Modeling control of low frequency plasma turbulence

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- Drift-wave turbulence in the Kiwi/Mirabelle device
- Autoresonance as a control method for nonlinear systems
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Motivation

Turbulence and the induced transport are key ingredients to a number of problems in building a working device for nuclear fusion:

- scaling of transport
- transport events: blobs, ELMs ...
- transport of impurities

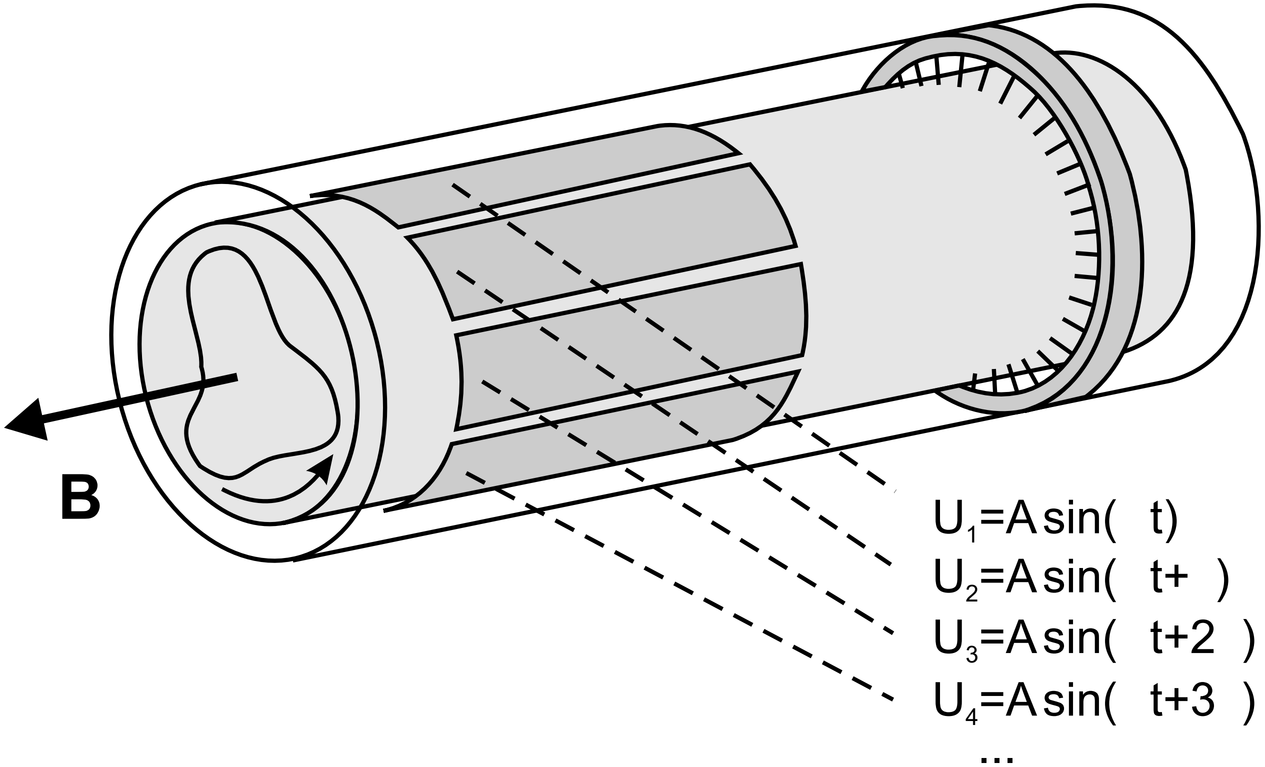
Thus effectively controlling or influencing turbulence is a mayor issue in fusion research.

There are clearly two ways to control a plasma:

- choose the right parameters and geometry
- do something actively

The Kiwi/Mirabelle device

Experimental buildup (Thomas Klinger this morning):

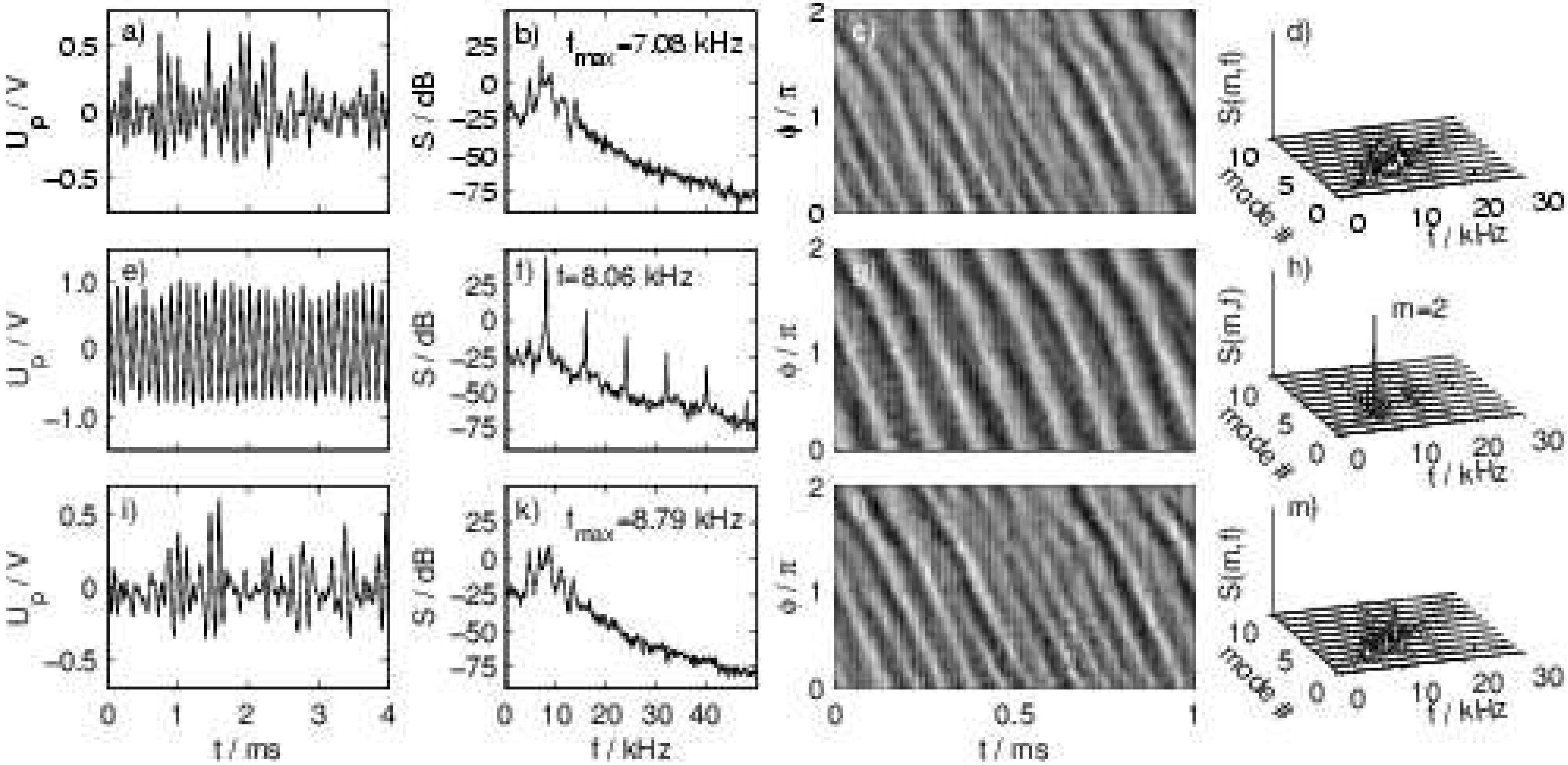


Magnetized cylinder, current driven instability of drift wave type, weak turbulence.

Control/influence by a spatiotemporal signal in resonance with naturally occuring 'linear' mode

The Kiwi/Mirabelle device

Experimental result:



Resonant drive with a $m = 2$ mode collapses turbulent spectrum.

Modeling the experiment

Idea:
Simplest system describing drift wave dynamics, reducing drift wave dynamics to its core
Hasegawa-Wakatani equations: (1987)

$$\frac{\partial}{\partial t}\nabla_\perp^2\phi + \vec{V}_{E\times B}\cdot\nabla\nabla_\perp^2\phi = \nabla_\| J_\| + \mu_w\nabla_\perp^4\phi$$

$$\frac{\partial}{\partial t}n + \vec{V}_{E\times B}\cdot\nabla\left(N_0 + n\right) = \nabla_\| J_\| + \mu_n\nabla_\perp^2 n$$

Balance divergences in ion-polarisation with parallel current to hold up quasi-neutrality.

Modeling the experiment

$$\frac{\partial}{\partial t}\nabla_\perp^2\phi + \vec{V}_{E\times B}\cdot\nabla\nabla_\perp^2\phi = \tilde{\sigma}(\phi - n) - S + \mu_w\nabla_\perp^4\phi$$

$$\frac{\partial}{\partial t}n + \vec{V}_{E\times B}\cdot\nabla(N_0(r) + n) = \tilde{\sigma}(\phi - n) - S + \mu_n\nabla_\perp^2 n$$

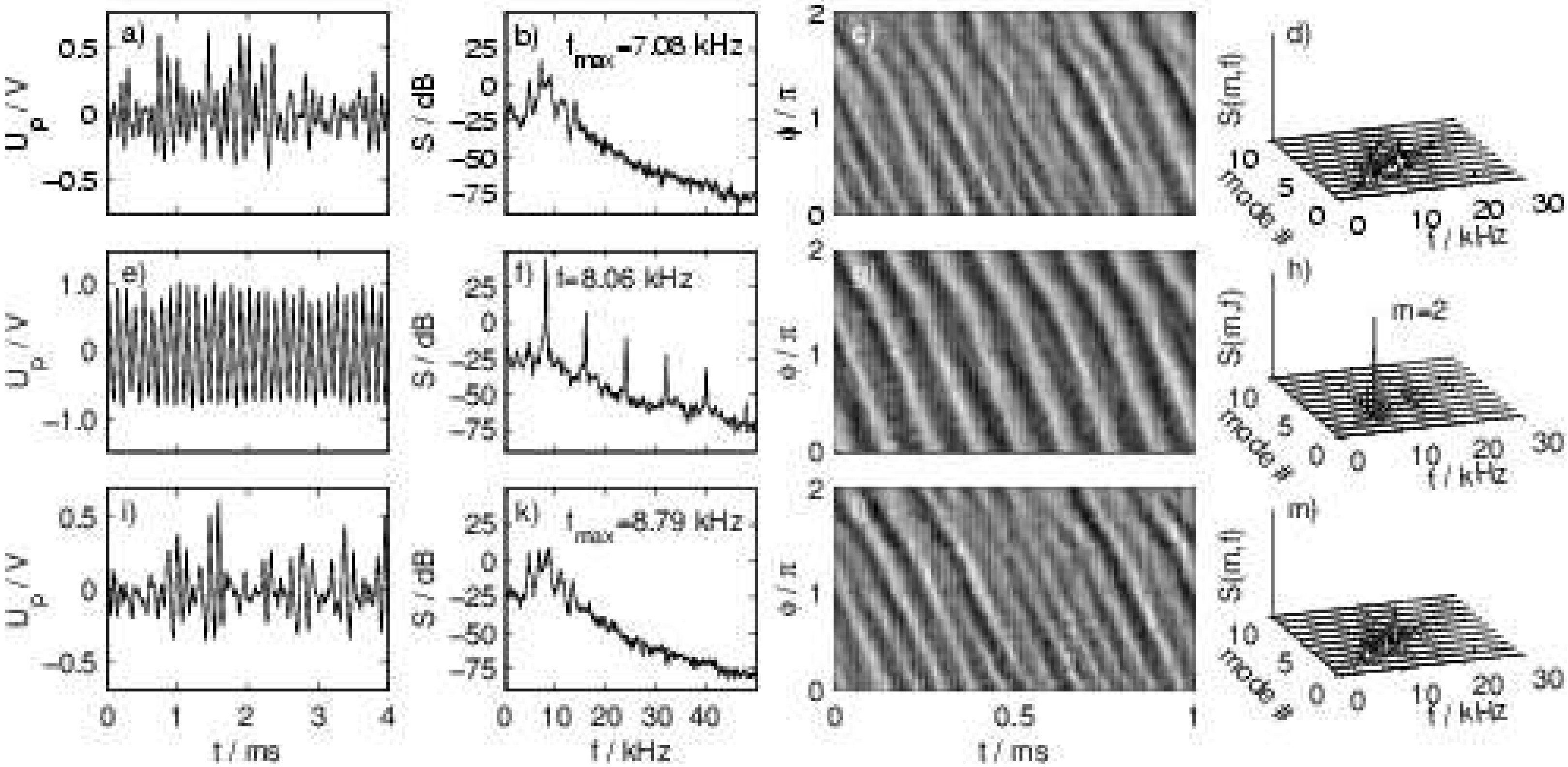
Driver term acts onto the source of instability:
Parallel current.

$$S = \nabla_\parallel J_{ext} = A\sin(\pi r/r_0)\sin(2\pi m_d\theta - \omega_d t)$$

Solved numerically on a disk (r,θ) .
Drive/Damping layers at around $r = 0$ and $r = r_{max}$ keep up density gradient N_0.

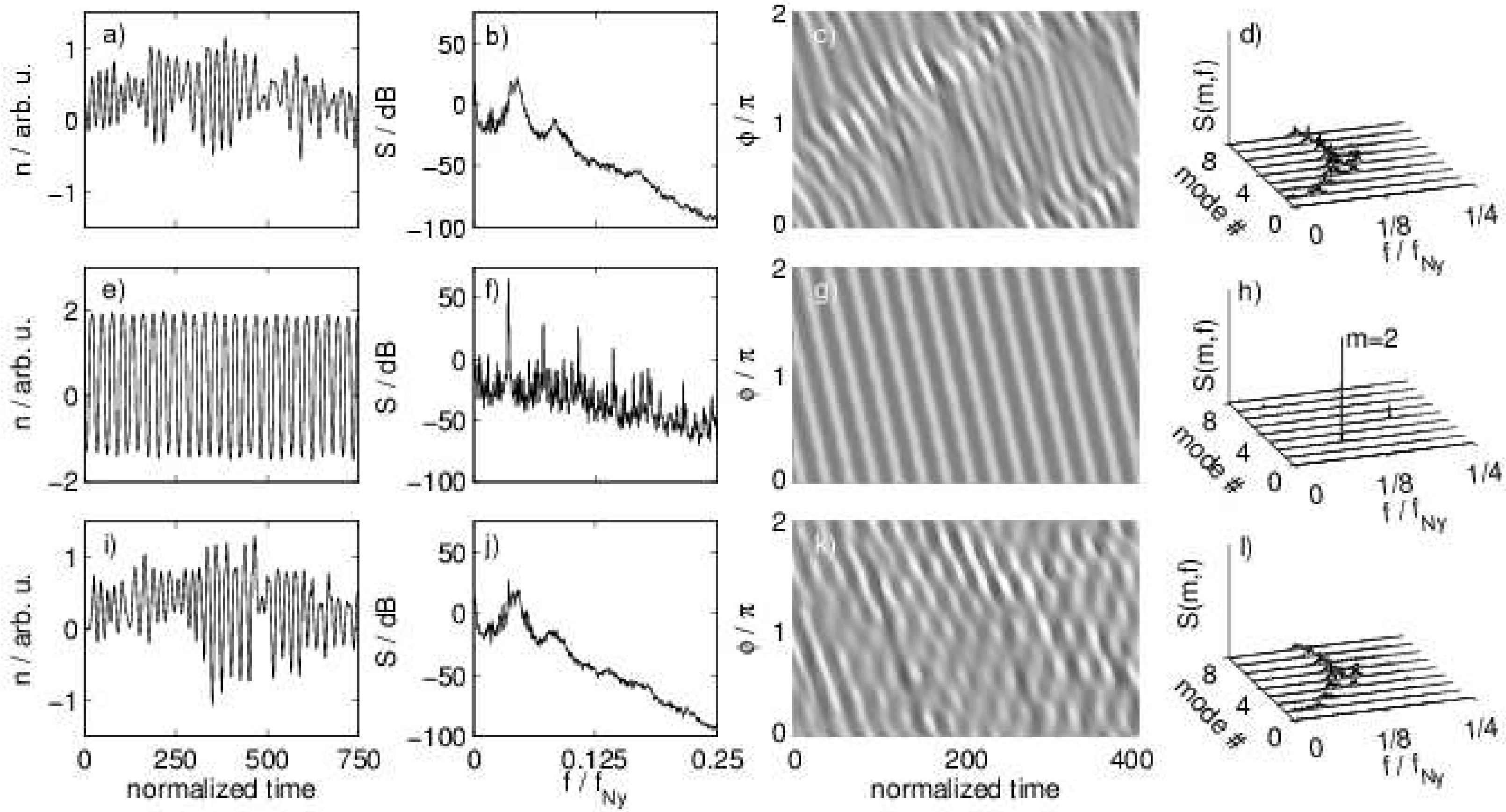
Modeling the experiment

Results: Experiment



Modeling the experiment

Results: Simulation

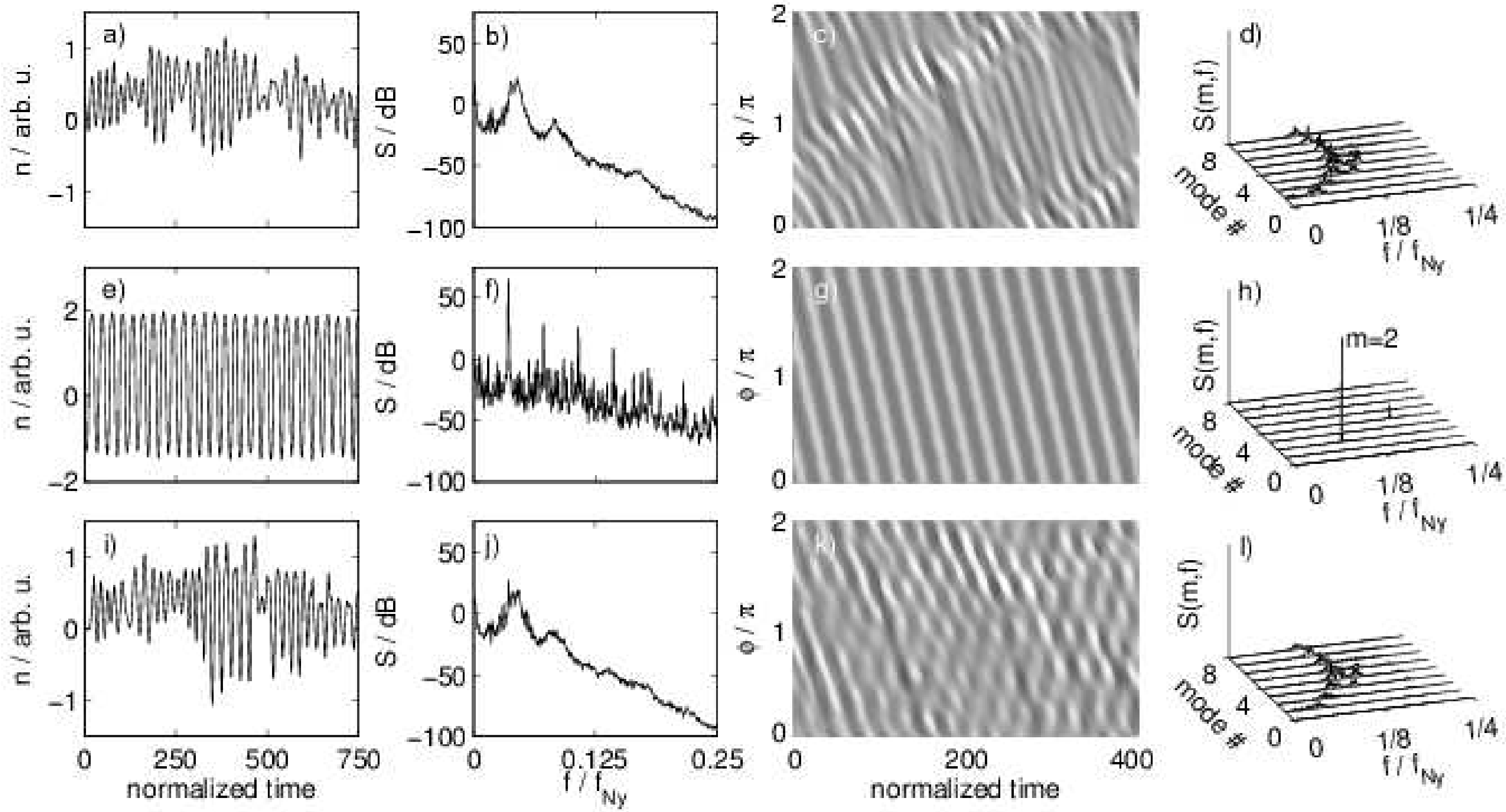


Excellent agreement between simulation and experiment.

Schröder et al. PRL 2001

Modeling the experiment

Results: Simulation



Excellent agreement between simulation and experiment.

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But: Is this the whole story?

No, modeling is merely a good starting point...

Controlling multidimensional PDEs

Control theory of multidimensional PDEs is an active field of research and not very well developed. It is not at all well developed for non-linear problems...

- linear control theory for low number of degrees of freedom ODE's well developed.
- nonlinear control theory for chaos (attractors)
- feedback, open loop control ....

Some identifications:

A possible resonance of a system helps to exert control at low control power....critical for fusion plasmas.

Autoresonance: Parametric Excitation

AUTORESONANCE (adiabatic nonlinear phase locking and synchronization) is a remarkable phenomenon of nonlinear physics when a driven nonlinear system stays in resonance with the driving oscillation or wave continuously, despite variation of system's parameters. (Fajans, Friedland)

Or: How do you drive a nonlinear oscillator or wave to high amplitude?

Autoresonance

Simple example: Pendulum

$$\frac{d^2}{dt^2}x + \sin(x) = A\cos(f(t))$$

Linear resonant frequency: $\omega = 1$.
At large oscillator amplitude resonant frequency changes:

Resonance lost

Solutions:

Autoresonance

Simple example: Pendulum

$$\frac{d^2}{dt^2}x + \sin(x) = A\cos(f(t))$$

Linear resonant frequency: $\omega = 1$.
At large oscillator amplitude resonant frequency changes:

Resonance lost

Solutions:
a) modify frequency accordingly
Inability to measure and to adjust makes this difficult for most non-trivial systems.

Autoresonance

Simple example: Pendulum

$$\frac{d^2}{dt^2}x + \sin(x) = A\cos(f(t))$$

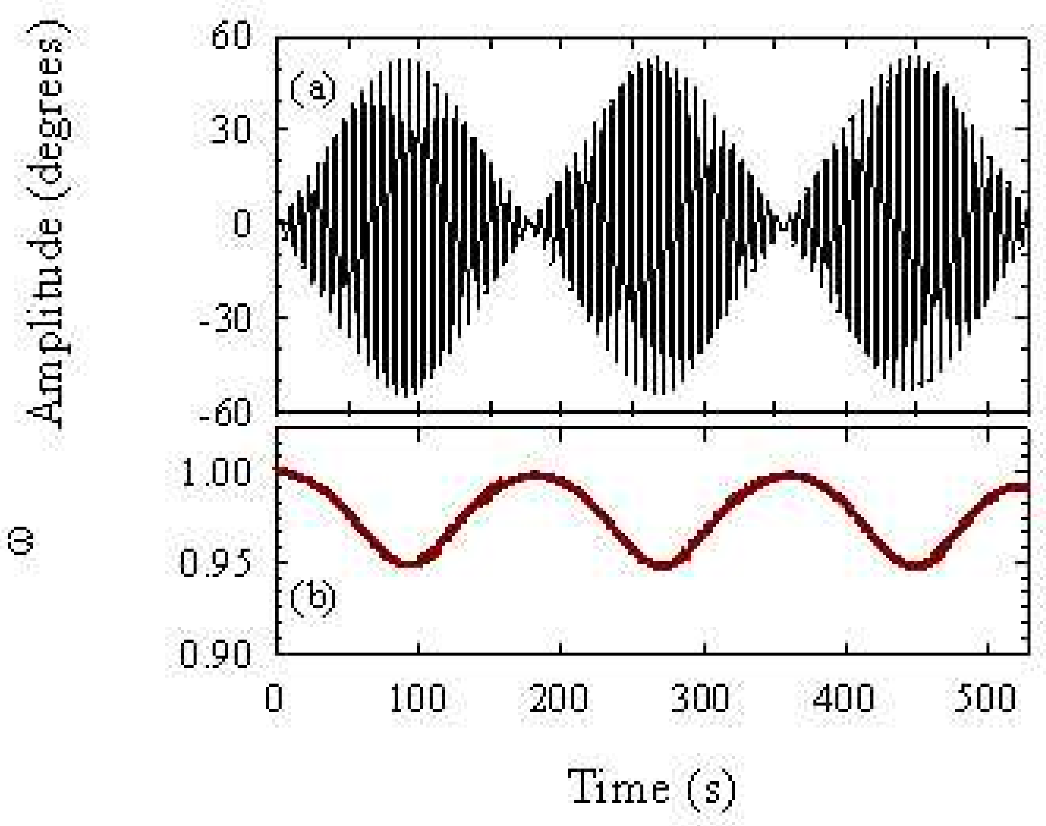
Linear resonant frequency: $\omega = 1$.
At large oscillator amplitude resonant frequency changes:

Resonance lost

Solutions:
b) sweep driver frequency slowly, assuming some monotonous frequency response
a (good) shot into the dark.

Autoresonance

Simple example: Pendulum



Note: Sweep must be slow compared to system frequency change!

L. Friedland, J. Fajans, and E. Gilson, Subharmonic autoresonance of the diocotron mode. Phys. Plasmas, 7:1712, 2000.

Autoresonance

AR and plasma turbulence?

No way, as turbulence

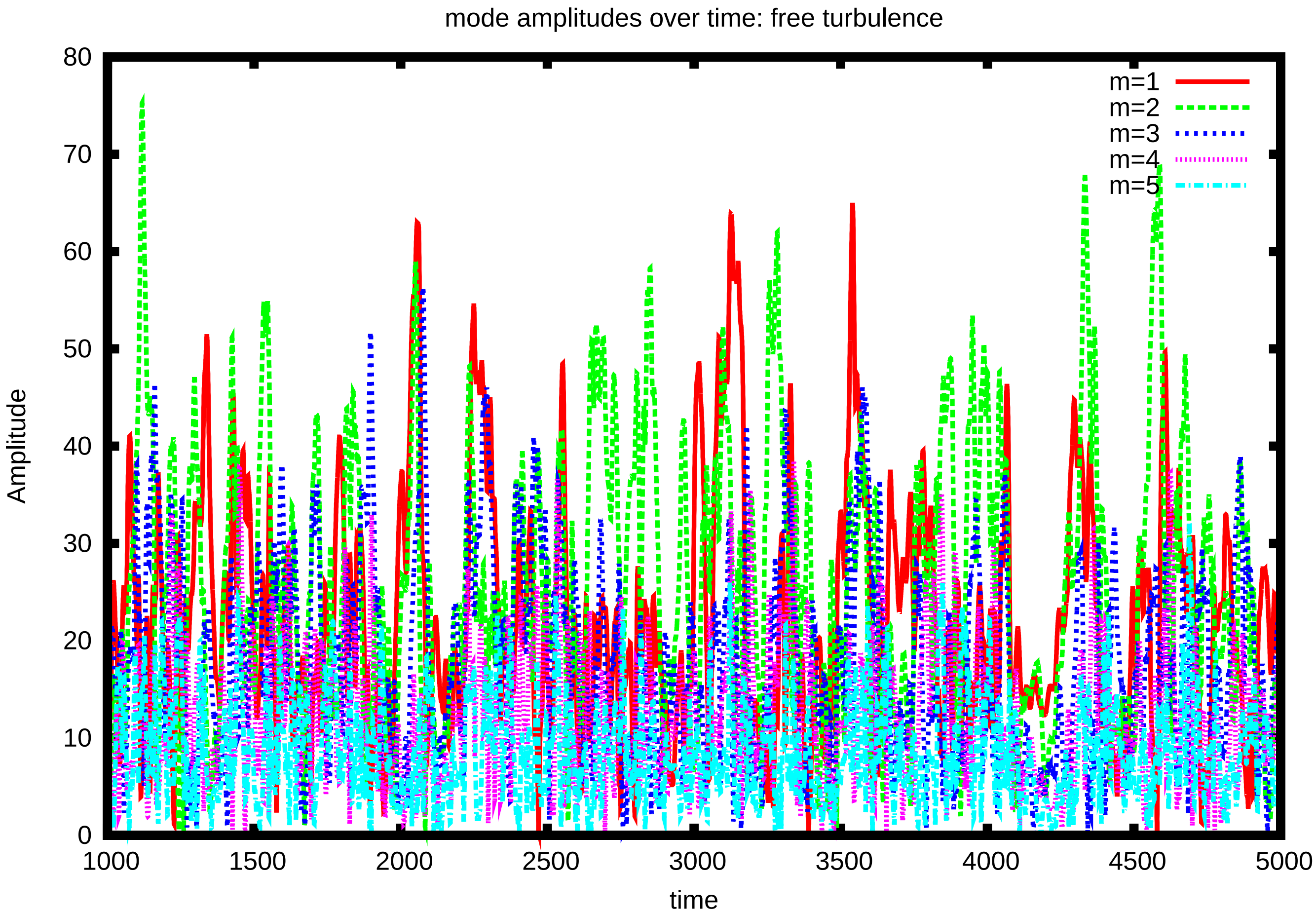
- is a complicated system, many degrees of freedom
- has a complicated frequency response
- exhibits instabilities
- ....

but....there are certain similarities:

- no good knowledge of system
- linear frequencies (waves)
- .....

Autoresonance

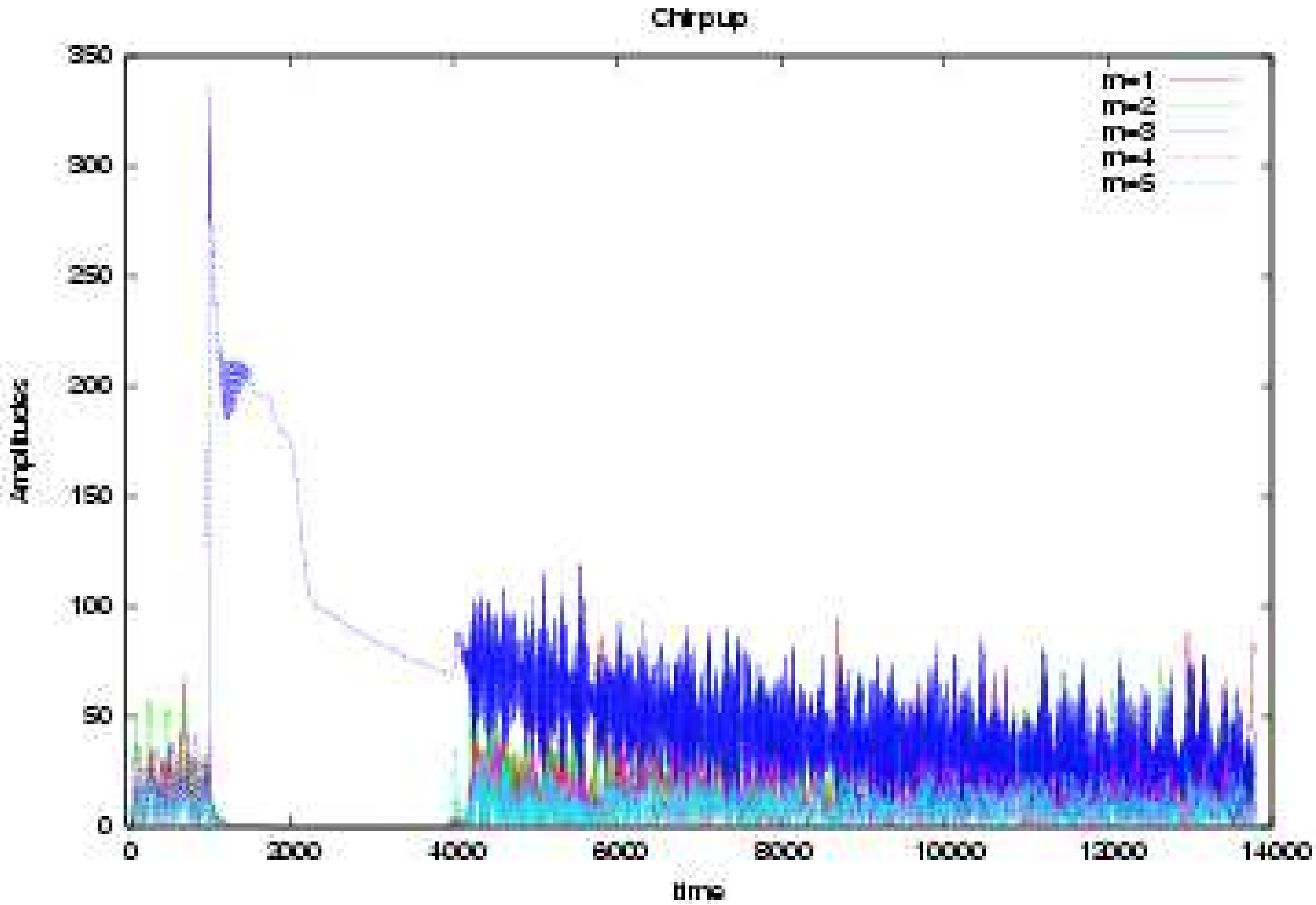
Results from simulations of Hasegawa- Wakatani System.



Amplitudes of modes $m = 1$ to $m = 5$. Free system, far from onset of instability, many unstable modes, turbulent.....

Autoresonance

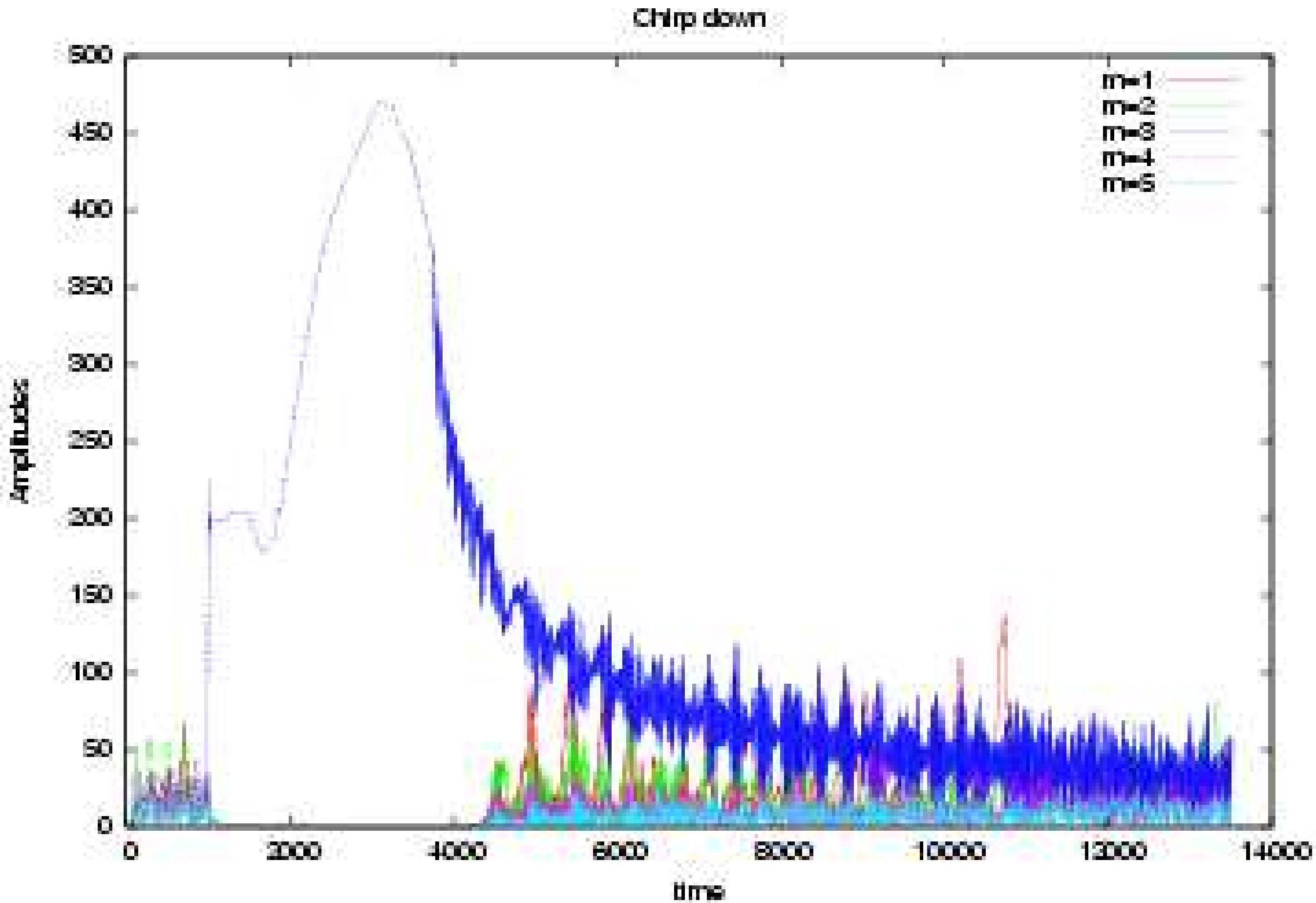
Results from simulations of Hasegawa- Wakatani System.



Driving mode $m = 3$ with driver frequency increasing over time.

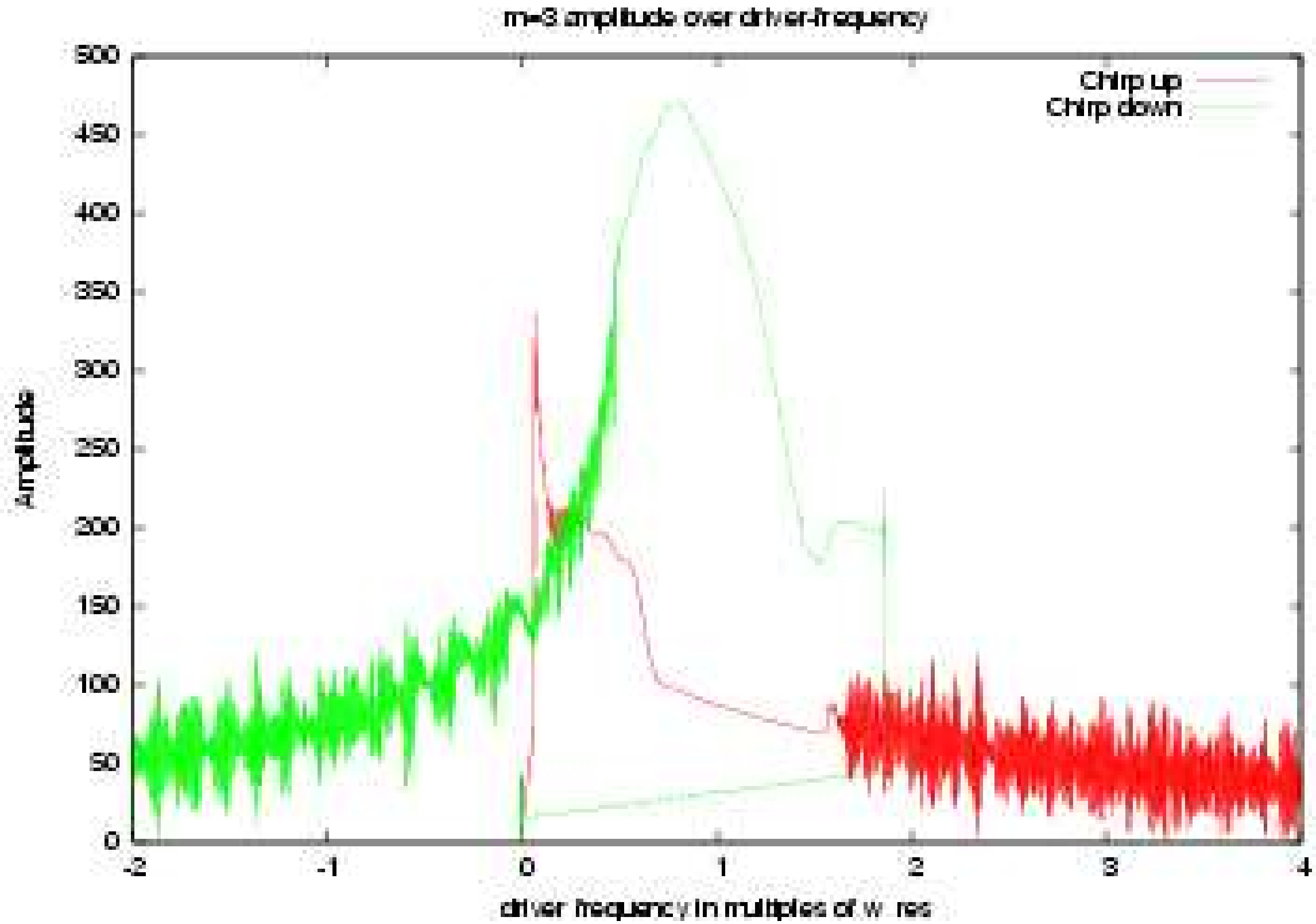
Autoresonance

Results from simulations of Hasegawa- Wakatani System.



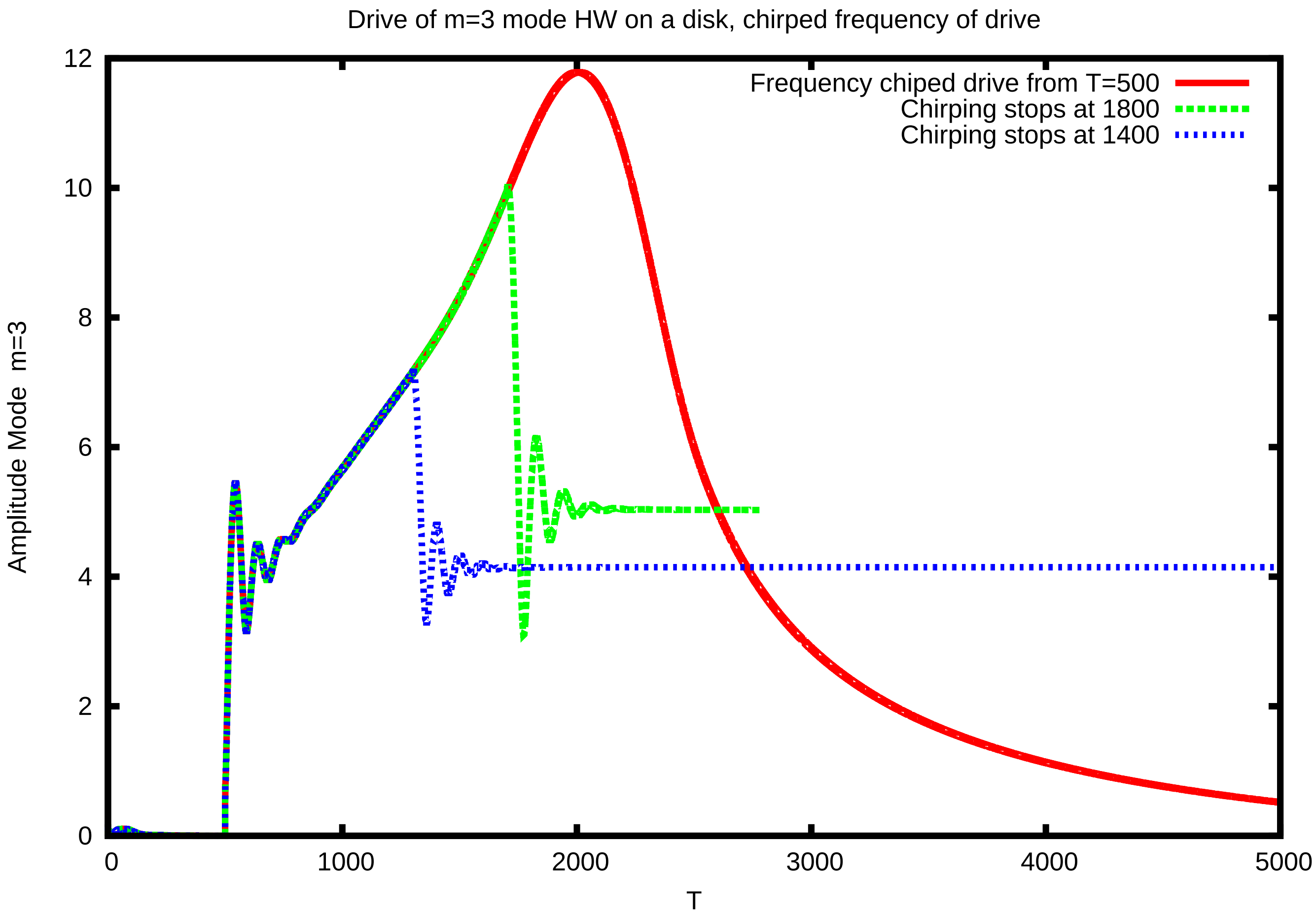
Driving mode $m = 3$ with driver frequency decreasing over time.

AR in turbulence



Amplitude of driven mode $m = 3$ over frequency, using increasing and decreasing driver frequency.

AR (linearly stable system)



Amplitude of driven mode $m = 3$ over frequency, using increasing driver frequency, sweep stopped at certain frequency.

Note: Linearly stable system.

Conclusion

- turblence control accomplished (aka GWB)
- modelled in simulations
- autoresonance as a simple control strategy
- some features of AR control demonstrated in 'taming turbulence'
- allows to optimize control power
- promising for future investigations

Thanks and Literature

Anders H. Nielsen, Jens Juul Rasmussen,
Thomas Klinger, Christiane Schröder,
Dietmar Block,
L. Friedland, A. Shagalov........

Autoresonance:

U. Rokni and L. Friedland Double Autoresonance in Two-Dimensional Dynamical Sysytems Phys. Rev. E59, 5242-5252 (1999)

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L. Friedland and A.G. Shagalov Excitation of Solitons by Adiabatic Multiresonant Forcing Phys. Rev. Lett. 81, 4357-4360 (1998)