THEORY OF QUANTUM NOISE DETECTORS BASED ON RESONANT TUNNELING

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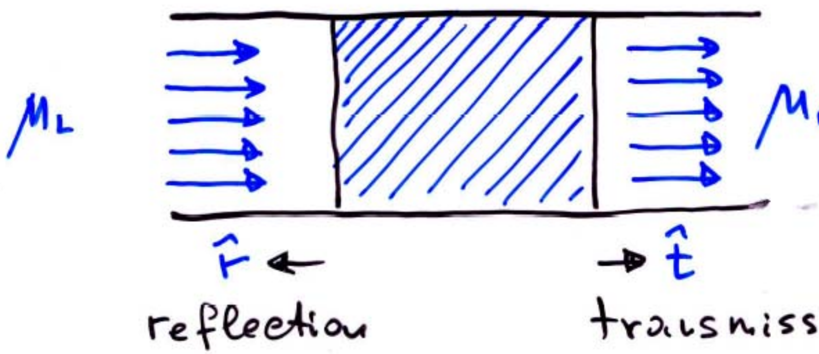
Outline:

1 Introduction to noise
2 Non-equilibrium FDT
3 Quantum noise detectors
4 Generalized P(E) - theory of tunneling
5 Application to double-dot detectors
6. Quantum and classical detection
7. Detection of third cumulant of current

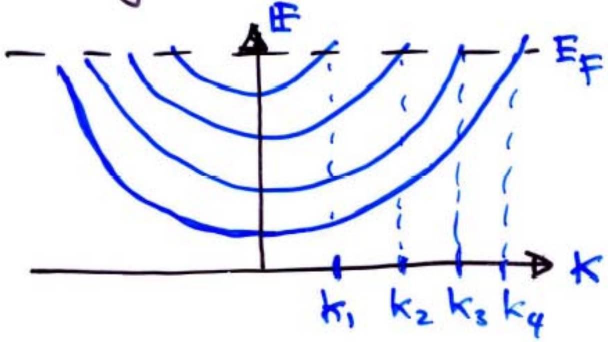
MARSEILLE, DECEMBER, 2009

1. Scattering theory

mesoscopic conductor:



Energy subbands:



$$(\hat{t}^+\hat{t}) \Rightarrow (T_1, T_2, T_3, \ldots, T_N)$$ – transmission eigenvalues

average current:

$$\langle I \rangle = \frac{e}{2\pi\hbar} \cdot \sum_{n=1}^{N} \int dE\, T_n(E)\, [f_L(E) - f_R(E)]$$

where $f_\alpha(E) = f_F(E - \mu_\alpha), \alpha = L, R$

INTRODUCTION TO NOISE 2

② Quantum noise : $[\hat{I}(t), \hat{I}(0)] \neq 0$

$$\Rightarrow \quad S^{+}(\omega) \equiv \int_{-\infty}^{+\infty} dt\, e^{i\omega t} \langle \delta I(t)\, \delta I(0) \rangle$$

$$S^{-}(\omega) \equiv \int_{-\infty}^{+\infty} dt\, e^{i\omega t} \langle \delta I(0)\, \delta I(t) \rangle$$

$$S(\omega) \equiv \frac{1}{2}\left[S^{+}(\omega) + S^{-}(\omega)\right], \quad S(0) = S^{+}(0)$$

Scattering theory

$$\Rightarrow \quad S^{\pm} = \frac{e^2}{h} \sum_n \int dE \left\{ T_n^2 \left(f_{LL}^{\pm} + f_{RR}^{\pm} \right) + T_n(1-T_n)\left(f_{LR}^{\pm} + f_{RL}^{\pm} \right) \right\}$$

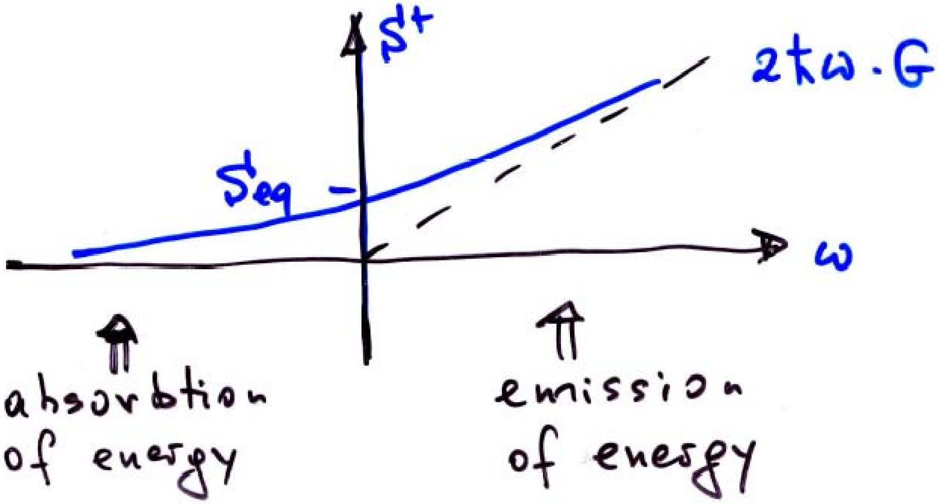
where $f_{\alpha\beta}^{+} \equiv f_\alpha(E)\left[1 - f_\beta(E+\hbar\omega)\right]$

$f_{\alpha\beta}^{-} = \left[1 - f_\alpha(E)\right] f_\beta(E+\hbar\omega)$

In equilibrium: $\mu_L = \mu_R$

$$\Rightarrow \boxed{S^+(\omega) = S^-(\omega) \cdot e^{\hbar\omega/k_BT}} \rightarrow \text{detailed balance}$$

$$S^+(\omega) = G \cdot \frac{2\hbar\omega}{1-e^{\beta\hbar\omega}}, \quad \beta = 1/k_BT$$



(3) Consequencies

1. Equilibrium noise, $\mu_L = \mu_R$

Use $f_L - f_R = \Delta\mu \cdot \partial_E f_F = \frac{\Delta\mu}{k_B T} \cdot f_F (1 - f_F)$

$\Rightarrow$ $\boxed{S(0) = 2 k_B T \cdot G}$, where $G \equiv \partial_V \langle I \rangle$

$\Rightarrow$ equilibrium FDT

2. Non-equilibrium noise,

$\Delta\mu \gg k_B T$

$\Rightarrow$ Fano factor: $\boxed{F \equiv S(0)/e\langle I \rangle = \frac{\sum_n T_n (1 - T_n)}{\sum_n T_n}}$

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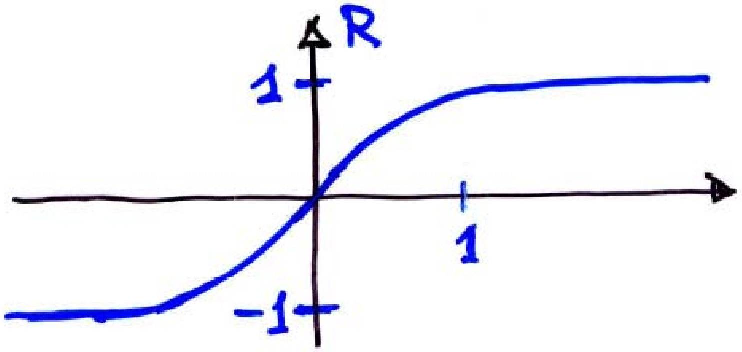
3. Tunneling noise,

$T_n \ll 1$

$$\Rightarrow \boxed{S(0) = e\langle I\rangle \cdot \coth\left(\frac{\Delta\mu}{2k_BT}\right)} \rightarrow$$ non-equilib. FDT !

$\Rightarrow$ Current-to-noise ratio:

$$R \equiv e\langle I\rangle / S(0) = \mathrm{th}\left(\frac{\Delta\mu}{2k_BT}\right)$$



$$x = \frac{\Delta\mu}{2k_BT}$$

NON-EQUILIBRIUM FDT

① Tunneling perturbation: $H_T = A + A^{+}$

② "Golden rule": tunneling rate

$$W^{+}_{nm} = \frac{2\pi}{\hbar} \cdot \delta(E_n - E_m + \Delta\mu)\, |A_{nm}|^2 \cdot \rho_m$$

$$W^{-}_{nm} = \frac{2\pi}{\hbar} \cdot \delta(E_n - E_m - \Delta\mu) \cdot |A_{nm}|^2 \cdot \rho_m$$

where $\rho_m = \frac{1}{Z} \cdot e^{-\beta E_m}$

NON-EQUILIBRIUM FDT 2

3. Tunneling current and noise:

$$\langle I \rangle = e \sum_{nm} \left(W_{nm}^{+} - W_{nm}^{-} \right)$$

$$S(0) = e^2 \sum_{nm} \left(W_{nm}^{+} + W_{nm}^{-} \right)$$

4. Non-equilibrium FDT

$$\rho(E + \Delta\mu) = e^{-\beta\Delta\mu} \cdot \rho(E) \Rightarrow W_{nm}^{+} = e^{\beta\Delta\mu} W_{nm}^{-}$$

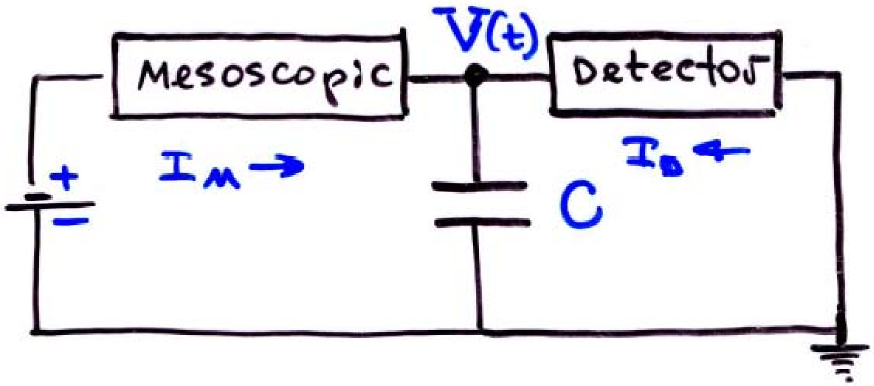
$$\Rightarrow \boxed{S(0) = e\langle I \rangle \cdot \coth\left(\frac{\Delta\mu}{2k_B T}\right)}$$

Universality: $\Rightarrow$ 1. arbitrary equilibrium leads

2. arbitrary interaction.

QUANTUM NOISE DETECTORS

1. Electrical circuit



Langevin equation:

$$C\dot{V} = -(G_M + G_D)\cdot V + J, \qquad J = I_M + I_D$$

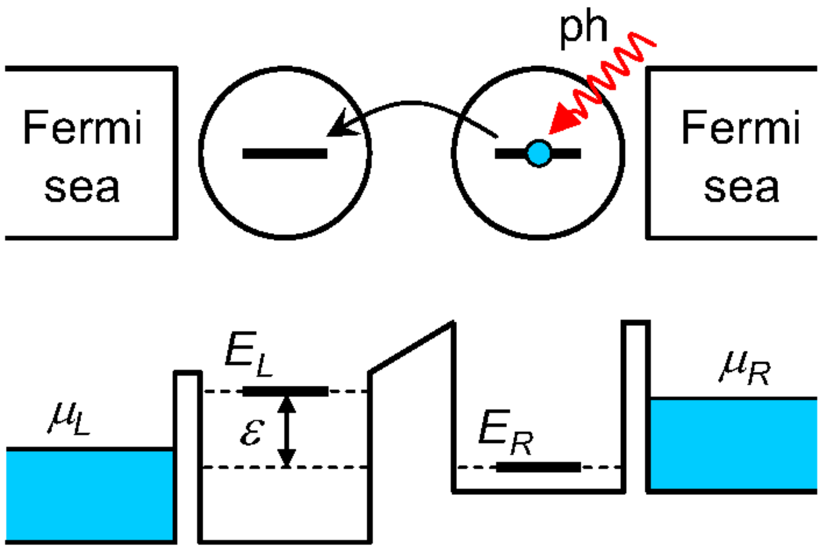
$$\Rightarrow \boxed{V(\omega) = Z(\omega)\cdot J(\omega)} \quad !$$

where $Z(\omega) = \dfrac{1}{G_M + G_D - i\omega C}$ → circuit impedance

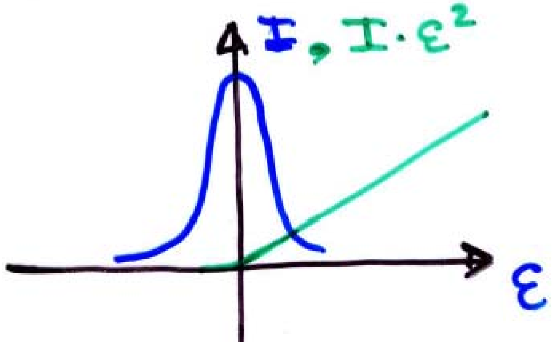
- Detector measures fluctuations $V(t)$

QUANTUM NOISE DETECTORS 2

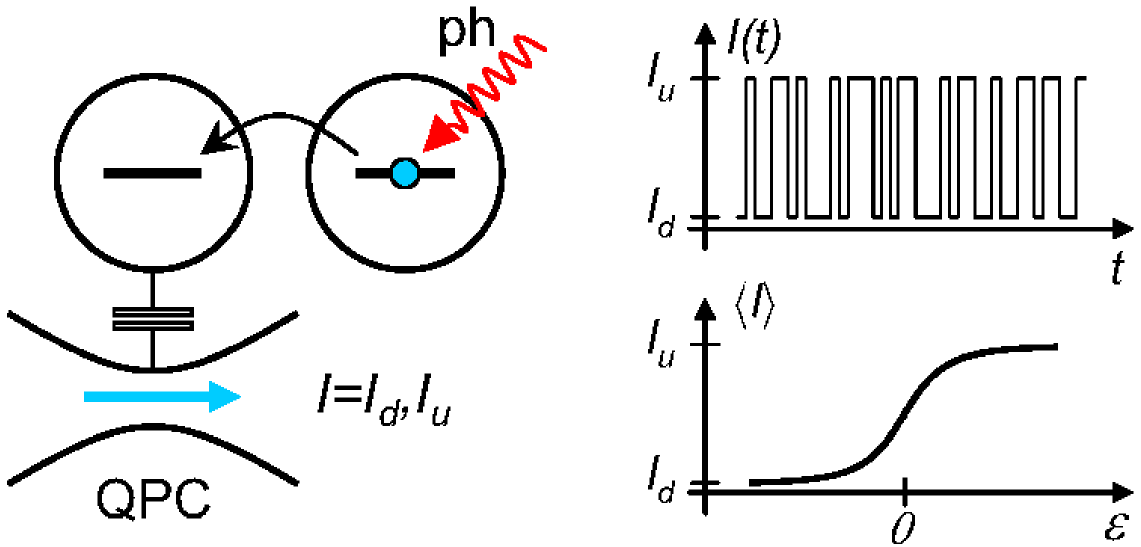
② Double-dot detector (DELFT)



$$I_{inel}(\varepsilon) \cong \frac{S^+(\varepsilon/\hbar)}{\varepsilon^2}$$



③ Double-dot detector (ZURICH)



IMPORTANT TIME SCALES AND PARAMETERS

1. Noise coupling constant: $\boxed{\alpha \equiv G_q/G \ll 1}$

   where $G = G_M + G_D = \frac{1}{R}$ → total conductance

2. Noise temperature: $\Omega \equiv S^I(0)/2G$

3. Level broadening: $\Gamma_\Omega \equiv \alpha\,\Omega$

4. Circuit response time: τ_{RC}

   → "fast" circuit: $\Gamma_\Omega \cdot \tau_{RC} \ll 1$

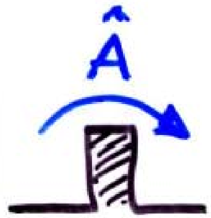
   → "slow" circuit: otherwise

5. Level splitting: ε → <u>detector bandwidth</u> !

GENERALIZED P(E) THEORY

① Tunneling Hamiltonian

$$\begin{cases} \hat{H} = \hat{H}_L + \hat{H}_R + \hat{H}_T \\ \hat{H}_L = \sum_k [\varepsilon_k + eV(t)]\, c_k^+ c_k \\ \hat{H}_R = \sum_k \varepsilon_k\, d_k^+ d_k \end{cases} , \quad \boxed{\hat{H}_T = \hat{A} + \hat{A}^+}$$



$$, \qquad \hat{A} = \sum_{kp} t_{kp}\, d_p^+ c_k$$

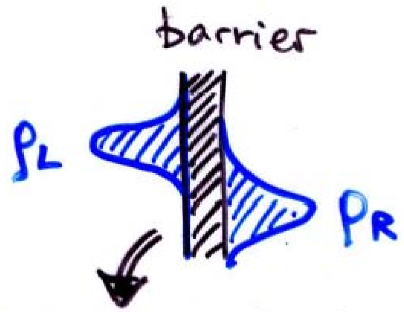
② Gauge transformation

$$\varepsilon_k + eV(t) \longrightarrow \varepsilon_k \quad \text{and} \quad \hat{A} \longrightarrow \hat{A} \cdot e^{i\varphi(t)}$$

where $\varphi(t) = e \cdot \int_{-\infty}^{t} dt'\, V(t')$

GENERALIZED P(E) THEORY 2

3 Average current and noise



$$S(0), \langle I \rangle = 2\pi |t|^2 \cdot \frac{e^2}{h} \cdot \iint dE\, dE'\, \rho_L(E)\, \rho_R(E')$$

$$\times \left\{ P_{LR}(E-E'+eV)\, f(E)\,[1-f(E')] \pm P_{RL}(E-E'+eV)\, f(E')\,[1-f(E)] \right\}$$

4. Probability of energy absorption (emission)

$$P_{LR}(E) = \frac{1}{2\pi\hbar} \int_{-\infty}^{+\infty} dt\, e^{iEt} \cdot \langle e^{i\varphi(t)} \cdot e^{-i\varphi(0)} \rangle$$

$$P_{RL}(E) = \frac{1}{2\pi\hbar} \int_{-\infty}^{+\infty} dt\, e^{iEt} \cdot \langle e^{-i\varphi(0)} \cdot e^{i\varphi(t)} \rangle$$

GENERALIZED P(E) THEORY 3

5 Cumulant expansion

$$\mathbb{P}(E) = \frac{1}{2\pi}\int dt\, e^{iEt - J_2(t) \pm J_3(t)}$$

where $J_2(t) = \frac{1}{2}\langle(\Delta\phi)^2\rangle + \frac{1}{2}\langle[\phi, \Delta\phi]\rangle$; $\Delta\phi \equiv \phi(t) - \phi(0)$

and $J_3(t) = \frac{1}{6}\langle(\Delta\phi)^3\rangle + \dots$

"quantum" noise

"classical" noise

6 Gaussian noise: $J_3 = 0$

$$\phi(t) = \frac{ie}{\hbar}\int \frac{d\omega}{2\pi\omega}\, e^{-i\omega t} \cdot Z(\omega) I(\omega)$$

$$\Rightarrow \boxed{J(t) = G_q \cdot \hat{\mathbb{P}} \int \frac{d\omega}{\omega^2}\, S^+(\omega)\, |Z(\omega)|^2 \left(1 - e^{-i\omega t}\right)}$$

where $S^+ = S^+_M + S^+_D$

APPLICATION TO QUANTUM DETECTORS

1. Average current and noise

$$S(0),\, e\langle I\rangle = \frac{2\pi e^2}{\hbar}\cdot|t|^2\cdot\Big\{P(\varepsilon)\, f(E_L)\,[1-f(E_R)]$$
$$\pm P(-\varepsilon)\, f(E_R)\,[1-f(E_L)]\Big\}$$
$$= \frac{2\pi e^2}{\hbar}\cdot|t|^2\cdot K(E_L,E_R)\cdot\left[P(\varepsilon)\pm P(-\varepsilon)\, e^{\beta\Delta E}\right]$$

where $\Delta E = E_L - E_R$

2. Current-to-noise ratio

$$\boxed{R \equiv \frac{e\langle I\rangle}{S(0)} = \frac{P(\varepsilon)-P(-\varepsilon)\cdot e^{\beta\Delta E}}{P(\varepsilon)+P(-\varepsilon)\, e^{\beta\Delta E}}}$$

QUANTUM DETECTION LIMIT

Weak coupling: $Z(\omega)/R_K \ll 1$!

$$\boxed{\begin{aligned} P(E) &= \frac{1}{2\pi\hbar} \int dt\, e^{iEt - J(t)} \\ &= \frac{1}{2\pi\hbar} \cdot \int dt\, e^{iEt} \left[1 - J(t)\right] \\ &= \frac{1}{2\pi R_K} \cdot \frac{|Z(E)|^2}{E^2} \cdot S_M^+(E/\hbar) \end{aligned}}$$

$$I \sim P(E) \sim \frac{S_M^+(E)}{E^2}$$

Inelastic transport:

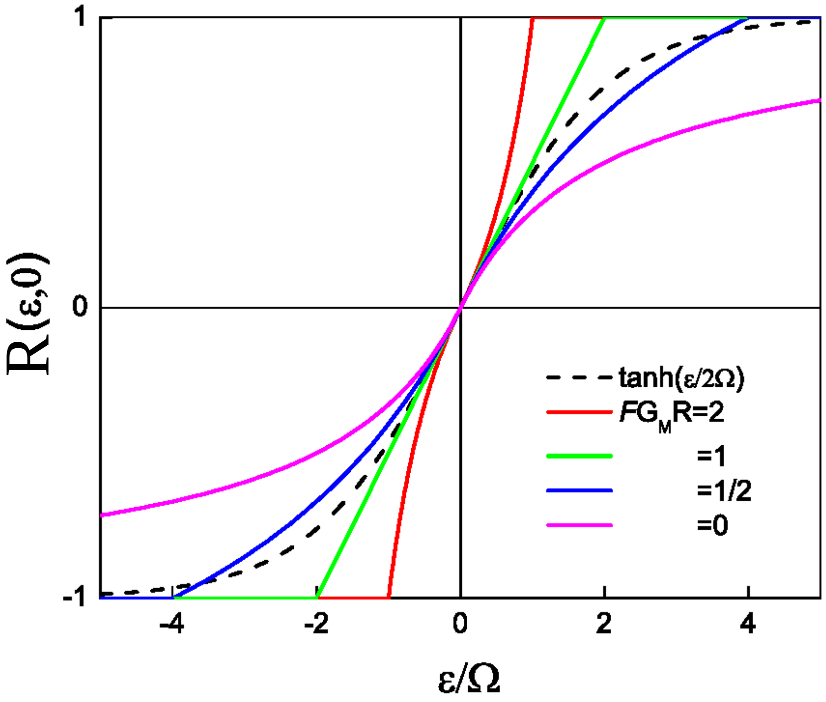
$$\Rightarrow \quad R \equiv \frac{e\langle I\rangle}{S(0)} = \frac{S_M^+(\varepsilon) - S_M^+(-\varepsilon)\, e^{\beta\Delta E}}{S_M^+(\varepsilon) + S_M^+(-\varepsilon)\, e^{\beta\Delta E}}$$

where $\Delta E = E_L - E_R$, $\varepsilon = eV + \Delta E$

QUANTUM LIMIT: RESULTS

Symmetric detector: $\Delta E = 0$

$$R(\varepsilon) = \frac{S^{+}(\varepsilon) - S^{+}(-\varepsilon)}{S^{+}(\varepsilon) + S^{+}(-\varepsilon)} \Rightarrow \boxed{\frac{\varepsilon}{F \cdot G_M R \cdot (eV_M - |\varepsilon|) + |\varepsilon|}}\ ; \ |\varepsilon| \leq eV_M$$



and $R(\varepsilon) = 1$ for $|\varepsilon| > eV_M$

1. Equilibrium: $S^{+}(\varepsilon) = e^{\beta\varepsilon} S^{+}(-\varepsilon)$

$$\Rightarrow R(\varepsilon) = \mathrm{th}\left(\frac{\varepsilon}{2k_BT}\right)$$

2. Low impedance: $G_M R \to 0$

$$\Rightarrow R(\varepsilon) = \frac{\varepsilon/\Omega}{2 + |\varepsilon/\Omega|}$$

where $\Omega \to$ noise temperature

CLASSICAL DETECTION LIMIT

1 Fast circuit: $\alpha \Omega \tau_{RC} \ll 1$

$$J(t) = 2\pi\alpha\Omega \left\{ |t| + i\partial_\omega S'(0)/S(0) \cdot \mathrm{sign}(t) \right\}$$

$$\Rightarrow \quad P(E) = \frac{2\alpha\Omega}{E^2 + (2\pi\alpha\Omega)^2} \cdot \left[1 + E \cdot \partial_\omega S(0)/S(0) \right]$$

2 Slow circuit: $\alpha \Omega \tau_{RC} \gg 1$

$$J(t) = \pi\alpha\left(\Omega/t_{RC}\right)^2 \cdot t^2 + i\pi\alpha\left(1/t_{RC}\right) \cdot t$$

$$\Rightarrow \quad P(E) = \frac{e^{-E^2/4E_C\Omega}}{\sqrt{4\pi E_C \Omega}} \cdot \left[1 + E \cdot \partial_\omega S(0)/S(0) \right]$$

3 Linear response: $\partial_\omega S(0) = 1/R \Rightarrow \frac{1}{2\Omega}$

$$\Rightarrow \quad \boxed{R(\varepsilon) = \frac{\varepsilon}{2\Omega}}$$

Universality ?

UNIVERSALITY OF CLASSICAL DETECTION

Sketch of proof

Long-time limit: $S(\omega) = S(0) + \omega \cdot \partial_\omega S(0)$

$\Rightarrow \quad J(t) = G_q \cdot S(0) \cdot H(t) + i G_q \partial_\omega S(0) \partial_t H(t)$

where $H(t) = \int \frac{d\omega}{\omega^2} |z(\omega)|^2 \cdot [1 - \cos(\omega t)]$ small at $dt\Omega \sim 1$

$$\Rightarrow \quad P(E) + P(-E) = \frac{1}{\pi} \int dt \, \exp[-G_q S(0) H(t)] \cdot \cos(Et)$$

$$\Rightarrow \quad P(E) - P(-E) = \frac{1}{\pi} \int dt \, \exp[-G_q S(0) H(t)] \times G_q \partial_\omega S(0) \partial_t H(t) \cdot \sin(Et)$$

$$\Rightarrow \quad \boxed{\frac{P(E) - P(-E)}{P(E) + P(-E)} = \frac{\partial_\omega S(0)}{S(0)} \cdot E}$$

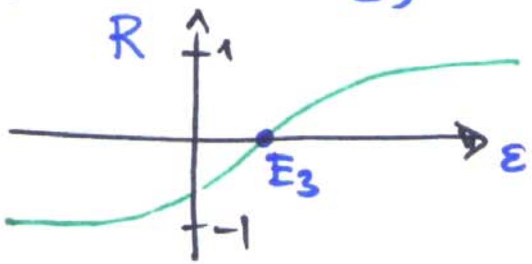
for $E \sim d\Omega \ll \Omega$
$\rightarrow$ classical limit.

DETECTION OF THIRD CUMULANT

① General property: $J_3(t) = -J_3(-t)$

$\Rightarrow$ breaks symmetry: $P(E) \to P(E \pm E_3)$

$\Rightarrow$ $R(\varepsilon) = \dfrac{\varepsilon - E_3}{2\Omega}$ $\Rightarrow$



② Fast circuit: $J_3(t) = e^3/6 \cdot \langle\langle V^3 \rangle\rangle \cdot t$ $\to$ Markovian limit

$\Rightarrow$ $\boxed{E_3 = (1/6)\cdot(2\pi\alpha/e)^3 \cdot \langle\langle I^3 \rangle\rangle}$

③ Slow circuit: $J_3(t) = e^3/6 \cdot \langle (\delta V)^3 \rangle \cdot t^3$

$\Rightarrow$ $\boxed{E_3 = \dfrac{(2\pi\alpha)^2}{6\tau_{RC}\Omega e^3} \cdot \langle\langle I^3 \rangle\rangle}$ Experiment is needed

CONCLUSION

1. Non-equilibrium noise → Breakdown of FDT !
2. Generalized P(E) theory of tunneling
3. Weak-coupling theory of resonant tunneling
   - → Quantum limit for coherent transport
   - → Classical limit: Universality !
4. Third cumulant detection
   - → fast circuit
   - → slow circuit