i i^2 \right\rangle = \left\langle i^3 \right\rangle_V + 3\left\langle i \; S_2(V(t)) \right\rangle \cong \left\langle i^3 \right\rangle_V - 3\left\langle i^2 \right\rangle (R // R_0) \frac{dS_2}{dV}$$

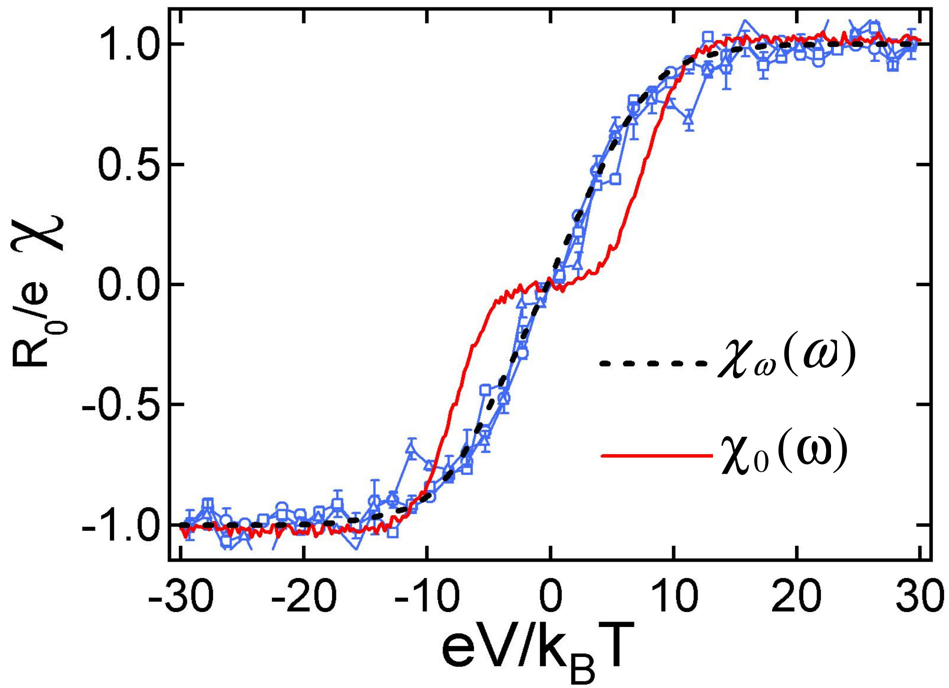
Feedback (even for T_{env}=0)

The same mechanism leads to Dynamical Coulomb Blockade

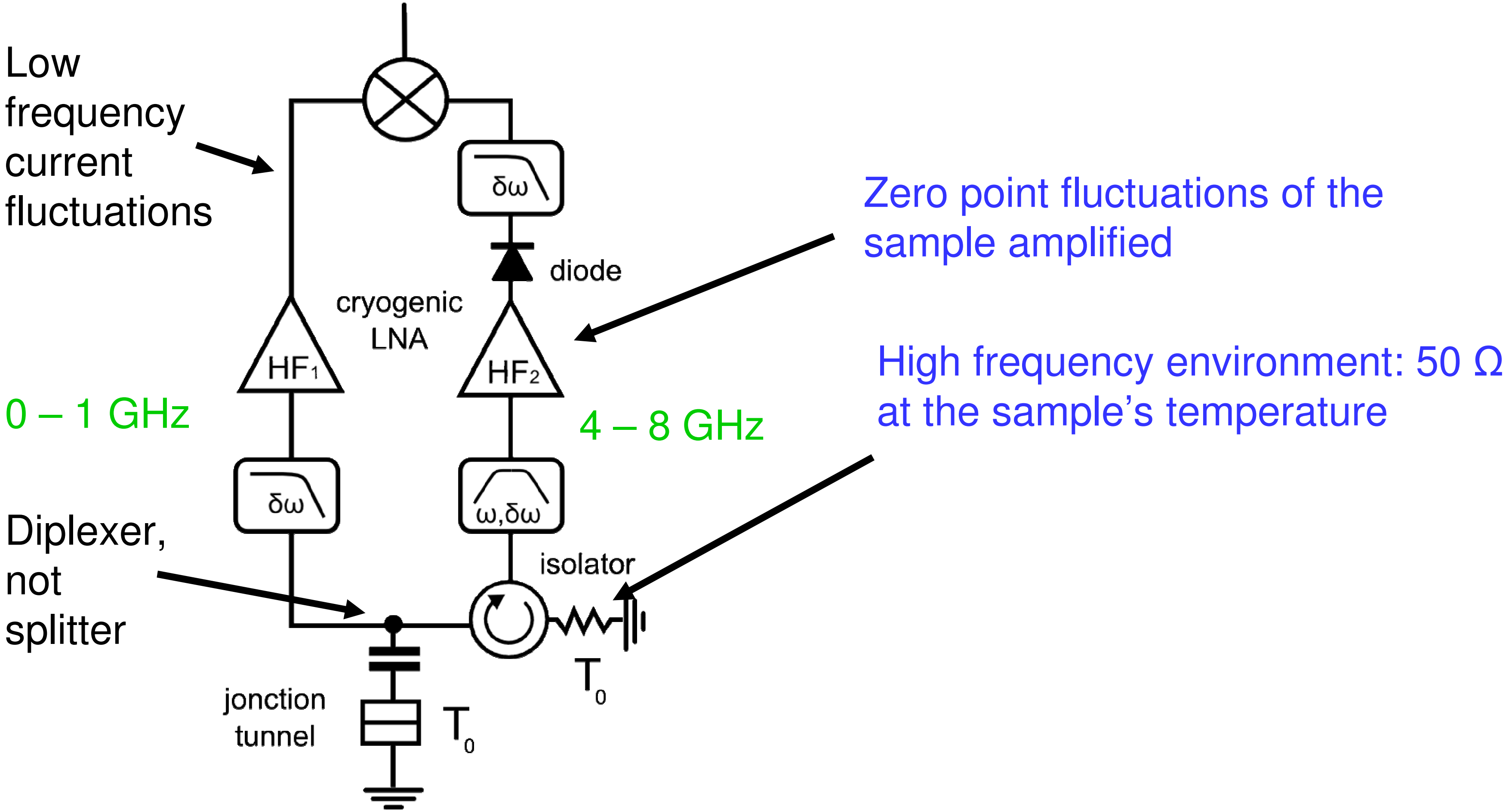
$$\delta I = \frac{\hbar}{e^2}\int K(-\omega)\omega\chi_\omega(\omega)d\omega$$

Correlation function of the environment

Noise susceptibility for $\omega_0=\omega$

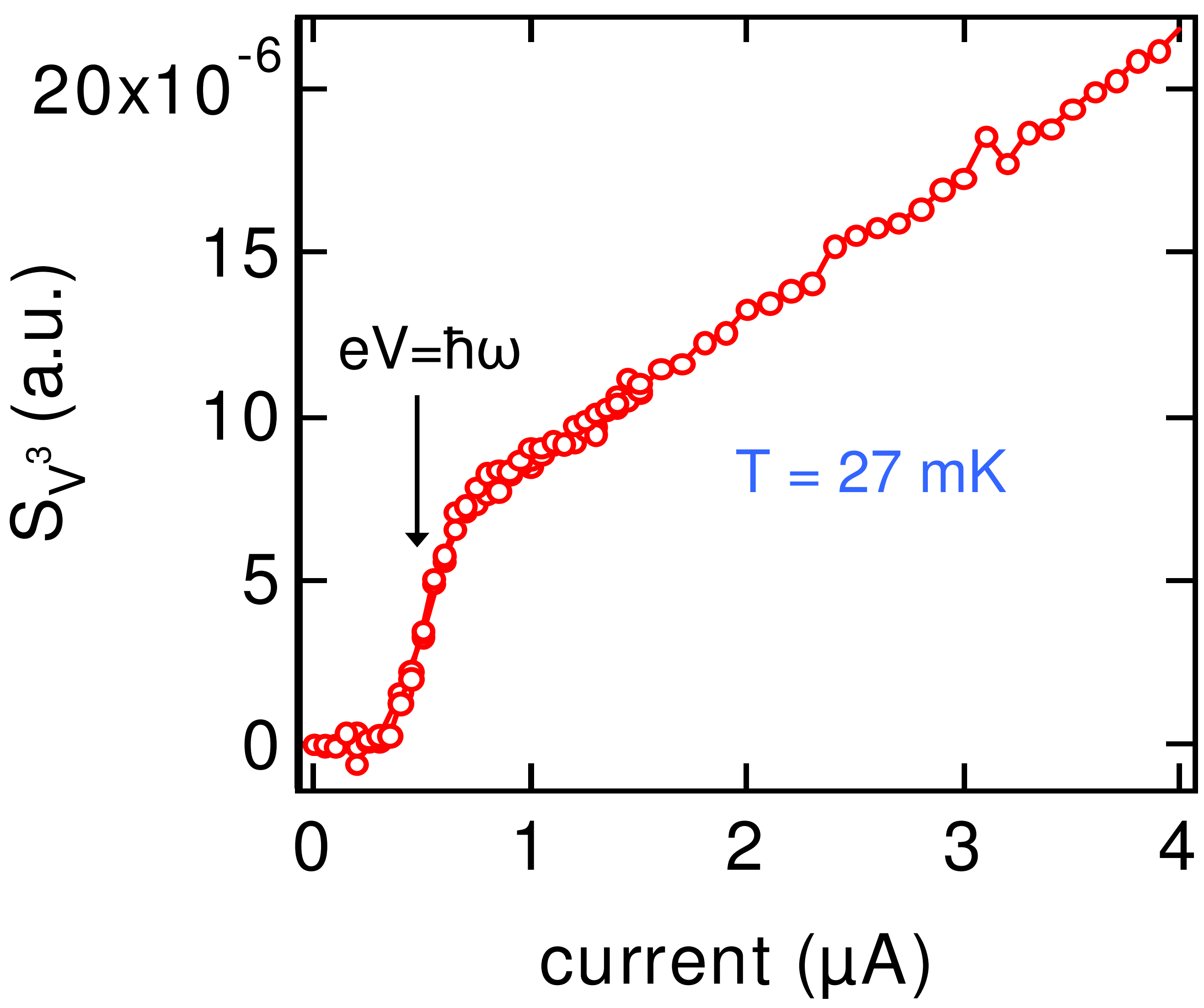


Experimental setup

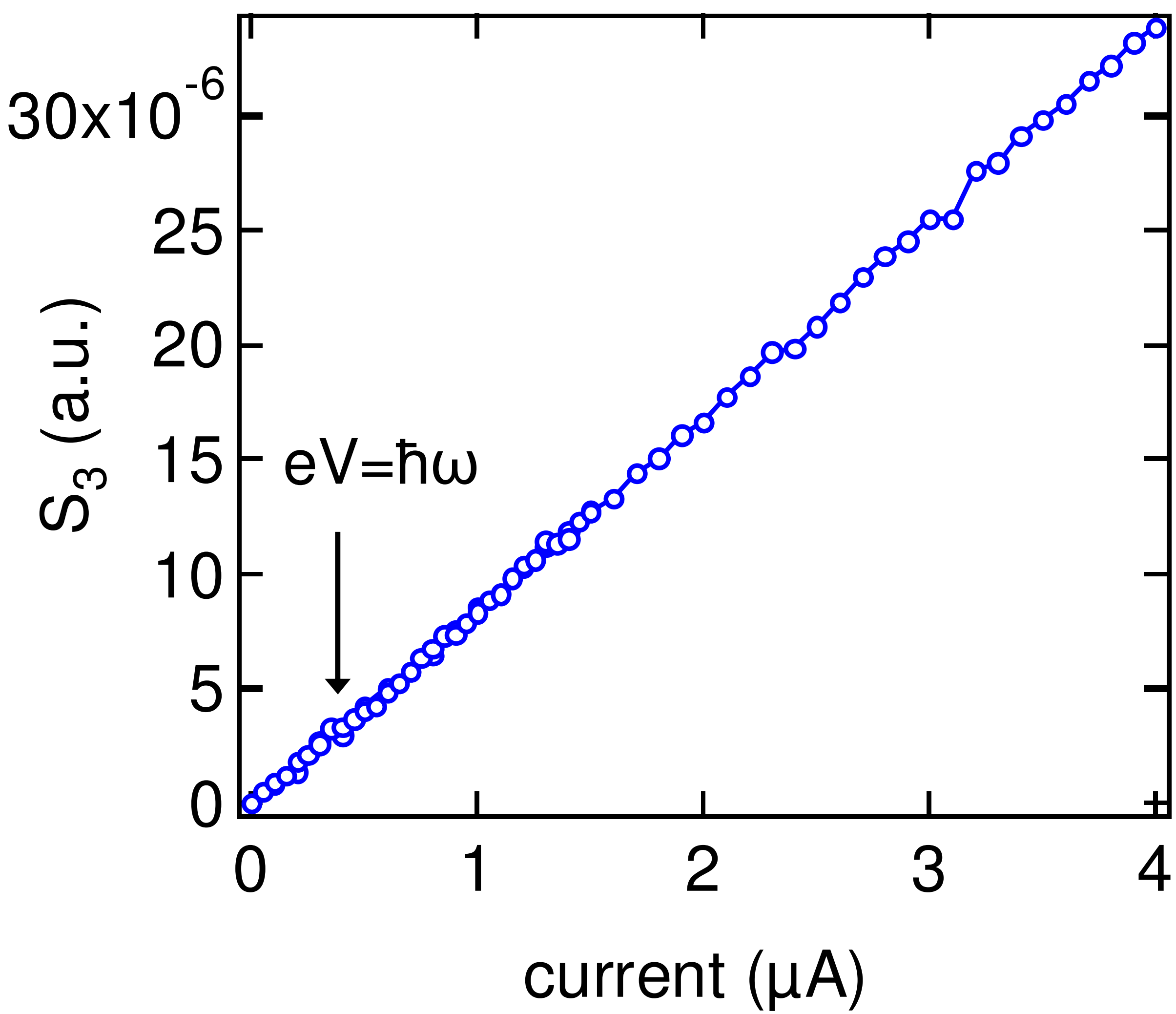


Environmental effects: only the low freq. environmental noise temp. is not well known.

Third cumulant of VOLTAGE



Third cumulant of CURRENT



$$e^2 I$$

Another way to measure $S_3(0,f)$?

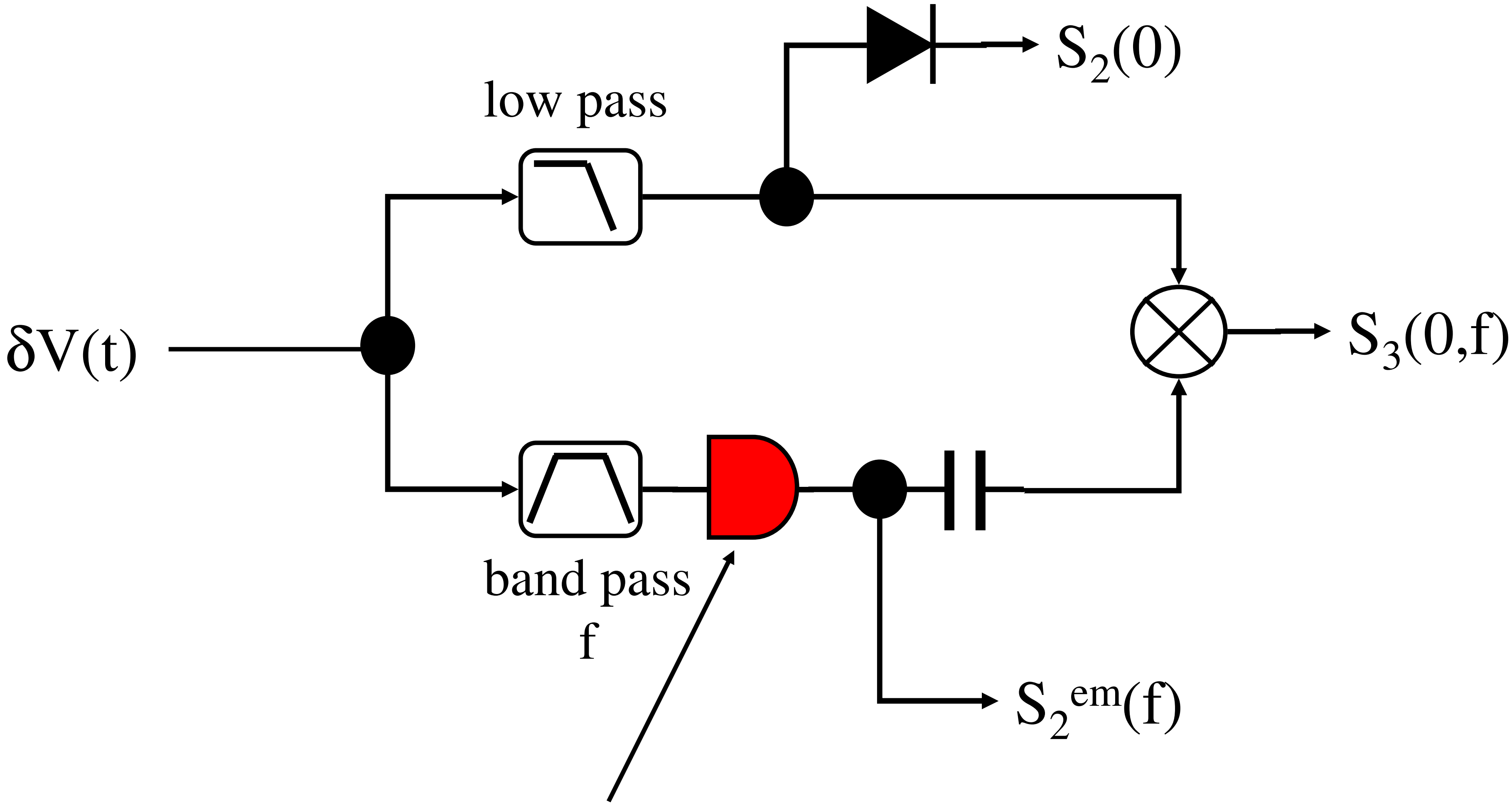
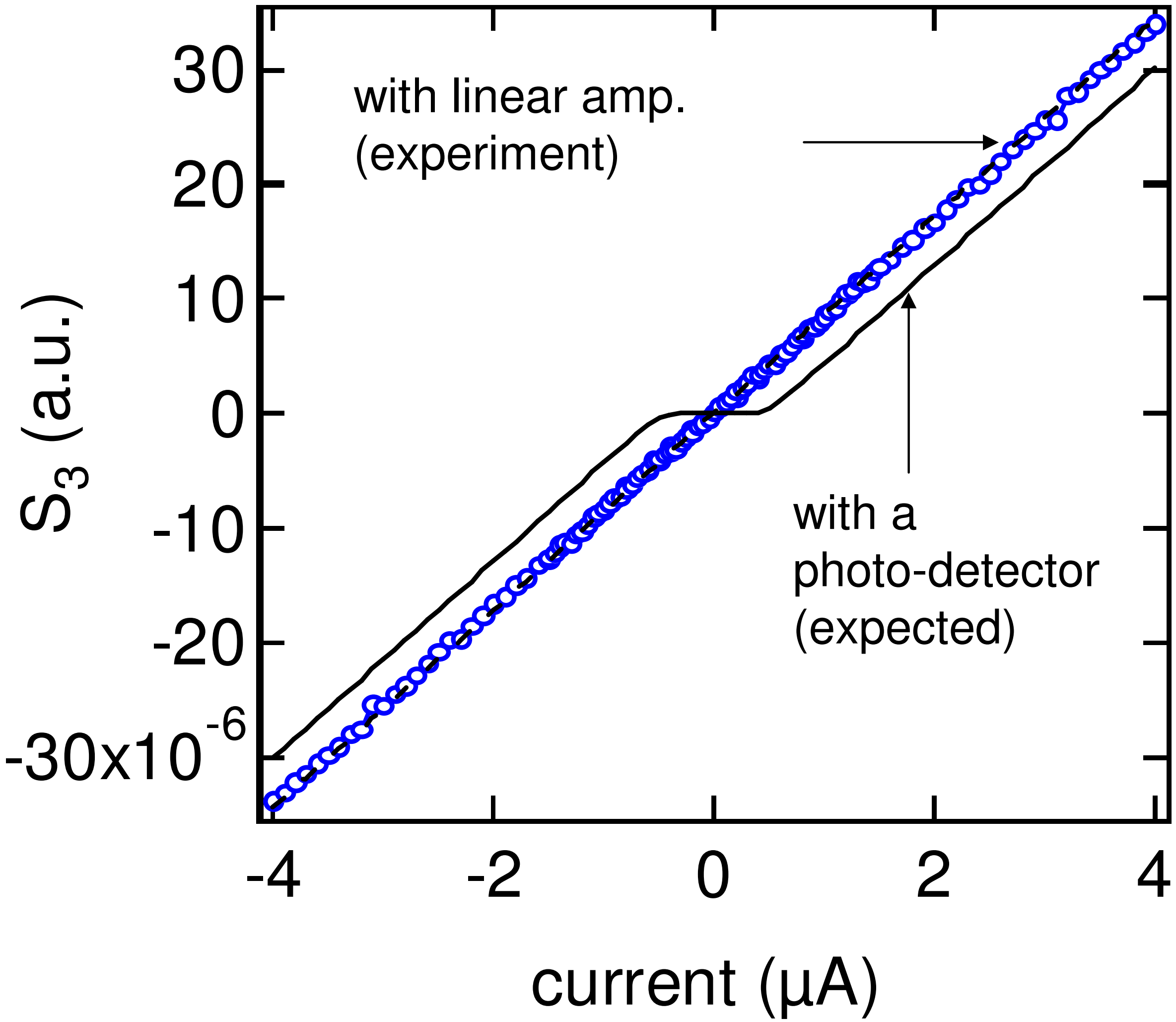


Photo-multiplier: absorbs photons

Gives zero for eV<hf: another ordering of the operators ?

Third cumulant of CURRENT



Conclusions

* A linear amplifier amplifies the zero point current fluctuations

* There is a correlation between low frequency current fluctuations and high frequency, zero point current fluctuations

* The third cumulant of current fluctuations measured with linear amplifiers is well described by the Keldysh order

$$S_3(0,\omega) = \left\langle I(0) I(\omega) I(-\omega) \right\rangle = e^2 \left\langle I \right\rangle$$

Some open questions

* What order of operators for a given experimental setup ? What experiment to perform to probe a given correlator ?

* We detect electromagnetic field: how correlators involving electric field (i.e., photons) in a coax cable are related to correlators of the current (i.e., electrons) in the samples ? What is the photon density matrix ?