



# Dynamical Band- Engineering of Spin- Polarized Edge States

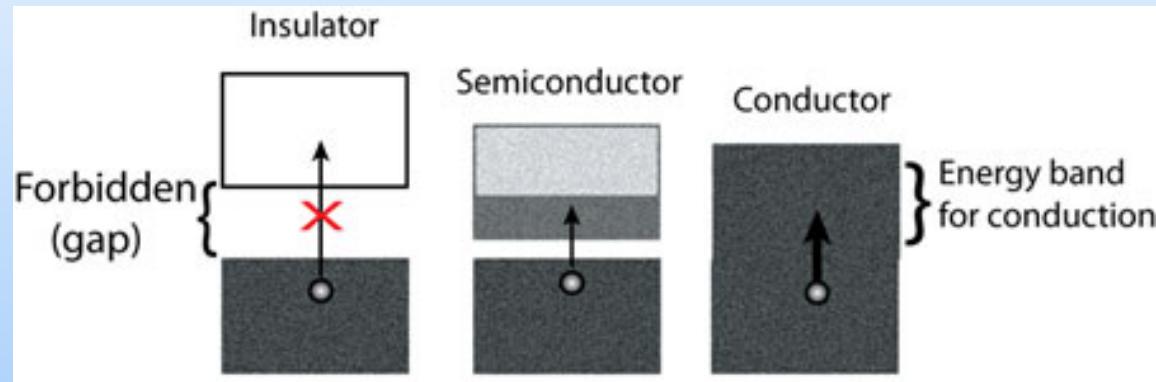
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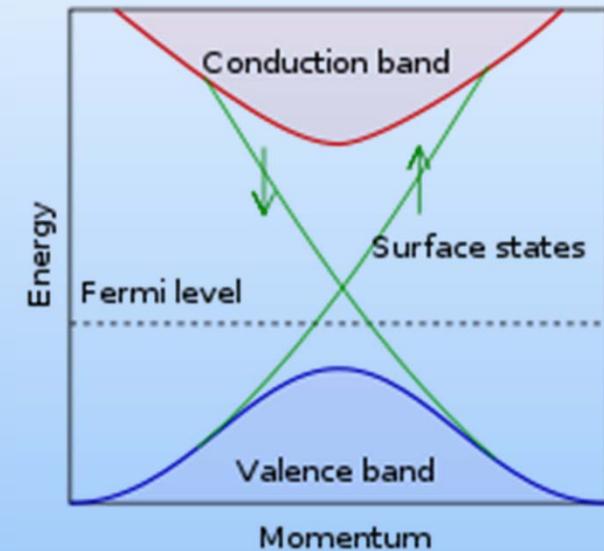
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# Band Theory

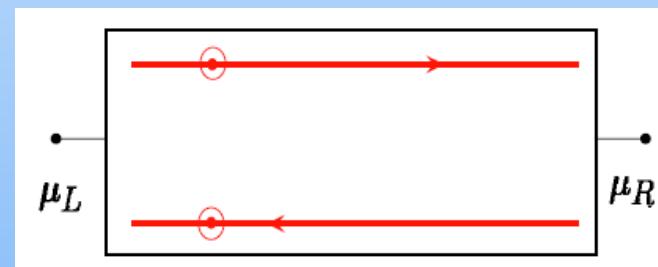
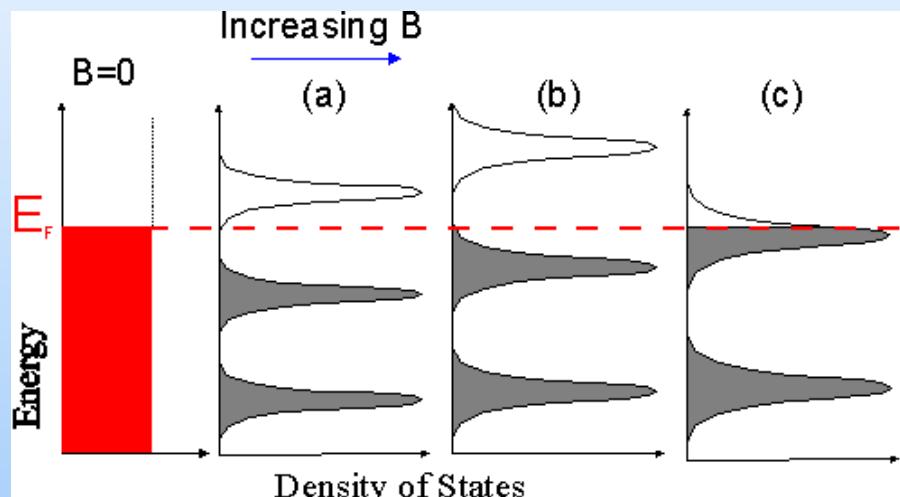


Insulator, Semiconductor and Conductor

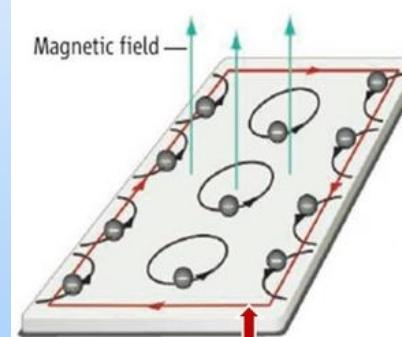


Topological Insulator

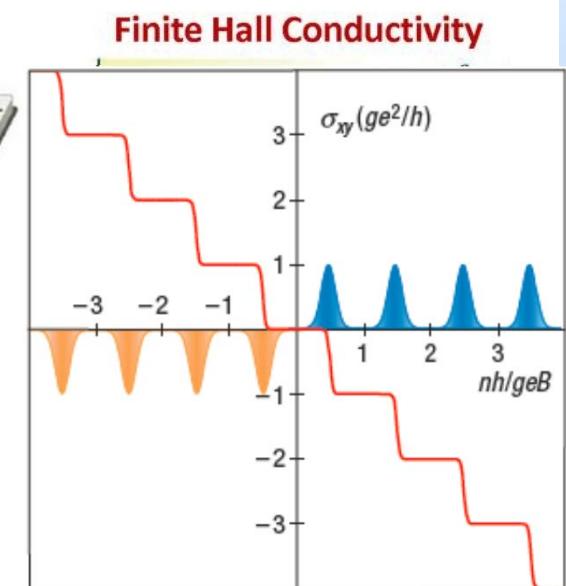
# Edge States: Hall effect



**Bulk Insulator** but **Conduction through the Edge**



Conduction through  
the boundary



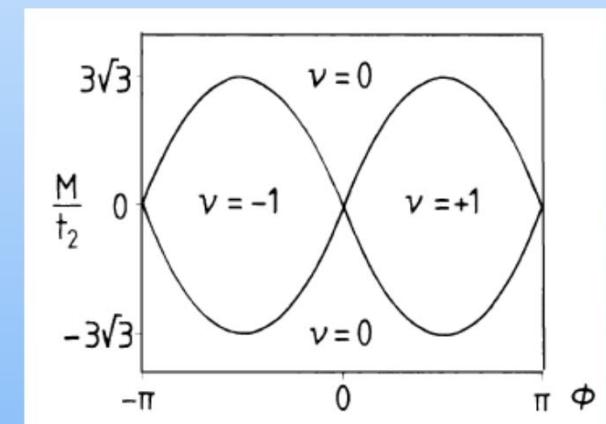
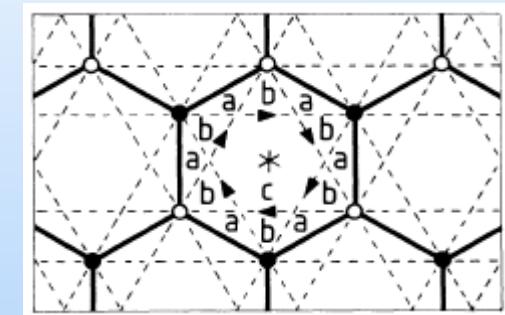
- Band: bulk gap due to Landau levels
- Conductivity: quantized Hall conductivity
- Chiral Edge states at the boundaries

# Haldane Model: Edge States without Magnetic field

This spinless model consists of tight-binding terms on the honeycomb lattice.

$$H = \sum_i t_0 c_i^\dagger c_i + \sum_{\langle i,j \rangle} t_1 c_i^\dagger c_j + \sum_{\langle\langle i,j \rangle\rangle} t_2 e^{i\phi_{ij}} c_i^\dagger c_j.$$

- The second term due to nearest-neighbor hopping on graphene gives rise to the Dirac dispersion.
- The first term describes a sublattice staggering and leads to the opening up of gap.
- The third term describes the next-nearest-neighbor hopping which is modified by a periodic vector potential and zero total magnetic-flux.



$$t_0 = \pm M$$

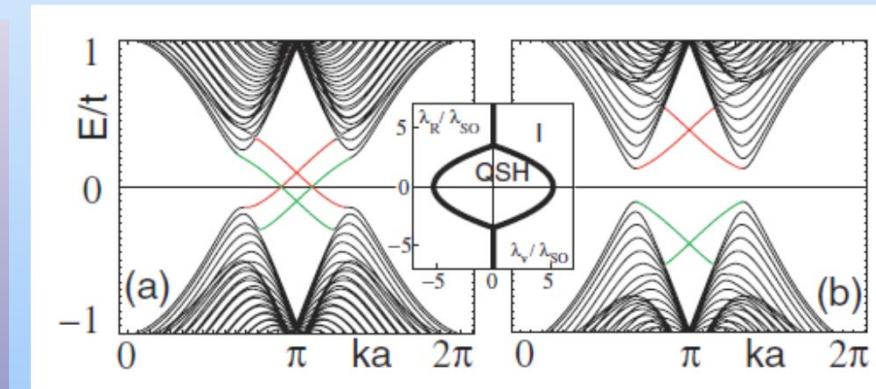
# Quantum Spin Hall Effect

Kane and Mele correlated spin with Haldane model and introduced the following Hamiltonian.

$$H_0 = -t \sum_{\langle ij \rangle} c_{i\sigma}^\dagger c_{j\sigma} + \lambda_v \sum_i \xi_i c_{i\sigma}^\dagger c_{i\sigma}$$

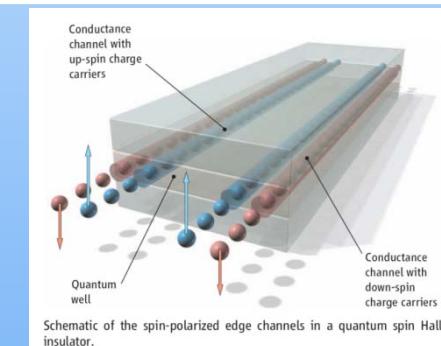
$$H' = i\lambda_{SO} \sum_{\langle\langle ij \rangle\rangle} v_{ij} c_i^\dagger s^z c_j + i\lambda_R \sum_{\langle ij \rangle} c_i^\dagger (\mathbf{s} \times \hat{\mathbf{d}}_{ij})_z c_j$$

Without Rashba term (second SO coupling), have two copies of Haldane's IQHE model. All physics is the same as IQHE physics.



The Kane-Mele model represents a new phase of materials:

Topological Insulator.

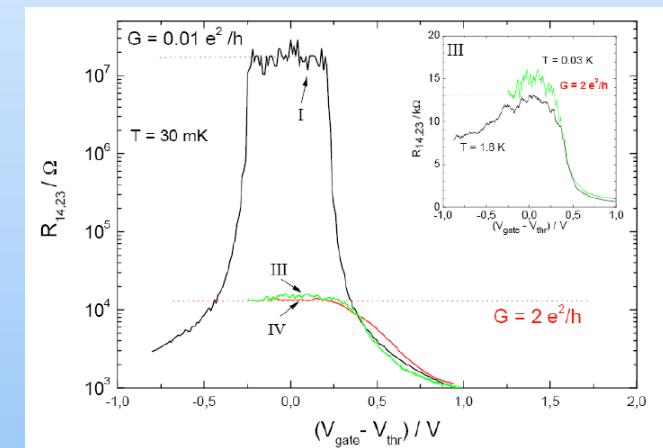
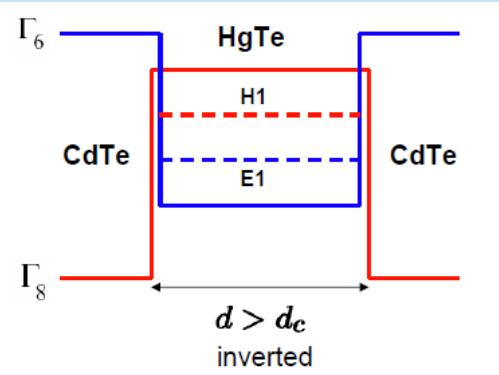
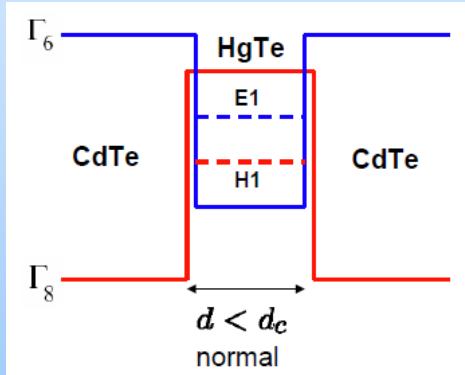


However, materials in nature which are classified as TI are rare!

C.L.Kane and E.J. Mele, Phys. Rev. Lett. 95, 146802 (2005).

# Artificial Topological Insulators: Band Engineering

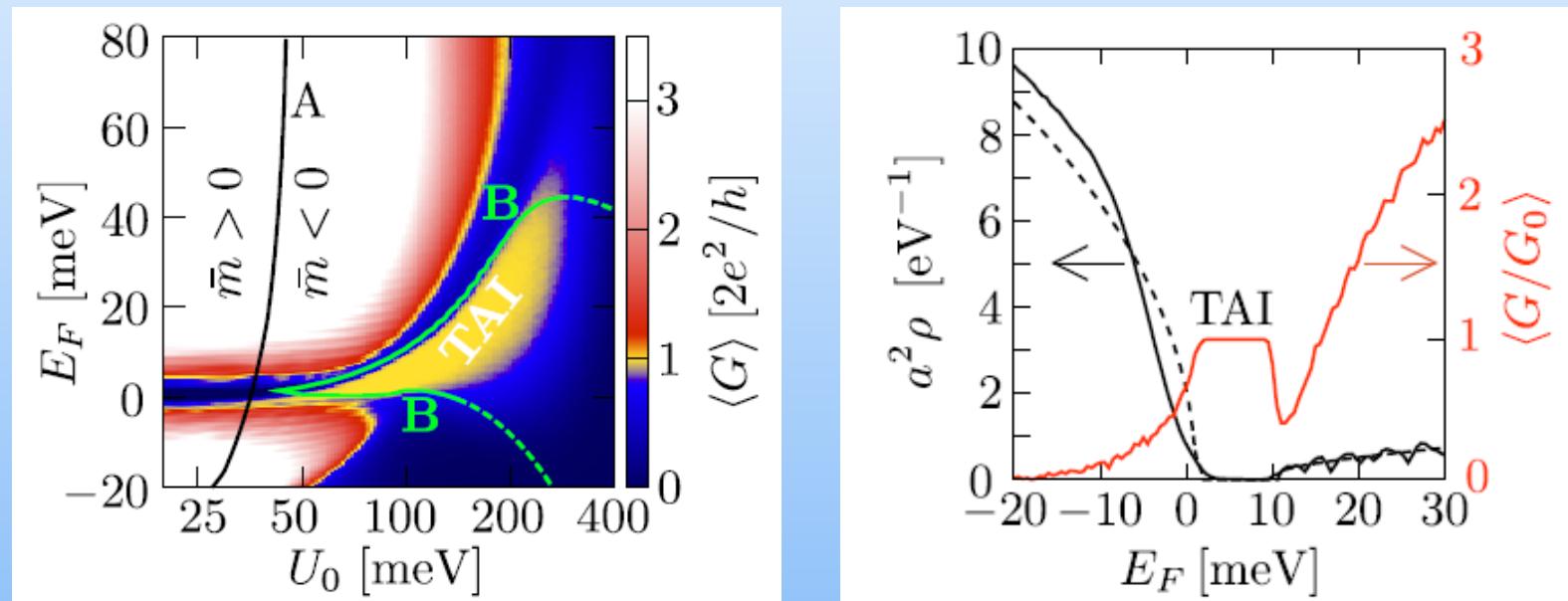
## ■ HgTe quantum wells



B.A Berneig et al., Science 314,1757 (2006)  
M. König, et al., Science 318, 766 (2007).

# Artificial Topological Insulators: Band Engineering

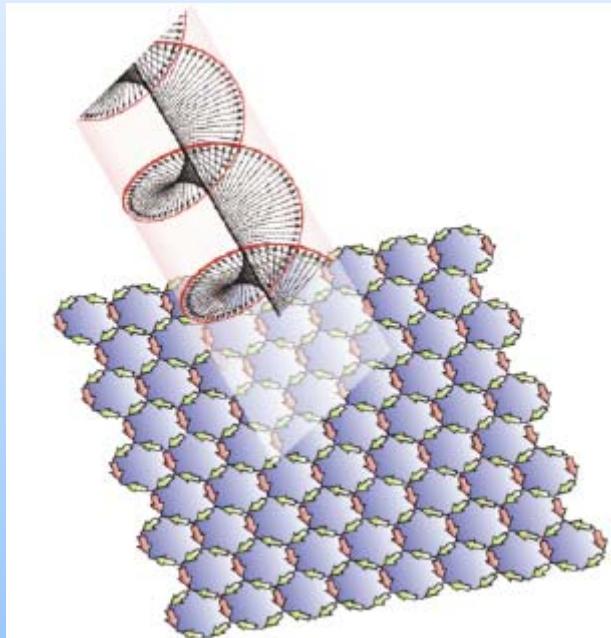
## ■ Topological Anderson Insulator



J. Li et al. Phys. Rev. Lett. 102, 136806 (2009).  
C. W. Groth et al. Phys. Rev. Lett. 103, 196805 (2009)

# Dynamical Band Engineering

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## Question:

What happens to the Dirac particles of graphene subjected to circularly polarized field?

A dynamical gap appears at the Dirac point of graphene making it a photo-induced quantum Hall state.

T. Oka and H. Aoki, Phys. Rev. B 79, 081406 (R) (2009)

# Floquet Theorem

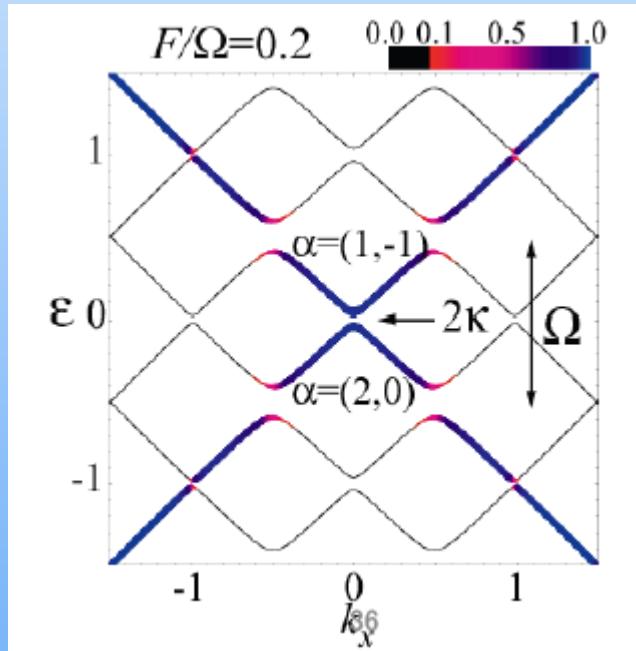
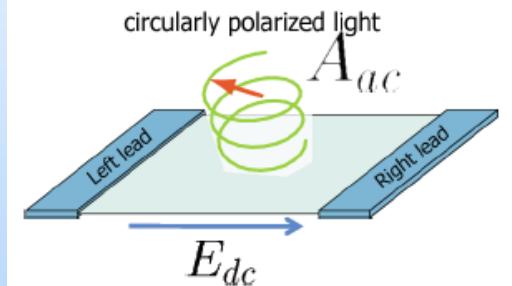
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## Floquet Theorem

If  $H(t) = H(t + T)$ , then there exists a complete set of solutions of the SE of the form  $\psi(t) = e^{-i\varepsilon t/\hbar} u(t)$ , with  $u(t + T) = u(t)$ .

- $\varepsilon$  is called Floquet quasi-energy; it is defined up to multiples of  $\hbar 2\pi/T$ .
- $u(t)$  is a solution of the problem  $[H(t) - i\hbar \frac{d}{dt}] u = \varepsilon u$ , or  $\sum_m (H_{n-m} + \hbar \frac{2\pi n}{T} \delta_{nm}) u_m = \varepsilon u_n$  in Fourier components.

# Photo-Induced Hall Effect ---- Floquet Topological Phase Transition



## Floquet Hamiltonian

$$H^{mn} = \frac{1}{T} \int_0^T dt H(t) e^{i(m-n)\Omega t} + m\delta_{mn}\Omega I$$

“Dynamical topological gap”

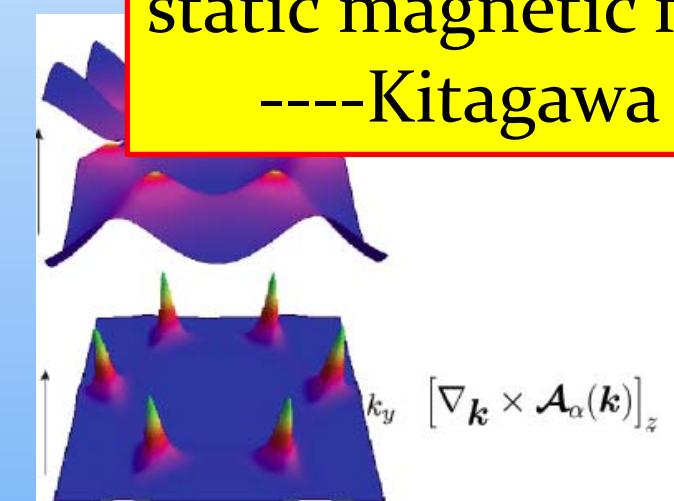
$$\kappa = \frac{\sqrt{4A^2 + \Omega^2} - \Omega}{2} \sim A^2/\Omega$$

# Photo-Induced Hall Effect ---- Floquet Topological Phase Transition

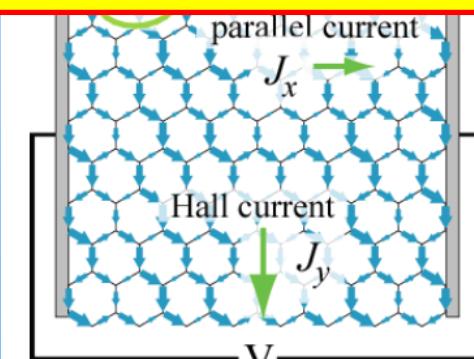
Extended Thouless-Kohmoto-Nightingale-Nijis formula for photo-induced Hall conductivity (photo-induced Chern form)

The application of circularly polarized light leads to an intriguing “Hall” effects without applying a static magnetic field!

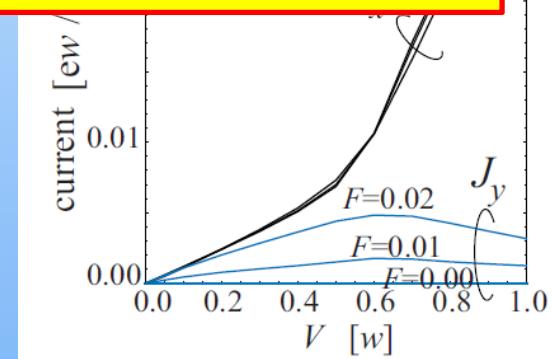
----Kitagawa et al. Phys. Rev. B 84, 235108 (2011)



Berry Curvature

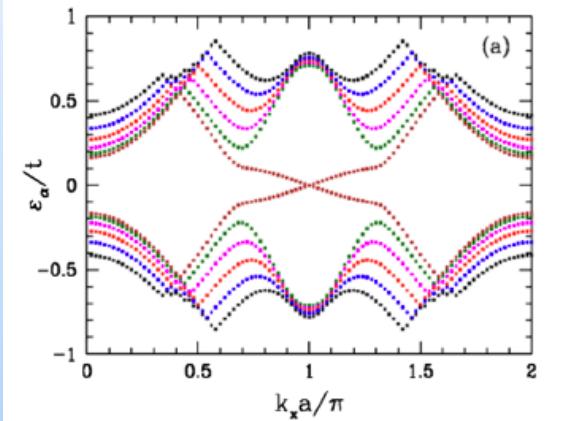
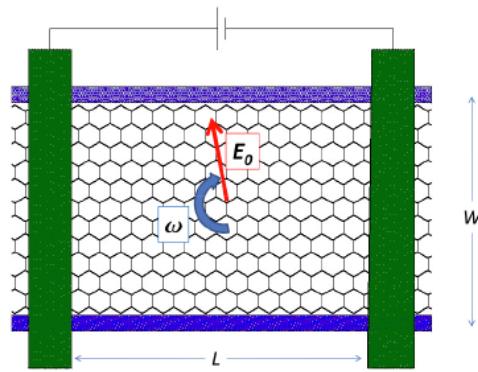


Current Distribution



I-V Curve

# Quantized Transport

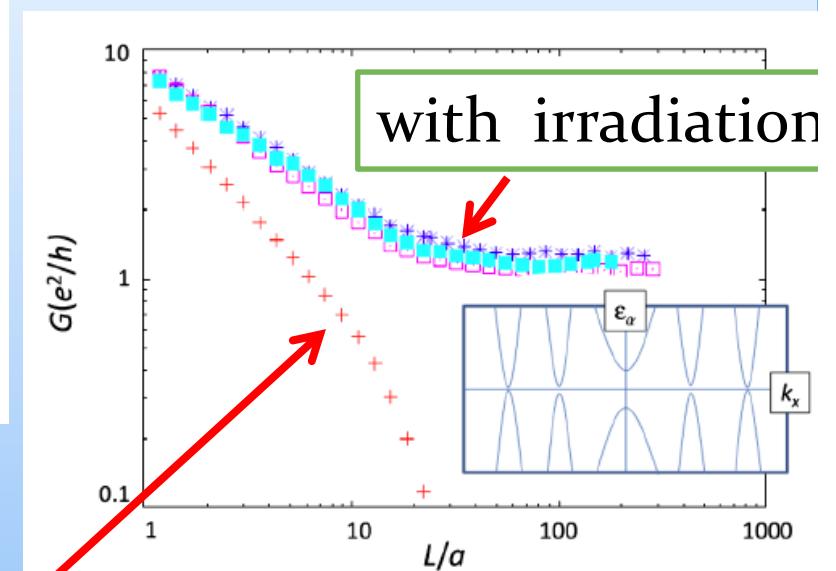


Graphene ribbon  
with open  
boundary  
condition

$$\hbar\Omega = 3t$$

Floquet spectrum  
of graphen ribbon

No irradiation

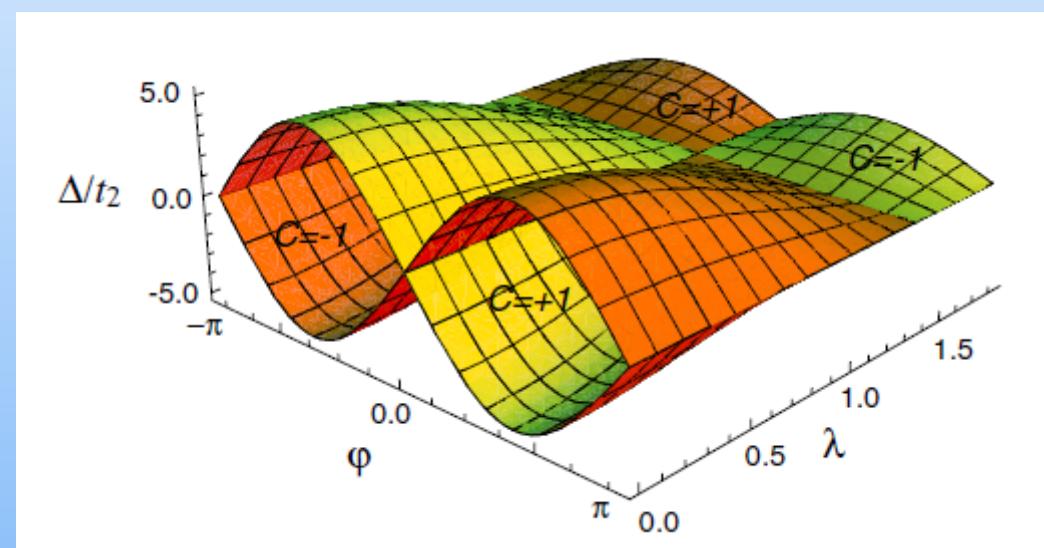
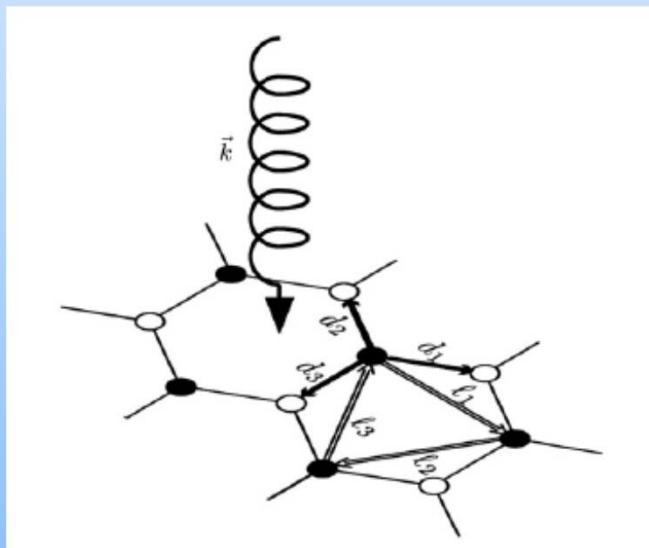


G~L relation. G levels off to quantized value when size is large, indicating the presence of edge states.

Z. Gu, et al. Phys. Rev. Lett. 107, 216601 (2011)

# Optical Tuning of Haldane Model

The phase diagram of Haldane model can be tuned by dynamical methods.



J. Inoue and A. Tanaka, Phys. Rev. Lett. 105, 017401 (2010).  
J. Inoue, J. of Luminescence.131, 482 (2011).

# Photo-induced Helical Edge States

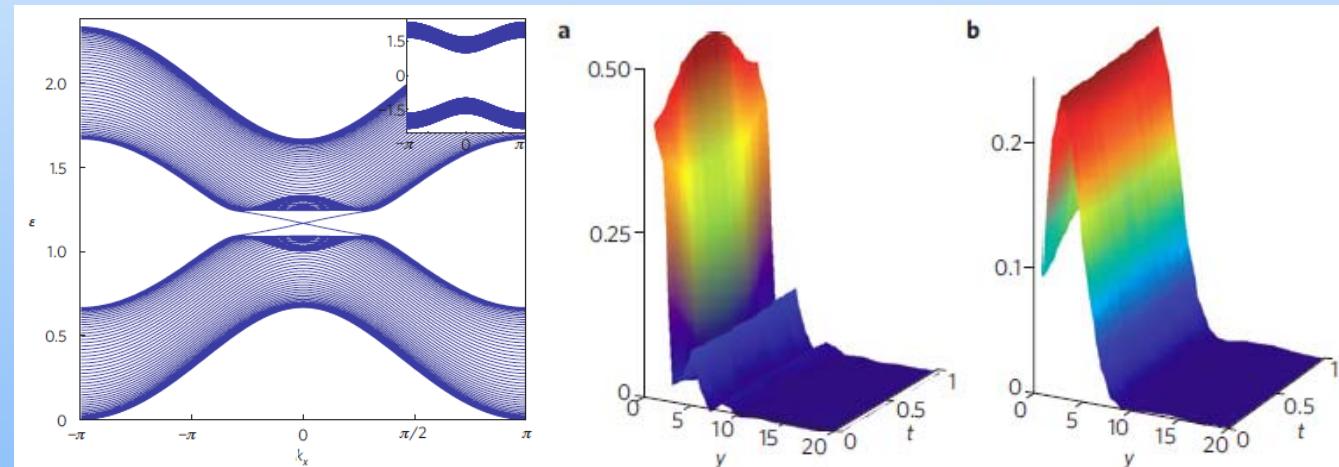
- By tuning the form of the periodic modulation, one can induce either chiral or helical edge states.
- Topological States with a pair of helical edge states can be turned on and off in HgTe/CdTe quantum wells by time-dependent perturbation.

Add time-dependent perturbation:

$$\check{V}(t) = \mathbf{V} \cdot \check{\sigma} \cos(\omega t)$$

with

$$V_0 \check{\sigma}_z \cos(\omega t)$$



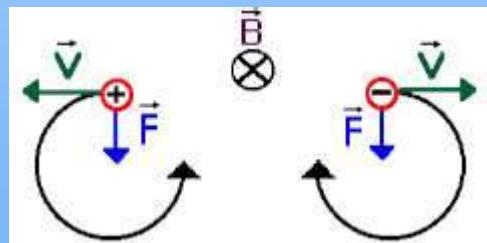
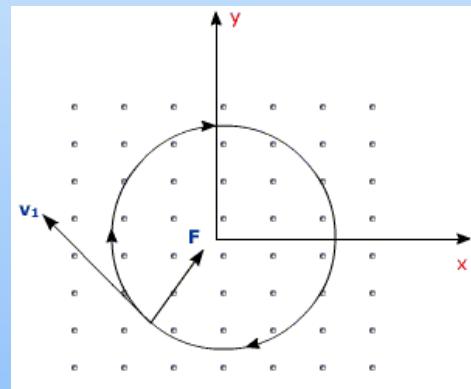
- Oscillating magnetic field – Zeeman effect
- Oscillating electric field – Orbital coupling
- Stress modulation – Material parameter

# Classical View: Edge States formation

- Hall effect

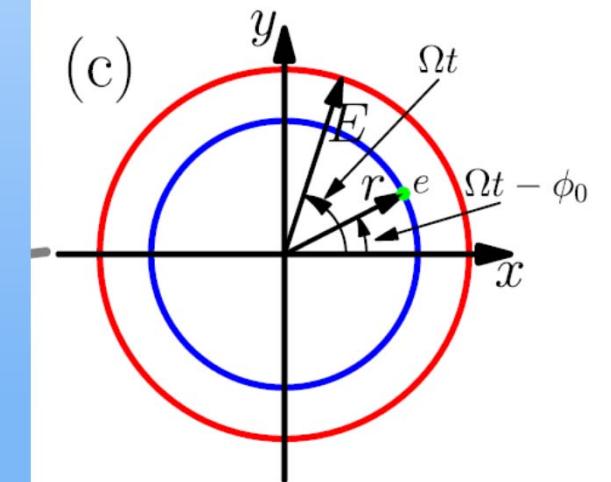
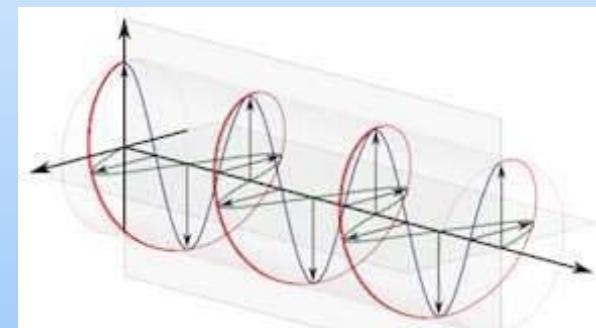
Magnetic field &  
Lorentz force:

$$\vec{F} = q \left( \vec{E} + (\vec{v} \times \vec{B}) \right)$$



- Optical Hall Effect

Circularly polarized field:



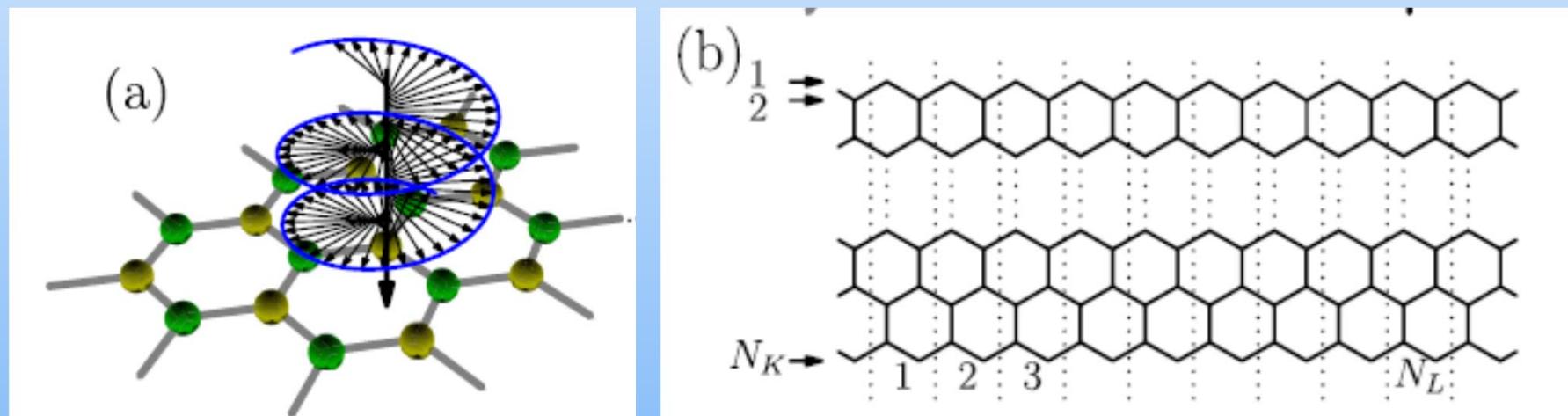
# Dynamical tuning the Kane-Mele Model?

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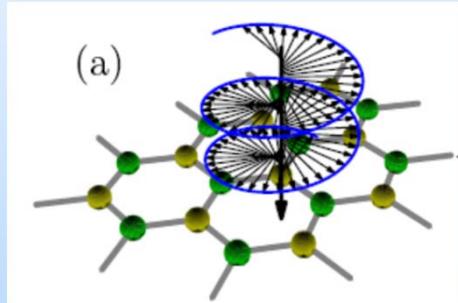
- Edge States Structure
  - Chiral or Helical ?
  - Spin-polarized or unpolarized?
- Tuning the topological insulating states via optical means.
  - Dynamical Band-Engineering

# Model System

- Nanoribbon of Kane-Mele model subjected to circular polarized field



# Hamiltonian



Model Hamiltonian:

$$H = \sum_{\langle ij \rangle} \tilde{t}_{ij} c_i^\dagger c_j + i \sum_{\langle\langle ij \rangle\rangle} \tilde{\lambda}_{SO,ij} \mu_{ij} c_i^\dagger \sigma^z c_j \\ + i \sum_{\langle ij \rangle} \tilde{\lambda}_{R,ij} c_i^\dagger (\boldsymbol{\sigma} \times \mathbf{d}_{ij})_z c_j + \lambda_v \sum_i \eta_i c_i^\dagger c_i,$$

Vector Potential:

$$\mathbf{A}(t) = A_0(\sin(\Omega t), \cos(\Omega t))$$

The hopping term between two lattice sites is modified with an extra phase:  $e^{iA_{ij}(t)}$

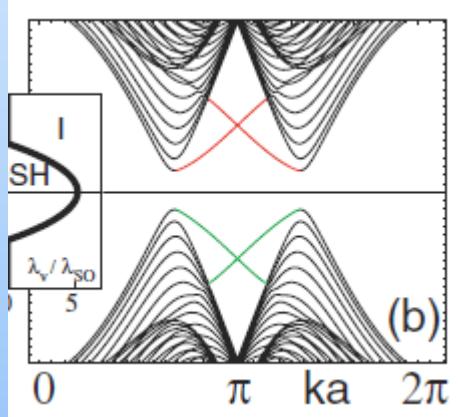
where  $A_{ij} = \frac{e}{\hbar}(\mathbf{r}_j - \mathbf{r}_i) \cdot \mathbf{A}(t)$

# Conductance Quantization

$$\lambda_{SO} = 0.06 t_0$$

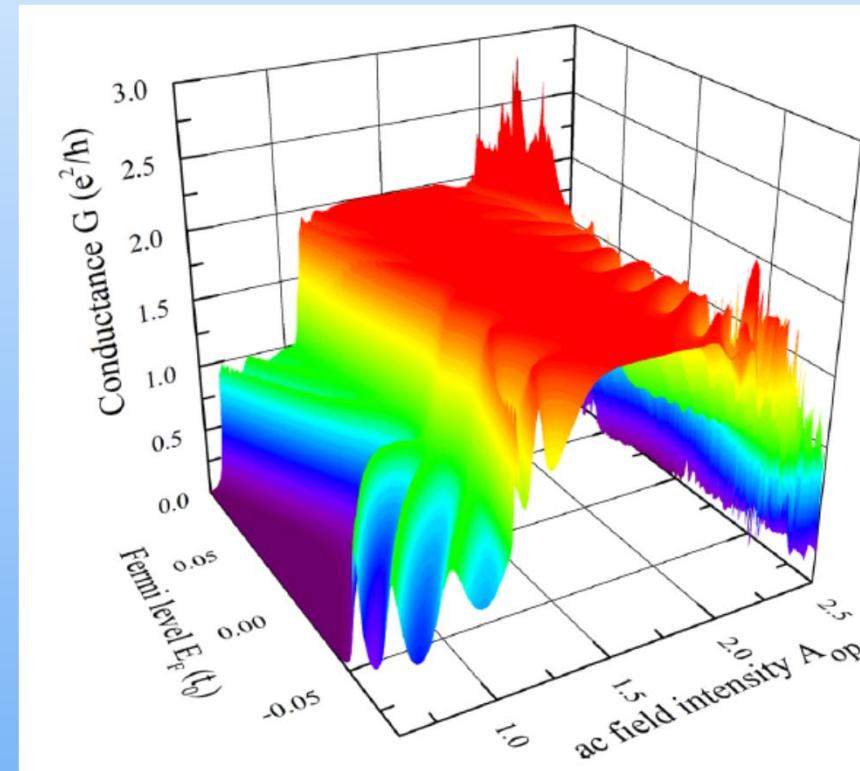
$$\lambda_R = 0.05 t_0$$

$$\lambda_v = 0.4 t_0$$



$$N_L = N_K = 36$$

$$\hbar\Omega = 5 t_0$$

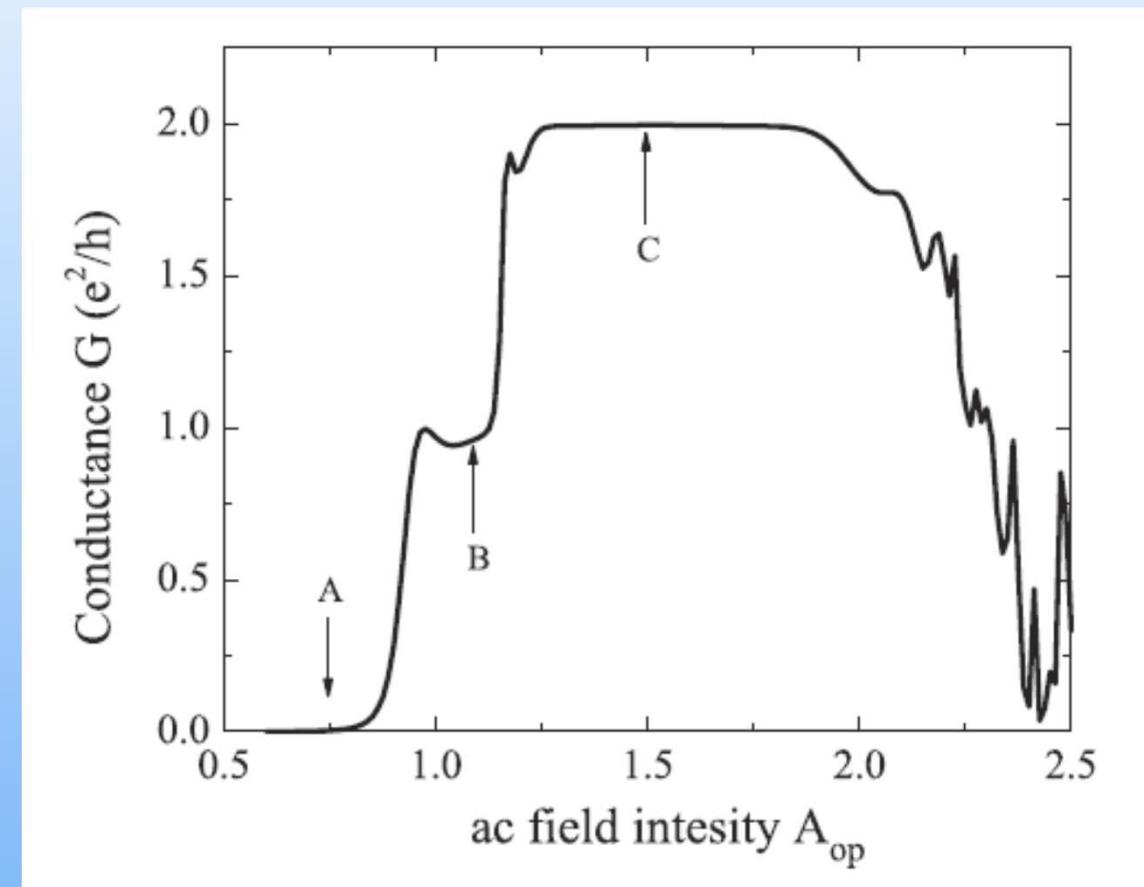


At static, the ribbon is an insulator as shown by Kane and Mele.

B. H. Wu, Q. Liu, X. Y. Jiang and J. C. Cao, Appl. Phys. Lett. 100, 203106 (2012)

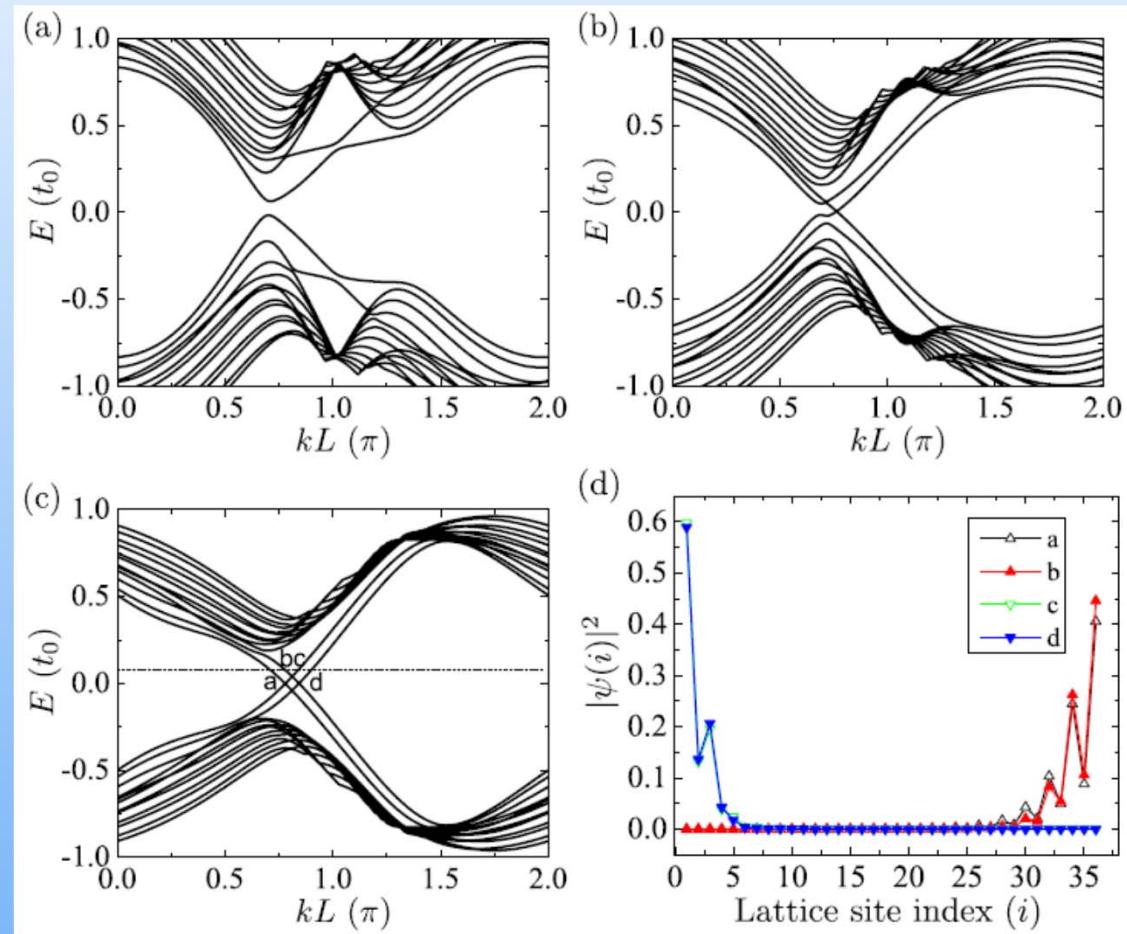
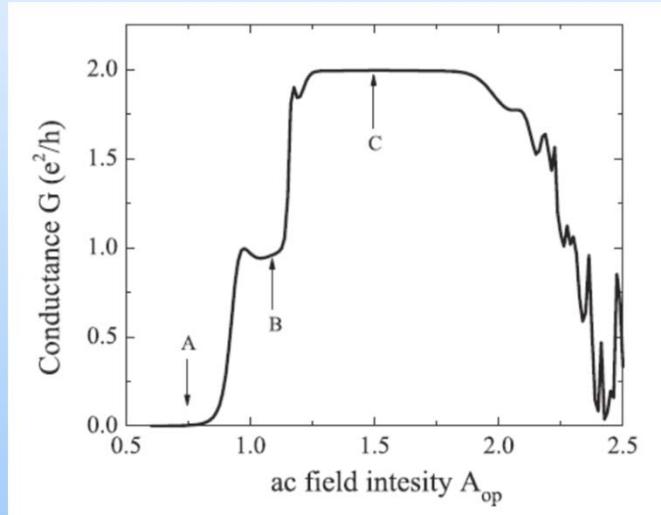
# Conductance Quantization

$$E_F = 0.04 t_0$$



B. H. Wu, Q. Liu, X. Y. Jiang and J. C. Cao, Appl. Phys. Lett. 100, 203106 (2012)

# Wave-function of Edge States



B. H. Wu, Q. Liu, X. Y. Jiang and J. C. Cao, Appl. Phys. Lett. 100, 203106 (2012)

# Conclusion

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- Dynamical methods provide useful tools to switch on and off edge states, enabling the dynamical band engineering of topological properties.
- The edge states of Kane-Mele model can be tuned by dynamical methods. With proper parameters, one can obtain spin-polarized edge states without external magnetic field, which may be useful for spin injection.

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Prof. Juncheng Cao

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**Thank you for your attention!**