Strongly interacting 1D bosons on a ring

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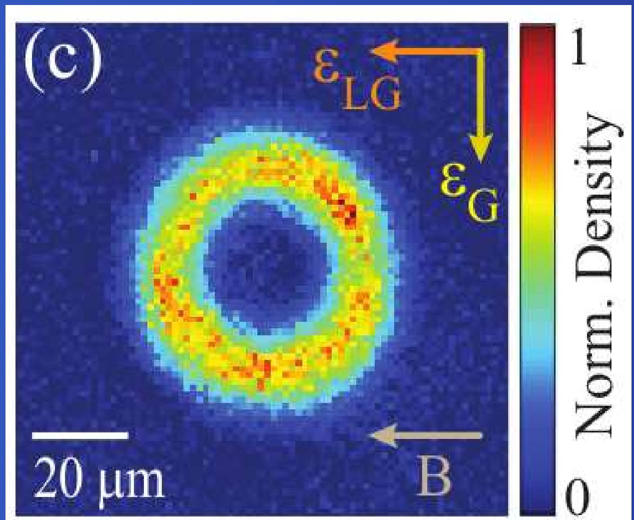
Plan

- Ultracold bosons on a 1D ring
- Probe of 1D superfluidity
- Generation of macroscopic superpositions of current states

Ultracold atoms on a ring

Recent experimental advances (talk H. Perrin)

- Realization of ring traps



- Possibility to set into *rotation* a barrier potential

Ultracold atoms on a ring

A novel topology

- vortex stability – *long lifetime of current states*

Ultracold atoms on a ring

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- artificial gauge fields – *rotation* $\Leftrightarrow$ *magnetic field*
  $\mathcal{H} = \frac{1}{2m}(i\hbar\nabla - m\vec{v})^2 + V_{ext}$

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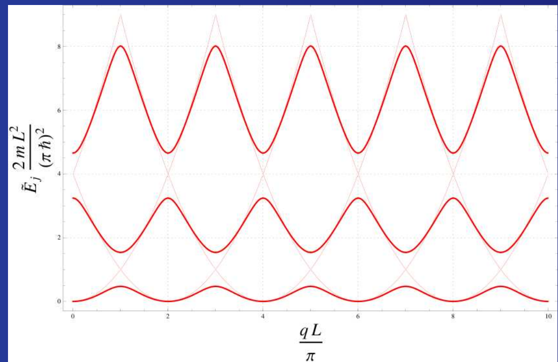


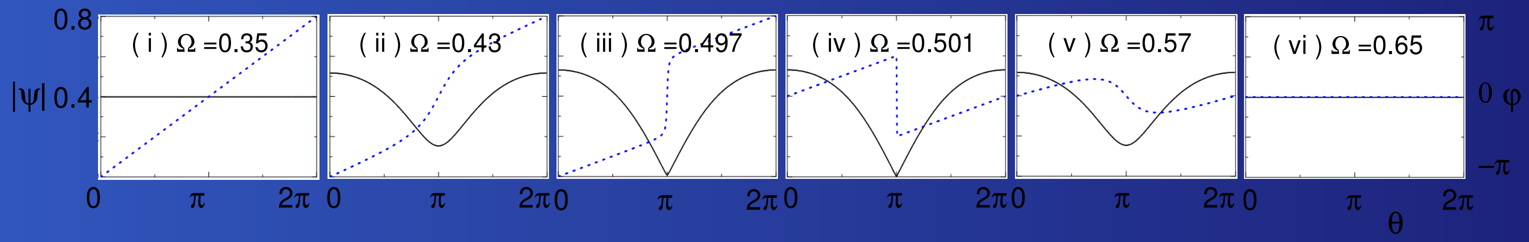
- mesoscopic effects
  Dependence on the Coriolis flux $\Phi = Lv$ L trap circumf.
  Periodicity in flux quantum $\Phi_0 = 2\pi\hbar/m$

The quasi-1D ring

Exploring fundamental questions

- vortices and solitons

  [R Kanamoto, LD Carr and M Ueda, PRL 100, 060401 (2008)]



- 1D superfluidity, phase slips

  [HP Buechler et al, PRL 87, 100403 (2001)]

- transport properties of correlated bosons

  [R Citro, AM, FWJ Hekking, PRB 79, 172505 (2009)]

The quasi-1D ring

Applications

- precision measurements –*atomic gyroscopes*

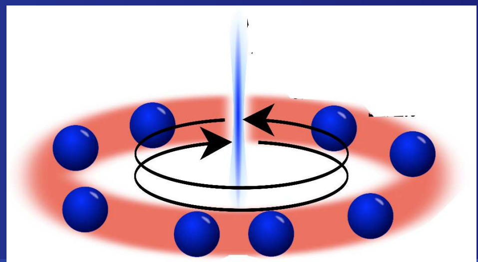
  [JP Dowling, PRA 57, 4736 (1998)]

- quantum state engineering –*creation of entangled states by tuning the Coriolis flux*

  [DW Hallwood et al, Phys. Rev. A 82, 063623 (2010), C Schenke, AM and FWJ Hekking, arXiv:1108.5075]

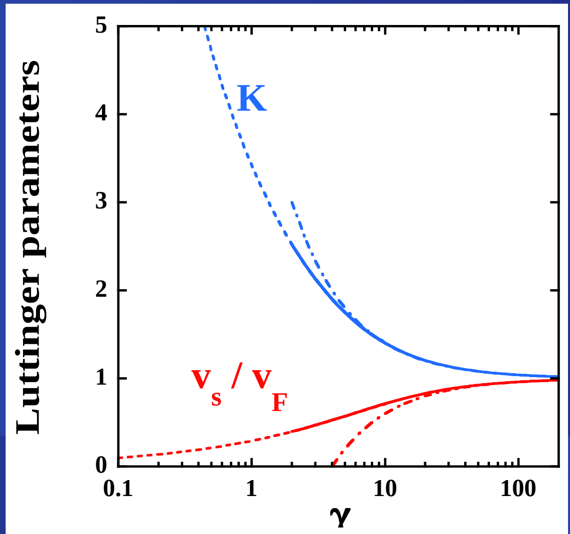
- use of nonclassical states to reach Heisenberg limit in interferometry

  [JJ Cooper et al PRA 81, 043624 (2010)]



1D interacting bosons

- Ultracold neutral atoms: short range interactions
  $v(x-x') = g\delta(x-x')$
- adimensional coupling strength
  $\gamma = E_{int}/E_{kin} = gm/n\hbar^2$
- 1D bosons are a Luttinger liquid; universal parameter K known from Bethe Ansatz



T=0 state diagram

- Crossover at increasing interactions
  - from quasicondensate – condensate with fluctuating phase, $K \gg 1$
  - to Tonks-Girardeau gas – $g \to \infty$, strong correlations, only $\sqrt{N}$ population of the lowest orbital, $K = 1$

Impenetrable bosons: special features

- For $g \to \infty$ the many-body wavefunction vanishes at contact

$$\Psi(...x_j = x_\ell) = 0$$

- Exact solution by mapping onto noninteracting fermions *[MD Girardeau,J. Math. Phys. 1, 516 (1960)]*

$$\Psi(x_1...x_N) = \Pi_{1\leq j<\ell\leq N}\mathrm{sign(x_j - x_\ell)}\frac{1}{\sqrt{N!}}\det(\psi_l(x_k))$$

with $\psi_l(x)$ single particle orbitals

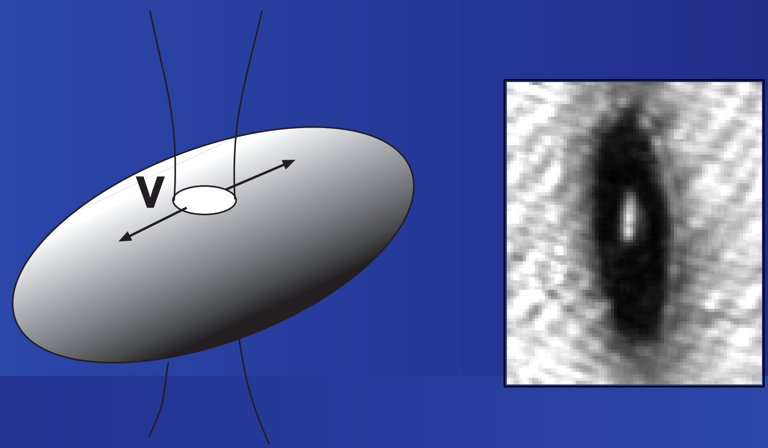
for arbitrary external potential, also time dependent

- *fermionization* $\Rightarrow$ impenetrable bosons are robust to two- and three-body particle losses

1D superfluidity

Superfluidity

- frictionless fluid flow past an obstacle at velocity $v < v_c$
  *** probe excitations $\epsilon(p)$ in a quantum fluid
  Landau criterion $\epsilon(p) - p \cdot v \leq 0 \Rightarrow v_c = \epsilon(p)/p$
  for a weakly interacting Bose gas, $v_c = v_{sound}$
- ... or moving obstacle in a fluid at rest: “stirring”
  experiments on 3D elongated Bose-Einstein condensates



a) b)

1D (bosonic) transport

- for a fermionic Luttinger liquid transport properties across a barrier dramatically change at $K = 1$ *[C.L. Kane and M.P.A. Fisher, PRL 68, 1220 (1992)]*, from RG:
  - for K>1 a small tunnel parameter yields a large conductance: superfluid
  - for K<1 even a small barrier yields zero conductance: insulator
- analysis valid also for 1D bosons *[A Schmid, PRL 51, 1506 (1983); R Citro, AM, FWJ Hekking, PRB 2009]*
- K=1 marginal under RG, a small tunnel parameter yields a small conductance – is the Tonks-Girardeau gas superfluid?

1D superfluidity?

- Drag force on a moving barrier $U(x,t)$; from energy dissipated $\partial_t E = \int d\omega \int dq\, \omega S(q,\omega)|U(\omega,q)|^2$

  [G. Astrakharchik and L. Pitaevskii PRA 70,013608 (2004), A.Yu Cherny, J.-S. Caux and J. Brand, arXiv:1106.6329]

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- bosons on an *infinite tube*

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- weakly interacting bosons on a *ring*: superfluid with critical velocity $v_c = \pi\hbar/mL$ *[HP Buechler et al, PRL 87, 100403 (2001)]*

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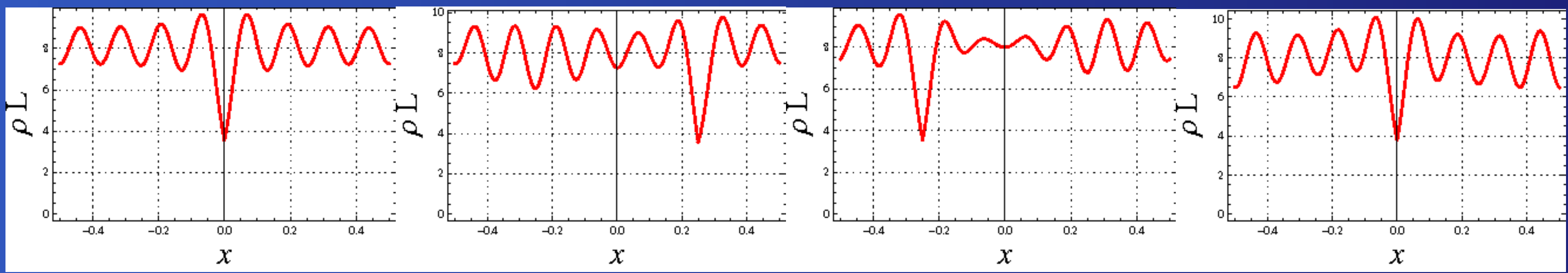
- weakly interacting bosons on a *ring*: superfluid with critical velocity $v_c = \pi\hbar/mL$ *[HP Buechler et al, PRL 87, 100403 (2001)]*
- superfluidity of TG bosons on a ring?

Stirring impenetrable bosons

TG bosons on a ring, stirred by $U(x,t) = U_0\delta(x - vt)$

- initial state: ground state of the static barrier problem
- *sudden quench* of the barrier velocity to its final value v
- exact solution of the quantum non-equilibrium problem



Details of the solution

- application of the unitary transformations $\mathcal{U}_1 = e^{-i\hat{p}vt/\hbar}$, $\mathcal{U}_2 = e^{imvx/\hbar}$ yields

$$\psi_l(x,t) = e^{imvx} e^{-imv^2t/2} \sum_j c_{jl} e^{-i\tilde{E}_j t} \tilde{\phi}_j(x - vt)$$

  with

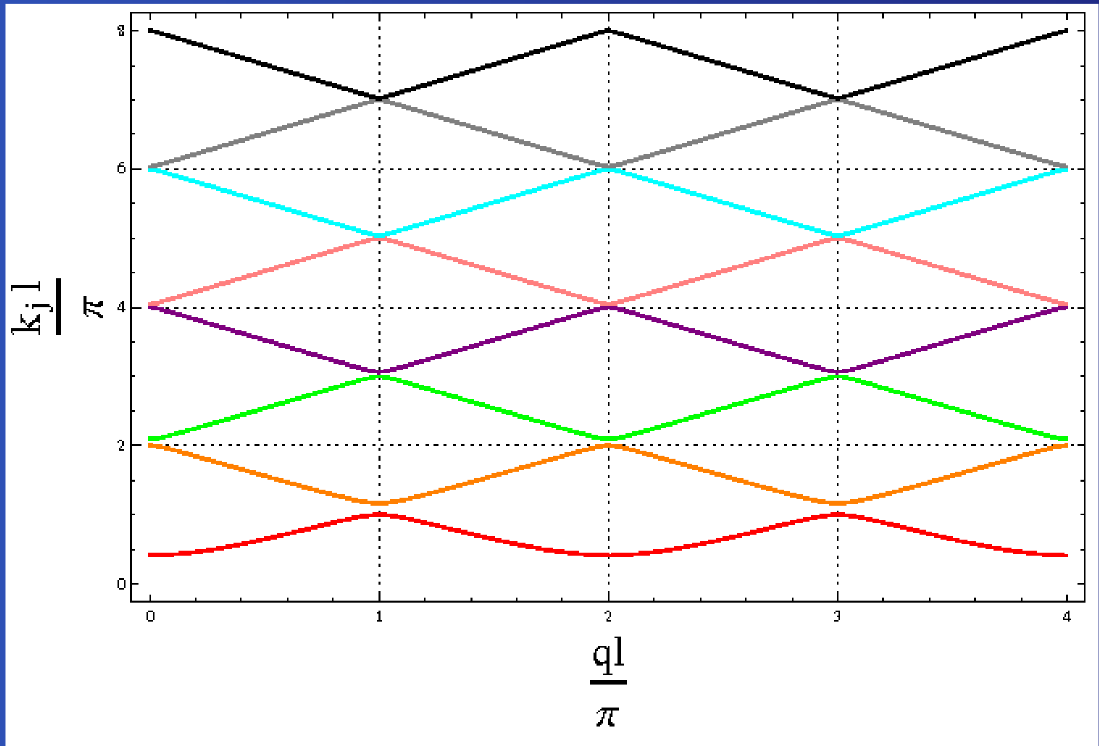
  - $\tilde{E}_j \tilde{\phi}_j = (-(\hbar^2/2m)\partial_x^2 + U_0\delta(x))\tilde{\phi}_j$
  - twisted boundary conditions $\tilde{\phi}_j(x+L) = e^{-imvx}\tilde{\phi}_j(x)$
  - time independent overlaps $c_{jl} = \langle \tilde{\phi}_j | e^{-imvx} | \phi_l \rangle$
  - $\tilde{E}_j = \hbar^2 k_j^2/2m$, transcendental equation for k_j
    $k_j = (mU_0/\hbar^2)\sin(k_j L)/(\cos(mvL) - \cos(k_j L))$

Out-of-equilibrium features

Recall $\Psi(x_1 ... x_N) = \mathcal{A}\frac{1}{\sqrt{N!}}\det(\psi_l(x_k))$ and

$\psi_l(x_k) =$superposition of plane waves with wavevectors k_j

Occupied k_j *vs* stirring velocity v, weak barrier $U_0 = \hbar^2/mL$



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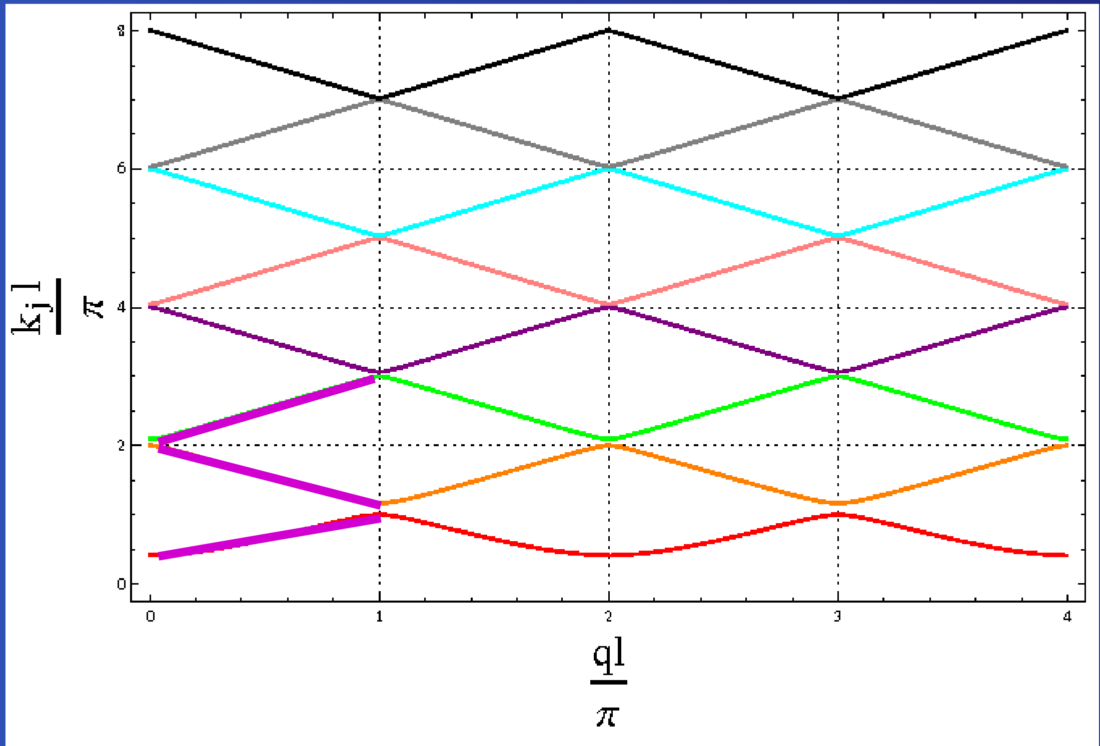

Fermi sphere for $N = 3$ particles, $0 < v < \pi\hbar/mL$

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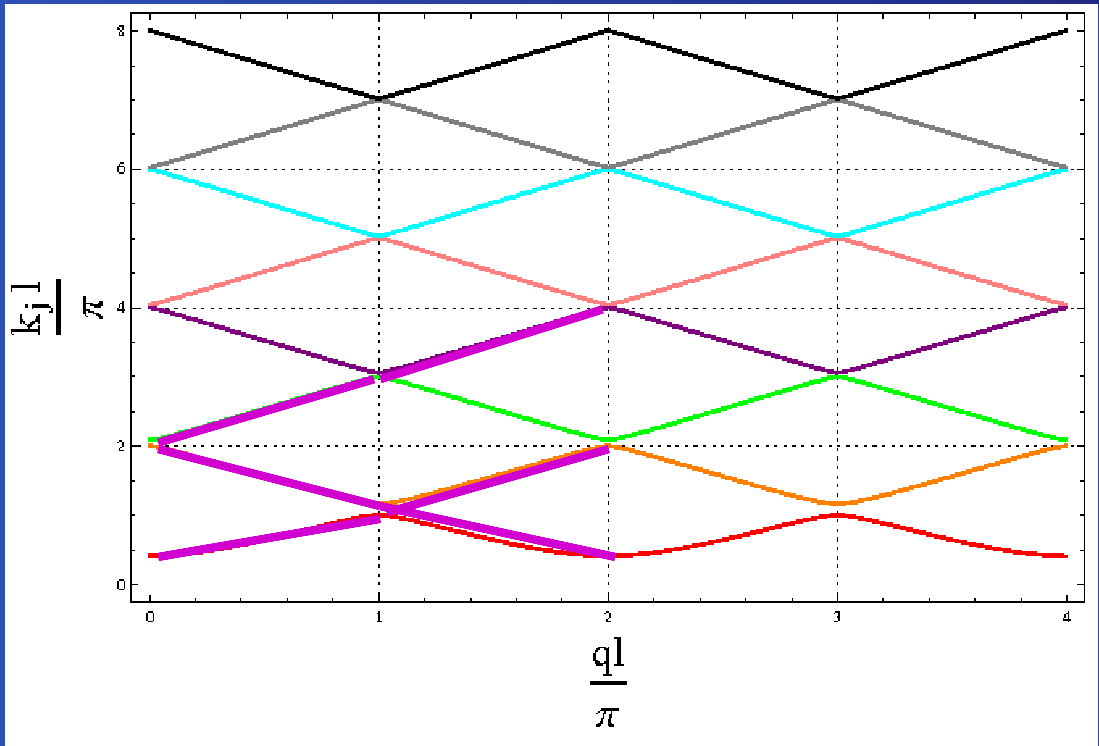

Fermi sphere for $N = 3$ particles, $\pi\hbar/mL < v < 2\pi\hbar/mL$

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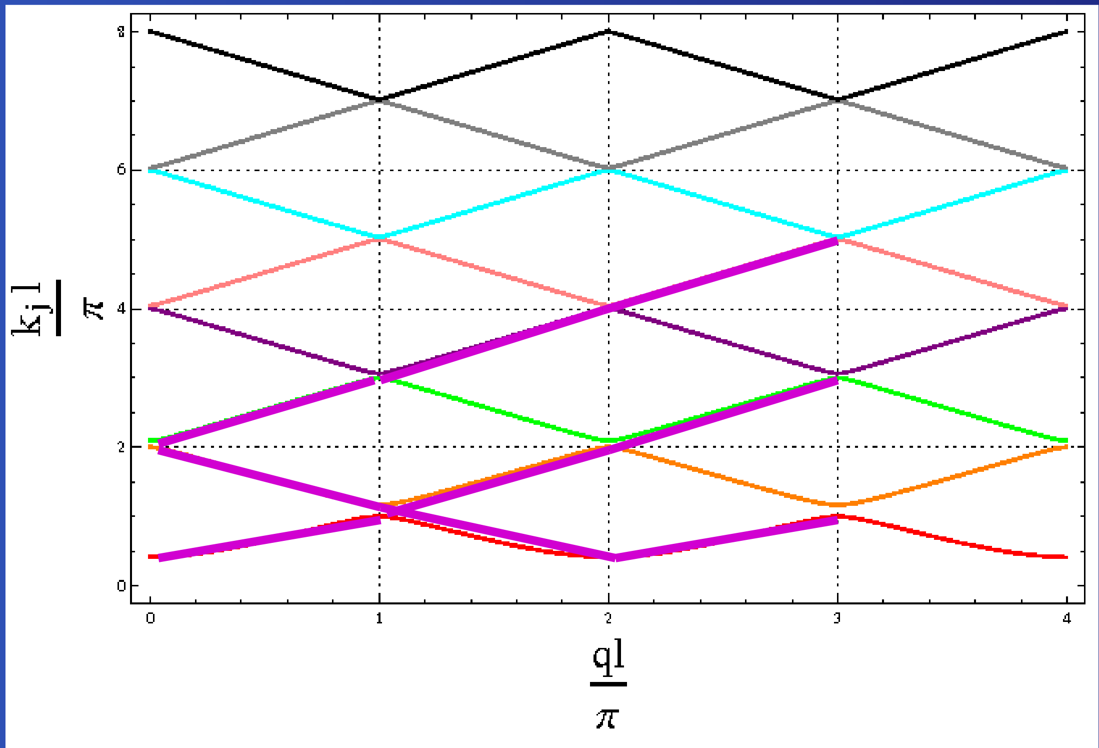

Fermi sphere for $N = 3$ particles, $2\pi\hbar/mL < v < 3\pi\hbar/mL$

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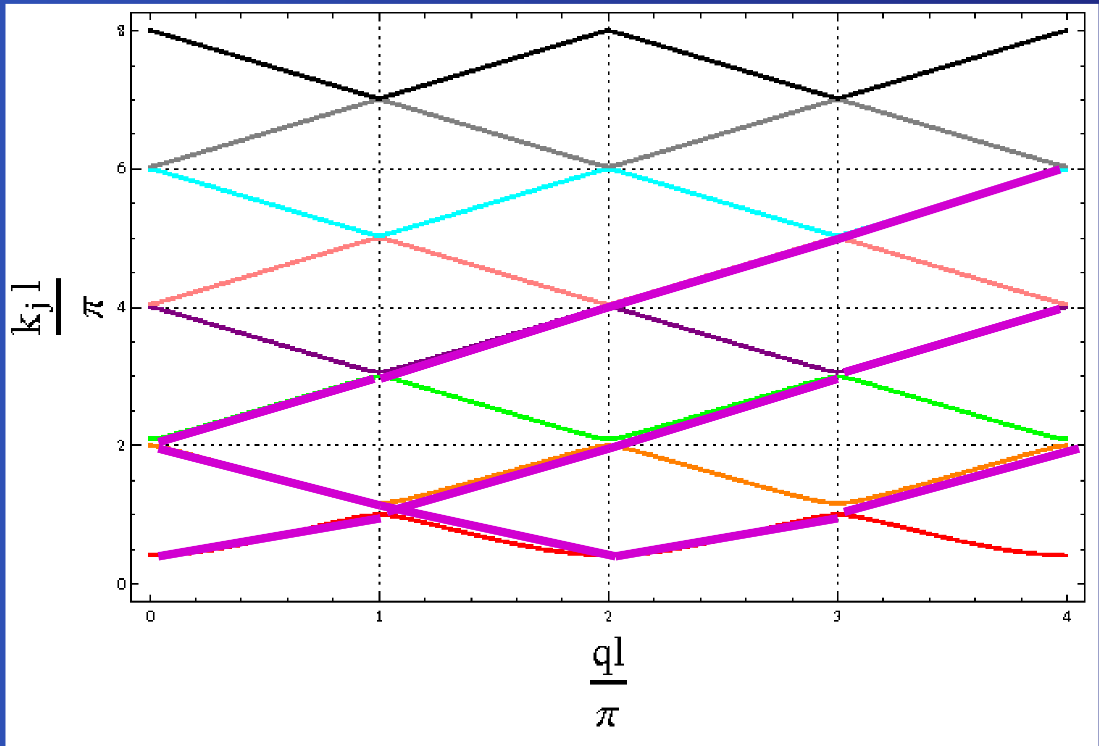
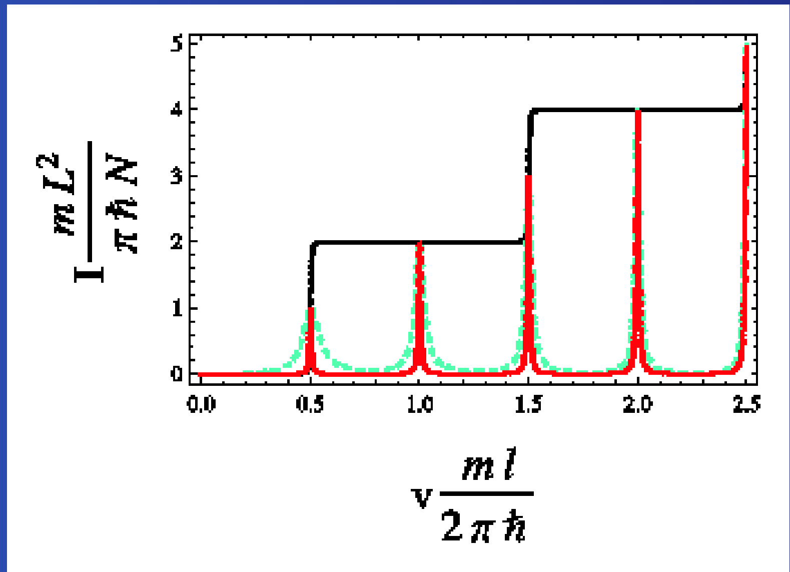


stirring : population of excited states

Effects of the stirring

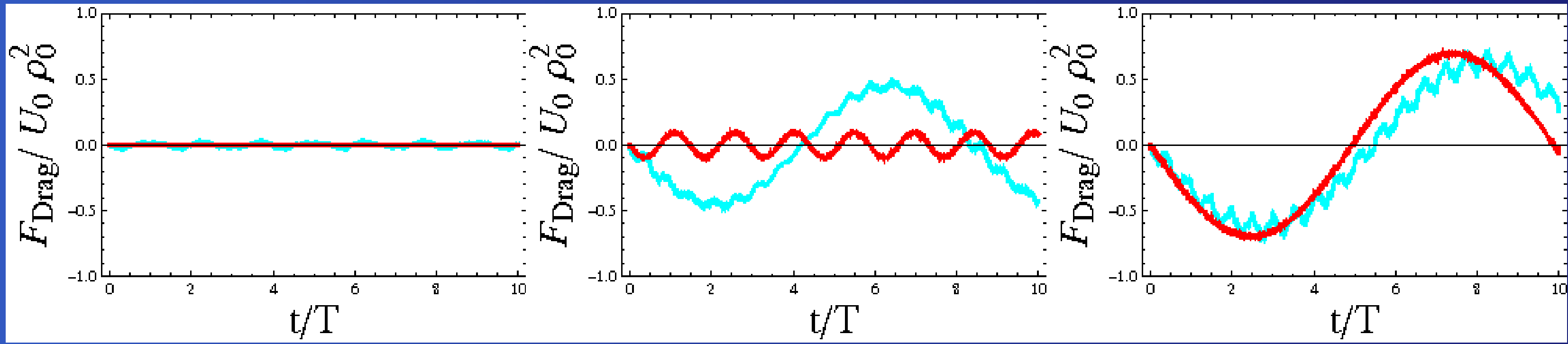
integrated particle current $I = \int dx j(x,t)$ induced by a moving barrier for adiabatic and sudden quench



- sudden quench $\Rightarrow$ narrow velocity intervals for transferring angular momentum
- a mesoscopic superfluid: critical velocity $v_c = \pi\hbar/Lm$

Drag force

for velocity $v = 0.25$, 0.925 and $1\ v_c$

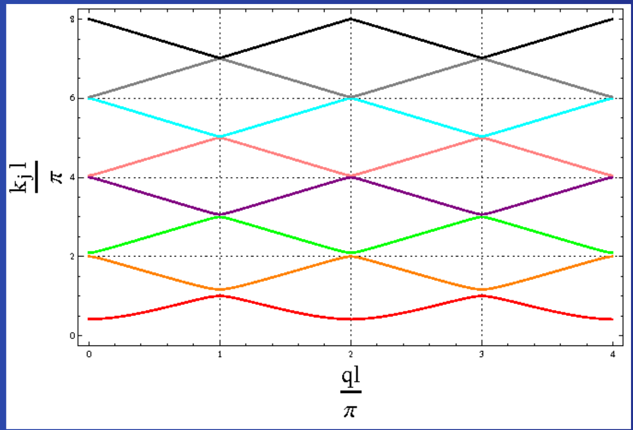


- (again) a mesoscopic superfluid
- the drag force depends on the interaction strength
  TG gas vs ideal bosons

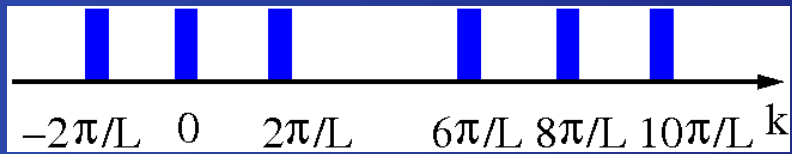
Macroscopic superpositions of current states

A zoom on special velocities

What happens for $v = n\pi/\hbar mL$?



- occupation number distribution:

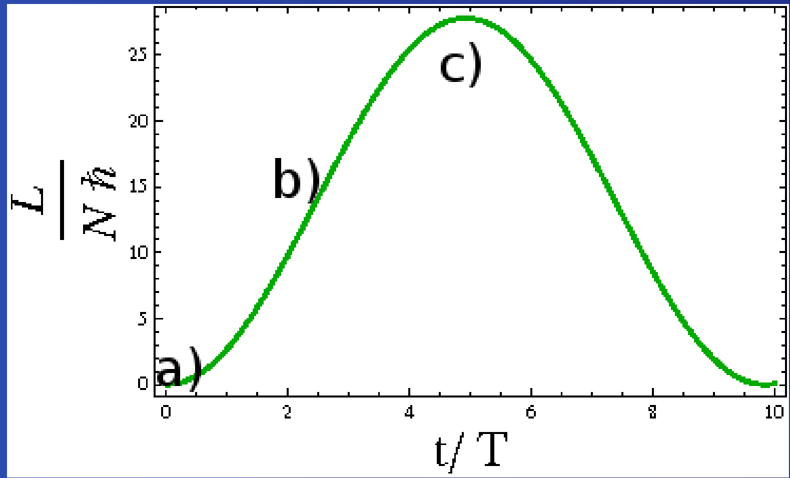


a superposition of two Fermi spheres

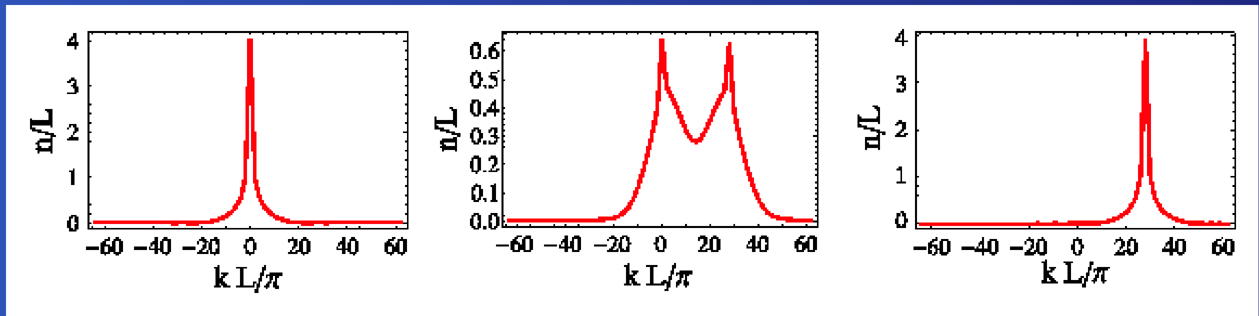
an entangled strongly correlated state

Momentum distribution

- integrated particle current *vs* time



- momentum distribution

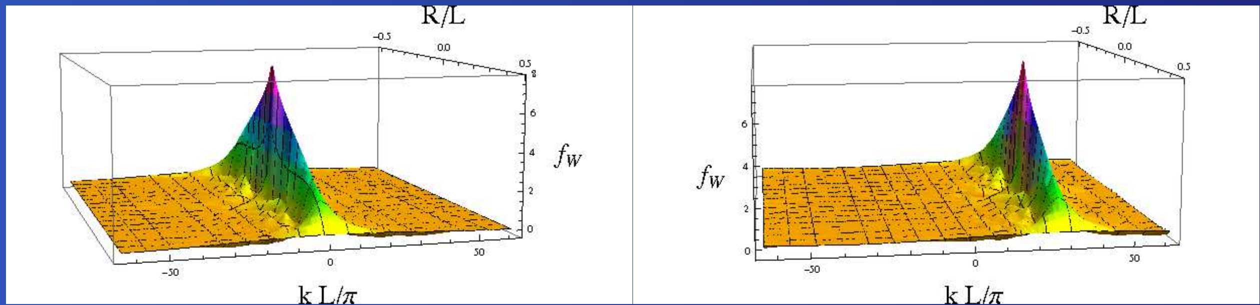


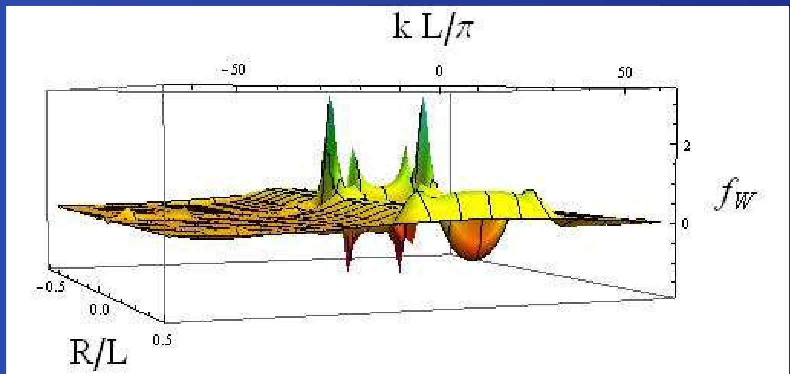
superposition of current states with velocity 0 and $2v$

Wigner function

really a nonclassical superposition?

- Wigner function



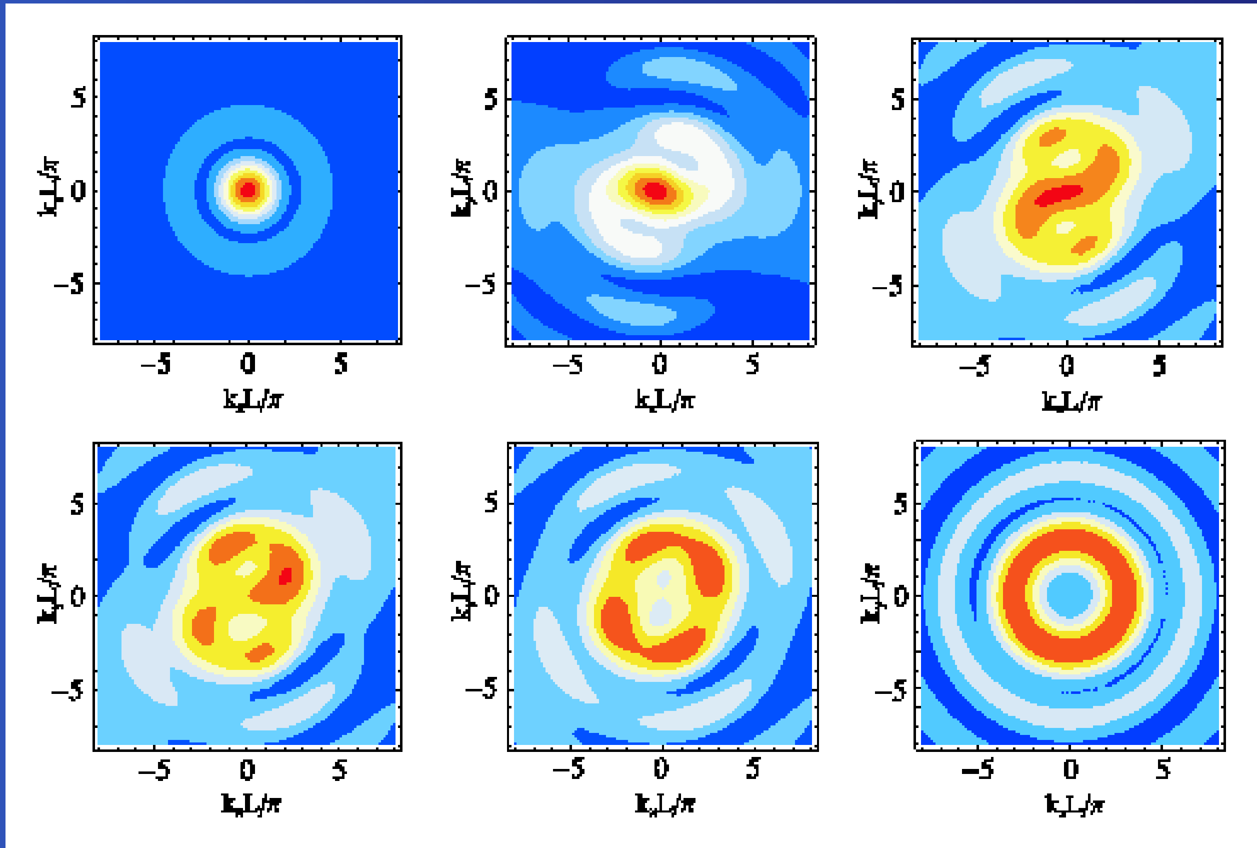


yes: negative regions in the Wigner function

[C Schenke, AM and FWJ Hekking, arXiv:1108.5075]

Ultracold atom observables

time-of-flight: expansion after sudden turn off of the trap

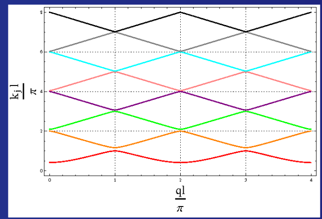


evidence of interference effects

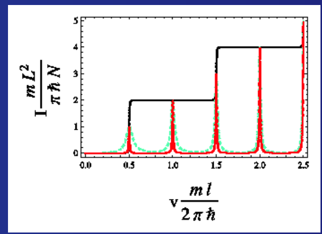
[C Schenke, AM and FWJ Hekking, arXiv:1108.5075]

Conclusions

- Exact, nonadiabatic solution for the stirring of a TG gas



- A mesoscopic superfluid



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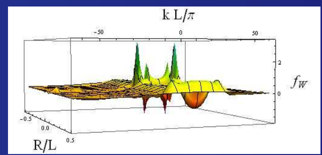

Grenoble – quantum gases

- Bose-Josephson junctions, entanglement, decoherence
  Frank Hekking, Giulia Ferrini, Dominique Spenher

- Anderson localization of Bose-Einstein condensates
  Sergey Skipetrov, Bart van Tiggelen

- Quasi 1D quantum gases
  Christoph Schenke,
  Frank Hekking