Distributions of waiting-times of dynamical single electron emitters

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Charge and heat dynamics in nano-systems
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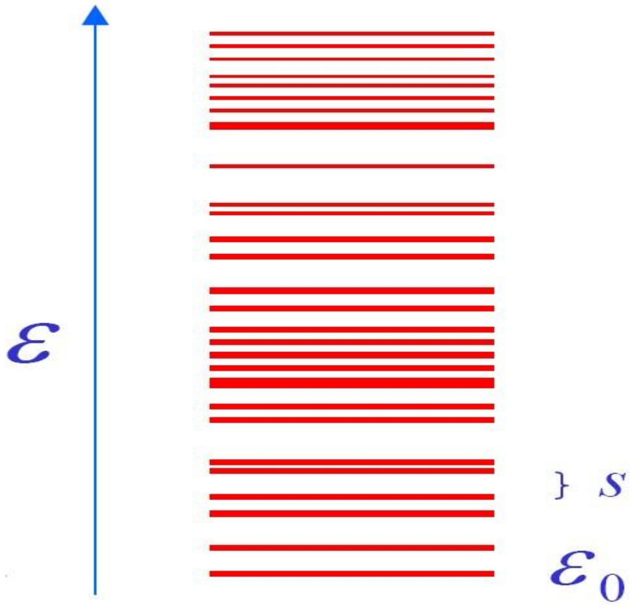
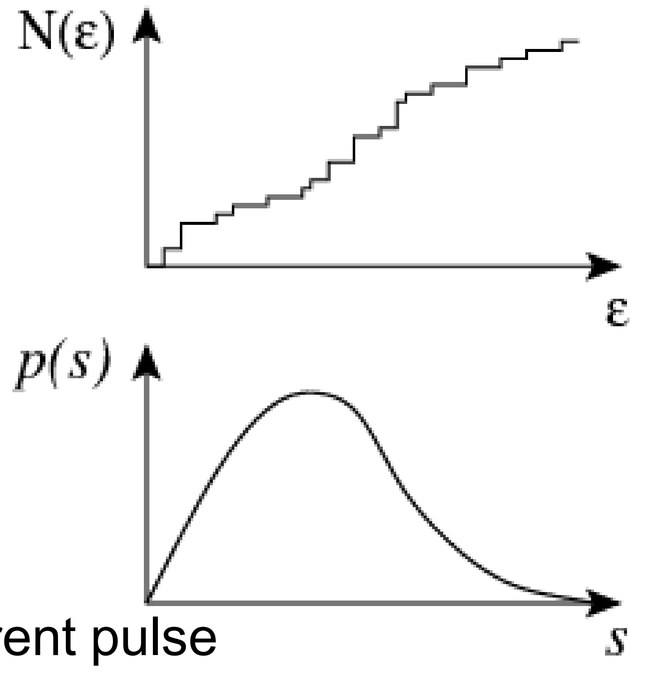
Level spacing statistics

Quantum chaos

Quantum scale: energy level spacing s

Classical enrgy scale: Thouless energy E_c





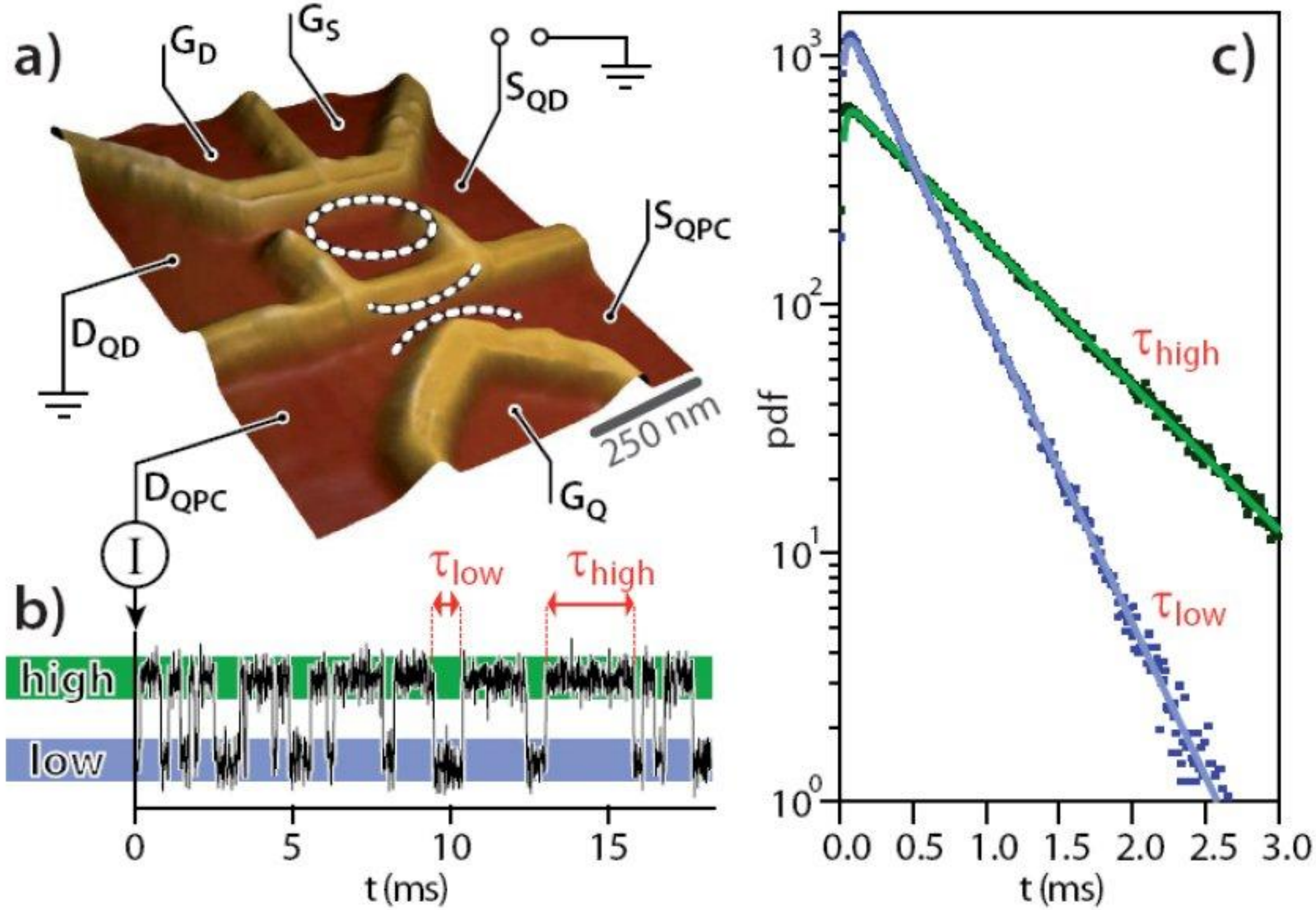
If $E \sim t$ and energy level = current pulse

$N(\epsilon)$ is the integrated current = tranferred charge investigated in FCS

$P(s)$ the Wigner distribution = distribution of waiting times

Waiting time distribution: Experiment[3]

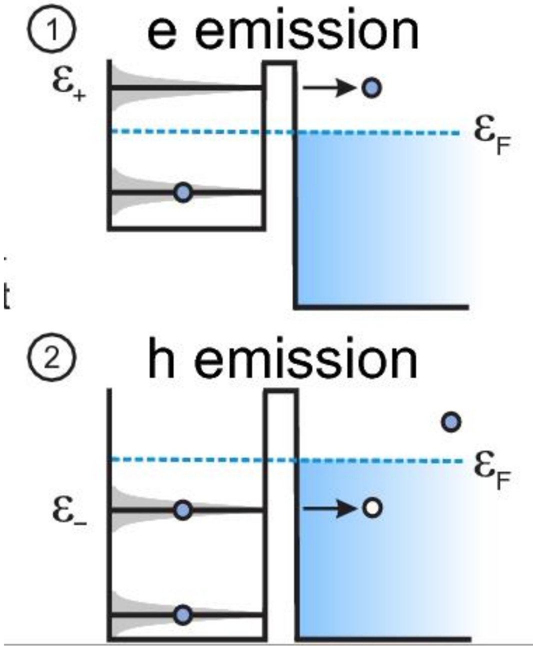
@Hannover

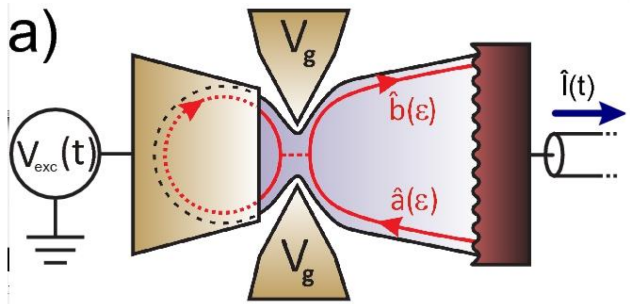


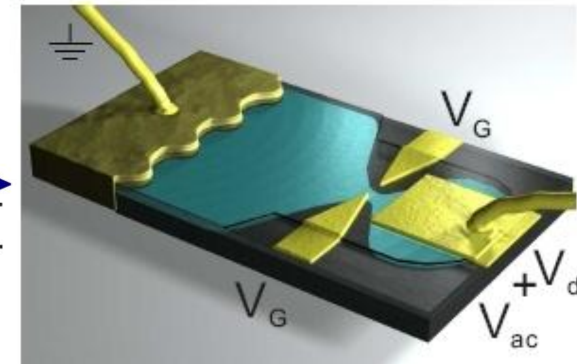
Distribution of waiting times is sufficent to derive FCS

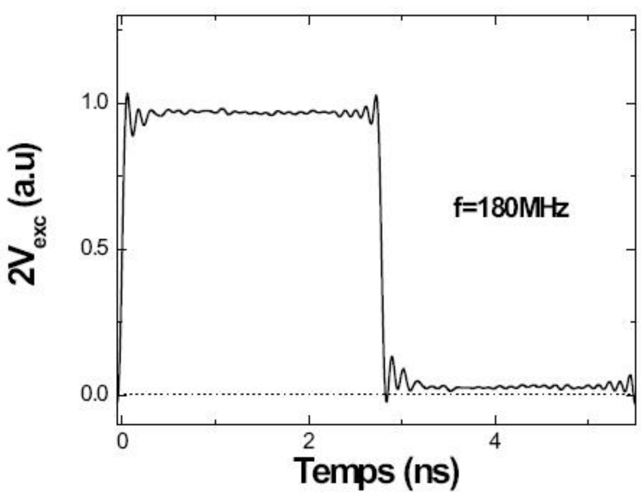
Single particle emitter

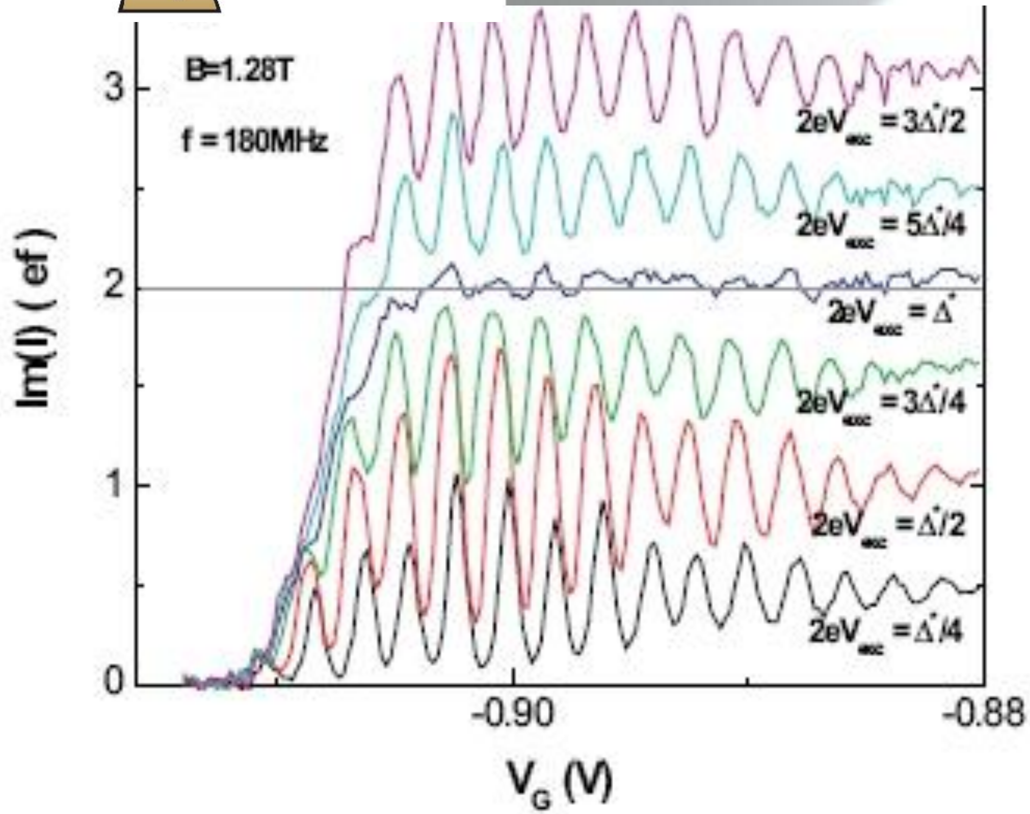
G.Fève, A. Mahé, J.-M.Berroir, T. Kottos, B.Plaçais, D. C., Glattli, A. Cavanna, B.Etienne, Y.Jin, SCIENCE 316, 1169 (2007)





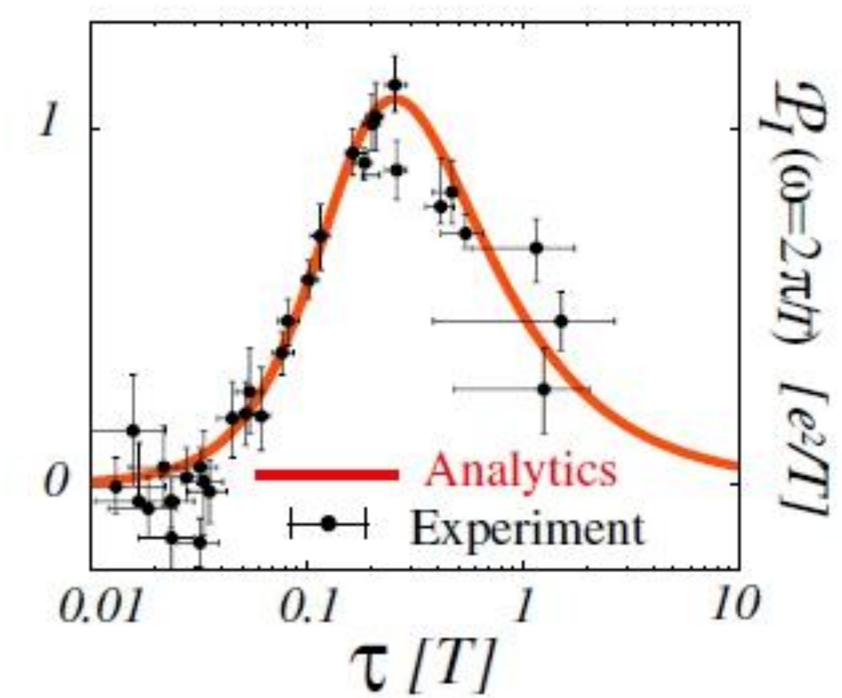


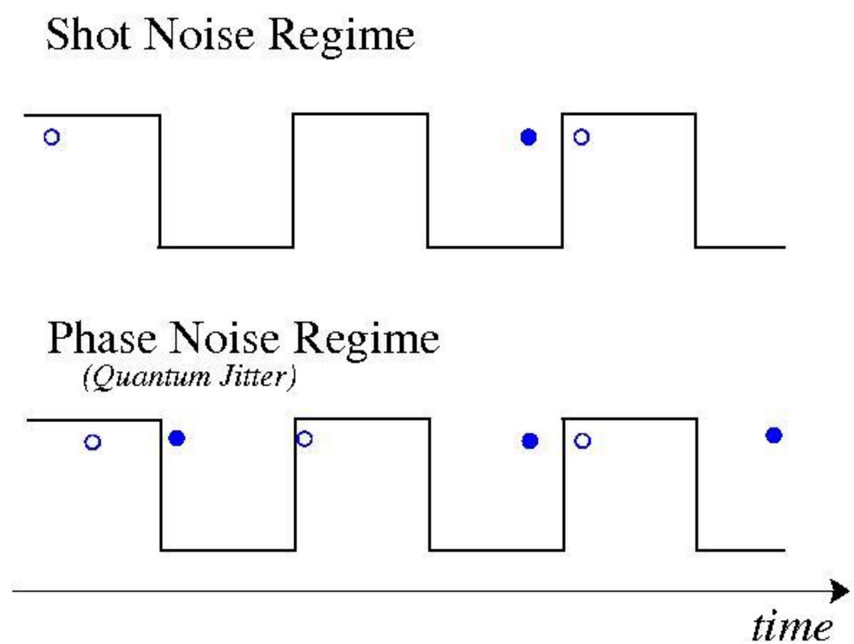




Shot- and phase-noise of a single particle emitter

Mahé, Parmentier, Bocquillon, Berroir, Glattli, Kontos, Plaçais,Fève, Cavanna, Jin, Phys. Rev. B 82, 201309 (2010).
Albert, Flindt, Buttiker, PRB 82, 041407 (2010).





$\tau >> T/2$ Shot noise limit : $P(\omega) = 1/2\tau$, $\omega\tau >> 1$

$\tau << T/2$ Phase noise limit: $P(\omega) = \frac{2}{T}\frac{\omega^2\tau^2}{1+\omega^2\tau^2}$, $\omega\tau > 1$

Noise is a universal function of lifetime τ and oscillation period T only
ω is the observation frequency

FCS for a single particle emitter

Mahé, Parmentier, Bocquillon, Berroir, Glattli, Kontos, Plaçais,Fève, Cavanna, Jin, Phys. Rev. B 82, 201309 (2010).
Albert, Flindt, Buttiker, PRB 82, 041407 (2010).
[Fabio Pistolesi, PRB 69, 245409 (2004)]

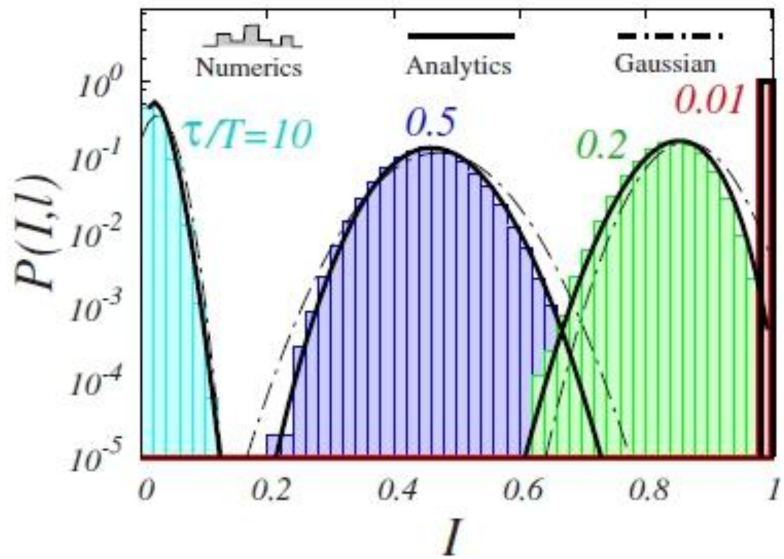
$$dP/dt = -\Gamma_E(t)\,P(t) + \Gamma_A(t)\,(1 - P(t))$$

$\Gamma_A(t) = \Gamma\ ,\ \Gamma_E(t) = 0$ during first half of period : absorption

$\Gamma_A(t) = 0\ ,\ \Gamma_E(t) = \Gamma$ during second half of period: emission

Consider only electrons (holes) emitted : $I = n/l\ ,\ l >> 1\ ,\ \epsilon = e^{-T/2\tau}$

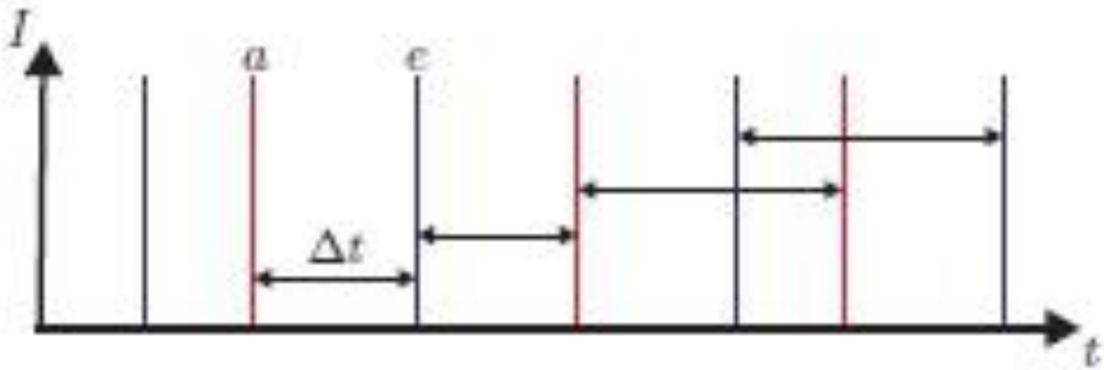
$$P(I,l) \simeq \frac{\left[\frac{\varepsilon(1+I)}{1-I}\right]^l \left[\frac{(1-I^2)(1-\epsilon)^2}{4\epsilon I^2}\right]}{\sqrt{\pi I(1-I^2)l}}$$



FCS: long time statistics: insentive to Quantum jitter (phase noise)!

Waiting time distribution

Albert, Flindt, Buttiker, Phys. Rev. Lett. 107, 086805 (2011)

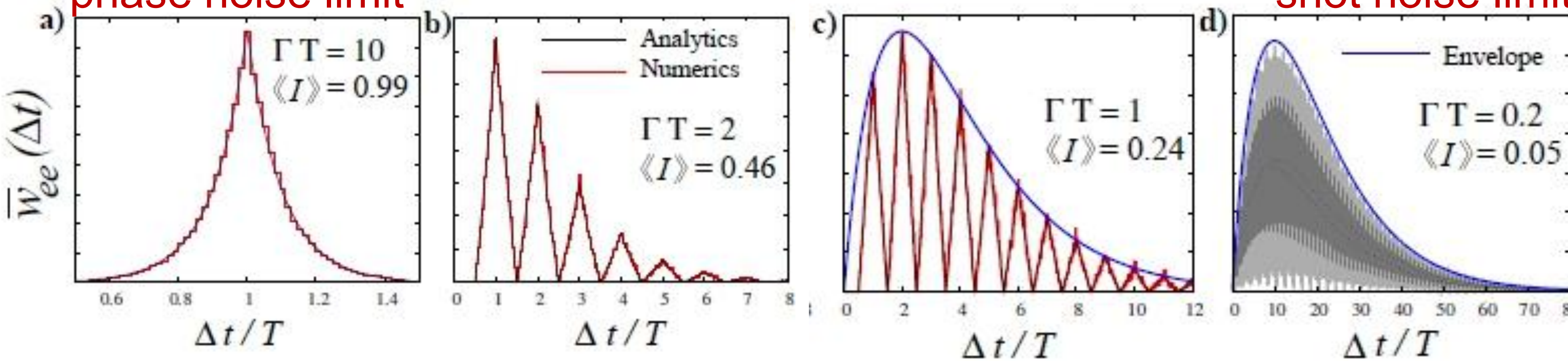


$$\overline{w}_{ee}(\Delta t) = \frac{\Gamma^{\lfloor \frac{\Delta t + T/2}{T} \rfloor} \varepsilon^{\lfloor \frac{\Delta t - T/2}{T} \rfloor}}{2} \left\{ e^{-|\xi_{ee}|} - \varepsilon^2 e^{|\xi_{ee}|} \right\}$$

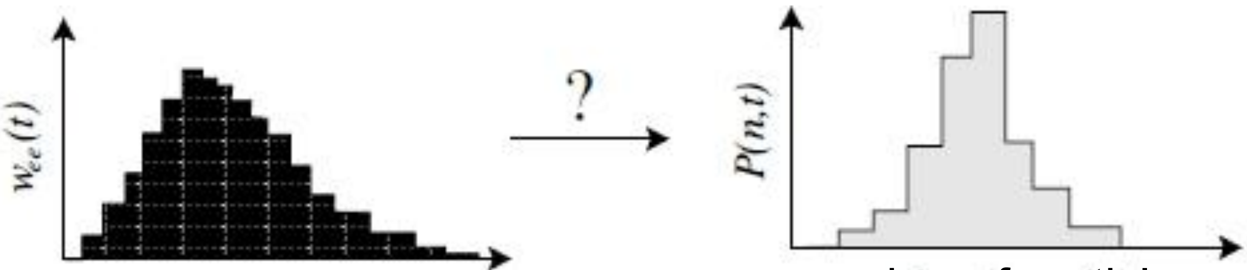
$\xi_{ee} = \Gamma[\Delta t - (\lfloor \Delta t/T \rfloor + 1)T]$,$[x]$ integer part of x

phase noise limit

shot noise limit



From WTD to FCS



waiting time

number of particles emitted in time t

$$p(n,t) = \int d\tau_1 ....d\tau_n \, w_{ee}(0,\tau_1).....w_{ee}(t-\tau_n,t)\, \delta(\sum_{l=1}^{l=n} \tau_l - t\,)$$

in the long time limit (average)

$$p(n,t) \simeq \int d\tau_1 ....d\tau_n \, \overline{w}_{ee}(\tau_1).....\overline{w}_{ee}(t-\tau_n)\, \delta(\sum_{l=1}^{l=n} \tau_l - t\,)$$

cumulant generating function of the WTD

$$G_{ee}(z) = ln \int_0^\infty d\tau \overline{w}_{ee}(\tau)\, e^{-z\tau}$$

cumulant generating function of the FCS

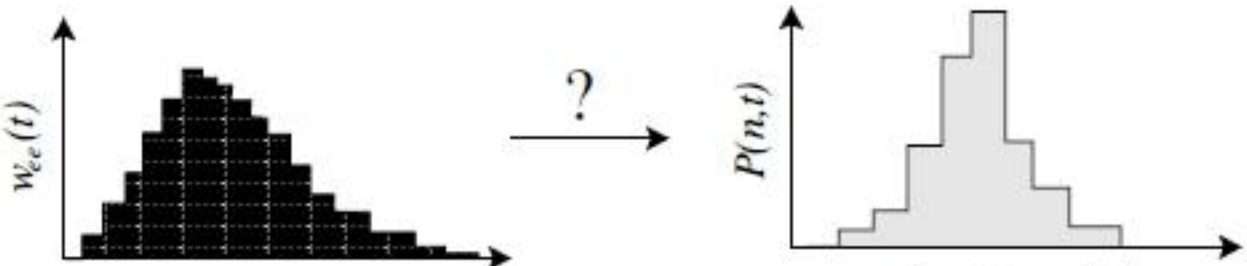
$$S_{ee}(\chi,t) = ln \sum_{n=1}^{\infty} p(n,t) e^{i\chi n}$$

in the long time limit

$$G_{ee}(S(\chi,t)) + i\chi = 0\,.$$

(For stationary problem, T. Brandes, Ann. Phys. 17, 477 (2008))

WTD and FCS: relation between cumulants



waiting time

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$$G_{ee}(z) = ln \int_0^\infty d\tau \overline{w}_{ee}(\tau)\, e^{-z\tau}$$

cumulant generating function of the FCS

$$S_{ee}(\chi, t) = ln \sum_{n=1}^{\infty} p(n,t) e^{i\chi n}$$

in the long time limit

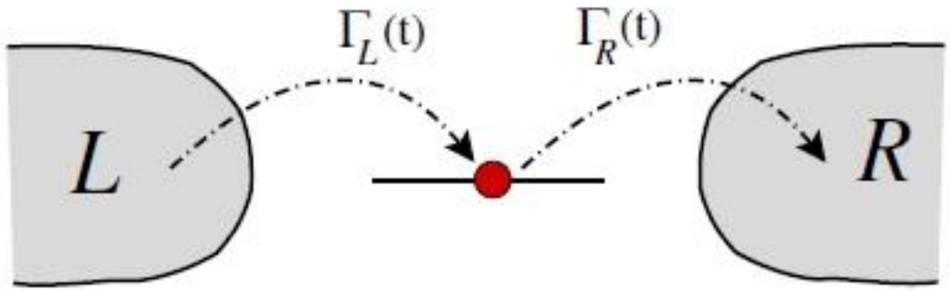
$$G_{ee}(S(\chi,t)) + i\chi = 0\,. \quad \Longrightarrow$$

Relations between cumulants

$$\langle\langle I \rangle\rangle = \frac{\langle\langle n \rangle\rangle}{t} = \frac{1}{\langle\langle \tau \rangle\rangle} \qquad F_2 = \frac{\langle\langle n^2 \rangle\rangle}{\langle\langle n \rangle\rangle} = \frac{\langle\langle \tau^2 \rangle\rangle}{\langle\langle \tau \rangle\rangle^2} \qquad F_3 = \frac{\langle\langle n^3 \rangle\rangle}{\langle\langle n \rangle\rangle} = 3\frac{\langle\langle \tau^2 \rangle\rangle^2}{\langle\langle \tau \rangle\rangle^4} - \frac{\langle\langle \tau^3 \rangle\rangle}{\langle\langle \tau \rangle\rangle^3}$$

For this class of systems knowing WTD $\Longrightarrow$ FCS

How to calculate WTD



Master equation $P(t) = \langle Q(t) \rangle$

$$dP/dt = -\Gamma_L(t)\, P(t) + \Gamma_R(t)\, (1 - P(t))$$

Incoming current $\langle I_{in}(t) \rangle = \Gamma_L(t)[1 - P(t)]$

Outgoing current $\langle I_{out}(t + \Delta t) \rangle = \Gamma_R(t + \Delta t) P_1^s(t + \Delta t)$

Survival porbability $dP_1^s(t + \Delta t)/dt = -\Gamma_R(t + \Delta t)\, P_1^s(t + \Delta t)$ with $P_1^s(t) = 1.$

$w_{ea}(t, t + \Delta t)$ waiting time is proportional to the probability that an electron is aborbed at time t and emitted at time $t + \Delta t$

Absorption at time t $\sim \langle I_{in}(t) \rangle$

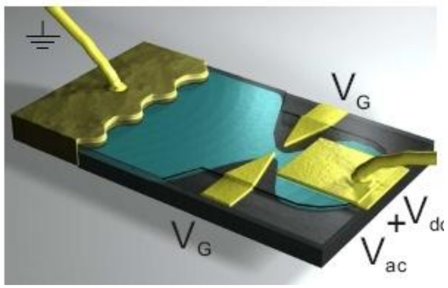
Emission at time $\sim \langle I_{out}^s(t + \Delta t) \rangle = \Gamma_R(t + \Delta t) P_1^s(t + \Delta t)$

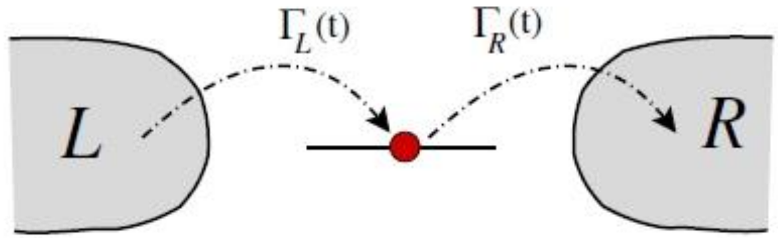
$$w_{ea}(t, t + \Delta t) = \mathcal{N} \, | \, \langle I_{in}(t) \rangle \, \langle I_{out}(t + \Delta t) \rangle \, |$$

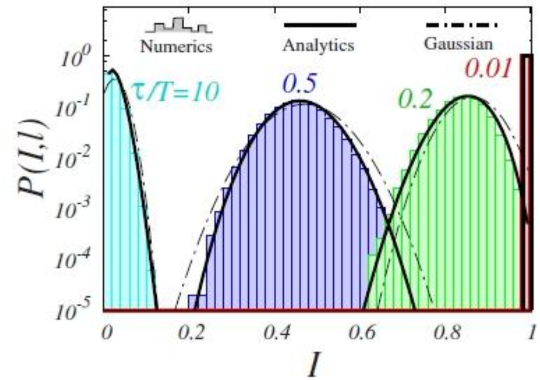
$$\overline{w}_{ea}(\Delta t) = \int_0^T \frac{dt}{T} w_{ea}(t, t + \Delta t)$$

Summary

Albert, Flindt, Buttiker, Phys. Rev. Lett. 107, 086805 (2011)







Distribution of waiting times

phase noise limit

shot noise limit