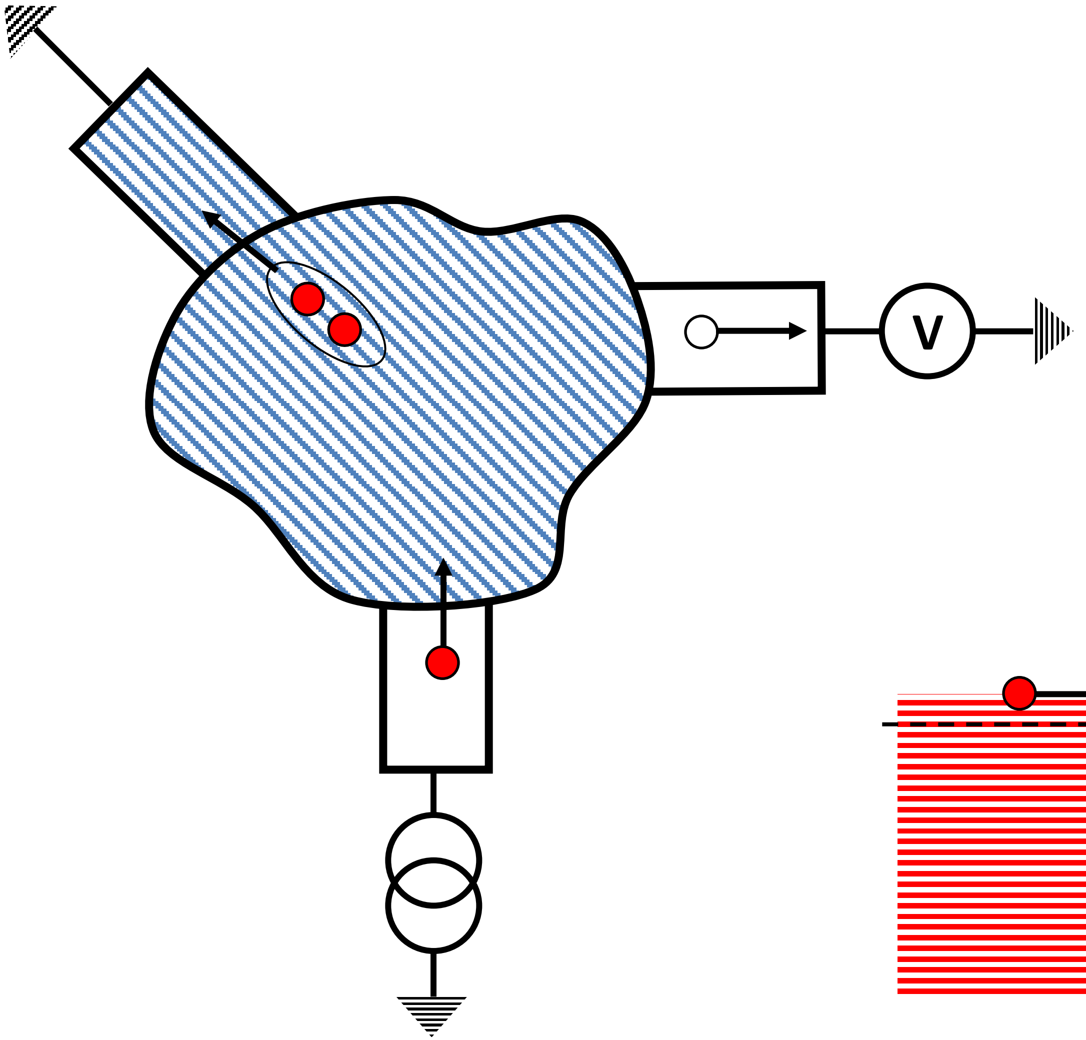
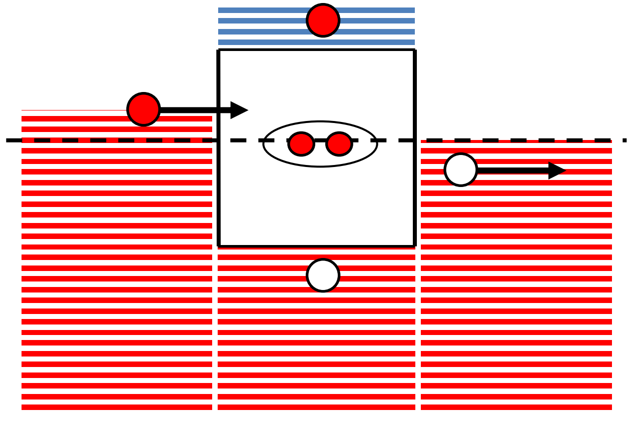
Cooper pair splitting in carbon nanotubes

Alfredo Levy Yeyati

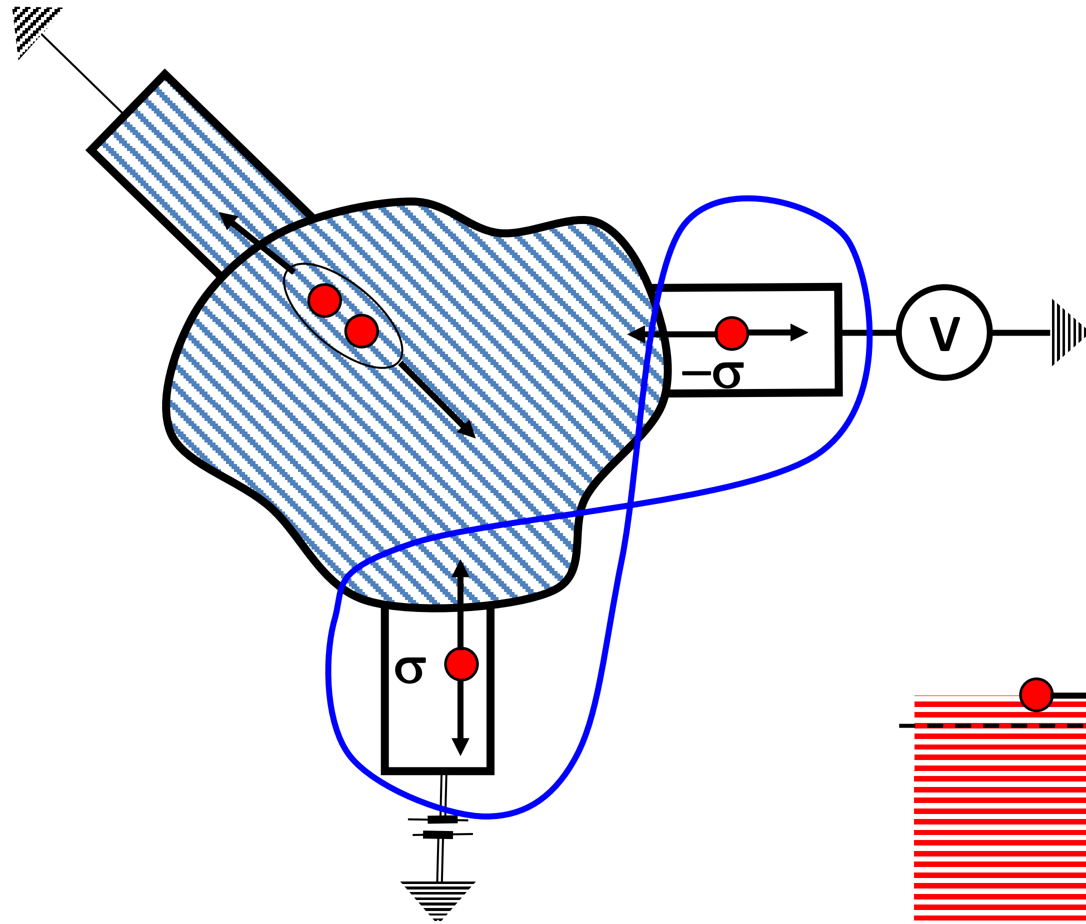
Theory: P. Burset, W. Herrera (UAM) A. Cottet (ENS-Paris)
Exps: T. Kontos (ENS-Paris) in collaboration with
L. Herrmann, C. Strunk (Regensburg),
P. Roche and F. Portier (CEA-Saclay).

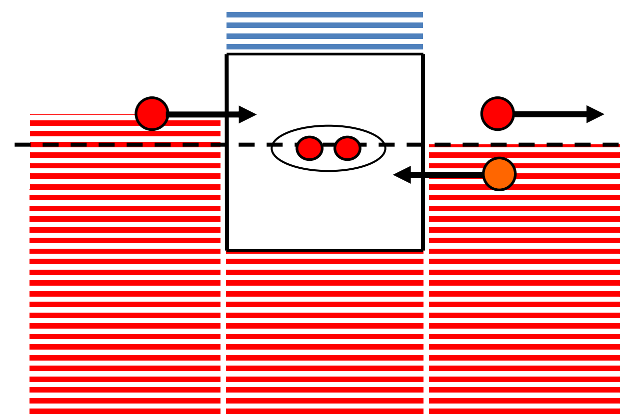
Crossed Andreev





Crossed Andreev: generation of entangled pairs

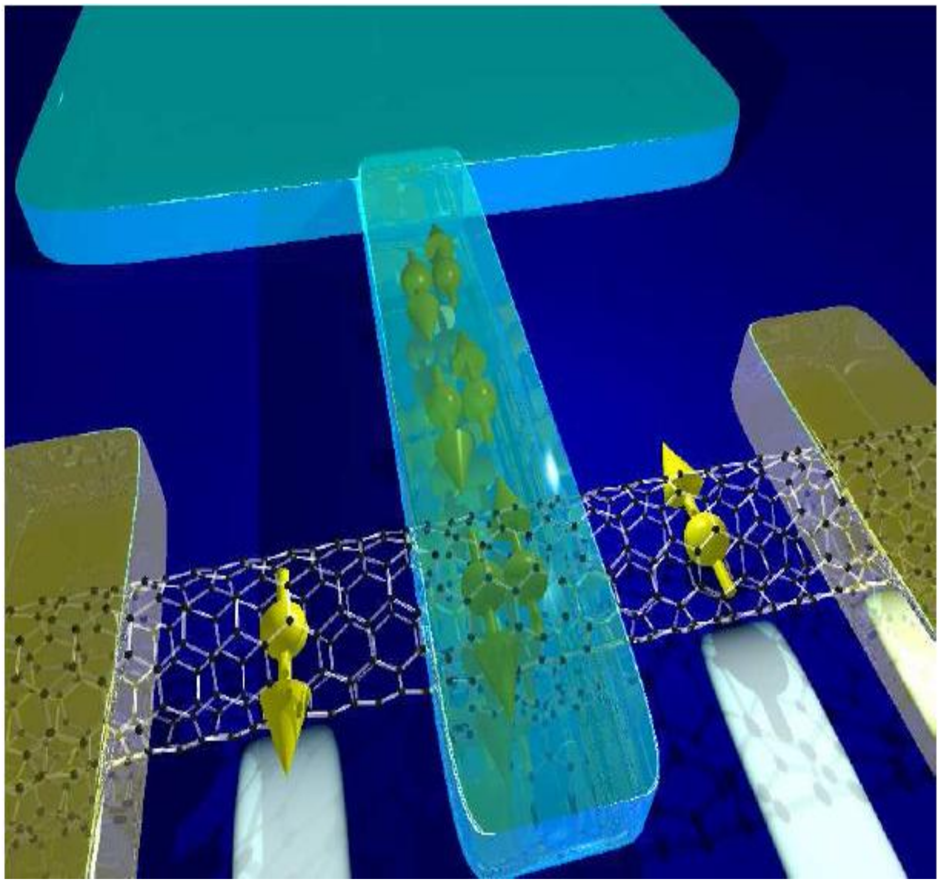




Carbon nanotubes as Cooper-pair beam splitters

L.Hermann, F. Portier, P. Roche, A. Levy Yeyati, T. Kontos and C. Strunk

PRL **104** (2010)



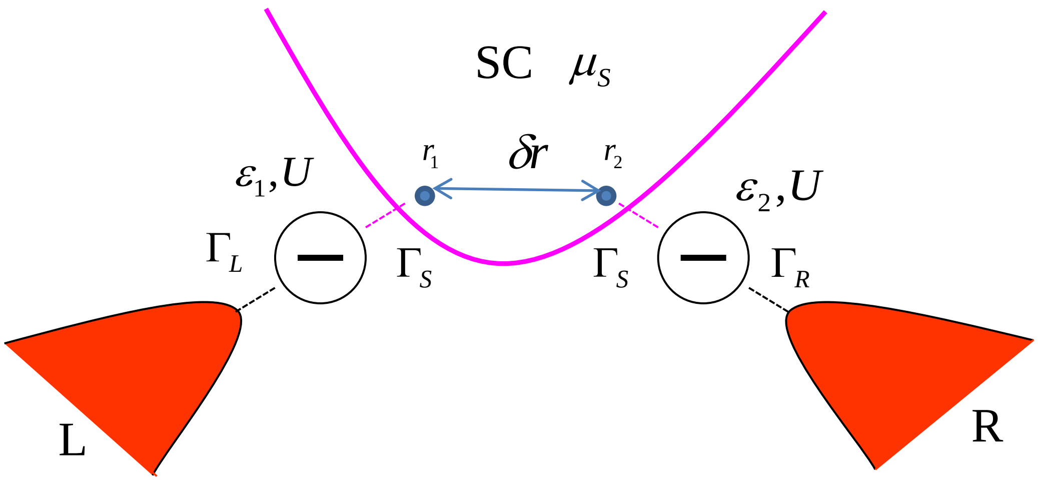
Physics
spotlighting exceptional research

Viewpoint

Carbon nanotubes help pairs survive a breakup

Similar exps. InAs nanowires, L. Hofstetter et al. Nature 461, 960 (2009)

The Recher-Sukhorukov-Loss proposal



Phys. Rev. B 63, 165314 (2001)

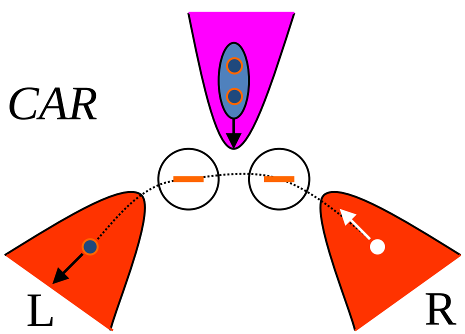
$$I_{CAR} \sim \left(\frac{\sin k_F \delta r}{k_F \delta r} \right)^2 e^{-2\delta r/\xi_0}$$

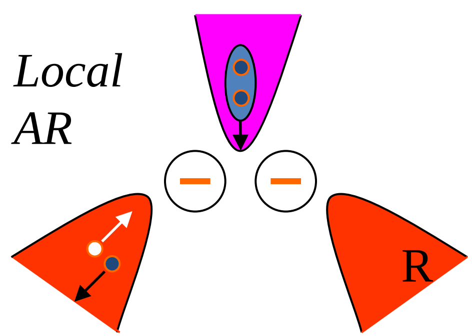
Falci , Feinberg, Hekking (2001)

$$\varepsilon_1 = \varepsilon_2 \simeq \mu_s$$

CAR would dominate when

$\varepsilon/\Gamma > k_F \delta r$ and $\delta r < \xi_0$



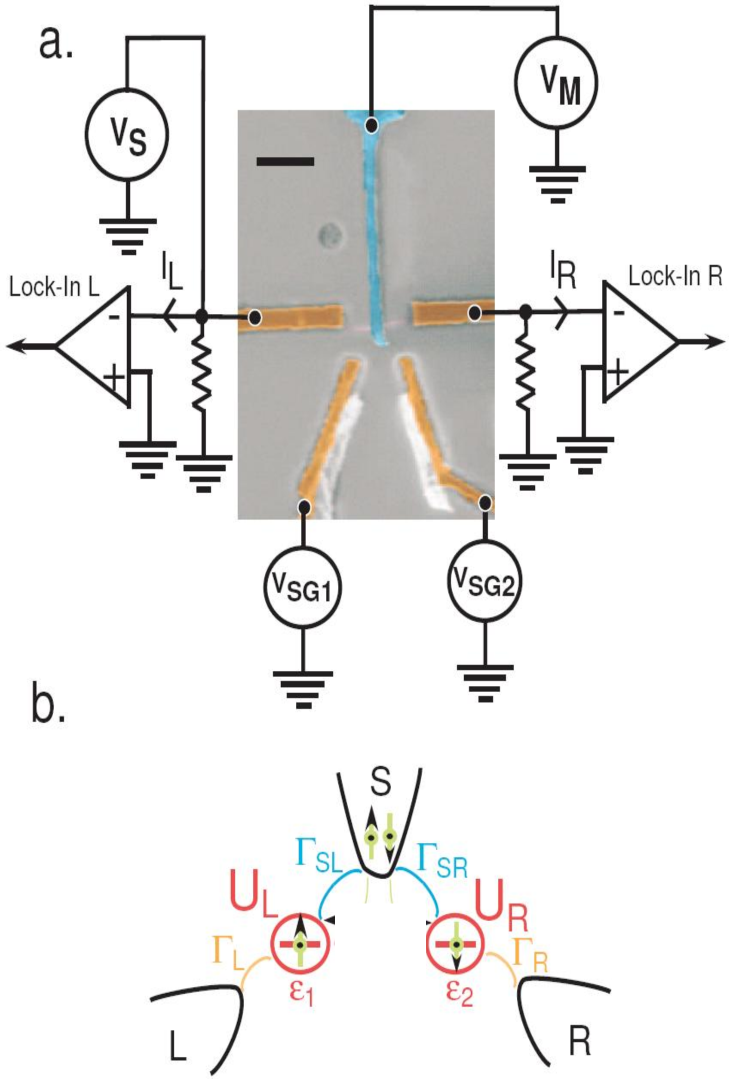


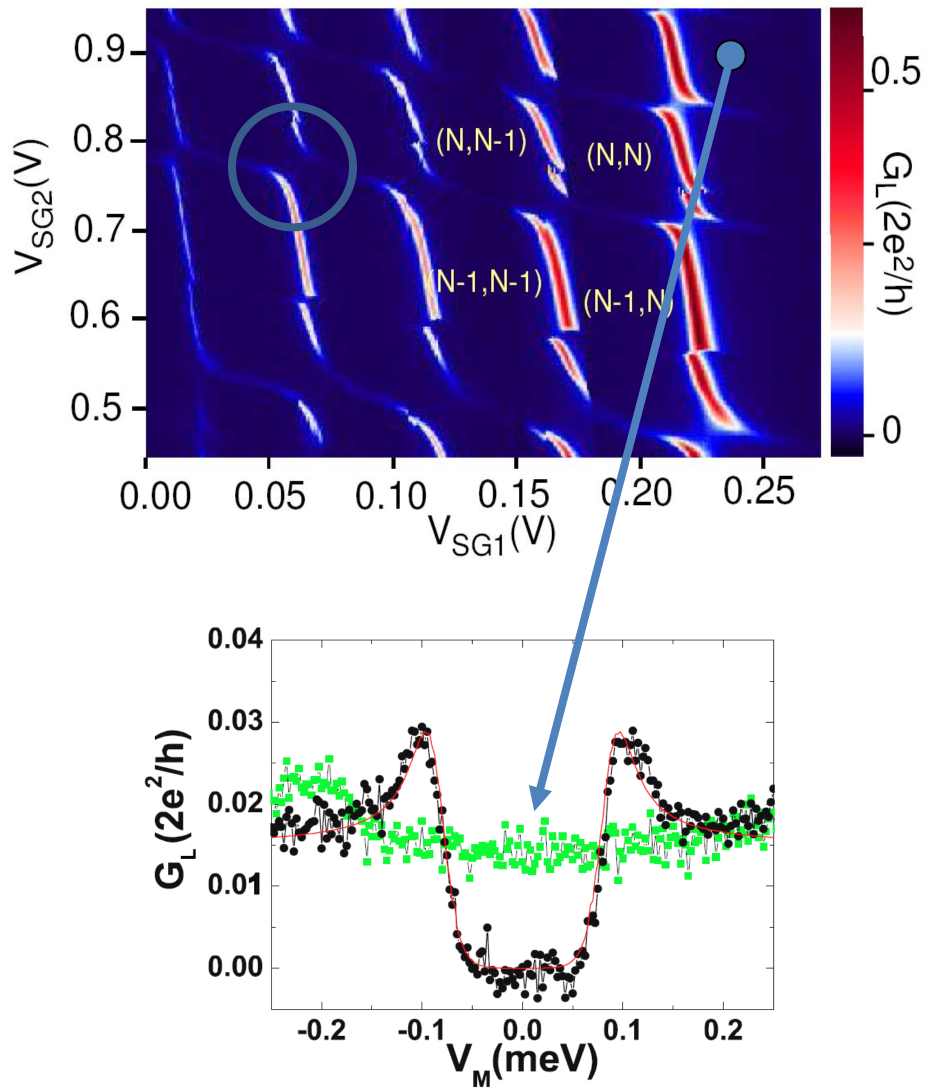
$$\frac{1}{\varepsilon} = \frac{1}{\pi\Delta} + \frac{1}{U}$$

$$\Gamma = \Gamma_L + \Gamma_R$$

See also, C. Bena et al. Phys. Rev. Lett. 89, 037901 (2002)

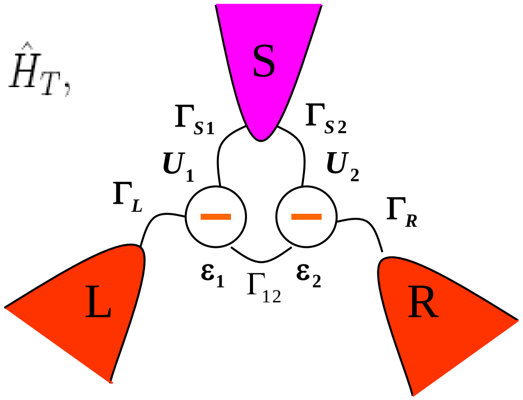
DQD based on CNT: tuning of double dot resonances





Phenomenological model and approximations

$$\hat{H} = \hat{H}_L + \hat{H}_R + \hat{H}_S + \sum_{i=1,2;\sigma} \epsilon_i \hat{n}_{i,\sigma} + \sum_{i=1,2} U_i \hat{n}_{i,\uparrow}\hat{n}_{i,\downarrow} + \sum_{\sigma} \Gamma_{12} \hat{d}^{\dagger}_{1,\sigma}\hat{d}_{2,\sigma} + \text{h.c.} + \hat{H}_T,$$



Retarded GF in 4x4 Nambu+DQD space

$$\hat{G}^r(\omega) = \left[\omega - \hat{h} - \hat{\Sigma}_U(\omega) - \hat{\Gamma}_N - \hat{\Gamma}_S\right]^{-1}$$

$$\hat{h} = \begin{pmatrix} \epsilon_1 & \Gamma_{12} \\ \Gamma_{21} & \epsilon_2 \end{pmatrix} \sigma_z \qquad \hat{\Gamma}_N = \begin{pmatrix} i\Gamma_L & 0 \\ 0 & i\Gamma_R \end{pmatrix} \sigma_0$$

$$\hat{\Gamma}_S = \begin{pmatrix} \Gamma_{SL} & 0 \\ 0 & \Gamma_{SR} \end{pmatrix} (g(\omega)\sigma_0 - f(\omega)\sigma_x),$$

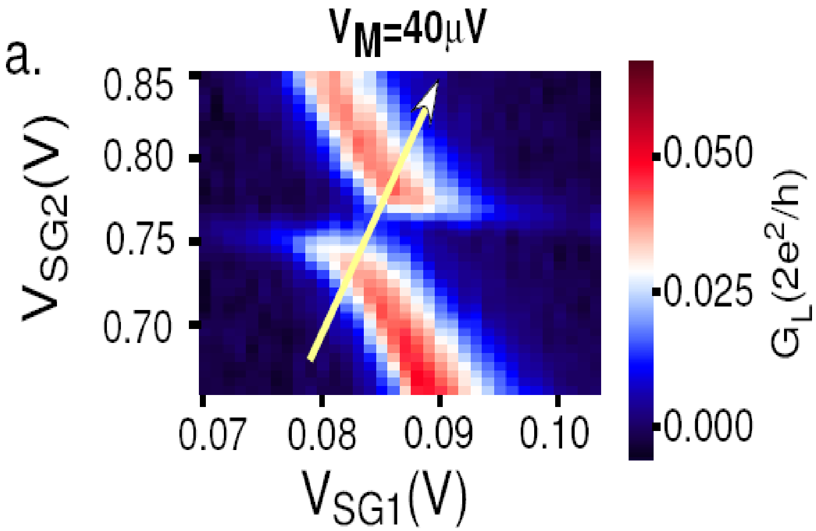
$$g(\omega) = \frac{-\omega}{\sqrt{\Delta^2 - \omega^2}} \qquad f(\omega) = \frac{\Delta}{\sqrt{\Delta^2 - \omega^2}}$$

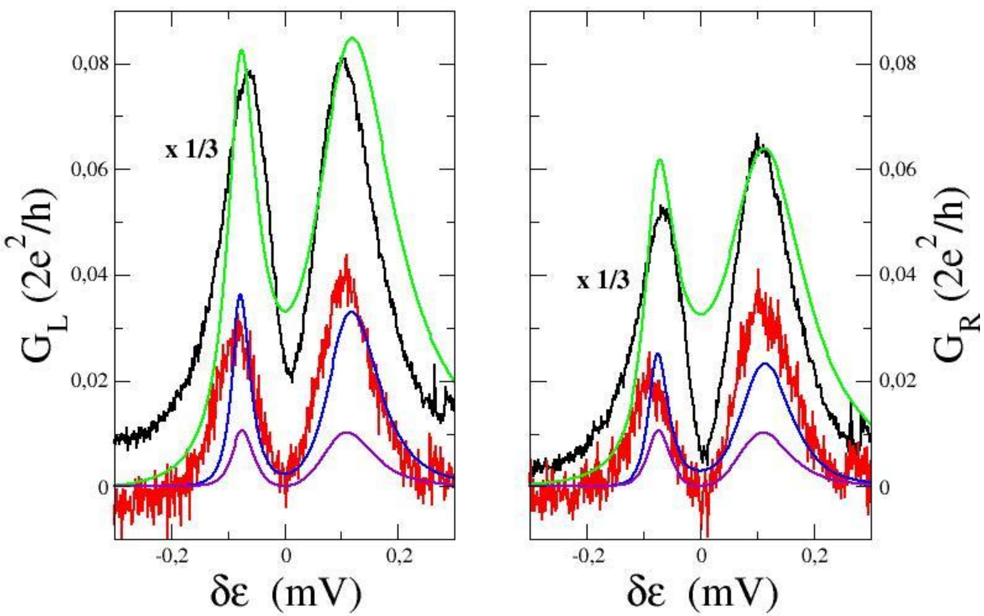
$\hat{\Sigma}_U(\omega)$ approximate self-energy, see Cuevas, ALY, Martín-Rodero PRB (2001)

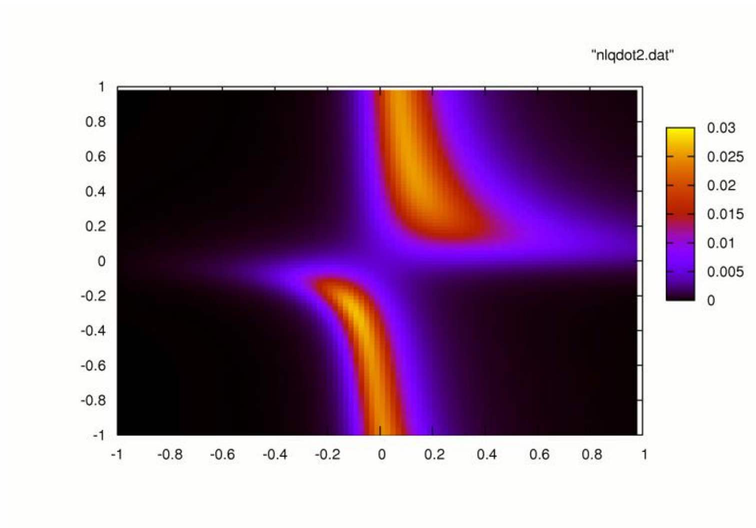
$$\epsilon_i \rightarrow \epsilon_i + U_i < n_i > \frac{\epsilon_i}{\epsilon_i + U_i(1- < n_i >)}$$

$$\Delta_i = U_i < d_{i,\uparrow} d_{i,\downarrow} >$$

Comparison between theory and exp. results







$$U_L \simeq 1.1meV \quad U_R \simeq 0.7meV \quad \Gamma_{12} \simeq 0.14meV$$

$$\Gamma_L \simeq 0.07meV \quad \Gamma_R \simeq 0.09meV \quad \Gamma_{LS,RS} \simeq 0.01meV$$

at resonance
$$\begin{cases} T_{CAR}/(R_R + T_{CAR}) & \simeq & 0.55 \\ T_{CAR}/(R_L + T_{CAR}) & \simeq & 0.35 \end{cases}$$

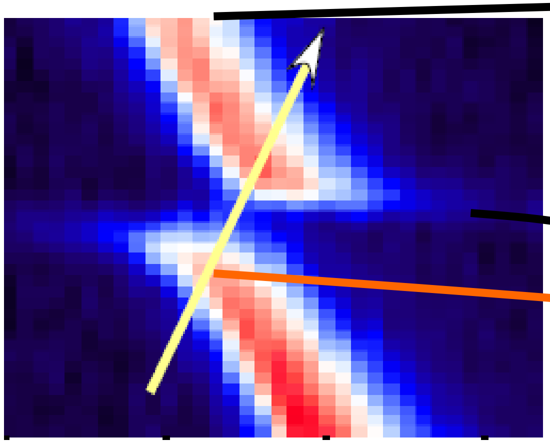
Asymmetry: theoretical prediction vs experimental results

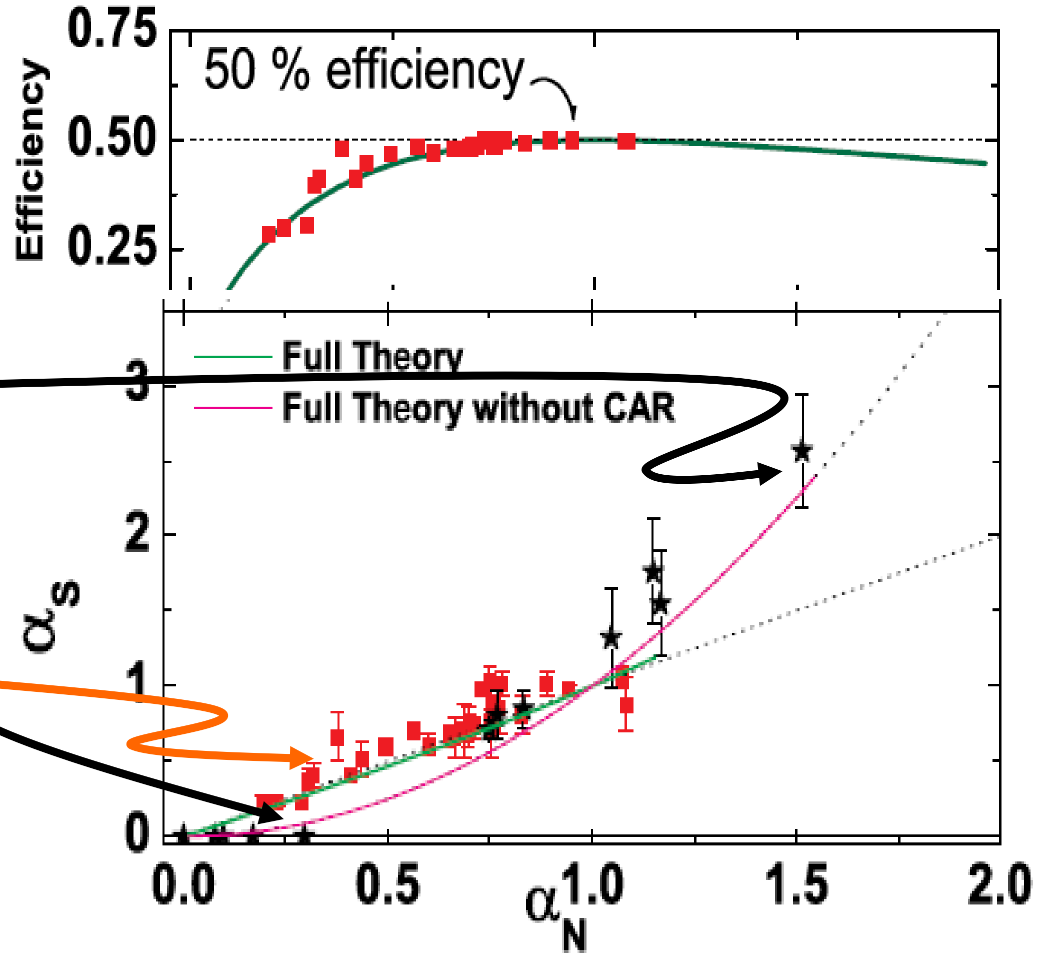
$$\alpha_{S,N} \equiv [G_L/G_R]_{S,N}$$

At resonance (max. CAR)

$$\alpha_S \approx \Gamma_L / \Gamma_R \approx \alpha_N$$

without CAR $\alpha_S \approx \alpha_N^2$





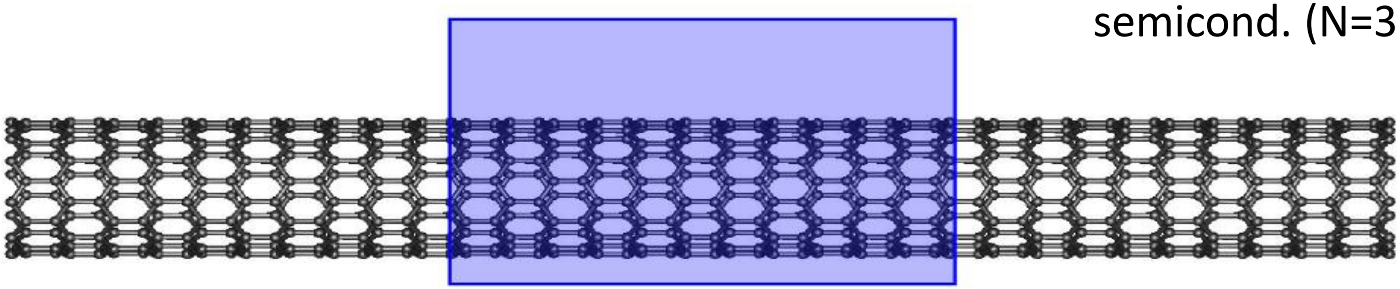
Define efficiency as

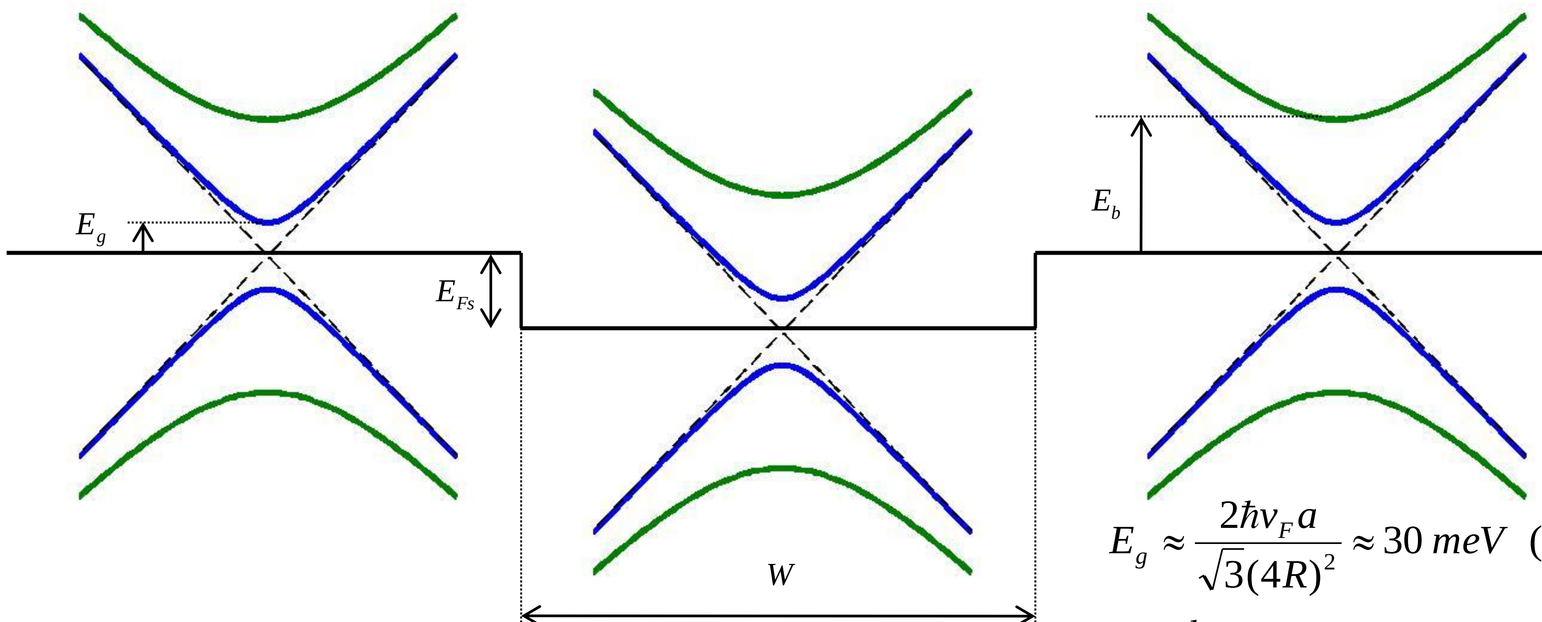
$$2T_{CAR}/(R_L + R_R + 2T_{CAR}) \approx 2/(2 + 1/\alpha_S + \alpha_S)$$

Microscopic analysis of CNT DQDs

P. Burset et al., Phys. Rev. B 84, 115448 (2011)

zig-zag CNT (N,0) metallic (N=3r)
semicond. (N=3r+1,3r-1)





$$E_g \approx \frac{2\hbar v_F a}{\sqrt{3}(4R)^2} \approx 30\ meV \quad (\mathrm{N}=9)$$

$$\frac{\hbar v_F}{3R} \approx 480\ meV \quad (\mathrm{N}=10)$$

$$E_b \approx \frac{\hbar v_F}{R} \approx 1.2\ eV \qquad R \approx 0.4\ nm$$

$$E_{Fs} \approx -0.5\ eV\ (\text{n-doping})$$

Al/Pd leads ab-initio
PRL 104, 076807 (10)

Effect of Spin Orbit Interaction

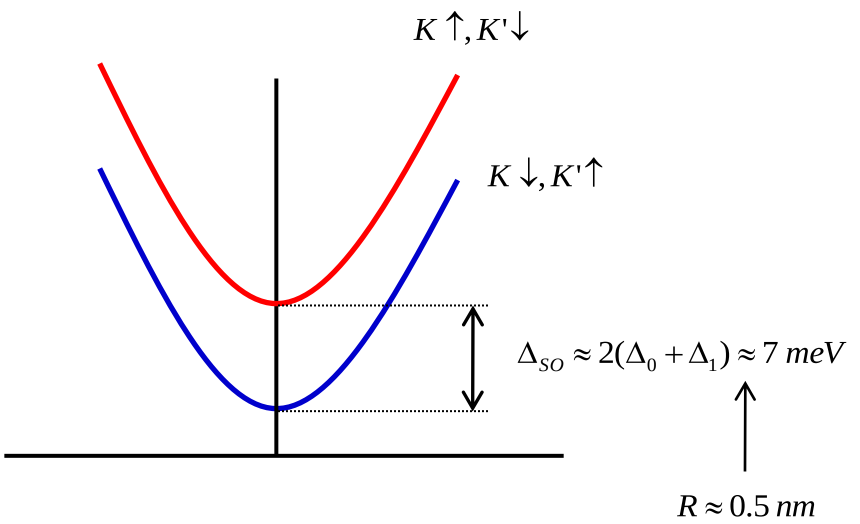
$$H_{SO} = \Delta_1 \tau_3 \sigma_1 s_3 + \Delta_0 s_3 \tau_3,$$

$\sigma_j \rightarrow$ sublattice

$\tau_j \rightarrow$ valley

$s_j \rightarrow$ spin

$$\Delta_0, \Delta_1 \propto \frac{1}{R}$$

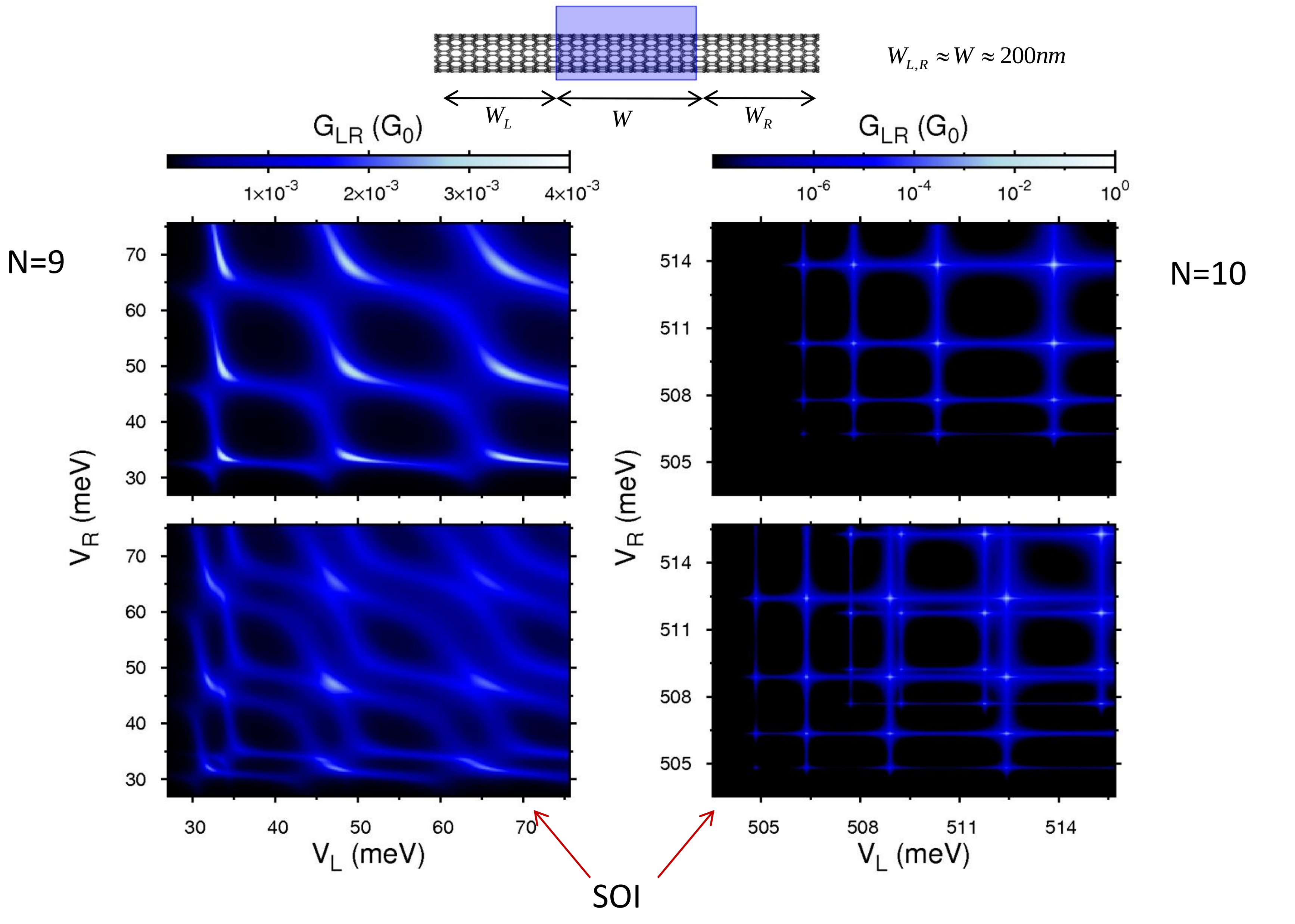


$$\Delta_0 = \frac{\delta_K}{R} = \frac{\lambda_{so} a(\varepsilon_s - \varepsilon_p)(V_{pp}^{\pi} + V_{pp}^{\sigma})}{12\sqrt{3} {V_{sp}^{\sigma}}^2} \frac{e^{-i\theta}}{R}$$

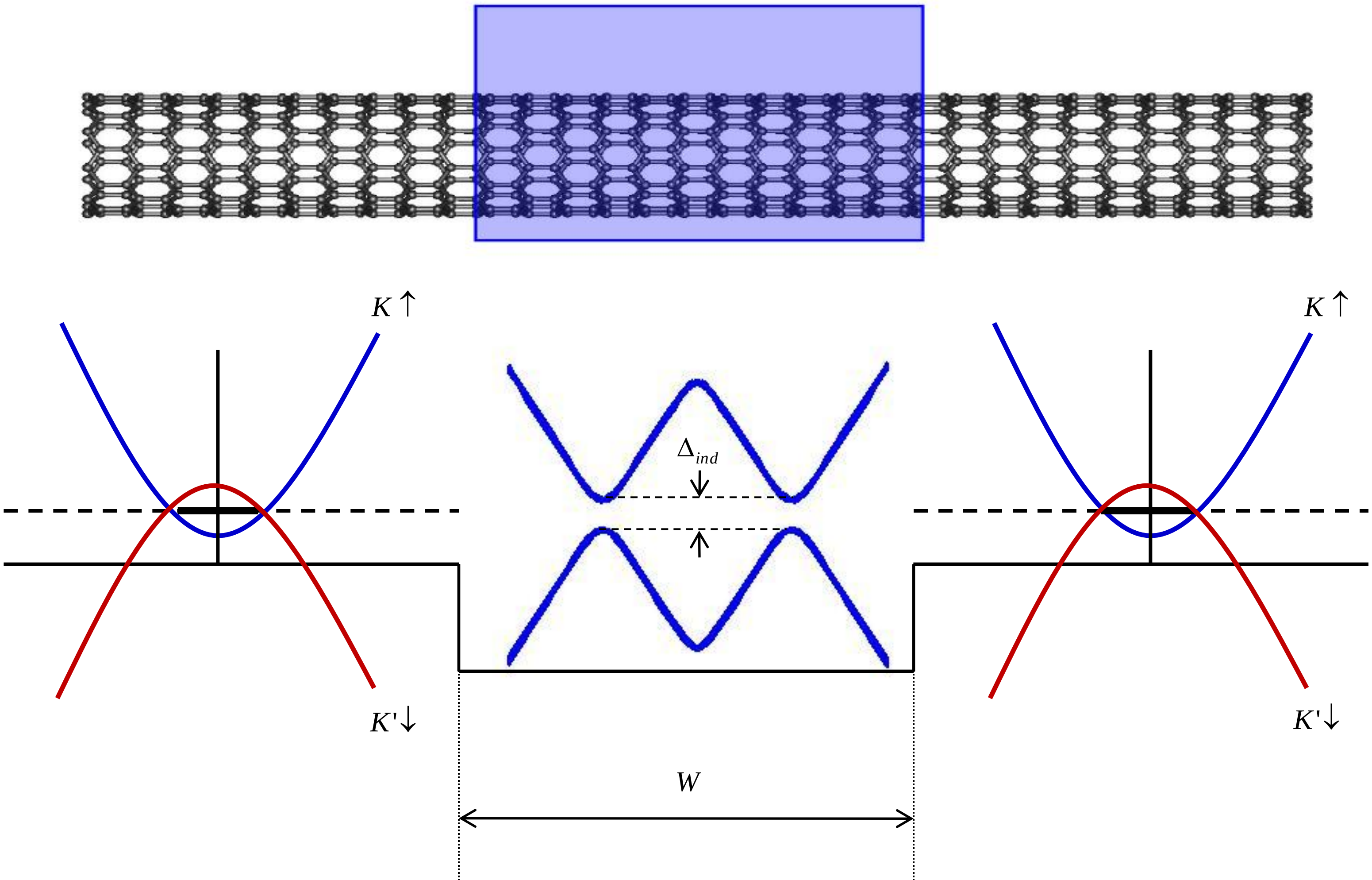
$$\Delta_1 = \frac{\delta'_K}{R} = \frac{\lambda_{so} a V_{pp}^{\pi}}{2\sqrt{3}(V_{pp}^{\sigma} - V_{pp}^{\pi})} \frac{\cos 3\theta}{R},$$

eg. Jeong et al PRB (09)

Normal conductance maps p-n-p' region



Superconducting state

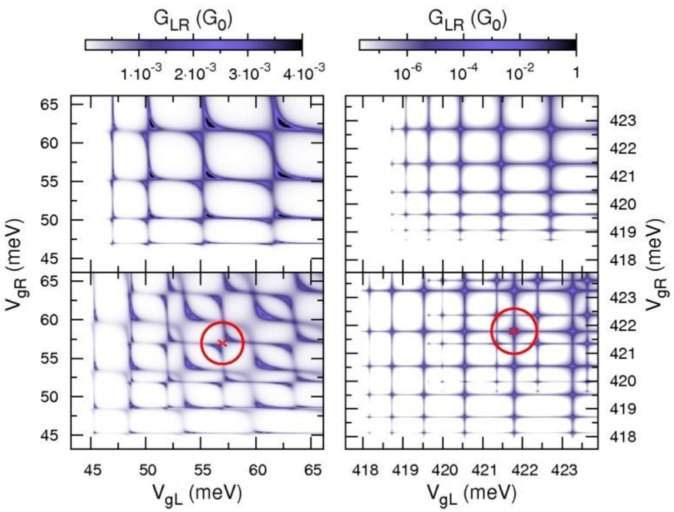


$\Delta_{ind} \approx \Gamma \approx 1\, meV$

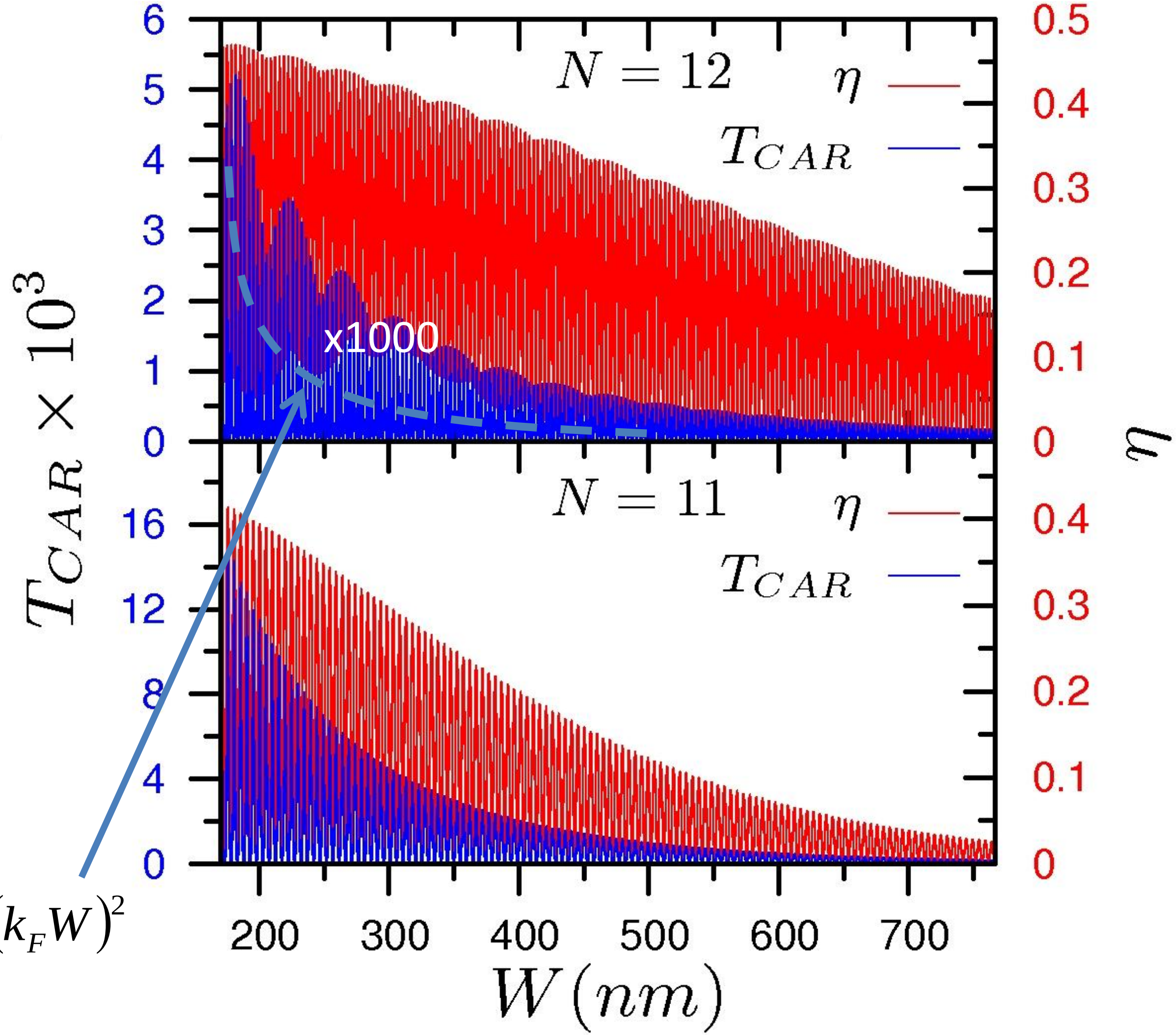
Al/Pd leads ab-initio
PRL 104, 076807 (10)

$\xi_{ind} \approx \frac{\hbar v_F}{\Gamma} \approx 300\, nm$

Efficiency: evolution with length W



$$\eta = \frac{2T_{CAR}}{(2T_{CAR} + R_{AL} + R_{AR})}$$



Overall decay $\approx e^{-2W/\xi}$

Fast oscillation $\lambda_F = \frac{hv_F}{E_{Fs}}$

Bulk BCS $T_{CAR} \propto \mathrm{e}^{-2W/\xi_{ind}} / (k_F W)^2$

Coupling self-energy from microscopic model

$$\hat{\Sigma}_{ab} \approx \frac{hv_F}{\sqrt{W_a W_b}} \sigma_z \left(\frac{F^1_{ab}}{\Omega} \left(E + \sigma_x \Delta_i\right) + \sigma_z F^2_{ab} \right) \sigma_z$$

$$\Omega = \sqrt{E^2 - \Delta_i^2}$$

$$a, b \equiv L, R$$

$\sigma_j \longrightarrow$ Nambu

$W >> \xi$ regime

$$\xi = \xi_0 \cos\alpha / \sqrt{1 - E^2/\Delta_i^2}$$

$$F^1_{LR} = -2ie^{-W/\xi} \cos\alpha \sin\left(k_0 W + \alpha\right)$$

$$F^2_{LR} = 2e^{-W/\xi} \cos\alpha \cos\left(k_0 W + \alpha\right)$$

$$k_0 = \sqrt{k_{Fs}^2 - q^2} = k_{Fs} \cos\alpha$$

$$q = q_0 \pm \Delta_1 / \hbar v_F$$

$$q_0 \approx \frac{2\pi}{9Na}$$

$$F^1_{LL} = F^1_{RR} = i \sin\alpha$$

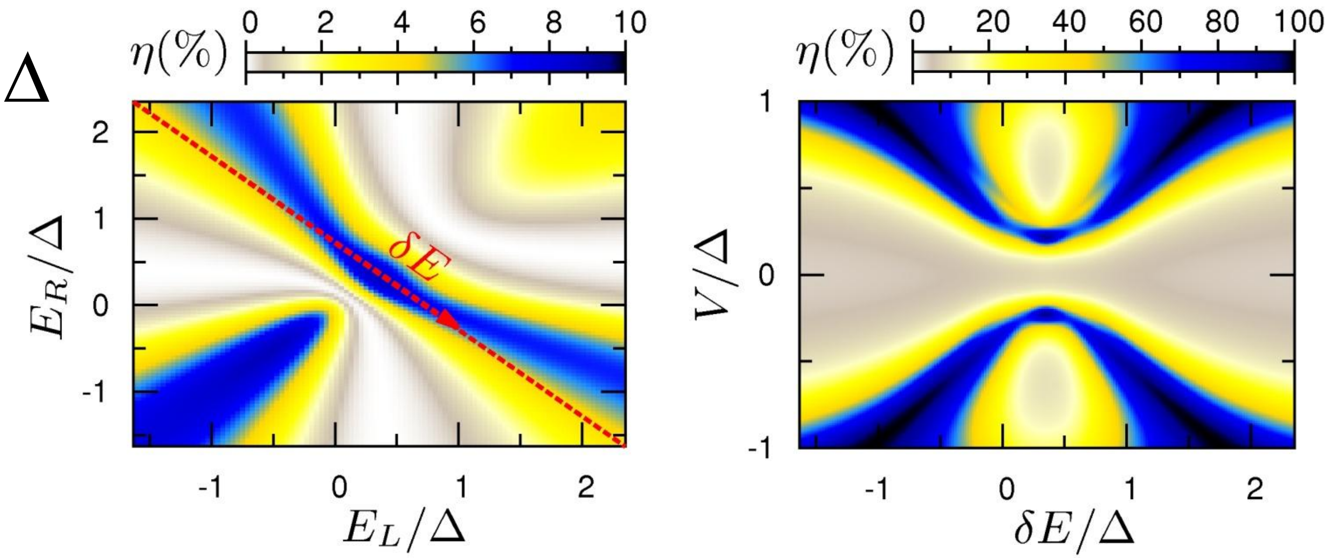
$$F^2_{LL} = F^2_{RR} = \cos\alpha$$

Minimal model: efficiency in the linear and non-linear regimes

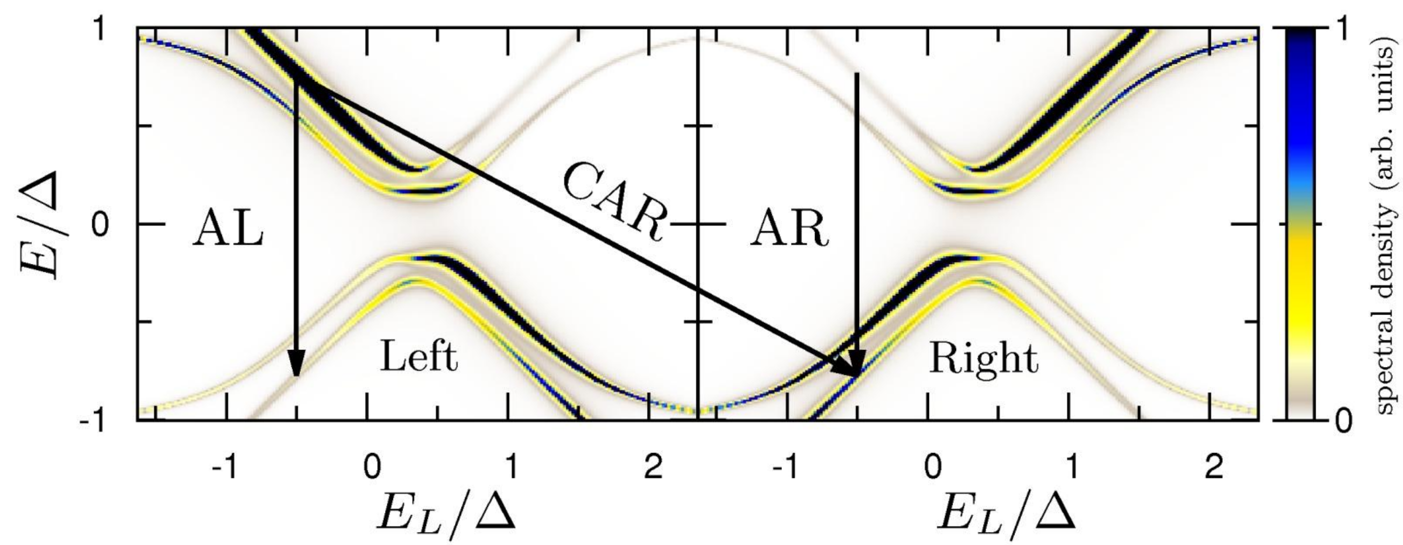
$W \approx 3\xi$

$\Gamma_{L,R} << \Delta$

linear



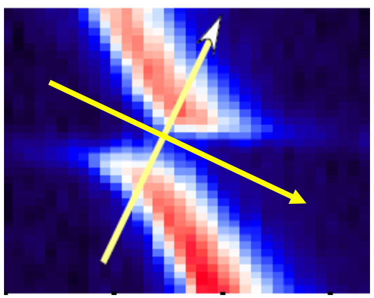
Non-linear

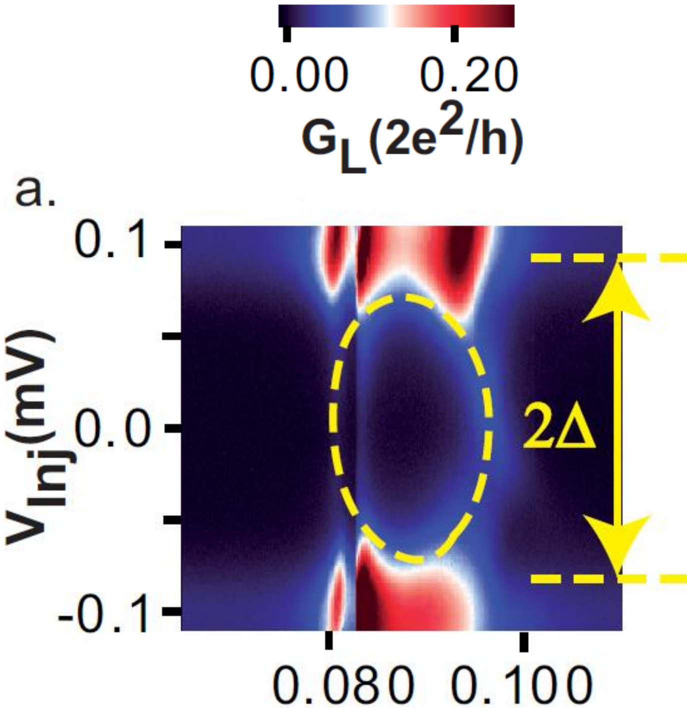


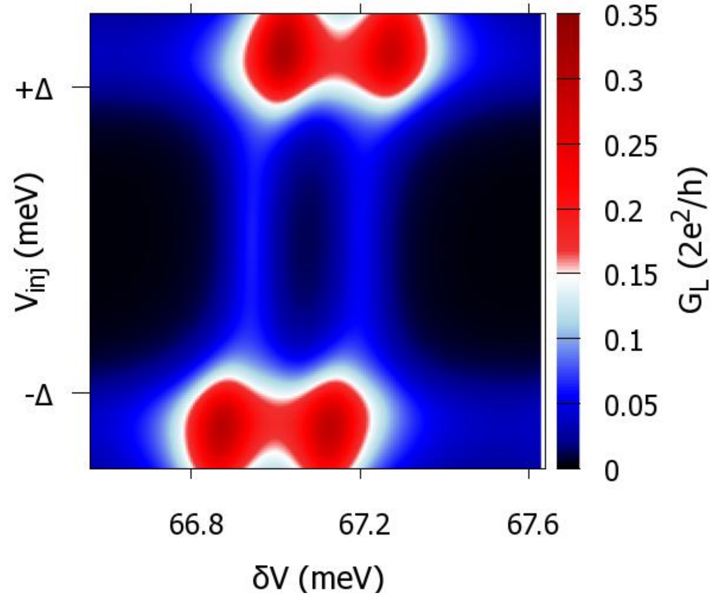
$W < \xi$

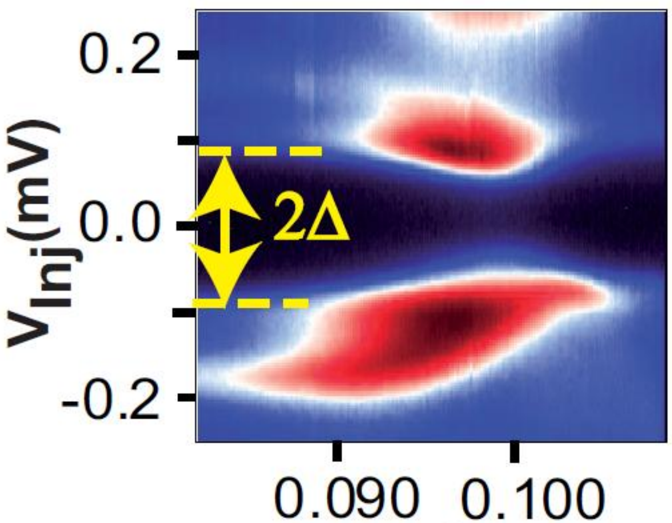
$\Gamma_{L,R} \approx \Delta$

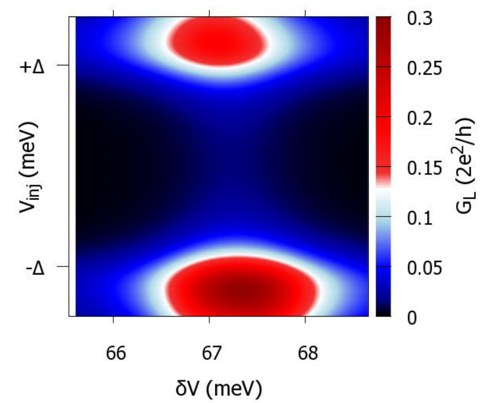
L. Hermmann , P Burset et al.
In preparation







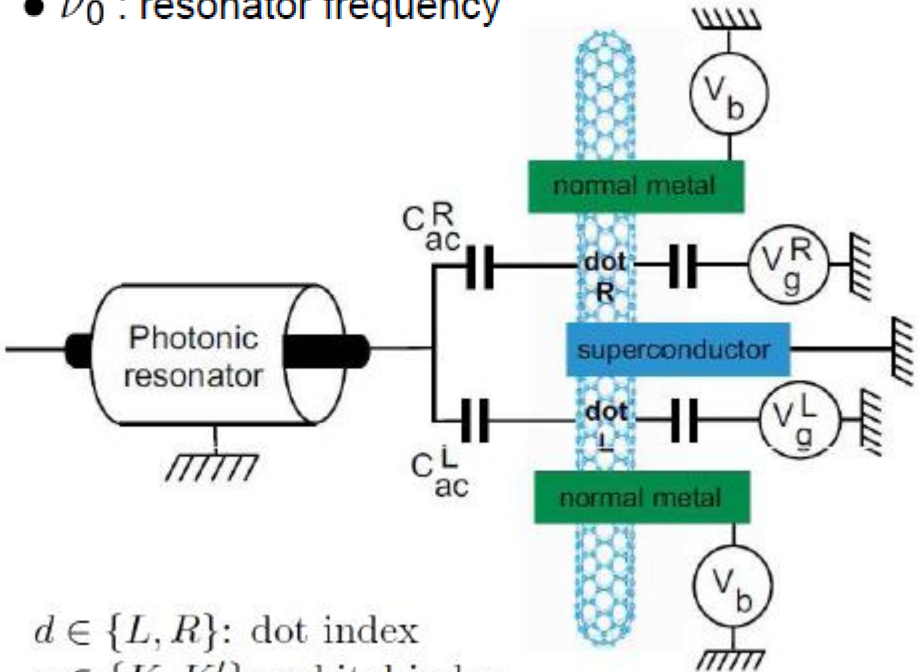




Probing coherence: splitter in a photonic cavity

A. Cottet et al.
In preparation

- ν_0 : resonator frequency



$d \in \{L, R\}$: dot index
$\tau \in \{K, K'\}$: orbital index
$\sigma \in \{\uparrow, \downarrow\}$: spin index

Spin/photon coupling due to SOI

$$\hat{H}_c = \sum_{d,\tau} \lambda_d^\tau (a + a^\dagger) d_{d\tau\sigma}^\dagger d_{d\tau\overline{\sigma}}$$

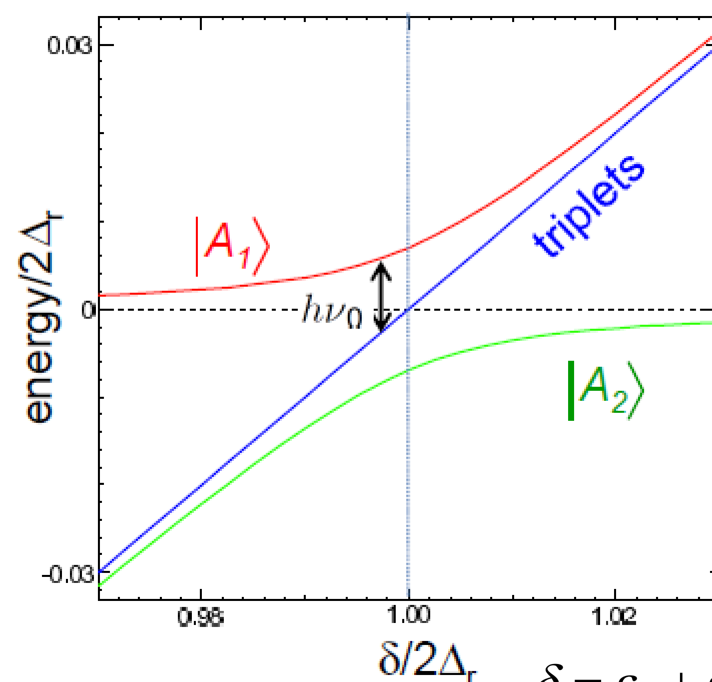
$$\lambda_d^\tau \approx 1MHz$$

- S=0 states

$$|A_i\rangle = \mu_i |0,0\rangle + v_i |\text{singlet}\rangle$$

- Triplet states: $|T_0\rangle$, $|T_+\rangle$, $|T_-\rangle$

$h\nu_0 > \sqrt{2} t_{eh}$

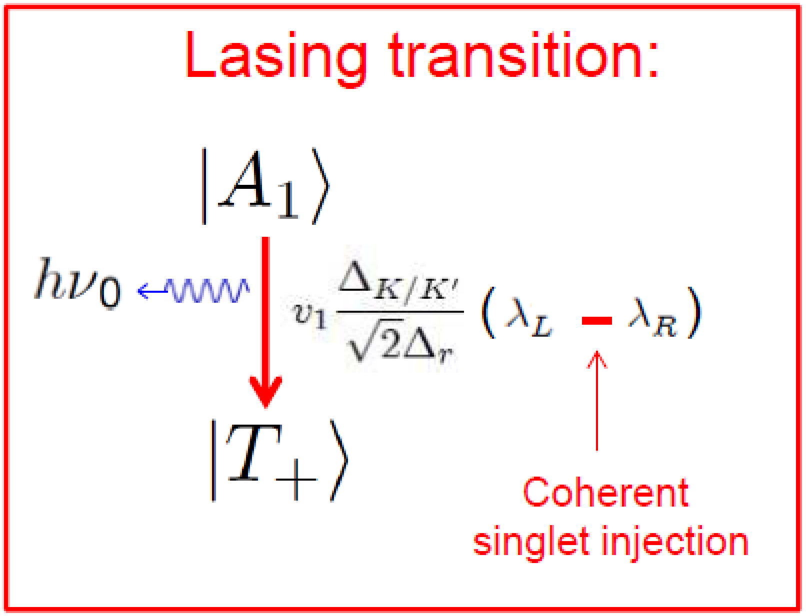


$$t_{eh} \approx \Sigma_{LR,12} \approx 10 \mu eV$$

$$\delta = \varepsilon_L + \varepsilon_R$$

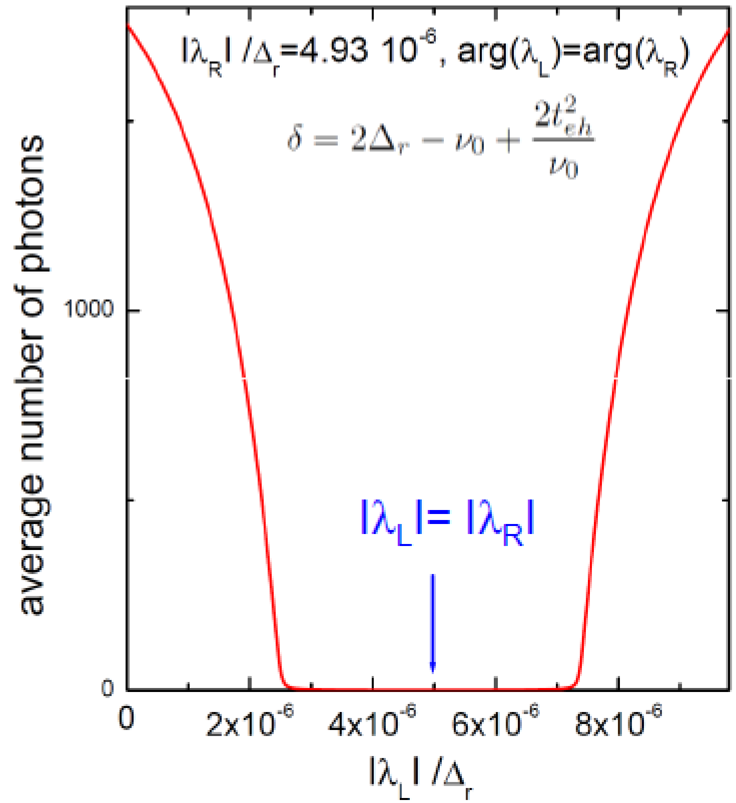
$$\Delta_r = \sqrt{\Delta_{so}^2 + \Delta_{K/K'}^2}$$

Subradiance of splitted Cooper pairs



$\lambda_L = \lambda_L^K + \lambda_L^{K\prime}$

$\lambda_R = \lambda_R^K + \lambda_R^{K'}$



Non-monotonic behavior of the average number of photons with $|\lambda_L|$

$\longrightarrow$ proof of coherent singlet injection

Conclusions

** Phenomenological model:*
evidence of CAR from conductance asymmetry
Maximum efficiency at resonance ~50%

** Microscopic analysis:*
Weak decay of CAR correlations with length of SC region
Efficiency can reach ~100% in non-linear regime

** Understanding of non-linear regime for the exp.*
results of Herrmann et al. (evidence of ABS formation)

**Theoretical proposal for detection of coherence from photon emision*

Future work:
Understanding non-linear properties of InAs splitter (Basel),
Other devices (graphene, topological superconductors, etc)