GETTING INFORMATION FROM THE ELECTRICAL–HEAT MIXED NOISE

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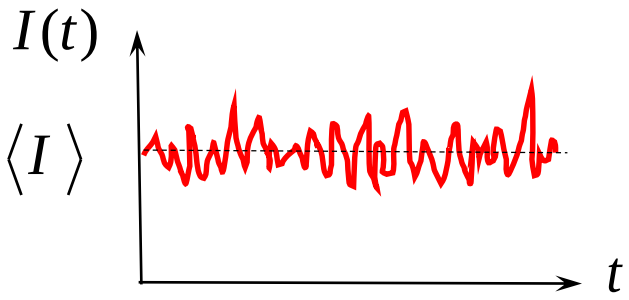
FLUCTUATIONS

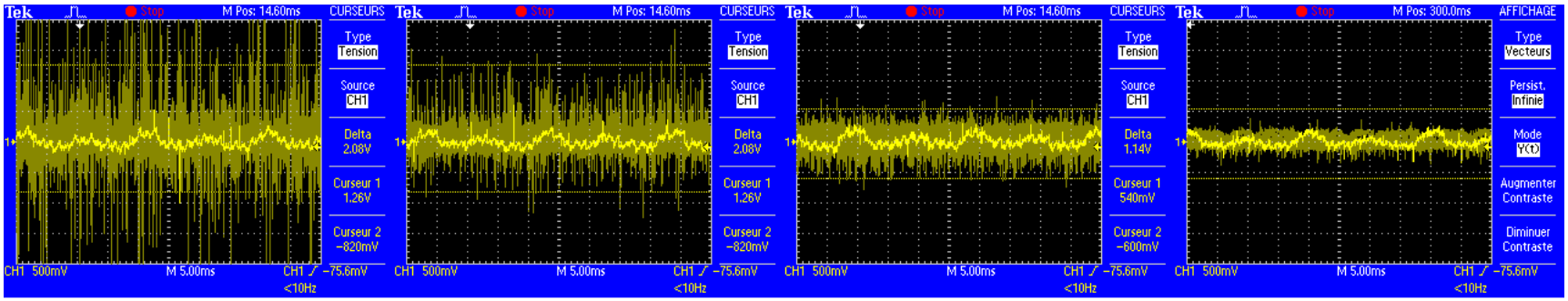
DC VOLTAGE



ORIGIN

- Thermal agitation
- Impurities/defects : 1/f noise
- Electromagnetic environment
- Probabilistic nature of transfer in quantum systems

FILTERING



No filtering component

2×2.2 nF ceramic between alimentation and structure

2×2.2 nF + 100 nF polyester between power supply leads

2×2.2 nF + 100 nF polyester + toroid filter

Noise

Condensed-matter physics

The noise is the signal

Rolf Landauer

Noise is not only a hindrance to signal detection. Advances in measurement techniques mean that it can now be used to probe the kinetics of electrons. That is because interactions between electrons can regulate their relative motion, and so reduce noise. These interactions include not only the Coulomb repulsion, but also the Pauli exclusion principle, which prevents two electrons from behaving alike. Several new noise measurements[1–4] in different systems have produced insights into how these interactions affect electronic behaviour.

Nature (1998)

CONTENTS

1. Getting information from **electrical noise**
2. Electrical noise spectrum
3. Mixed noise spectrum
4. Getting information from **mixed noise**

1. GETTING INFORMATION FROM ELECTRICAL NOISE

NON-SYMMETRIZED ELECTRICAL NOISE

DEFINITION

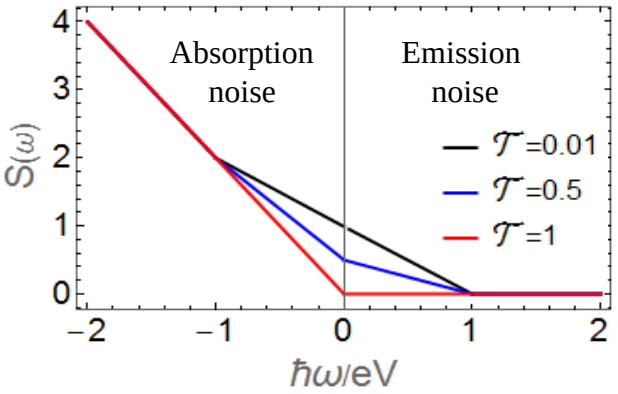
$$S_{\alpha\beta}(\omega) = \int_{-\infty}^{+\infty} dt\ e^{-i\omega t} \left\langle \Delta\hat{I}_\alpha(t)\ \Delta\hat{I}_\beta(0) \right\rangle$$

JETP 1997 – Lesovik, Loosen

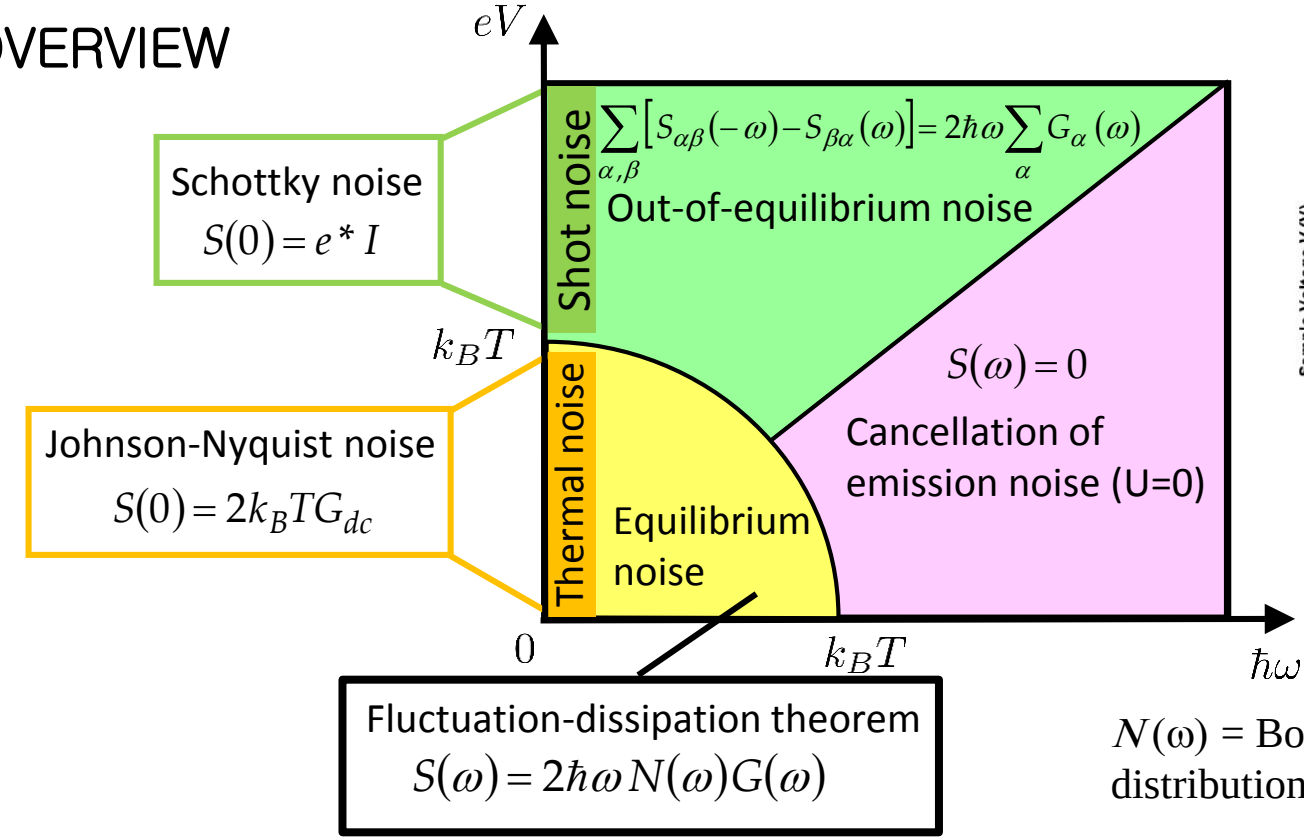
$\omega > 0$: Emission noise

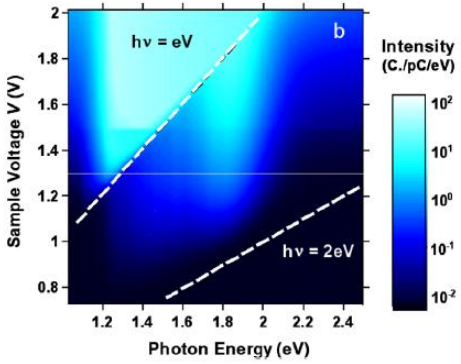
$\omega < 0$: Absorption noise

low temperature



OVERVIEW





PRL 2009 – Schull et al.
PRB 2016 – Xu et al.

$N(\omega)$ = Bose-Einstein distribution function

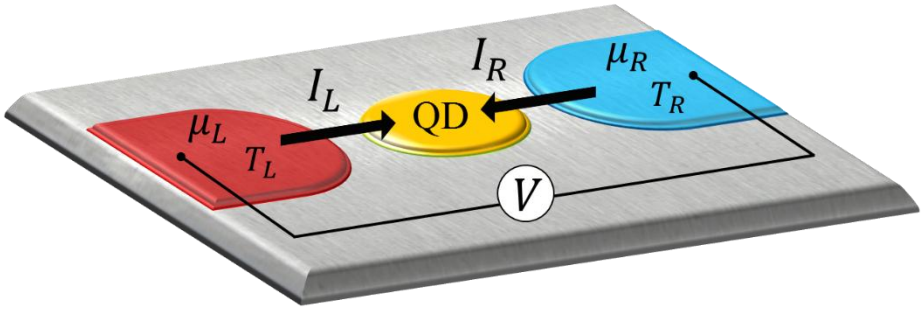
2. ELECTRICAL NOISE SPECTRUM

$$S_{\alpha\beta}(\omega) = \int_{-\infty}^{+\infty} dt\ e^{-i\omega t} \left\langle \Delta \hat{I}_{\alpha}(t)\ \Delta \hat{I}_{\beta}(0) \right\rangle$$

Electrical currents

QUANTUM DOT

SYSTEM



Single-level non-interacting QD connected to 2 reservoirs

$$eV = \mu_L - \mu_R$$
$$\Delta T = T_L - T_R$$

HAMILTONIAN

$$H = \sum_{k\in L} \varepsilon_k c_k^+ c_k + \sum_{k\in R} \varepsilon_k c_k^+ c_k + \varepsilon_0 d^+ d + \sum_{k\in L,R} V_k c_k^+ d + h.c.$$

dot energy **hopping energy**

ELECTRICAL CURRENT

$$\hat{I}_\alpha = -e\frac{d\hat{N}_\alpha}{dt} = -\frac{e}{i\hbar}\left[\hat{N}_\alpha, H\right] \qquad \hat{N}_\alpha = \sum_{k\in\alpha} c_k^+ c_k$$

$$\Rightarrow \hat{I}_\alpha = \frac{ie}{\hbar}\sum_{k\in\alpha}\left(V_k c_k^+ d - V_k^* d^+ c_k\right)$$

CONSERVATION LAWS

$$\left\langle \hat{I}_L \right\rangle + \left\langle \hat{I}_R \right\rangle = 0$$ **Particle conservation**

$$\sum_{\alpha\beta} S_{\alpha\beta}(\omega = 0) = 0$$ **Fluctuations conservation**

OUR WORK

METHOD

Equation of motion technique for the non-equilibrium Keldysh Green function

- Initially developed for the calculations of time-dependent current

  PRB 1994 – Jauho, Wingreen, Meir

- Used for the calculations of symmetrized noise only

  PRB 2008 – Souza, Jauho, Egues

ADVANTAGES

- Relation between $t(\varepsilon)$ and $G^r(\varepsilon)$ appears explicitly: $t(\varepsilon) = i\,\Gamma\,G^r(\varepsilon)$
- Calculations can be performed at any frequency, temperature, voltage and barriers strength
- Response to time-dependent excitation can be calculated
- Interactions can be included to some extent (Hartee level)

CALCULATIONS OF

- **Electrical noise spectrum** (as a test to validate the method)
- **Mixed electrical-heat noise spectrum**

OUT-OF-EQUILIBRIUM TECHNIQUE

NOISE

$$S_{\alpha\beta}(\omega)=\int_{-\infty}^{+\infty} dt\, e^{-i\omega t}\left\langle \Delta\hat{I}_{\alpha}(t)\;\Delta\hat{I}_{\beta}(0)\right\rangle \qquad \text{where}\quad \Delta\hat{I}_{\alpha}(t)=\hat{I}_{\alpha}(t)-\left\langle\hat{I}_{\alpha}\right\rangle$$

$$S_{\alpha\beta}(t,t')=\frac{e^2}{\hbar^2}\sum_{k\in\alpha,k'\in\beta}\left[\,V_kV_{k'}G_1^{cd,>}(t,t')-V_kV_{k'}^{*}G_2^{cd,>}(t,t')-V_k^{*}V_{k'}G_3^{cd,>}(t,t')+V_k^{*}V_{k'}^{*}G_4^{cd,>}(t,t')\,\right]-\left\langle\hat{I}_{\alpha}\right\rangle\left\langle\hat{I}_{\beta}\right\rangle$$

GREEN FUNCTIONS

$$G_1^{cd,>}(t,t')=-\left\langle\, c_k^{+}(t)\,d(t)\,c_{k'}^{+}(t')\,d(t')\,\right\rangle \qquad G_3^{cd,>}(t,t')=-\left\langle\, d^{+}(t)\,c_k(t)\,c_{k'}^{+}(t')\,d(t')\,\right\rangle$$

$$G_2^{cd,>}(t,t')=-\left\langle\, c_k^{+}(t)\,d(t)\,d^{+}(t')\,c_{k'}(t')\,\right\rangle \qquad G_4^{cd,>}(t,t')=-\left\langle\, d^{+}(t)\,c_k(t)\,d^{+}(t')\,c_{k'}(t')\,\right\rangle$$

ASSUMPTIONS

- Two-particles dot Green functions decoupling
- Time-independent excitation
- Wide-band limit
- Symmetrical barriers strength

$$\Gamma=\Gamma_L=\Gamma_R=2\pi\left|V\right|^2\rho(\varepsilon_F)$$

RESULTS

ELECTRICAL NOISE SPECTRUM

$$S_{\alpha\beta}(\omega)=\frac{e^2}{h}\sum_{\gamma\delta}\int_{-\infty}^{+\infty}d\varepsilon\, M_{\alpha\beta}^{\gamma\delta}(\varepsilon,\omega)\, f_\gamma(\varepsilon)\left[1-f_\delta(\varepsilon-\hbar\omega)\right]$$

Auto-correlators = reals & positives
Cross-correlators = complex quantities

Transmission amplitude

$$t(\varepsilon)=i\Gamma G^r(\varepsilon)$$

Transmission coefficient

$$\mathcal{T}(\varepsilon)=t(\varepsilon)\,t^*(\varepsilon)$$

MATRIX

$M_{\alpha\beta}^{\gamma\delta}(\varepsilon,\omega)$	$\gamma=\delta=L$	$\gamma=\delta=R$	$\gamma=L,\ \delta=R$	$\gamma=R,\ \delta=L$
$\alpha=L$ $\beta=L$	$\mathcal{T}(\varepsilon)\mathcal{T}(\varepsilon-\hbar\omega)$ $+\lvert t(\varepsilon)-t(\varepsilon-\hbar\omega)\rvert^2$	$\mathcal{T}(\varepsilon)\mathcal{T}(\varepsilon-\hbar\omega)$	$\mathcal{T}(\varepsilon-\hbar\omega)[1-\mathcal{T}(\varepsilon)]$	$\mathcal{T}(\varepsilon)[1-\mathcal{T}(\varepsilon-\hbar\omega)]$
$\alpha=R$ $\beta=R$	$\mathcal{T}(\varepsilon)\mathcal{T}(\varepsilon-\hbar\omega)$	$\mathcal{T}(\varepsilon)\mathcal{T}(\varepsilon-\hbar\omega)$ $+\lvert t(\varepsilon)-t(\varepsilon-\hbar\omega)\rvert^2$	$\mathcal{T}(\varepsilon)[1-\mathcal{T}(\varepsilon-\hbar\omega)]$	$\mathcal{T}(\varepsilon-\hbar\omega)[1-\mathcal{T}(\varepsilon)]$
$\alpha=L$ $\beta=R$	$t(\varepsilon)t^*(\varepsilon-\hbar\omega)$ $\times[(1-t^*(\varepsilon))(1-t(\varepsilon-\hbar\omega))-1]$	$t^*(\varepsilon)t(\varepsilon-\hbar\omega)$ $\times[(1-t(\varepsilon))(1-t^*(\varepsilon-\hbar\omega))-1]$	$t(\varepsilon)t(\varepsilon-\hbar\omega)[1-t^*(\varepsilon)]$ $\times[1-t^*(\varepsilon-\hbar\omega)]$	$t^*(\varepsilon)t^*(\varepsilon-\hbar\omega)[1-t(\varepsilon)]$ $\times[1-t(\varepsilon-\hbar\omega)]$
$\alpha=R$ $\beta=L$	$t^*(\varepsilon)t(\varepsilon-\hbar\omega)$ $\times[(1-t(\varepsilon))(1-t^*(\varepsilon-\hbar\omega))-1]$	$t(\varepsilon)t^*(\varepsilon-\hbar\omega)$ $\times[(1-t^*(\varepsilon))(1-t(\varepsilon-\hbar\omega))-1]$	$t^*(\varepsilon)t^*(\varepsilon-\hbar\omega)[1-t(\varepsilon)]$ $\times[1-t(\varepsilon-\hbar\omega)]$	$t(\varepsilon)t(\varepsilon-\hbar\omega)[1-t^*(\varepsilon)]$ $\times[1-t^*(\varepsilon-\hbar\omega)]$

CHARGE NOISE

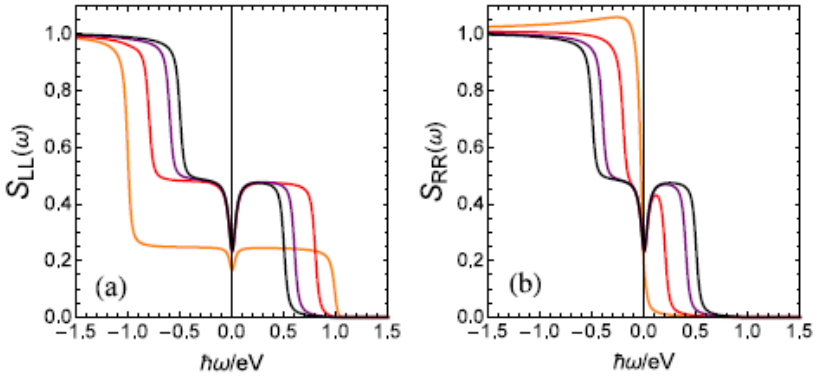
$$\sum_{\alpha\beta}S_{\alpha\beta}(\omega)=\frac{e^2}{h}\int_{-\infty}^{+\infty}d\varepsilon\left|t(\varepsilon)-t(\varepsilon-\hbar\omega)\right|^2\sum_{\gamma\delta}f_\gamma(\varepsilon)\left[1-f_\delta(\varepsilon-\hbar\omega)\right]=\omega^2 S_{QQ}(\omega)$$

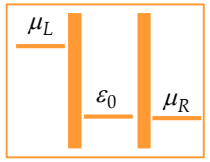
In agreement with *Blanter, Büttiker (2000)*

Cancel at zero-frequency or for energy independent transmission

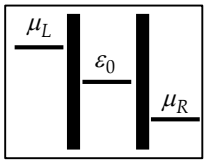
MAKE A DISTINCTION BETWEEN S_{LL} AND S_{RR}

NON-SYMMETRIC POTENTIAL PROFILE



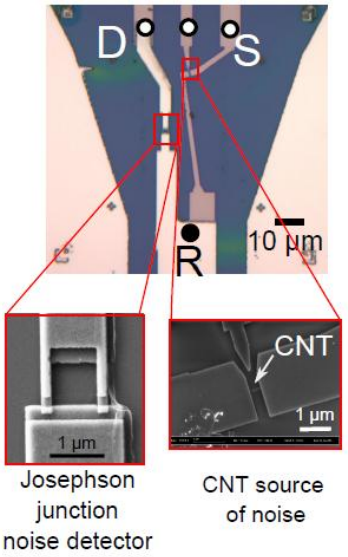


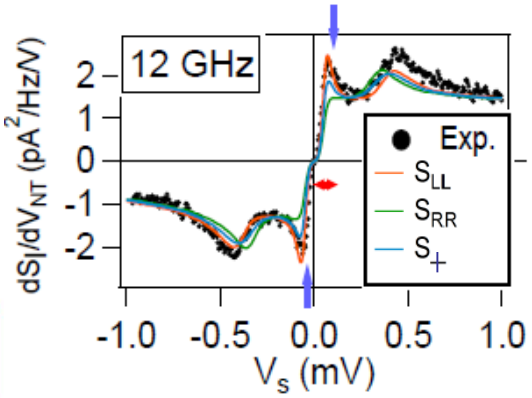
$$\Rightarrow S_{LL}(\omega) \neq S_{RR}(\omega)$$



$$\Rightarrow S_{LL}(\omega) = S_{RR}(\omega)$$

NANOTUBE QUANTUM DOT

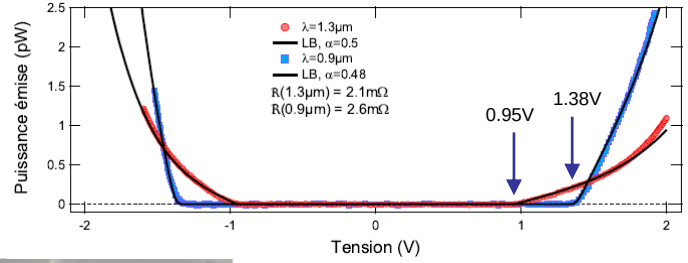


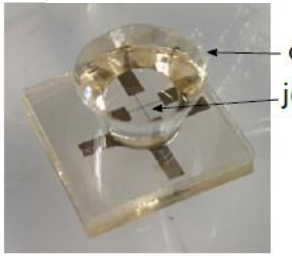


R. Delagrange, PhD thesis (2016)

TUNNEL JONCTION

$$P = \alpha^2 S_{LL}(\omega) + (1-\alpha)^2 S_{RR}(\omega) - 2\alpha(1-\alpha) S_{LR}(\omega)$$





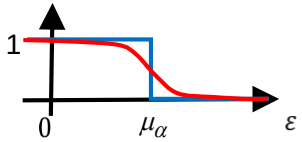
P. Février, PhD thesis (2017)

S_{LL} EMISSION NOISE

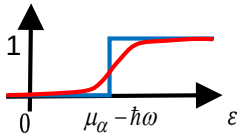
FULL EXPRESSION

$$S_{LL}(\omega) = \frac{e^2}{h}\int_{-\infty}^{+\infty} d\varepsilon \Big[f_R(\varepsilon)\,[1-f_R(\varepsilon-\hbar\omega)]\,\mathcal{T}(\varepsilon)\,\mathcal{T}(\varepsilon-\hbar\omega) + f_R(\varepsilon)\,[1-f_L(\varepsilon-\hbar\omega)]\,\mathcal{T}(\varepsilon)\,[1-\mathcal{T}(\varepsilon-\hbar\omega)] + f_L(\varepsilon)\,[1-f_R(\varepsilon-\hbar\omega)]\,[1-\mathcal{T}(\varepsilon)]\,\mathcal{T}(\varepsilon-\hbar\omega) + f_L(\varepsilon)\,[1-f_L(\varepsilon-\hbar\omega)]\,\Big[\mathcal{T}(\varepsilon)\,\mathcal{T}(\varepsilon-\hbar\omega) + \big|t\,(\varepsilon)-t\,(\varepsilon-\hbar\omega)\big|^2\Big] \Big]$$

Fermi-Dirac distribution function for **electrons**



Distribution function for **holes**

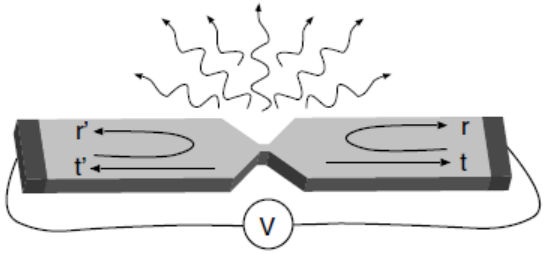


This term is **present** in *Büttiker (1992)* and *Hammer, Belzig (2011)* but **absent** in *Nazarov, Blanter (2009)*

→ **The physical processes which contribute to the noise involved an electron-hole pair**

RADIATION PRODUCED BY CURRENT FLUCTUATIONS

Glauber (1951) : "A classical current produces photons with Poisson statistics"



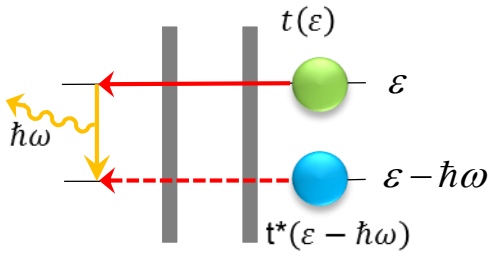
QUANTUM SYSTEMS

PRL 2001 – Beenakker, Schomerus
PRL 2004 – Gabelli, Reydellet, Fève, Berroir, Plaçais, Roche, Glattli
CR Phy 2016 – Forgues, Gasse, Lupien, Reulet

→ **Interpretation in terms of photoemission ?**

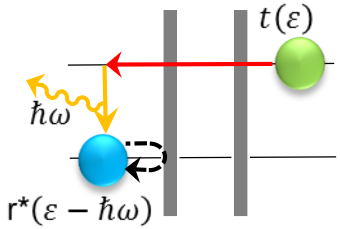
S_{LL} EMISSION NOISE

CONTRIBUTION $\propto f_R(\varepsilon)\left[1-f_R(\varepsilon-\hbar\omega)\right]$



$$\Rightarrow \left| t(\varepsilon)\, t^*(\varepsilon-\hbar\omega) \right|^2 = \mathcal{T}(\varepsilon)\, \mathcal{T}(\varepsilon-\hbar\omega)$$

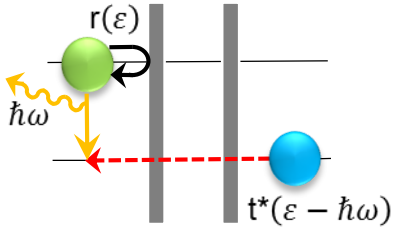
CONTRIBUTION $\propto f_R(\varepsilon)\left[1-f_L(\varepsilon-\hbar\omega)\right]$



$$\Rightarrow \left| t(\varepsilon)\, r^*(\varepsilon-\hbar\omega) \right|^2 = \mathcal{T}(\varepsilon)\, \left[1-\mathcal{T}(\varepsilon-\hbar\omega)\right]$$

Reflection coefficient : $\mathcal{R}(\varepsilon-\hbar\omega) = 1-\mathcal{T}(\varepsilon-\hbar\omega)$

CONTRIBUTION $\propto f_L(\varepsilon)\left[1-f_R(\varepsilon-\hbar\omega)\right]$



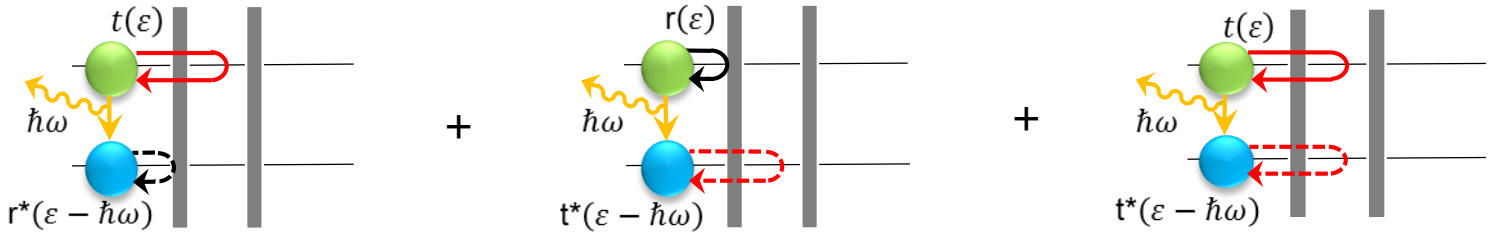
$$\Rightarrow \left| r(\varepsilon)\, t^*(\varepsilon-\hbar\omega) \right|^2 = \left[1-\mathcal{T}(\varepsilon)\right] \mathcal{T}(\varepsilon-\hbar\omega)$$

LEGEND

- Electron
- Hole
- Recombination
- Emission
- Transmission
- Reflection

S_{LL} EMISSION NOISE

CONTRIBUTION $\propto f_L(\varepsilon)[1-f_L(\varepsilon-\hbar\omega)]$



$t_1(\varepsilon,\omega)=t(\varepsilon)\,r^*(\varepsilon-\hbar\omega)$ $t_2(\varepsilon,\omega)=r(\varepsilon)\,t^*(\varepsilon-\hbar\omega)$ $t_3(\varepsilon,\omega)=t(\varepsilon)\,t^*(\varepsilon-\hbar\omega)$

When more than one process are involved : one has to take the coherent superposition

$$\Rightarrow \left|t_1(\varepsilon,\omega)+t_2(\varepsilon,\omega)+t_3(\varepsilon,\omega)\right|^2=\mathcal{T}(\varepsilon)\,\mathcal{T}(\varepsilon-\hbar\omega)+\left|t(\varepsilon)-t(\varepsilon-\hbar\omega)\right|^2$$

The presence of this term is thus confirmed

→ **Powerful method to get each contribution to S_{LL} (ω) without heavy calculation !**

PRB 2016 – R. Zamoum, M. Lavagna, A. Crépieux

3. MIXED NOISE SPECTRUM

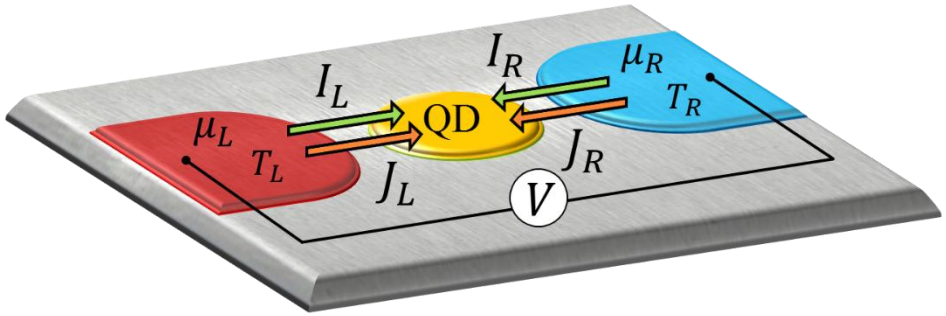
$$S^{IJ}_{\alpha\beta}(\omega) = \int_{-\infty}^{+\infty} dt\ e^{i\omega t} \left\langle \Delta \hat{I}_{\alpha}(t)\ \Delta \hat{J}_{\beta}(0) \right\rangle$$

Electrical current

Heat current

QUANTUM DOT

SYSTEM



Single-level non-interacting QD connected to 2 reservoirs

HAMILTONIAN

$$H = \sum_{k\in L} \varepsilon_k c_k^+ c_k + \sum_{k\in R} \varepsilon_k c_k^+ c_k + \varepsilon_0\, d^+ d + \sum_{k\in L,R} V_k\, c_k^+ d + h.c.$$

HEAT CURRENT

$$\hat{J}_\alpha = -\frac{d\left(\hat{H}_\alpha - \mu_\alpha \hat{N}_\alpha\right)}{dt} \qquad \hat{H}_\alpha = \sum_{k\in\alpha} \varepsilon_k c_k^+ c_k$$

$$\Rightarrow\ \hat{\mathrm{J}}_\alpha = \frac{i}{\hbar} \sum_{k\in\alpha} \left(\varepsilon_k - \mu_\alpha\right) \left(V_k c_k^+ d - V_k^* d^+ c_k \right)$$

CONSERVATION LAWS

$$\left\langle \hat{I}_L \right\rangle + \left\langle \hat{I}_R \right\rangle = 0$$ **Particle conservation**

$$\left\langle \hat{\mathrm{J}}_L \right\rangle + \left\langle \hat{\mathrm{J}}_R \right\rangle = V \left\langle \hat{I}_R \right\rangle \neq 0$$ **Heat is not conserved**

$$\int_{-\infty}^{\infty} \left\langle P^{el}(t) P^{el}(0) \right\rangle dt = \int_{-\infty}^{\infty} \left\langle P^{heat}(t) P^{heat}(0) \right\rangle dt$$ **Power conservation**

with $P^{el} = V\,\hat{I}_L$ and $P^{heat} = \hat{\mathrm{J}}_L + \hat{\mathrm{J}}_R$

ZERO-FREQUENCY MIXED NOISE

LANDAUER/BUTTIKER-LIKE EXPRESSION

$$S_{LR}^{I\,J}(\omega=0) = -\frac{e}{h}\int_{-\infty}^{+\infty} d\varepsilon\ (\varepsilon-\mu_R)\Big[\mathcal{T}^2(\varepsilon)\left(f_L(\varepsilon)\left[1-f_L(\varepsilon)\right]+f_R(\varepsilon)\left[1-f_R(\varepsilon)\right]\right) + \mathcal{T}(\varepsilon)\left[1-\mathcal{T}(\varepsilon)\right]\left(f_L(\varepsilon)\left[1-f_R(\varepsilon)\right]+f_R(\varepsilon)\left[1-f_L(\varepsilon)\right]\right)\Big]$$

Additional factor

JPCM 2015 – A. Crépieux, F. Michelini

AT EQUILIBRIUM (V→0)

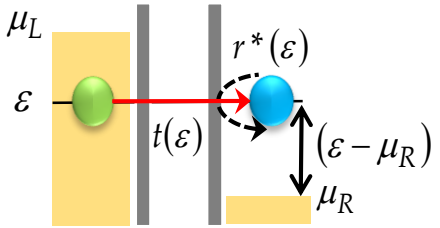
$$S_{LR}^{I\,J}(\omega=0) = 2k_B T^2\, S_T\, G$$

Seebeck coefficient

→ Mixed noise gives the Seebeck coefficient

ZERO-TEMPERATURE LIMIT (T→0)

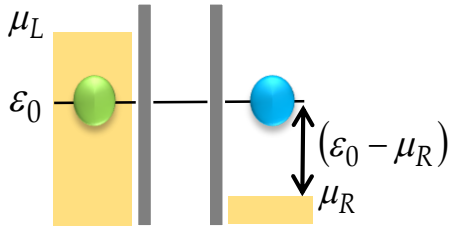
$$S_{LR}^{I\,J}(\omega=0) = -\frac{e}{h}\int_{\mu_R}^{\mu_L} d\varepsilon\ (\varepsilon-\mu_R)\,\mathcal{T}(\varepsilon)\,\left[1-\mathcal{T}(\varepsilon)\right]$$



→ Mixed noise gives the energy lost during the transfer

SCHOTTKY LIMIT ($\mathcal{T} << 1$)

$$S_{LR}^{I\,J}(\omega=0) = (\varepsilon_0-\mu_R)\, I_R$$



→ Mixed noise gives the energy difference (ε_0-μ_R)

FINITE-FREQUENCY MIXED NOISE

RESULT

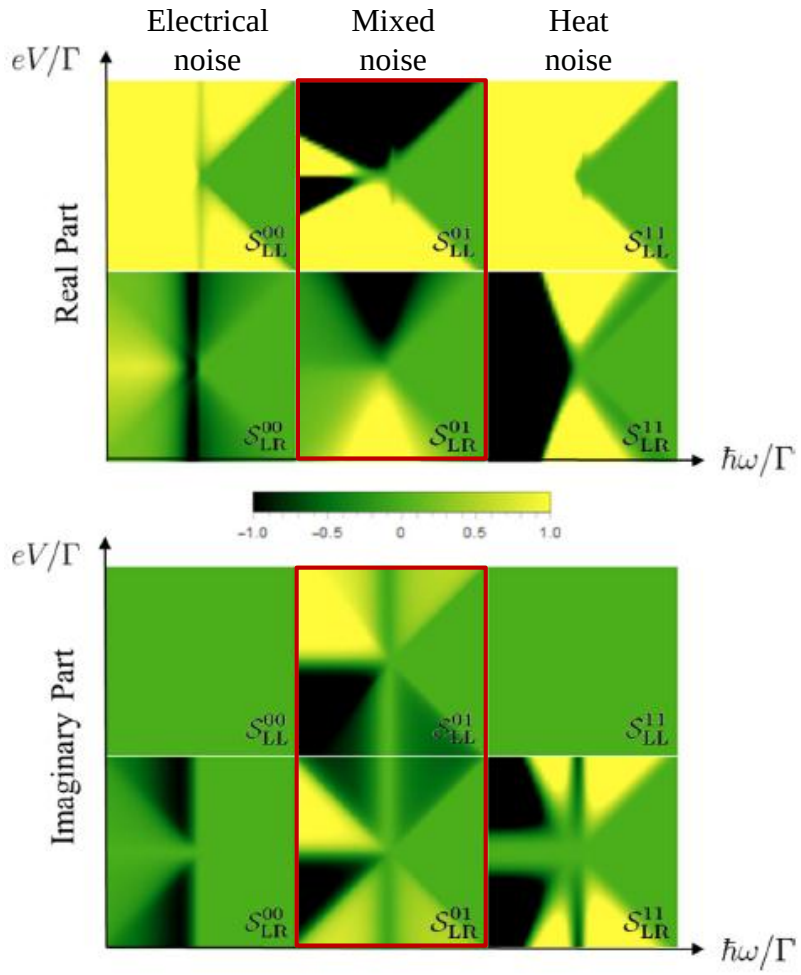
$$S^{I\,J}_{\alpha\beta}(\omega) = \frac{e}{h}\int_{-\infty}^{+\infty} d\varepsilon \;\Big[\big(\varepsilon-\mu_\beta\big)\big[\mathcal{A}_{\alpha\beta}(\varepsilon,\omega)+\mathcal{B}^{*}_{\beta\alpha}(\varepsilon,\omega)\big] + \big(\varepsilon-\hbar\omega-\mu_\beta\big)\big[\mathcal{B}_{\alpha\beta}(\varepsilon,\omega)+C_{\alpha\beta}(\varepsilon,\omega)\big]\Big]$$

$$\mathcal{A}_{\alpha\beta}(\varepsilon,\omega) = \mathcal{T}(\varepsilon-\hbar\omega)f^{h}_{M}(\varepsilon-\hbar\omega)\big[\mathcal{T}(\varepsilon)f^{e}_{M}(\varepsilon) + [\delta_{\alpha\beta}-t(\varepsilon)]f^{e}_{\alpha}(\varepsilon) + [\delta_{\alpha\beta}-t^{*}(\varepsilon)]f^{e}_{\beta}(\varepsilon)\big],$$

$$\mathcal{B}_{\alpha\beta}(\varepsilon,\omega) = t(\varepsilon)t(\varepsilon-\hbar\omega)\big[f^{e}_{\alpha}(\varepsilon)-t^{*}(\varepsilon)f^{e}_{M}(\varepsilon)\big] \times\big[f^{h}_{\beta}(\varepsilon-\hbar\omega)-t^{*}(\varepsilon-\hbar\omega)f^{h}_{M}(\varepsilon-\hbar\omega)\big],$$

$$\mathcal{C}_{\alpha\beta}(\varepsilon,\omega) = \mathcal{T}(\varepsilon)f^{e}_{M}(\varepsilon)\big[\mathcal{T}(\varepsilon-\hbar\omega)f^{h}_{M}(\varepsilon-\hbar\omega) + [\delta_{\alpha\beta}-t^{*}(\varepsilon-\hbar\omega)]f^{h}_{\alpha}(\varepsilon-\hbar\omega) + [\delta_{\alpha\beta}-t(\varepsilon-\hbar\omega)]f^{h}_{\beta}(\varepsilon-\hbar\omega)\big],$$

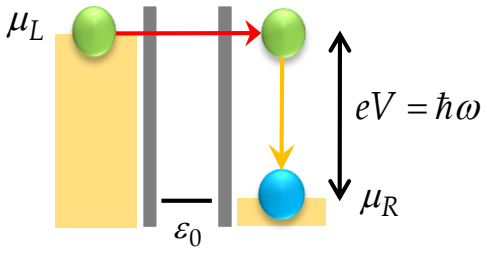
PRB 2016 – Eyméoud, Crépieux

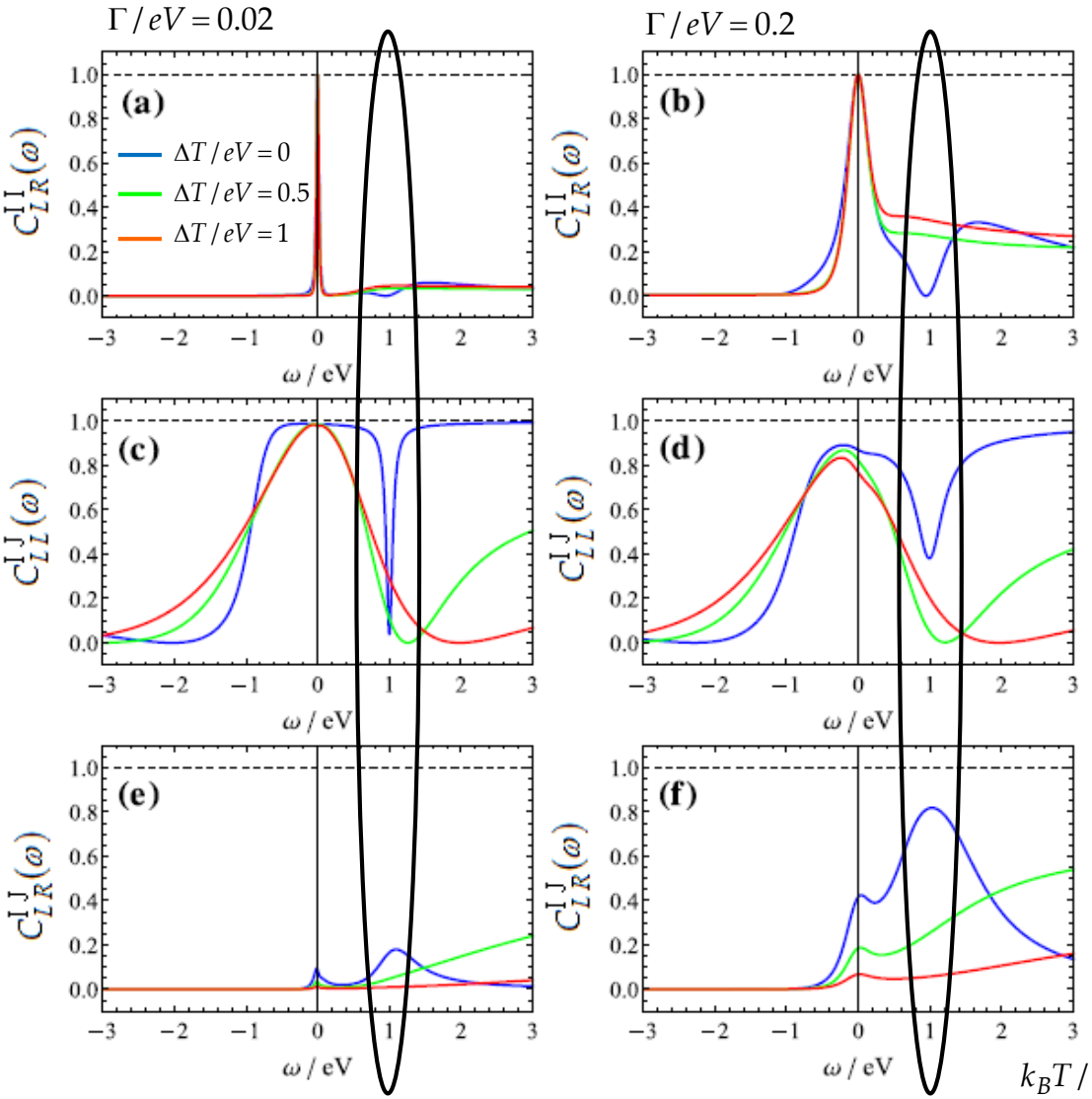


SPECTRAL COHERENCE

$$C^{I\,J}_{\alpha\beta}(\omega) = \frac{\left| S^{I\,J}_{\alpha\beta}(\omega) \right|^2}{S^{I\,I}_{\alpha\alpha}(\omega)\, S^{J\,J}_{\beta\beta}(\omega)}$$

$$\begin{cases} C^{I\,J}_{\alpha\beta}(\omega) = 1 \;\Rightarrow\; \text{max. of coherence} \\ C^{I\,J}_{\alpha\beta}(\omega) = 0 \;\Rightarrow\; \text{no coherence} \end{cases}$$





PRB 2016 – Eyméoud, Crépieux

Electricity to heat conversion

4. GETTING INFORMATION FROM MIXED NOISE

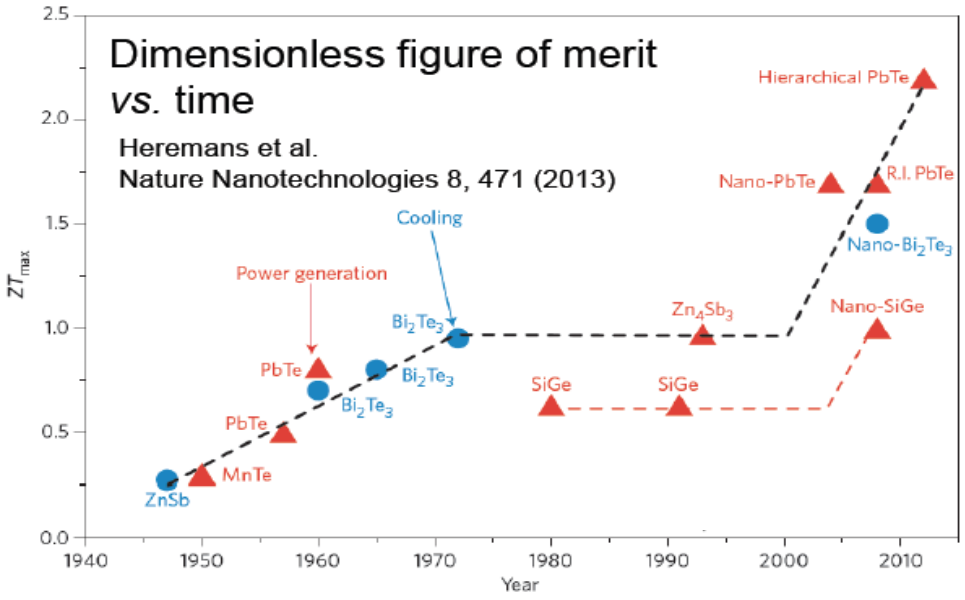
THERMOELECTRIC CONVERSION

FIGURE OF MERIT

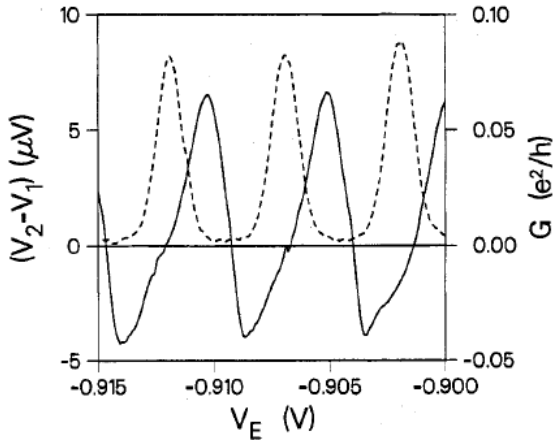
$$ZT = \frac{S_T^2 T G}{K} \qquad S_T = -\frac{V}{\Delta T}\Big|_{I=0}$$

G = Electrical conductance
S_T = Seebeck coefficient
K = Thermal conductance
T = Average temperature

$$\eta_{\max} = \eta_C \frac{\sqrt{ZT+1}-1}{\sqrt{ZT+1}+1} \approx 0.17\eta_C$$



MEASUREMENT OF THERMOPOWER IN QD



EPL 1993 – Staring, Molenkamp, Alphenaar, van Houten, Buyk, Mabesoone, Beenakker, Foxen

For a review, see: *CR Phy 2016 – Svilans, Leijnse, Linke*

LINEAR RESPONSE

$$V \approx \Delta T \approx \omega \approx 0$$

CONDUCTANCE MATRIX

$$\begin{matrix}\text{Electrical current} \longrightarrow \\ \text{Heat current} \longrightarrow\end{matrix} \begin{pmatrix} I \\ J \end{pmatrix} = \begin{pmatrix} G & GS_T \\ G\Pi & K \end{pmatrix} \begin{pmatrix} V \\ \Delta T \end{pmatrix}$$

$$\Pi = S_T T$$

$$K = \tilde{K} - S_T^2 T G$$

EXPRESSIONS FOR NOISES

$$S^{II} = 2k_B T G$$

$$S^{JJ} = 2k_B T^2 \tilde{K}$$

$$S^{IJ} = 2k_B T^2 S_T G$$

Fluctuation-dissipation theorem applies for any kind of noises

FIGURE OF MERIT

$$ZT = \frac{S_T^2 T G}{K}$$

$$ZT = \frac{\left(S^{IJ}\right)^2}{S^{II} S^{JJ} - \left(S^{IJ}\right)^2}$$

$$\begin{cases} ZT = 0 \quad \text{when} \quad S^{IJ} = 0 \\ S^{II} S^{JJ} > \left(S^{IJ}\right)^2 \quad \text{due to Cauchy – Swartz inequality} \end{cases}$$

Electrical noise Heat noise Mixed noise

JPCM 2015 – A. Crépieux, F. Michelini

SCHOTTKY REGIME

WEAK TRANSMISSION $\mathcal{T} \ll 1$

$$S^{II}_{LR} = e\, I_R$$
$$S^{JJ}_{LR} = (\varepsilon_0 - \mu_L)\, J_R$$
$$S^{IJ}_{LR} = e\, J_R = (\varepsilon_0 - \mu_R)\, I_R$$

$$J_R = (\varepsilon_0 - \mu_R)\, I_R$$

Noises are proportional to currents (electric and heat)

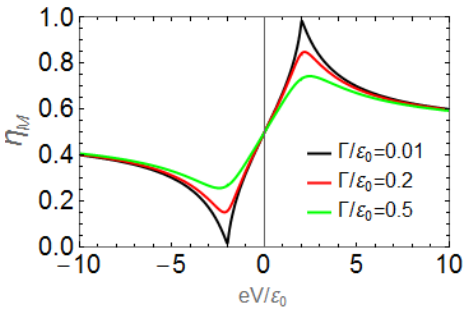
MACROSCOPIC EFFICIENCY

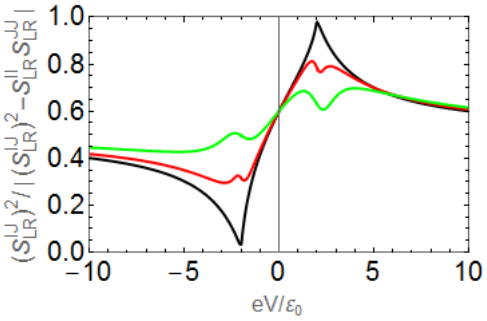
$$\eta_M = \frac{P^{out}}{P^{in}} = \left| \frac{J_R}{I_L\, V} \right|$$

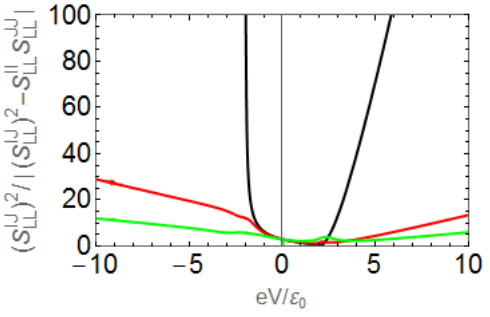
$$\eta_M = \frac{\left(S^{IJ}_{LR}\right)^2}{\left| S^{II}_{LR} S^{JJ}_{LR} - \left(S^{IJ}_{LR}\right)^2 \right|}$$

JPCM 2015 – A. Crépieux, F. Michelini
JSTAT 2016 – A. Crépieux, F. Michelini

NUMERICAL CHECK



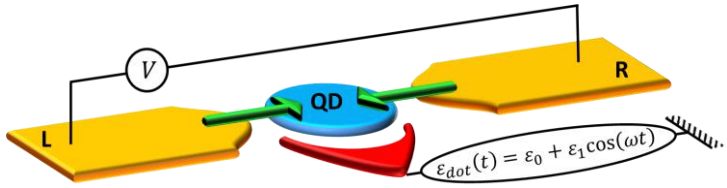




FLUCTUATION–DISSIPATION THEOREM

TIME–MODULATED HEAT CURRENT

$$\mathrm{J}_\alpha(t) = \mathrm{J}_\alpha^{(0)} + 2\sum_{N=1}^{\infty} \mathrm{Re}\left\{ \mathrm{J}_\alpha^{(\mathrm{N})}(\omega) e^{-iN\omega t} \right\}$$



THERMOELECTRIC AC CONDUCTANCE

$$\mathrm{X}_\alpha(\omega) = \mathrm{Re}\left\{ \frac{d\mathrm{J}_\alpha^{(1)}(\omega)}{d\varepsilon_1} \right\}$$

OUT–OF–EQUILIBRIUM

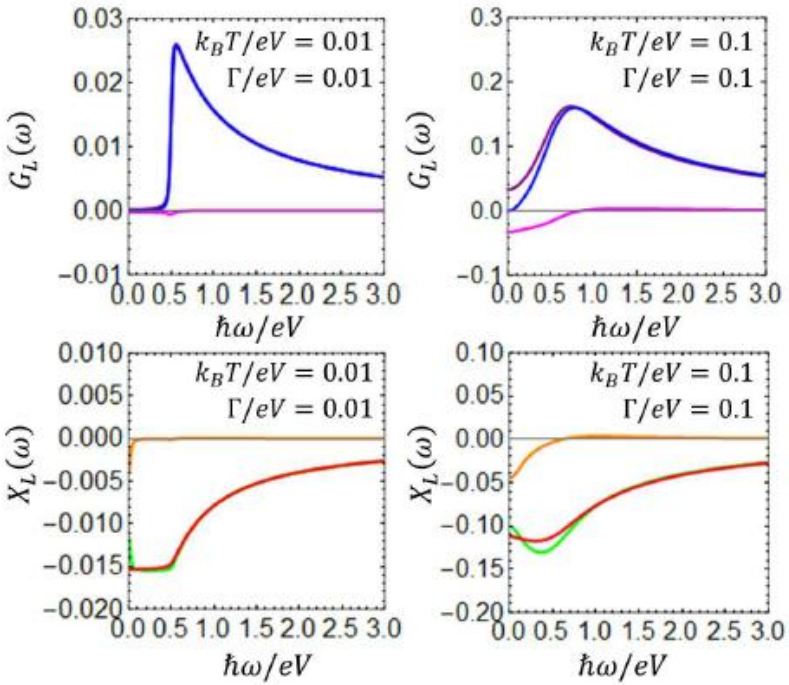
$$2\hbar\omega\,\mathrm{X}_\alpha(\omega) \approx \mathrm{Re}\left\{ S_{\alpha\alpha}^{\mathrm{IJ}}(-\omega) - S_{\alpha\alpha}^{\mathrm{JI}}(\omega) \right\}$$

Provided that one of the characteristic energies dominates over the others

AT EQUILIBRIUM (V=0)

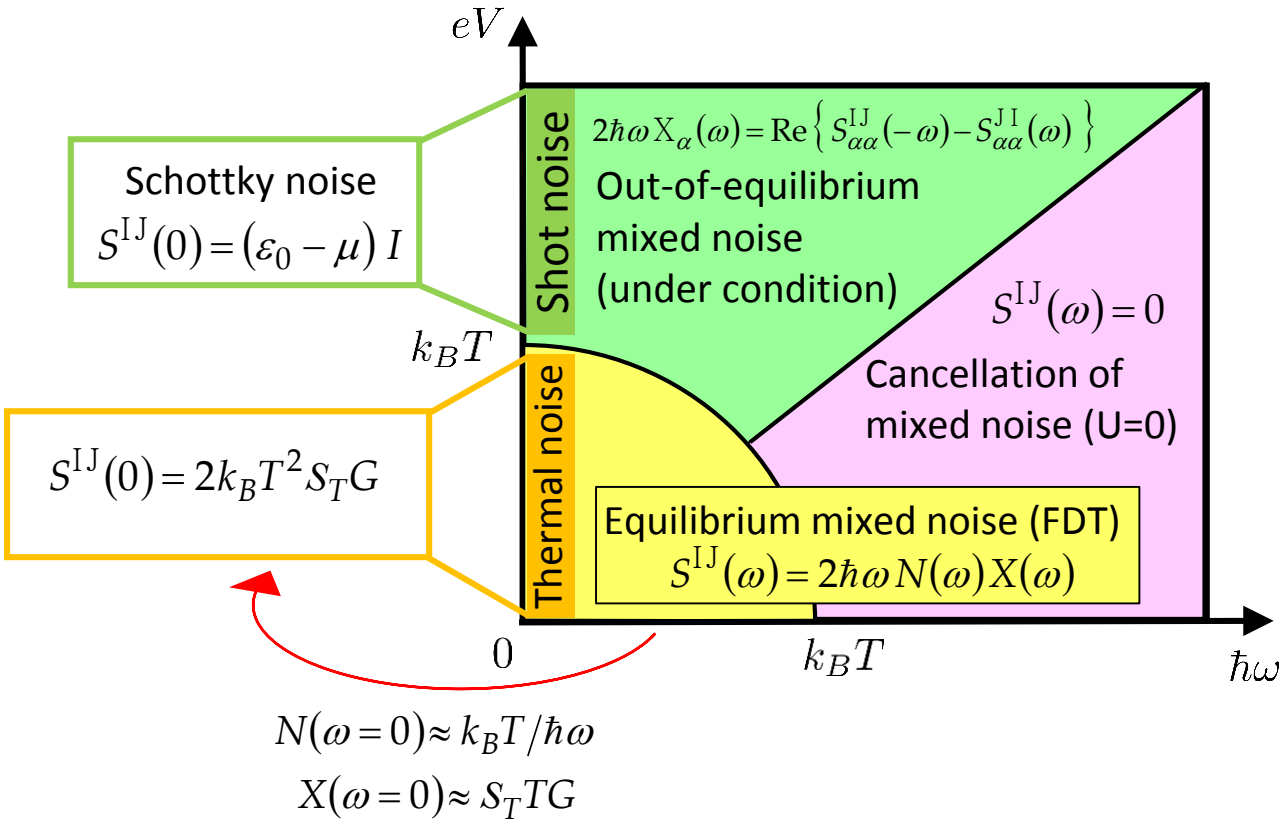
$$S^{\mathrm{IJ}}(-\omega) = e^{\hbar\omega/k_B T} S^{\mathrm{IJ}}(\omega)$$ *Kubo-Martin-Schwinger*

$$\mathrm{Re}\left\{ S^{\mathrm{IJ}}(\omega) \right\} = 2\hbar\omega N(\omega) \mathrm{X}(\omega)$$



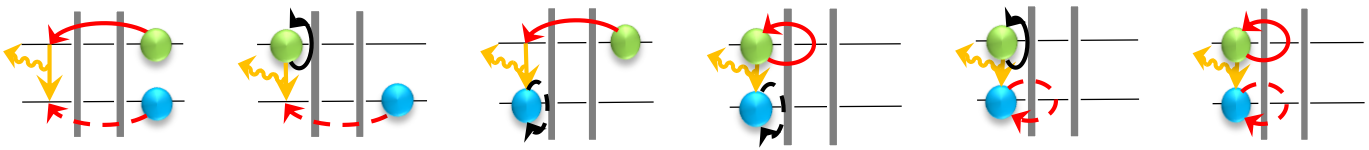
AdP 2017 – Crépieux

MIXED NOISE OVERVIEW



CONCLUSION

- Noise calculations based on the non-equilibrium Keldysh Green functions
- Each contribution to electrical noise is interpreted in terms of **energy exchange with environment**



- More than one physical process : take **coherent superposition of scattering paths**
- Zero-frequency mixed noise gives the **figure of merit** in linear response regime
- Zero-frequency mixed noise gives the **thermoelectric efficiency** in Schottky regime
- Finite-frequency mixed noise gives the **thermoelectric ac conductance**

Thanks to

M. GUIGOU
R. DEBLOCK
R. DELAGRANGE
F. PORTIER
R. WHITNEY
T. MARTIN
J. GABELLI
P. FEVRIER
Y.M. BLANTER
Y.V. NAZAROV

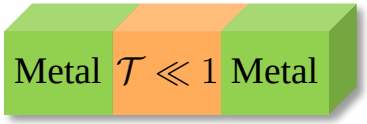
END

ZERO-FREQUENCY ELECTRICAL NOISE

SCHOTTKY REGIME

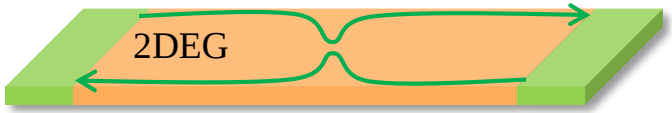
$$S_{\alpha\alpha}(\omega=0) = F \left|\left\langle \hat{I} \right\rangle\right|$$

F = Fano factor



$F = e$

$F = 2e$

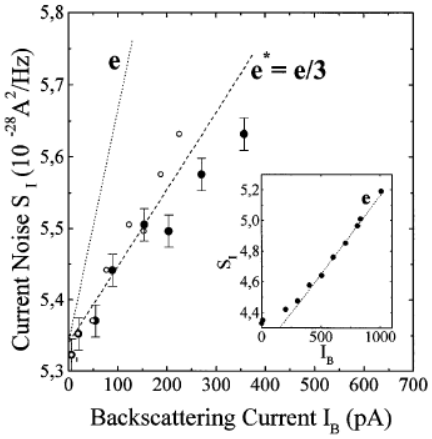


$F = e/3$

FRACTIONAL CHARGE IN 2DEG

$$F = \frac{e}{3}$$

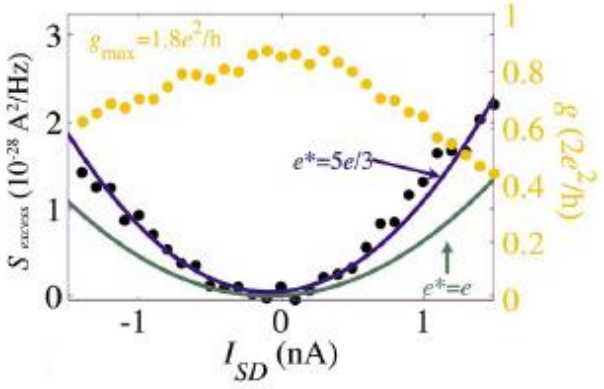
PRL 1997 – Saminadayar et al.
Nature 1997 – De Picciotto et al.

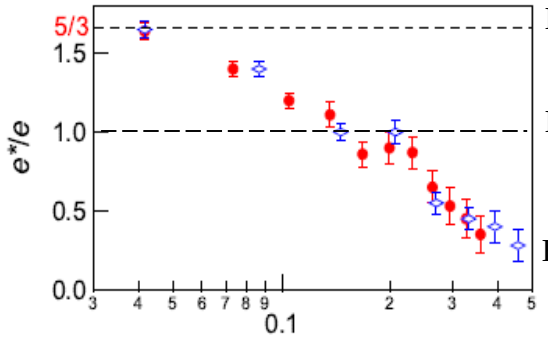


KONDO QUANTUM DOT

$$F = \frac{5e}{3}$$

PRL 2006 – Sela et al.
PRB 2008 – Zarchin et al.
Nature Physics 2016 – Ferrier et al.

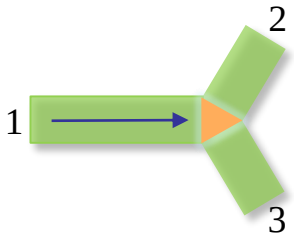




$T_K \approx 1.6$ K

SIGN OF CROSS-CORRELATION

Y-GEOMETRY

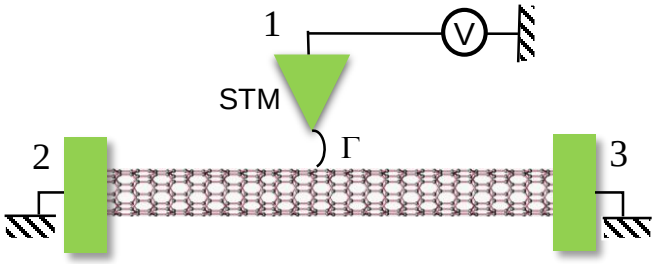


Fermions (anti-bunching) : $S_{23}(\omega=0)<0$

Bosons (bunching) : $S_{23}(\omega=0)>0$

PRL 1992 – Martin, Landauer

CARBON NANOTUBE + STM TIP

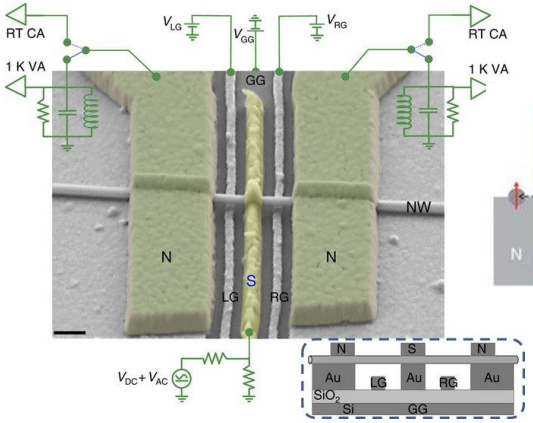


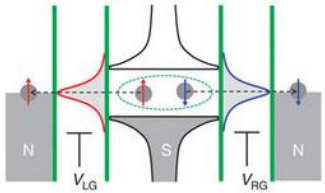
$$S_{23}(\omega=0)=\frac{1-K^2}{2}e\left|\langle I\rangle\right|>0$$

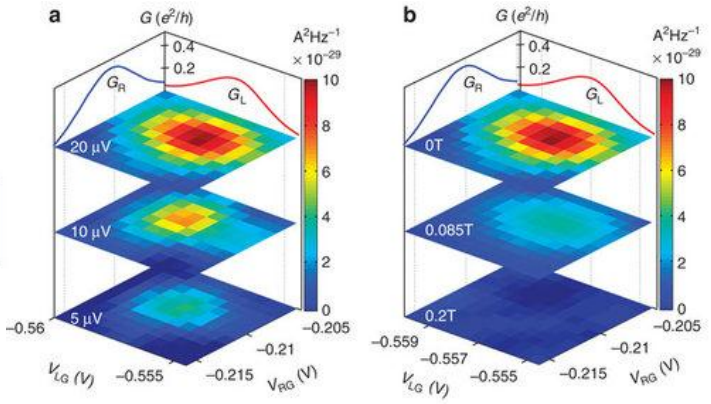
Entangled bosonic excitations

PRB 2003 – Crépieux, Guyon, Devillard, Martin

COOPER PAIR SPLITTING







Nature Com 2012 – Das et al.

Positive cross-correlations ⇒ entangled electron pairs

FINITE-FREQUENCY NOISE

DEFINITION

$$\mathcal{S}(\omega) = \int_{-\infty}^{+\infty} \langle \Delta\hat{I}(t)\Delta\hat{I}(0)\rangle e^{-i\omega t} dt$$

Nonsymmetrized noise

ω > 0 : Emission noise

ω < 0 : Absorption noise

JETP 1997 – Lesovik, Loosen

AC CONDUCTANCE

$$G(\omega) = \frac{\mathcal{S}(-\omega) - \mathcal{S}(\omega)}{2\hbar\omega}$$

Generalized Kubo formula

AT EQUILIBRIUM

$$\mathcal{S}(-\omega) = e^{\hbar\omega/k_B T}\mathcal{S}(\omega)$$

Kubo-Martin-Schwinger relation

$$\mathcal{S}(\omega) = 2\hbar\omega N(\omega) G(\omega)$$

Fluctuation dissipation theorem

Bose-Einstein function

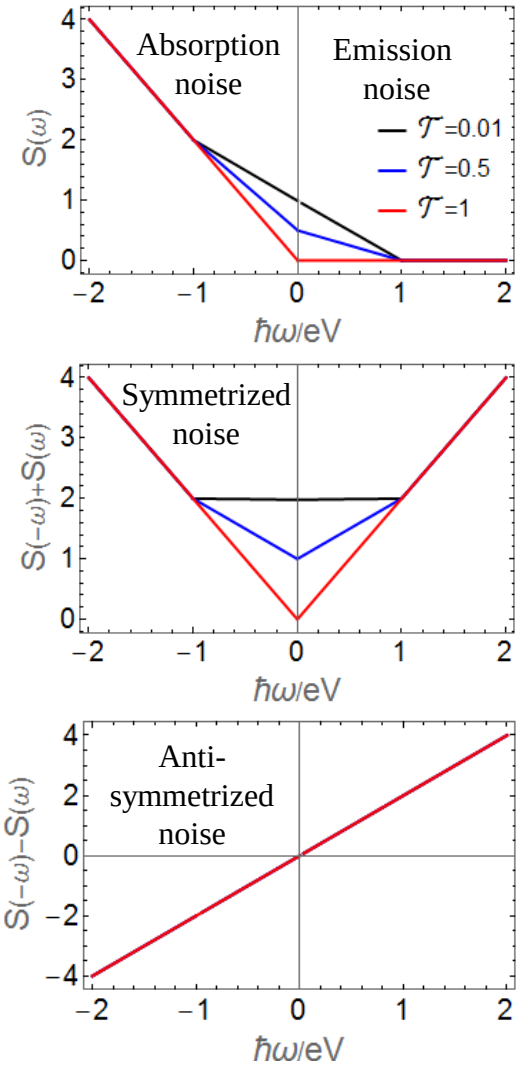
ac-conductance

OUT-OF-EQUILIBRIUM (small transmission)

$$\mathcal{S}(\omega) = \frac{e^2\mathcal{T}}{h}\sum_{\pm}(\hbar\omega \pm eV)N(\omega \pm eV/\hbar)$$

Out-of-equilibrium fluctuation dissipation relation

low temperature



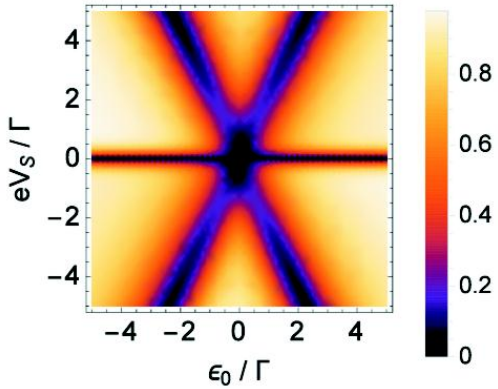
DIFFERENTIAL FANO FACTOR

$$F(V_S,\omega)=\frac{dS_{LL}/dV|_{V_S}}{edI_L/dV|_{V_S-\hbar\omega/e}}$$

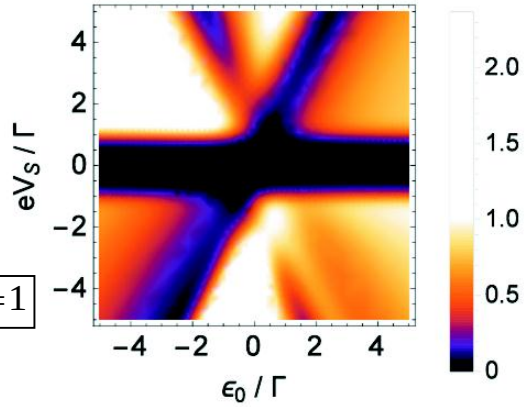
- F vanishes for voltage smaller than frequency
- At high frequency, F can take values larger than 1

THEORETICAL RESULT

$$\mathcal{T}(\varepsilon)=\frac{\Gamma^2}{(\varepsilon-\varepsilon_0)^2+\Gamma^2}$$

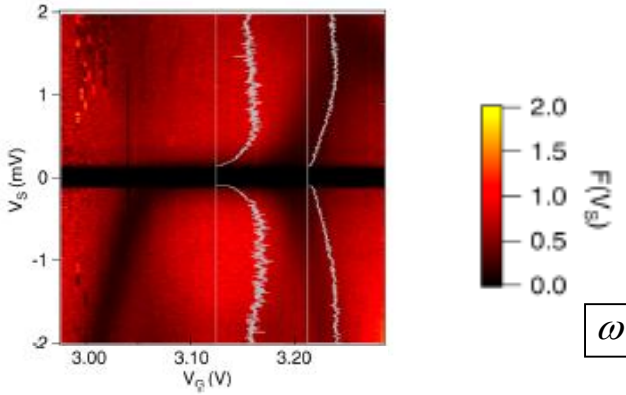


$\hbar\omega/\Gamma=0.1$

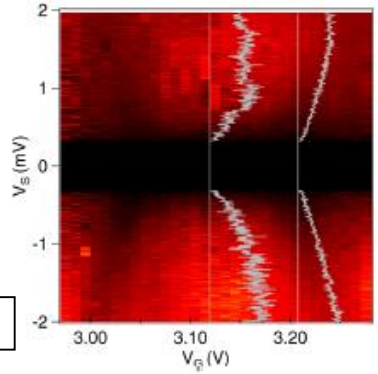


$\hbar\omega/\Gamma=1$ $\quad k_BT/\Gamma=0.1$

MEASUREMENT IN KONDO REGIME



$\omega=29.5\text{GHz}$



$\omega=78\text{GHz}$ $\quad T=20\text{mK}$

PRL 2012 – Basset, Kasumov, Moca, Zarand, Simon, Bouchiat, Deblock

MEASUREMENT OF ELECTRICAL NOISE

SIS JUNCTION

2003 – Science – Deblock, Onac, Gurevich, Kouwenhoven
2006 – PRL – Billangeon, Pierre, Bouchiat, Deblock
2010 – PRL – Basset, Bouchiat, Deblock

CARBON NANOTUBE QD

2006 – PRL – Onac, Balestro, Trauzettel, Lodewijk, Kouwenhoven
2012 – PRL – Basset, Kasumov, Moca, Zarand, Simon, Bouchiat, Deblock

QPC

2006 – PRL – Onac, Balestro, van Beveren, Hartmann, Nazarov, Kouwenhoven
2007 – PRL – Zakka-Bajjani, Ségala, Portier, Roche, Glattli

QUANTUM HALL DEVICE

2012 – PRB – Parmentier, Bocquillon, Berroir, Glattli, Plaçais, Fève, Albert, Flindt, Büttiker

TUNNEL JUNCTION

2014 – PRL – Altimiras, Parlavecchio, Joyez, Vion, Roche, Esteve, Portier
2015 – PRL – Parlavecchio, Altimiras, Souquet, Simon, Safi, Joyez, Vion, Roche, Esteve, Portier

CALCULATION OF ELECTICAL NOISE

SCATTERING THEORY

2009 – PRB – Rothstein, Entin-Wohlman, Aharony
2011 – PRB – Hammer, Belzig
2014 – PRB – Rothstein, Horovitz, Entin-Wohlman, Aharony

PERTURBATIVE CALCULATIONS

2004 – PRL – Engel, Loss
2014 – EPL – Malkoc, Bergenfeldt, Samuelsson
2016 – PRB – Roussel, Degiovanni, Safi

LUTTINGER LIQUID THEORY

2005 – PRB – Lebedev, Crépieux, Martin
2007 – PRB – Bena, Safi
2008 – PRB – Safi, Bena, Crépieux

REFERMIONIZATION

2012 – PRB – Zamoum, Crépieux, Safi

RENORMALIZATION GROUP APPROACH

2011 – PRB – Moca, Simon, Chung, Zarand
2013 – PRB – Müller, Pletyukhov, Schuricht, Andergassen