



école normale  
supérieure de Lyon

**ENS**<sub>Lyon</sub>

# Quantum tomography of single electron excitations in quantum Hall edge channels

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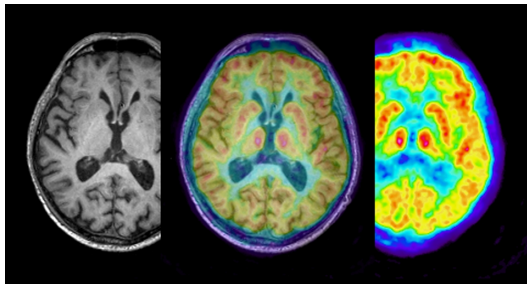
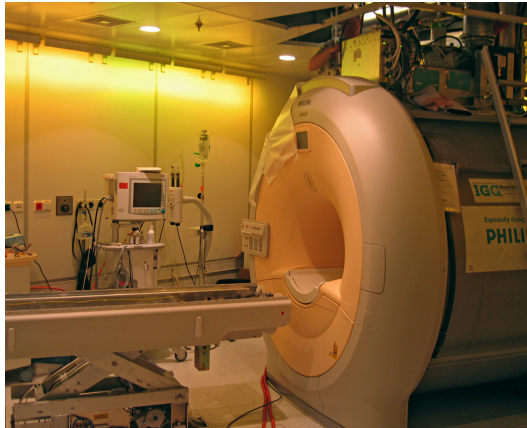
C. Altimiras



- Introduction and motivation
- Quantum tomography...
- and its applications
- Conclusion

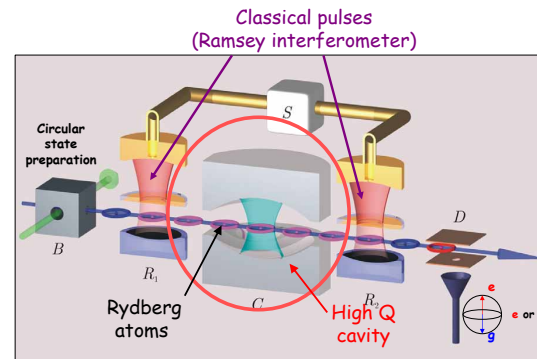
# What is quantum tomography ?

## Classical tomography

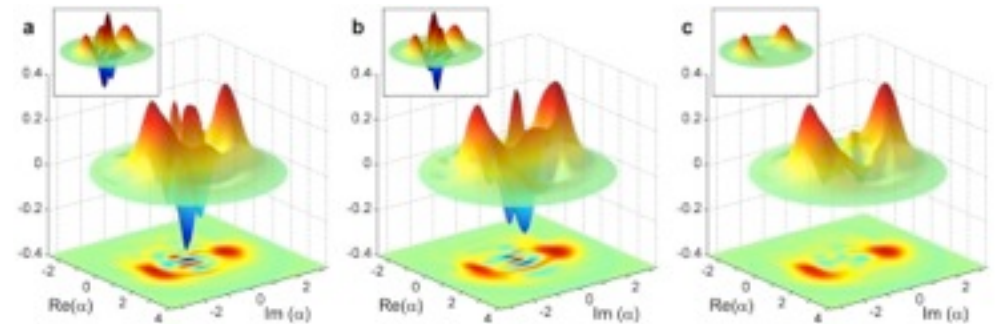


Density reconstruction from many measurements on a single system.

## Quantum tomography

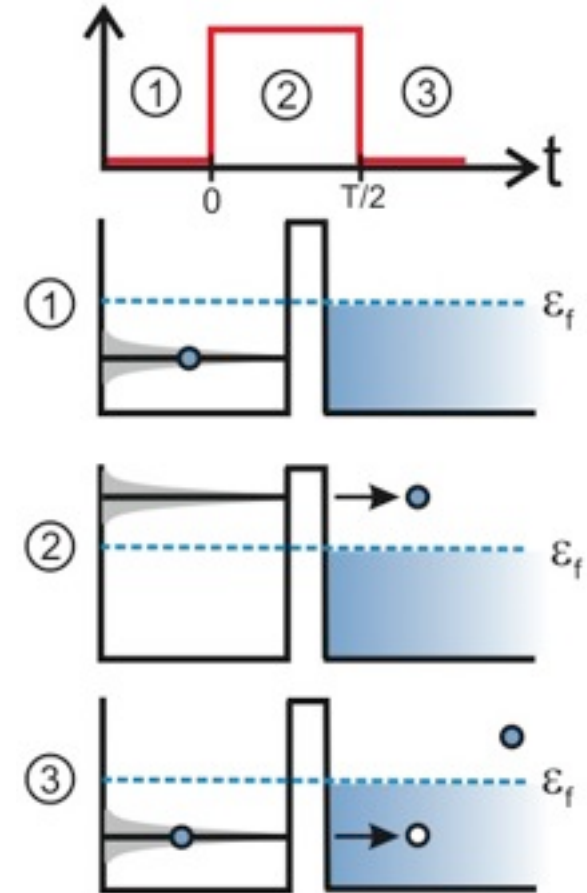
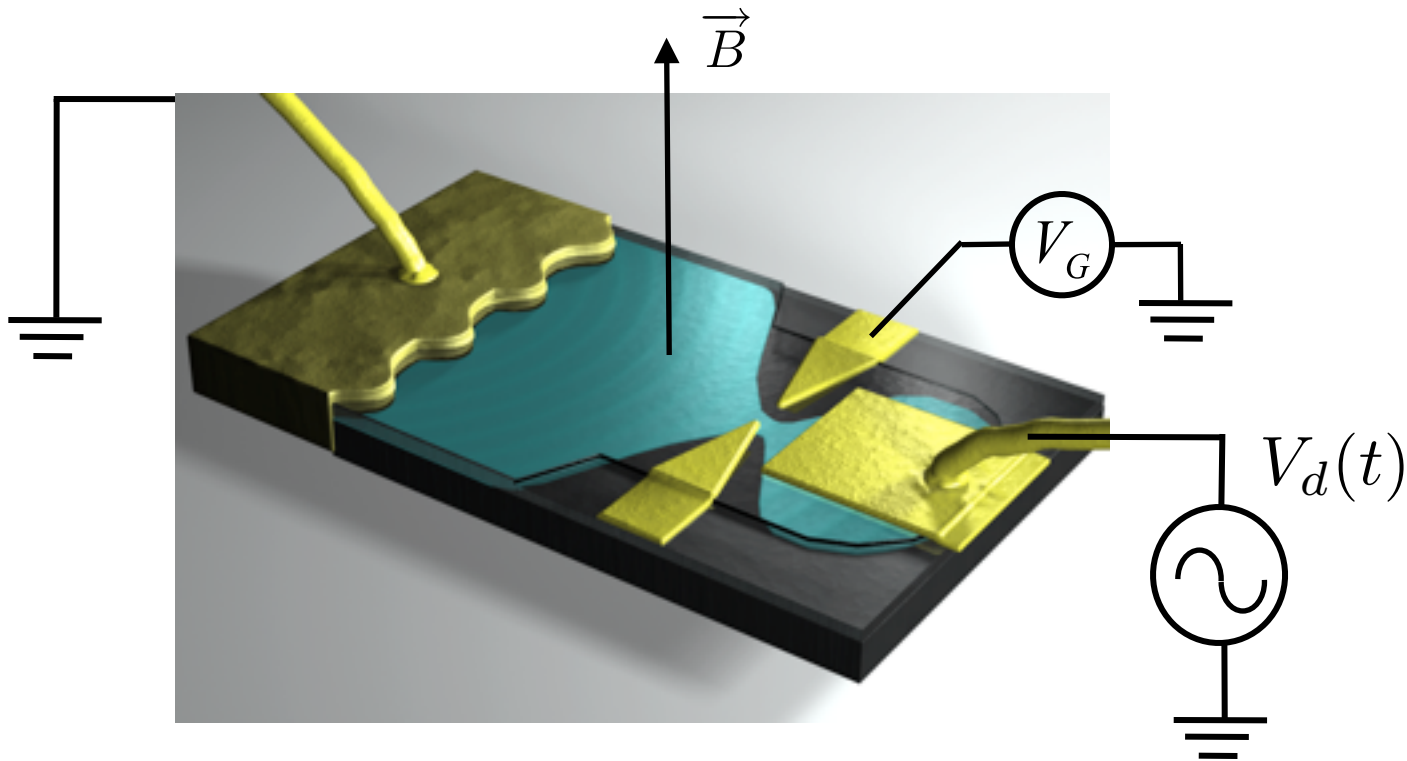


High Q cavity



It requires many realizations of the state (*no cloning theorem*).

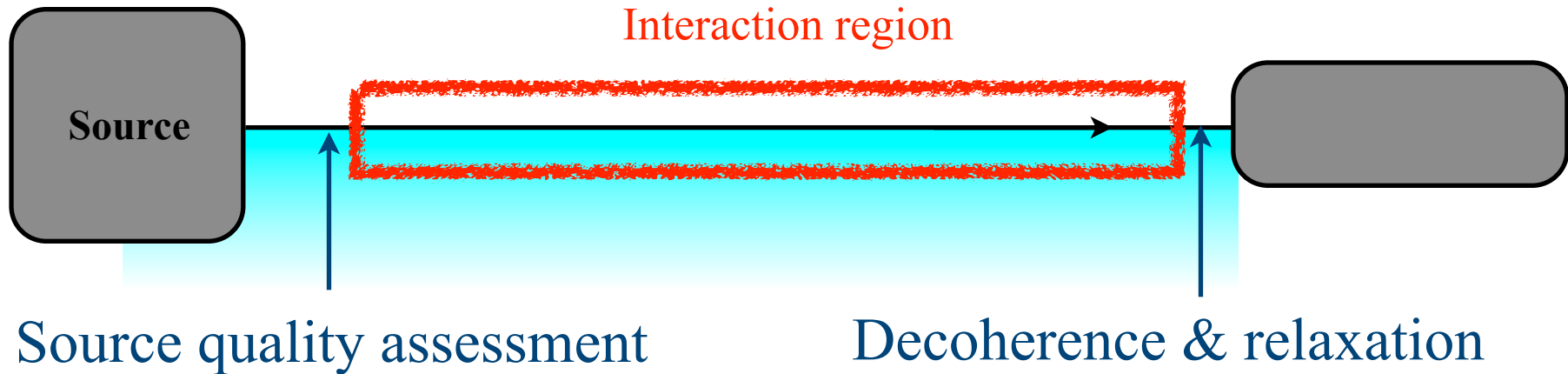
# The single electron source



Emits single electron excitations!

G. Fève *et al*, Science **316**, 1169 (2007)  
A. Mahé *et al*, Phys. Rev. B **82**, 201309 (2010)

See M. Büttiker's talk and M. Albert *et al*,  
Phys. Rev. B **82**, 041407 (2010) and Phys. Rev. Lett. **107**, 086805 (2011)



**Statistical properties:** average current, noise, full counting statistics

*See Ch. Glattli and M. Büttiker talks*

**Coherence properties:** can we access single electron wave packets ?

Single electron coherence:  $\mathcal{G}_\rho^{(e)}(x, \tau | x', \tau') = \text{Tr}(\psi(x, \tau) \rho \psi^\dagger(x', \tau'))$

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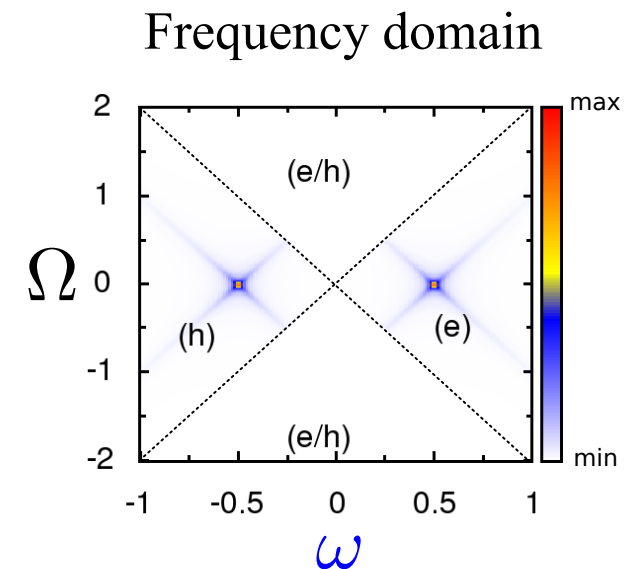
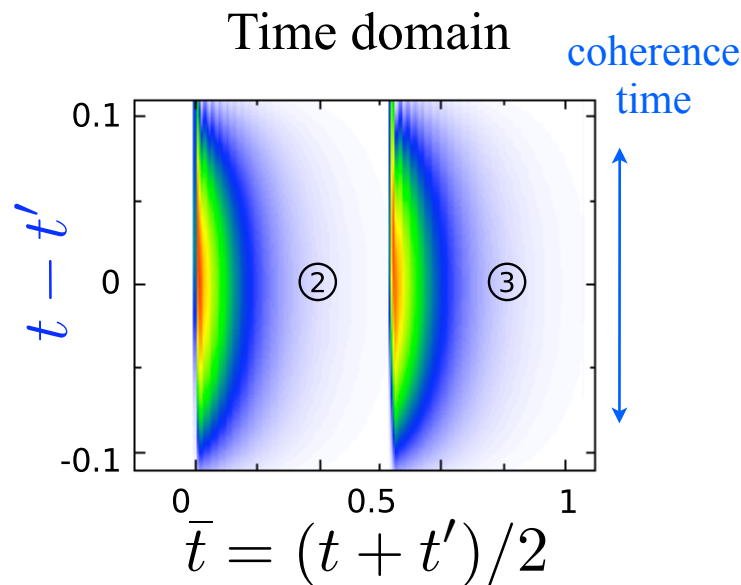
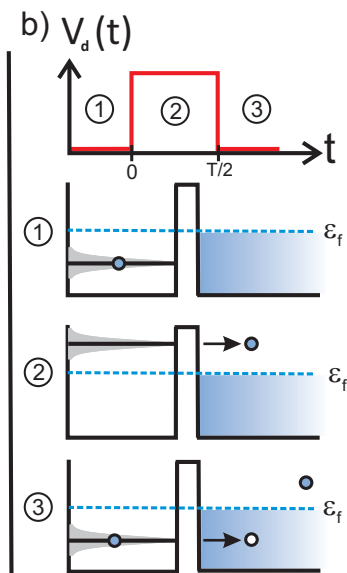
Electronic analogue of Glauber's correlators

$$\mathcal{G}_\rho^{(1)}(x, t | x' t') = \text{Tr}(E^+(x, t) \rho E^-(x' t'))$$

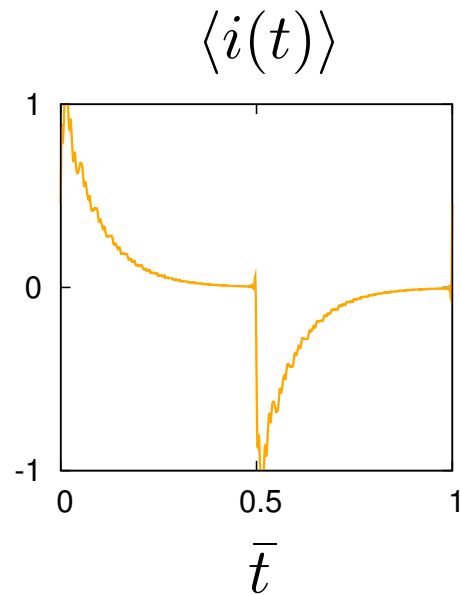
The Quantum Theory of Optical Coherence\*

ROY J. GLAUBER  
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts  
(Received 11 February 1963)

Phys. Rev. **130**, 2529 (1963)

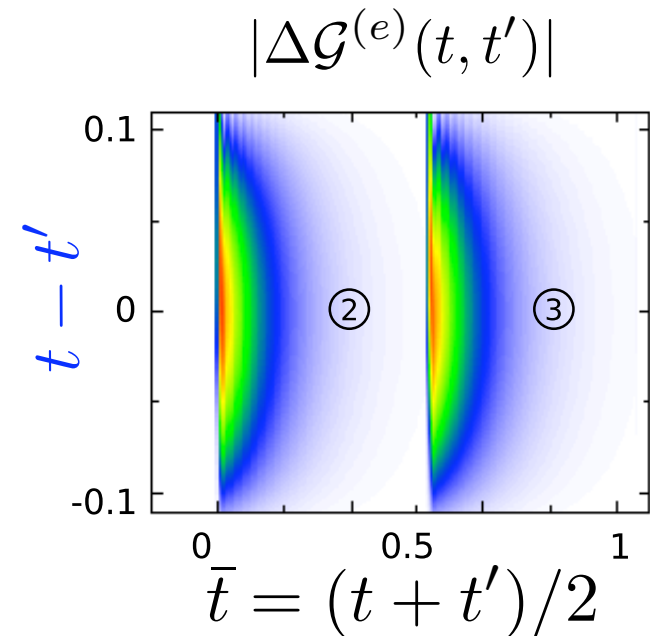


## Classical signal



Measured directly

## Quantum signal



How to measure it ?

Ch. Grenier *et al*, New Journal of Physics **13**, 093007 (2011)

- Introduction and motivation
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Single electron coherence:  $\mathcal{G}_\rho^{(e)}(x, \tau | x', \tau') = \text{Tr}(\psi(x, \tau) \rho \psi^\dagger(x', \tau'))$

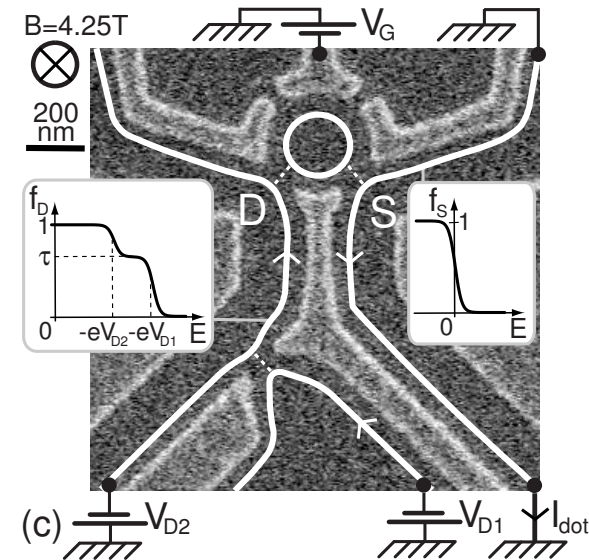
*First idea:* spectroscopy experiment

$$\bar{I} = e \int (n_S - n_D)(\omega) g_d(\omega) \frac{d\omega}{2\pi}$$

$$n_D(\omega) = \int \overline{\mathcal{G}^{(e)}(x, t | x, t')}^{\bar{t}} e^{i\omega(t-t')} d(t-t')$$

$$\bar{t} = (t + t')/2$$

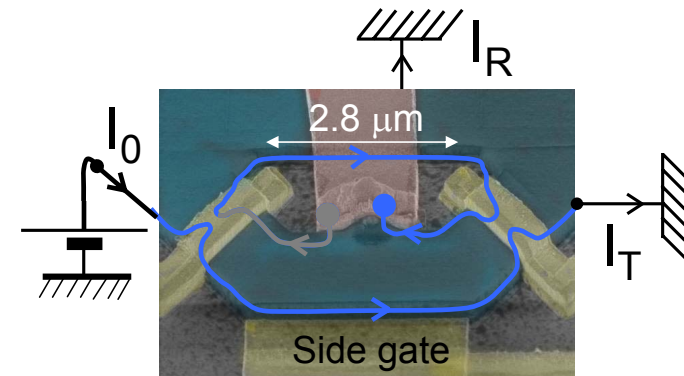
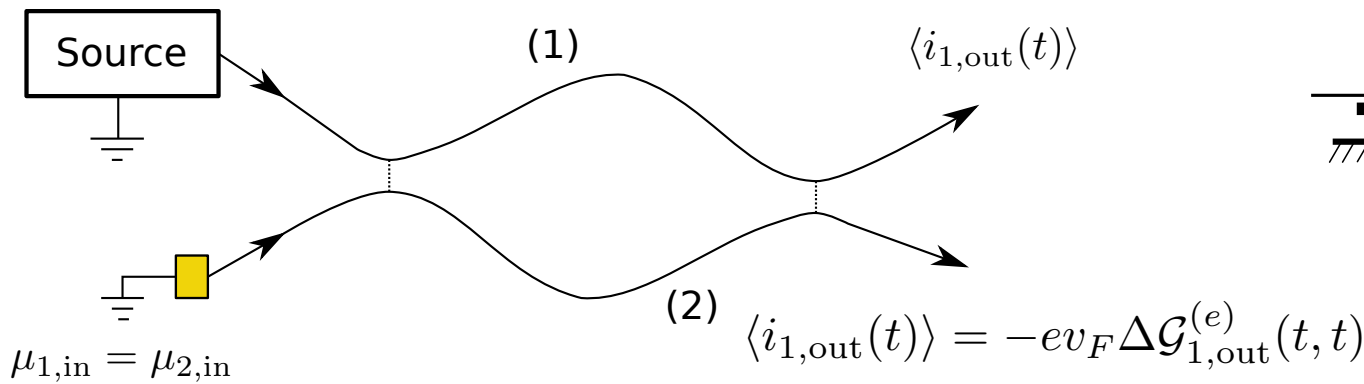
C. Altimiras *et al*, Nature Physics **6**, 34 (2010)



Only captures the stationary part: well suited to stationary sources...

Used to study relaxation of a non equilibrium energy distribution: [Phys. Rev. B \*\*81\*\*, 121302\(R\) \(2010\)](#)  
[Phys. Rev. Lett. \*\*105\*\*, 056803 \(2010\)](#)  
[Phys. Rev. Lett. \*\*105\*\*, 226804 \(2010\)](#)

## Second idea: Mach Zehnder interferometry



Courtesy P. Roche

$$\Delta \mathcal{G}_{1,out}^{(e)}(t, t') = \sum_{\alpha, \beta} \mathcal{A}_{\alpha, \beta} \Delta \mathcal{G}_{1,in}^{(e)}(t - \tau_{\alpha}, t' - \tau_{\beta})$$

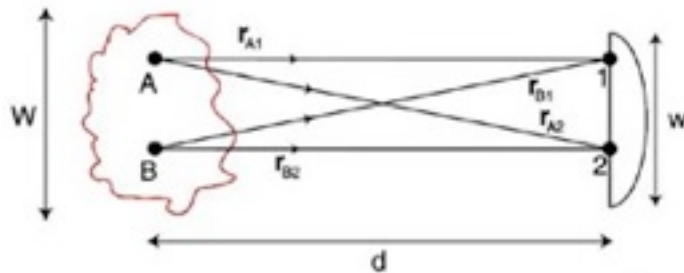
→ Classical contributions  $\alpha = \beta$   
→ Quantum contributions  $\alpha \neq \beta$

Quantum contributions to  $\langle i_{1,out}^{(dc)} \rangle \simeq \overbrace{\mathcal{A}_{12} \Delta \mathcal{G}_{1,in}^{(e)}(t - \tau_1, t - \tau_2)}^t + \overbrace{\mathcal{A}_{21} \Delta \mathcal{G}_{1,in}^{(e)}(t - \tau_2, t - \tau_1)}^t$

G. Haack *et al*, Phys. Rev. B **84**, 081303(R) (2011)

Relies on a simple measurement but decoherence within the MZI may be a problem!

*Our idea:* HBT interferometry

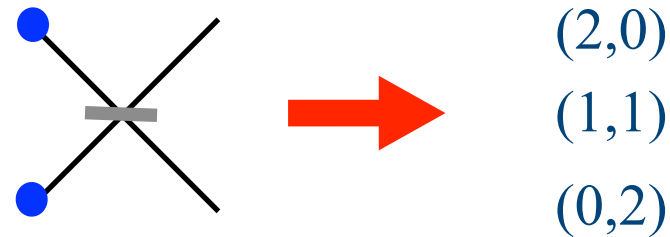


$$P = |\psi_A e^{i\phi_{A1}} \psi_B e^{i\phi_{B2}} \pm \psi_A e^{i\phi_{A2}} \psi_B e^{i\phi_{B1}}|^2$$

Nature **178**, 1046 (1956)

Two particle interference interpretation

U. Fano, Am. J. Phys. **29**, 539 (1961)



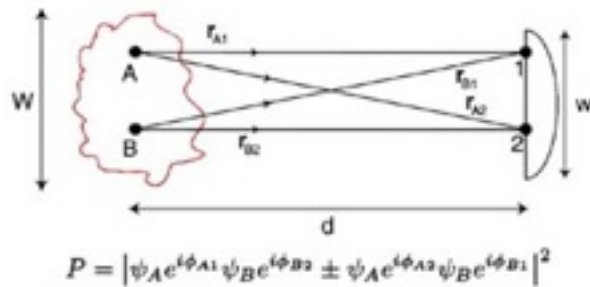
	Classical	Bosons	Fermions
	(0,2) (1,1) (2,0)	(0,2) (2,0)	(1,1)

HBT effect with electrons:

Liu *et al*, Nature **391**, 263 (1998)  
 Henny *et al*, Science **284**, 396 (1999)  
 Oliver *et al*, Science **284**, 299 (1999)

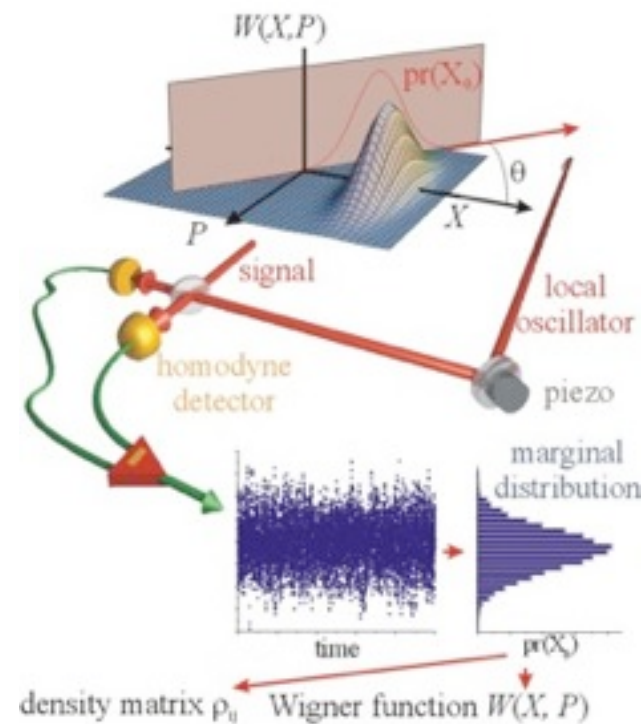
# Homodyne tomography

From stellar diameter measurements...



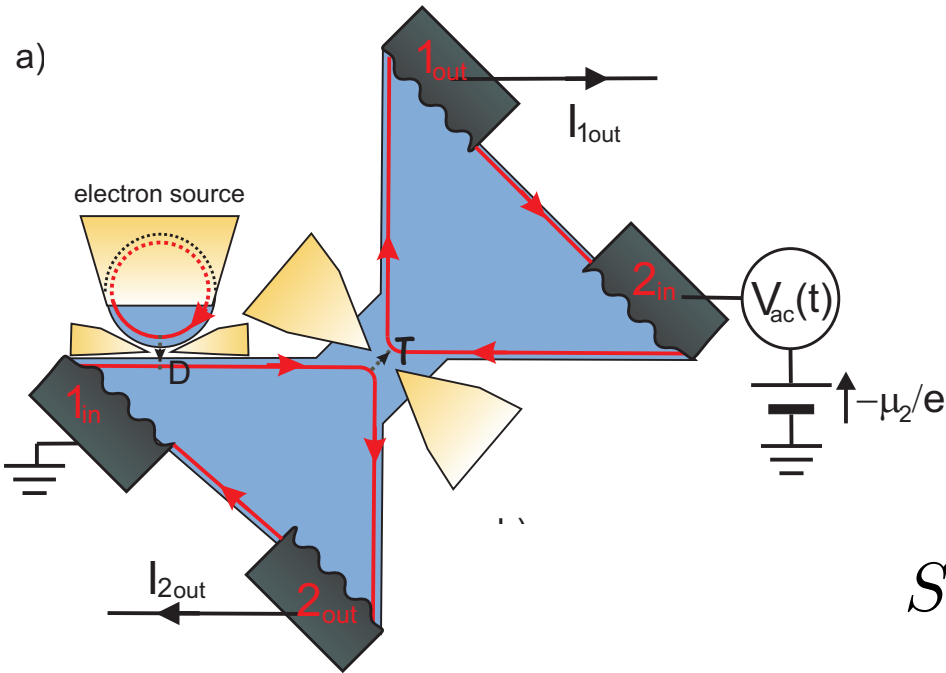
Nature **178**, 1046 (1956)

to quantum tomography in quantum optics!



Smithey *et al*, Phys. Rev. Lett. **70**, 1244 (1993)

Lvovsky & Raymer, Rev. Mod. Phys. **81**, 299 (2009)



Input: two sources

Beam splitter: QPC with

$$G = (e^2/h)\mathcal{T}$$

What can we measure ?

$$S_{\alpha\beta}^{\text{exp}} = 2 \int \overline{S_{\alpha,\beta}^{\text{out}}(\bar{t} + \tau/2, \bar{t} - \tau/2)}^{\bar{t}} d\tau$$

$$S_{11}^{\text{out}} = \mathcal{T}^2 S_{11}^{\text{in}} + \mathcal{R}^2 S_{22}^{\text{in}} + \mathcal{RT} Q_0$$

Classical terms (partitioning)

Quantum terms (HBT & HOM)

$$Q_0 = (ev_F)^2 \int \overline{(\mathcal{G}_1^{(e)} \mathcal{G}_2^{(h)} + \mathcal{G}_1^{(h)} \mathcal{G}_2^{(e)})(t, t')}^{\bar{t}} d(t - t')$$

Overlap between single particle coherences!

Comparing sources through HBT correlations: see Moskalets and Büttiker, Phys. Rev. B **83**, 035316 (2011)

# What will we reconstruct ?

$$\mathcal{G}^{(e)}(t, t') = \underbrace{\mathcal{G}_F^{(e)}(t - t')}_{\text{Fermi sea}} + \underbrace{\Delta\mathcal{G}^{(e)}(t, t')}_{\text{Source contribution}}$$

Fermi sea

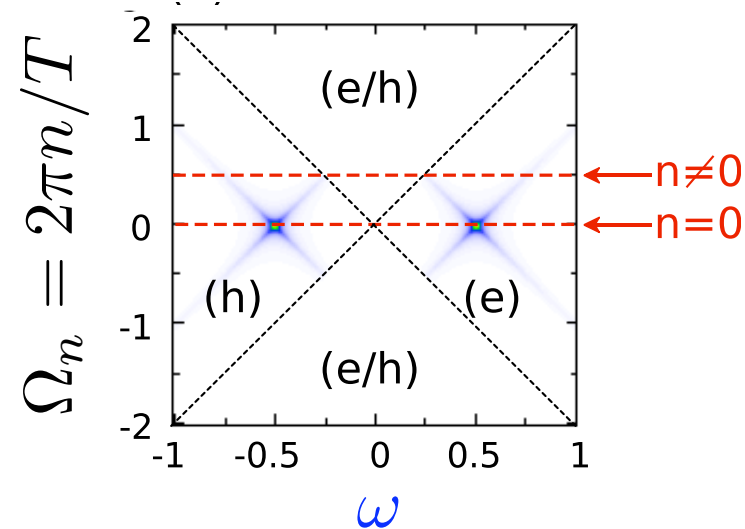
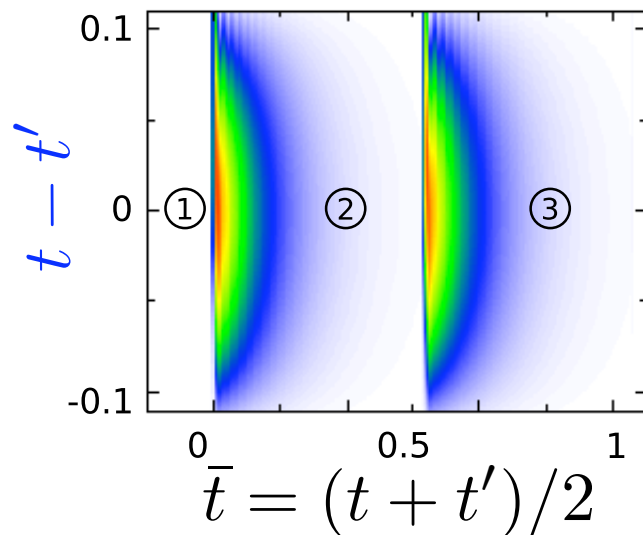
Source contribution

Time domain

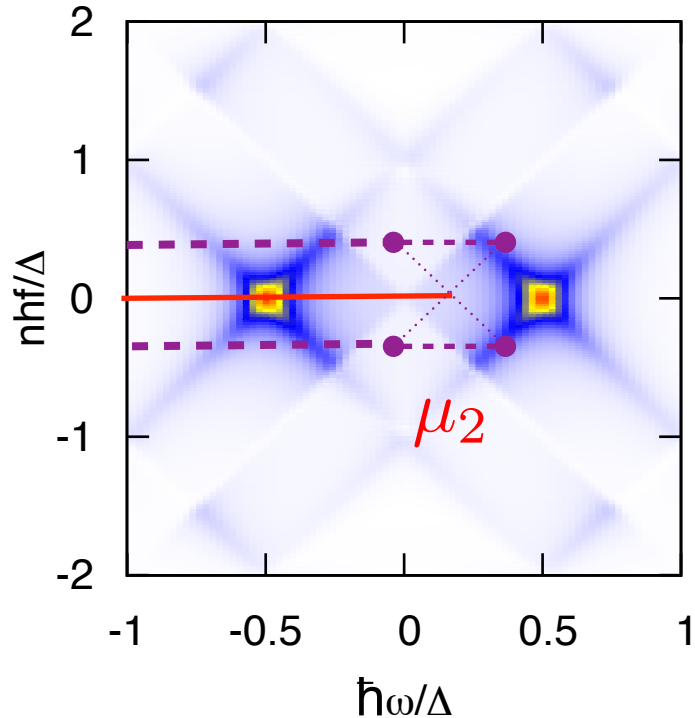
Frequency domain

$$|\Delta\mathcal{G}^{(e)}(t, t')|$$

$$|\Delta\mathcal{G}_n^{(e)}(\omega)|$$



$$\Delta\mathcal{G}^{(e)}(t, t') = \sum_{n \in \mathbb{Z}} e^{-2\pi i n \bar{t}/T} \int \Delta\mathcal{G}_n(\omega) e^{-i\omega(t-t')} \frac{d\omega}{2\pi}$$



DC bias:  $V_{\text{dc}} = -\mu_2/e$

AC drive:  $V_{\text{ac}}(t) = V \cos(2\pi nft + \phi)$

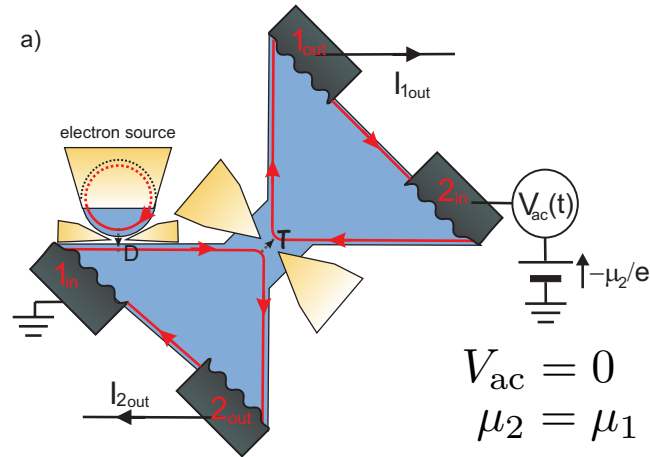
Current noise vs DC bias:  $\Delta\mathcal{G}_{n=0}^{(e)}(\omega)$

Current noise response to AC drive:  $\Delta\mathcal{G}_{n\neq 0}^{(e)}(\omega)$

«*The noise is the signal*» (R. Landauer 1998)

No AC drive, no DC bias:

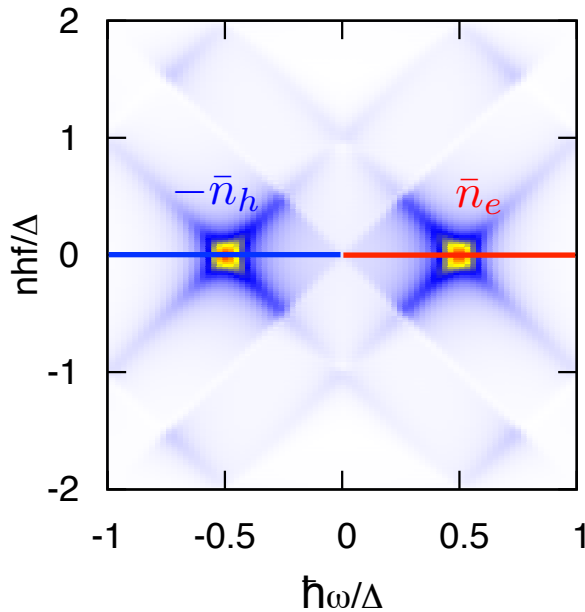
$$\Delta Q_0 = e^2 \int \underbrace{\tanh\left(\frac{\hbar\omega}{2k_B T_2}\right)}_{\text{Known source}} v_F \underbrace{\Delta \mathcal{G}_{1,0}^{(e)}(\omega)}_{\text{Source to be characterized}} \frac{d\omega}{2\pi}$$



Vanishing temperature ( $T_2 = 0$  K)

$$\Delta Q_0(\mu_2 = 0) = e^2 f(\bar{n}_e + \bar{n}_h)$$

$\bar{n}_{e/h} = \#$  of electron (resp. hole) excitations per period

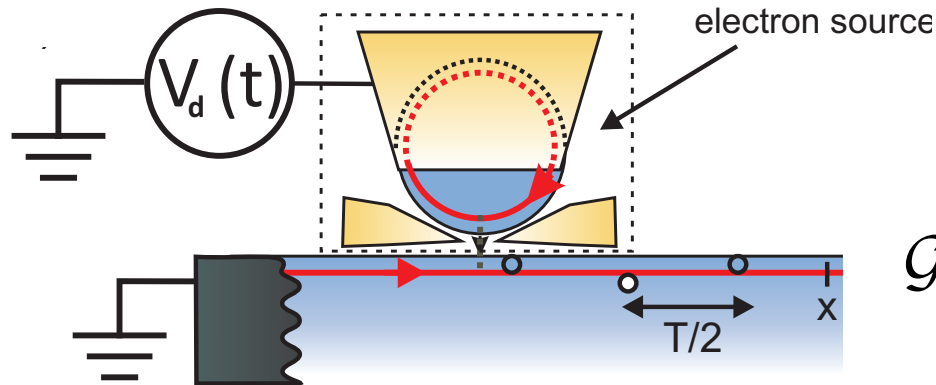


Non zero temperature

Excitations below  $k_B T_2$  antibunch with thermal excitations from the reservoir #2.

See E. Bocquillon's talk!

## The single electron source



Controls:

QPC transparency  $D$

Driving frequency  $f = T^{-1}$

Drive amplitude  $V_d$

$2eV_d = \Delta$  (level spacing)

$$\mathcal{G}^{(e)}(t, t') = ?$$

## Modeling the source

Floquet scattering approach

Büttiker & Moskalets,

Phys. Rev. B **66**, 205320 (2002)

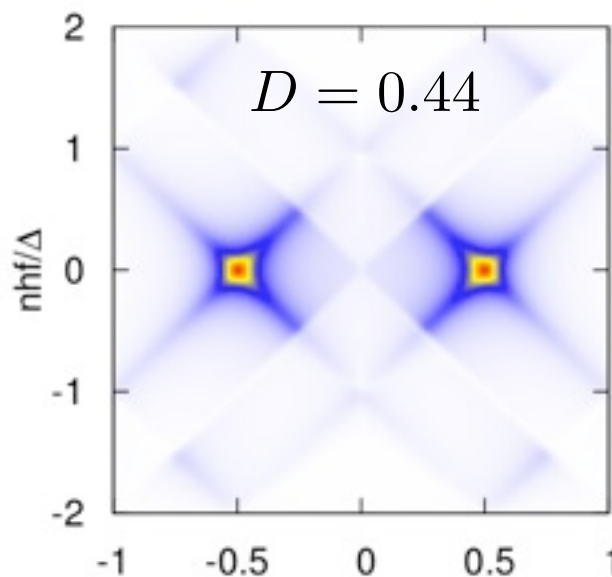
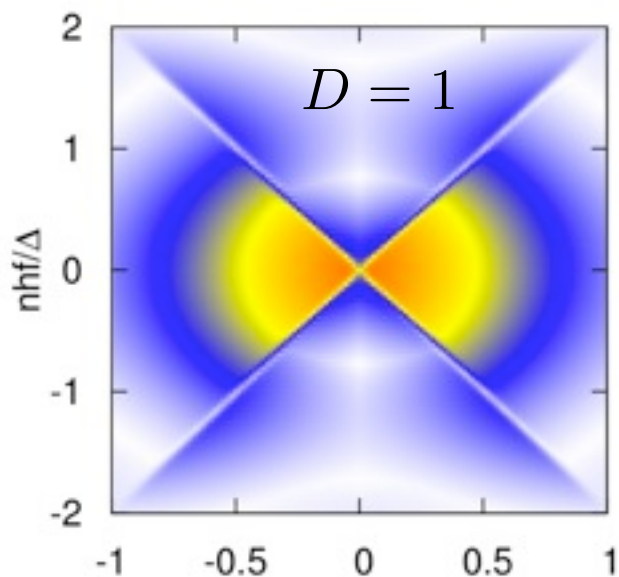
Phys. Rev. Lett. **100**, 086601 (2008)

*Neglects interactions within the dot (experimentally OK)*

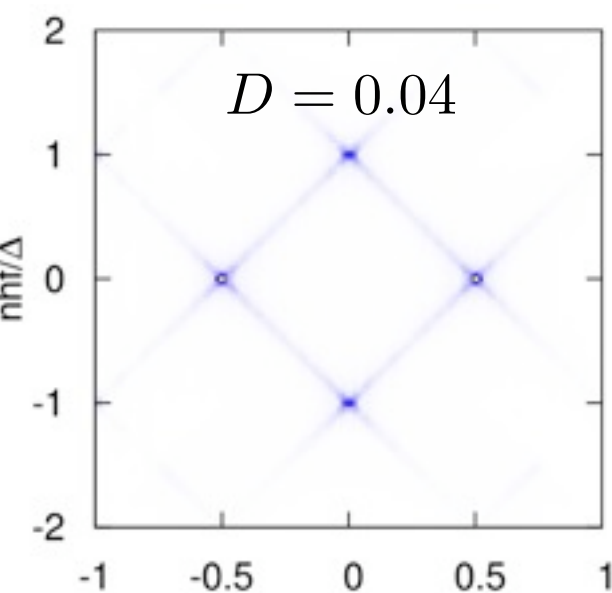
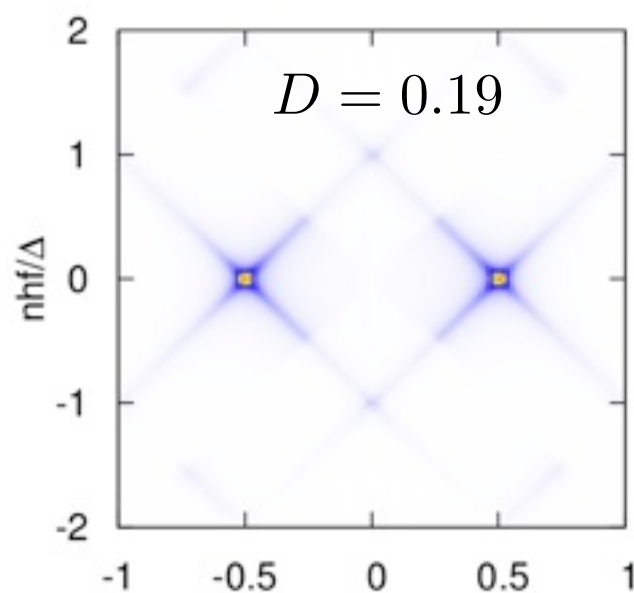
Evaluation of  $\Delta \mathcal{G}_n^{(e)}(\omega)$  and of the excess HBT contribution of the source to the noise (*expected experimental signal*)

# Single electron coherence

$f = 3$  GHz  
 $T_{\text{el}} = 40$  mK



$|v_F \Delta \mathcal{G}_n^{(e)}(\omega)|$



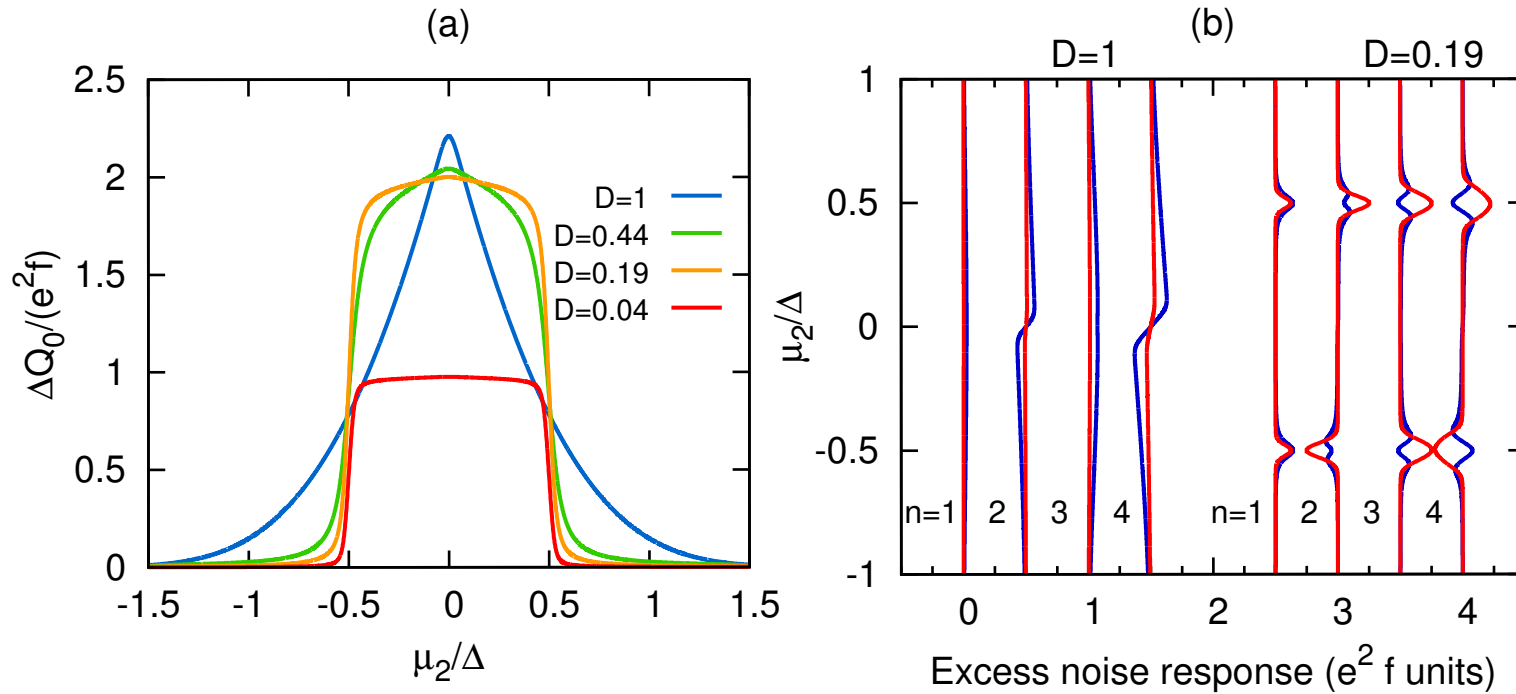
$\hbar\omega/\Delta$

$\hbar\omega/\Delta$

# Expected experimental signals

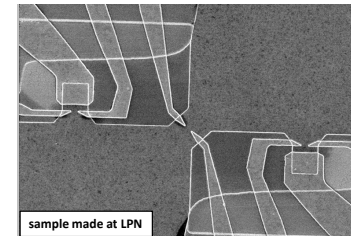
$f = 3$  GHz

$T_{el} = 40$  mK



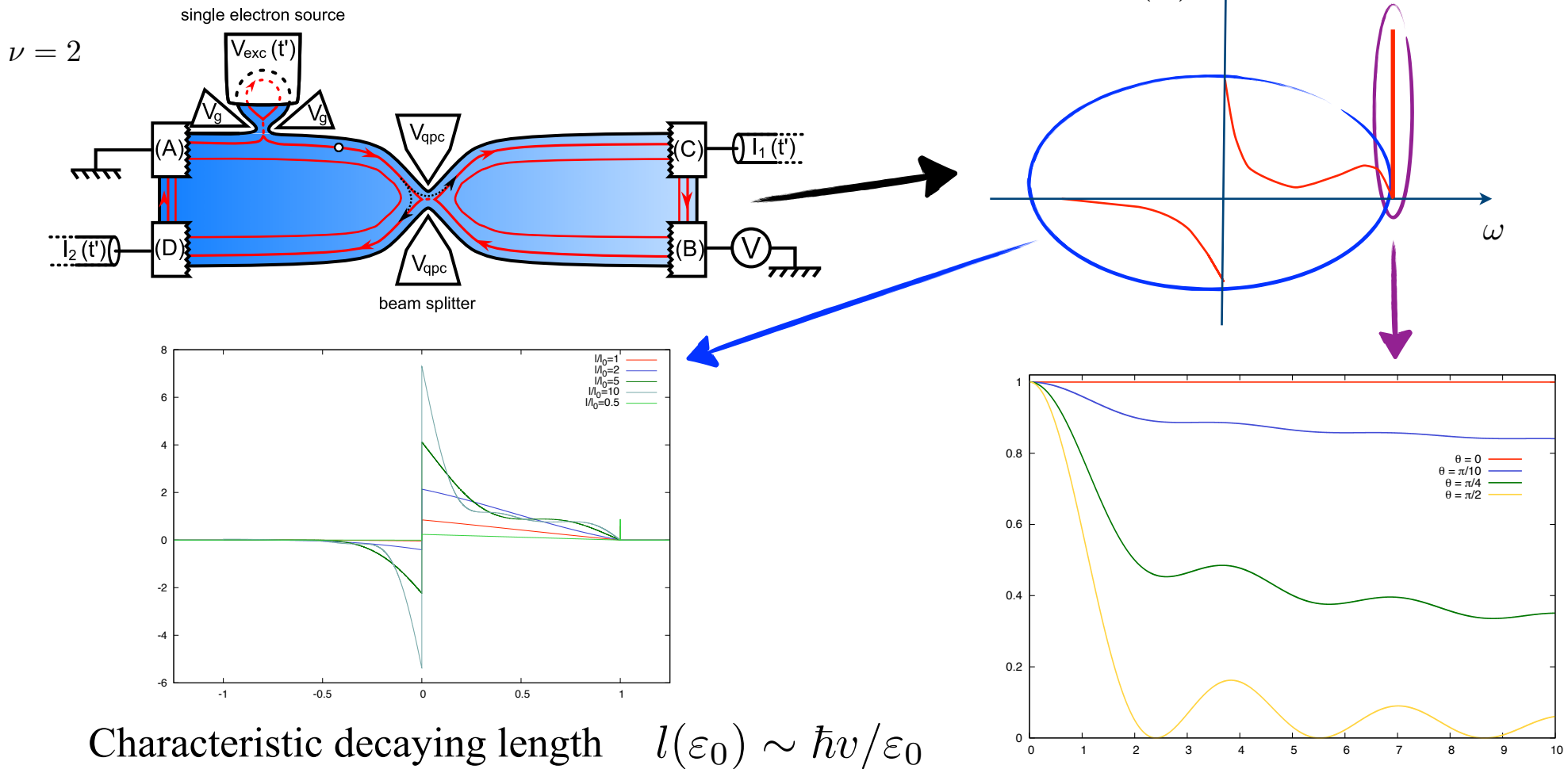
Exp. requirement: sensitivity  $\simeq 0.05 \times e^2 f = 4 \times 10^{-30} \text{ A}^2/\text{Hz}$

Ongoing experimental effort at ENS Paris:  
see Erwann Bocquillon's talk!



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## Quasiparticle relaxation

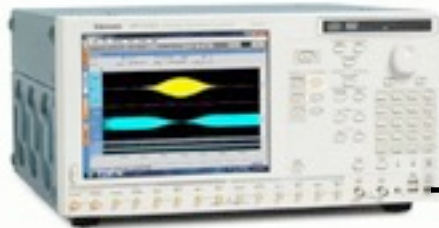


No well defined quasi particle!

P. Degiovanni, Ch. Grenier, G. Fève,  
 Phys. Rev. B **80**, 241307(R) (2009)

# Fractionalization of Lorentzian pulses

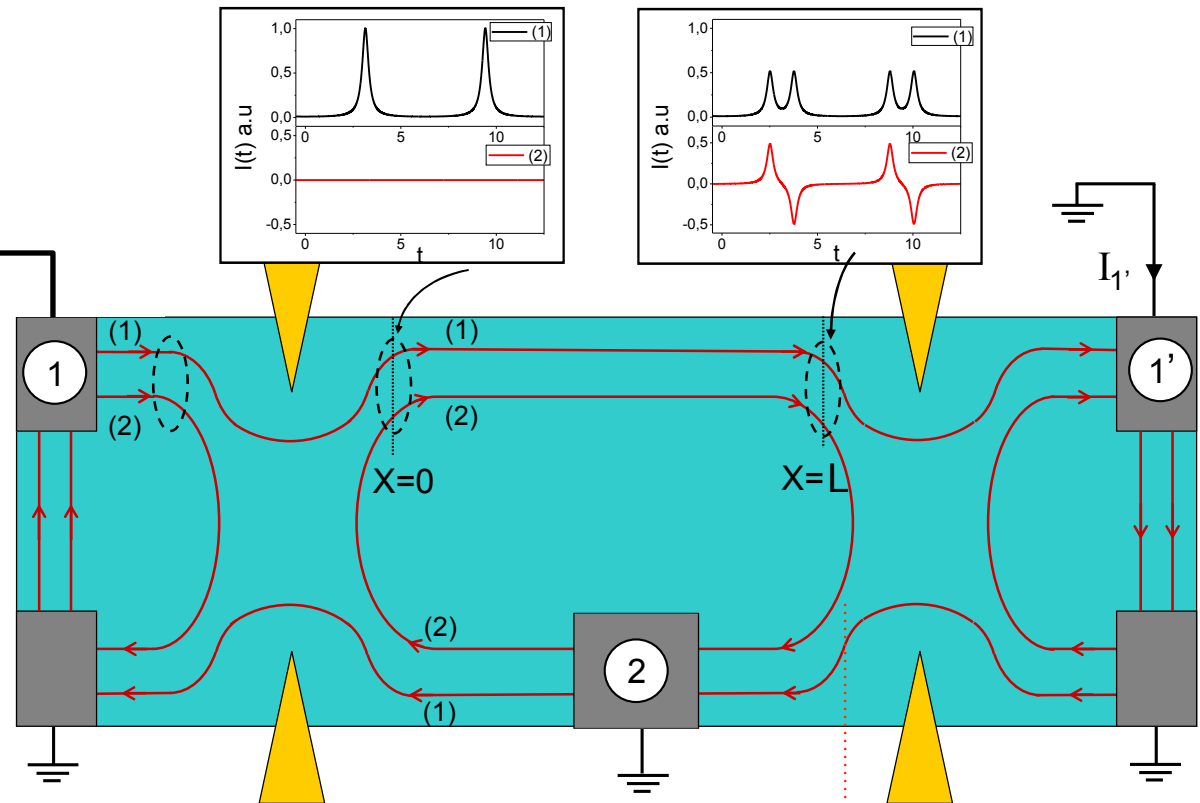
Lorentzian voltage pulse



Periodic train of quantized pulses

$$\int eV_d(t) dt = h$$

Ch. Glattli, ERC project



Stroboscopic coherence revivals: dispersionless magnetoplasmon eigenmodes

Fractionalization: e/h pair production vs charge...

Ch. Grenier, G. Fève et P. Degiovanni, *in preparation*

## Electron quantum optics

Coherence properties of the single electron source

Single electron quantum tomography protocol

[New Journal of Physics \*\*13\*\*, 093007 \(2011\)](#)

Plasmon scattering role in electric and energy transport along chiral edges

Applications to energy relaxation in edge channels at  $\nu=2$ .

[Phys. Rev. B \*\*81\*\*, 121302\(R\) \(2010\)](#)

Non perturbative solution to the Landau problem in chiral edge channels (*relaxation of a single electron excitation above the Fermi level*).

[Phys. Rev. B \*\*80\*\*, 241307\(R\) \(2009\)](#)

Quantum detection of electronic flying qubits

[Phys. Rev. B \*\*77\*\*, 035308 \(2008\)](#)

## Case of $n=0$ harmonics

Control parameters:  $\mu_2$  (DC bias)

$$\frac{\partial(\Delta Q_0)}{\partial\mu_2} [\omega = 0, \mu_2, V_{ac}(t) = 0] = -\frac{2e^2}{h} v_F \Delta \mathcal{G}_{1,n=0}^{(e)}(\mu_2/\hbar)$$

## Higher harmonics

Control parameters:  $\mu_2$  and  $\phi$ , the phase of the AC drive  $V_{ac}(t) = V \cos(2\pi nft + \phi)$

Susceptibility: 
$$\bar{\chi}_n(\mu_2, \phi) = \left( \frac{\partial(\Delta Q_0)}{\partial(eV/nhf)} \right)_{\omega=0, V=0}$$

$$\frac{\partial\bar{\chi}_n}{\partial\mu_2}(\mu_2, \phi) = \frac{e^2}{h} \Re \left[ e^{i\phi} \left( v_F \Delta \mathcal{G}_{1,n}^{(e)} \left( \frac{\mu_2}{\hbar} + \pi n f \right) - v_F \Delta \mathcal{G}_{1,n}^{(e)} \left( \frac{\mu_2}{\hbar} - \pi n f \right) \right) \right]$$