Majorana and Andreev bound states in topological wires in the proximity of superconductors

D. Chevallier

LPS, Université Paris Sud 11, Orsay

IPHT, CEA Saclay

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Collaborators

D. Sticlet
(Mpipks Dresden)

P. Simon (1)

C. Bena (2)

LPS [1]

CEA-IPHT [2]

Outline

- Introduction to Majorana fermions in 1D systems
- How can we locate these Majorana states?
- Interplay between Majorana and Andreev states in NS and SNS junctions
- Conclusion

A Majorana fermion is a fermion
which is its own anti-particle

$$\Gamma = \Gamma^{\dagger}$$

Dirac equation : $(i\gamma^{\mu}\partial_{\mu} - m)\Psi = 0$

<u>Complex</u> solution Ψ

Dirac (1928)

Ψ particle $\neq \Psi^{*}$ antiparticle

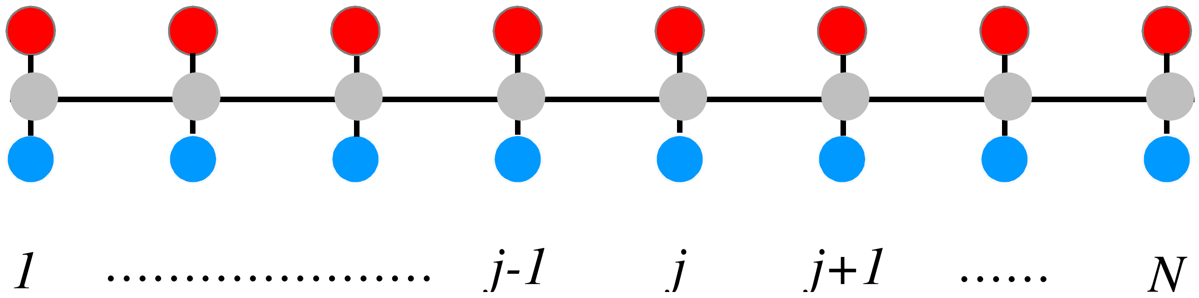
<u>Real</u> solution Ψ

Majorana (1937)

Ψ particle $= \Psi^{*}$ antiparticle

E. Majorana, Nuovo Cimento **14**, 171 (1937)

Majorana in 1D : Kitaev model (spinless p-wave superconductor)



$$H = \sum_j [t(c_j^\dagger c_{j+1} + c_{j+1}^\dagger c_j) + \Delta(c_j c_{j+1} + c_{j+1}^\dagger c_j^\dagger) - \mu(c_j^\dagger c_j - \frac{1}{2})]$$

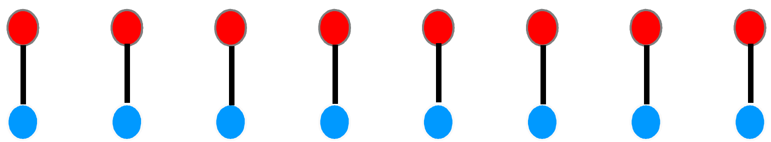
$$\begin{cases} \gamma_j^A = c_j + c_j^\dagger \\ \gamma_j^B = i(c_j - c_j^\dagger) \end{cases}$$

$$(\gamma_j^\alpha)^\dagger = (\gamma_j^\alpha)$$
$$\left\{\gamma_i^\alpha, \gamma_j^\beta\right\} = 2\delta_{ij}\delta_{\alpha\beta}$$

$$H = \sum_j [i\mu\gamma_j^A\gamma_j^B + i(t-\Delta)\gamma_j^B\gamma_{j+1}^A - i(t+\Delta)\gamma_j^A\gamma_{j+1}^B]$$

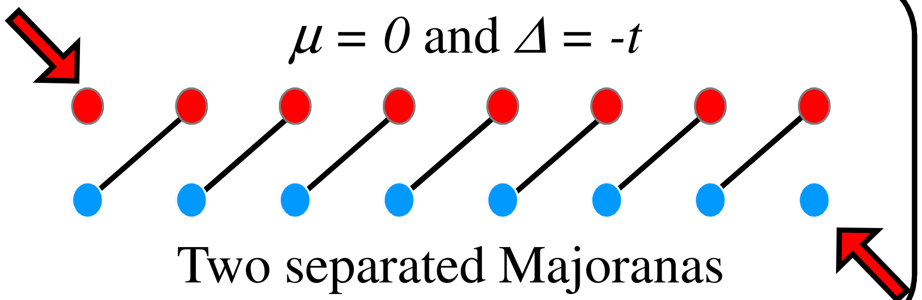
<u>Two extreme cases:</u>

Δ = t =0



No Majorana

μ = 0 and *Δ = -t*



Two separated Majoranas

Kitaev 2001, Fleury GDR Aussois

Solution

a standard superconductor — in proximity to — a material with strong spin-orbit coupling to avoid spin-degeneracy

↓

S-wave superconductor — in proximity to — InAs nanowire in presence of Zeeman field

Sau et al, PRL (2010); J. Alicea, PRB (2010); Oreg, Refael & Von Oppen, PRL(2010)

How to locate these Majorana states?

Majorana Polarization!!!

Sticlet et al, PRL ***108****, 096802 (2012)*

How to locate these Majorana states??
Majorana Polarization
(Analogy with real spin)

General wavefunction

$$u\,c^{\dagger} + v\,c = (u+v)/\sqrt{2}\,M_{\uparrow} + (u-v)/i\sqrt{2}\,M_{\downarrow}$$

Project this wavefunction in the Majorana basis

$$P_{x\uparrow} = |u+v|^2/2$$

$$P_{x\downarrow} = |u-v|^2/2$$

Majorana polarization

$$P_x = P_{x\uparrow} - P_{x\downarrow} = 2\,\mathrm{Re}(u\,v^*)$$

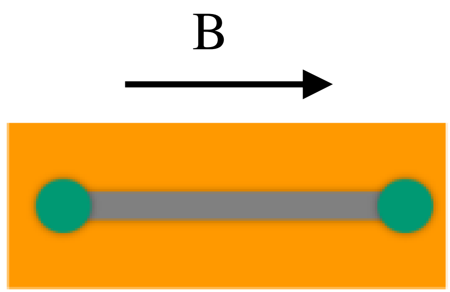
New fermionic operators

$$M_{\uparrow} = (c^{\dagger} + c)/\sqrt{2}$$

$$M_{\downarrow} = i(c^{\dagger} - c)/\sqrt{2}$$

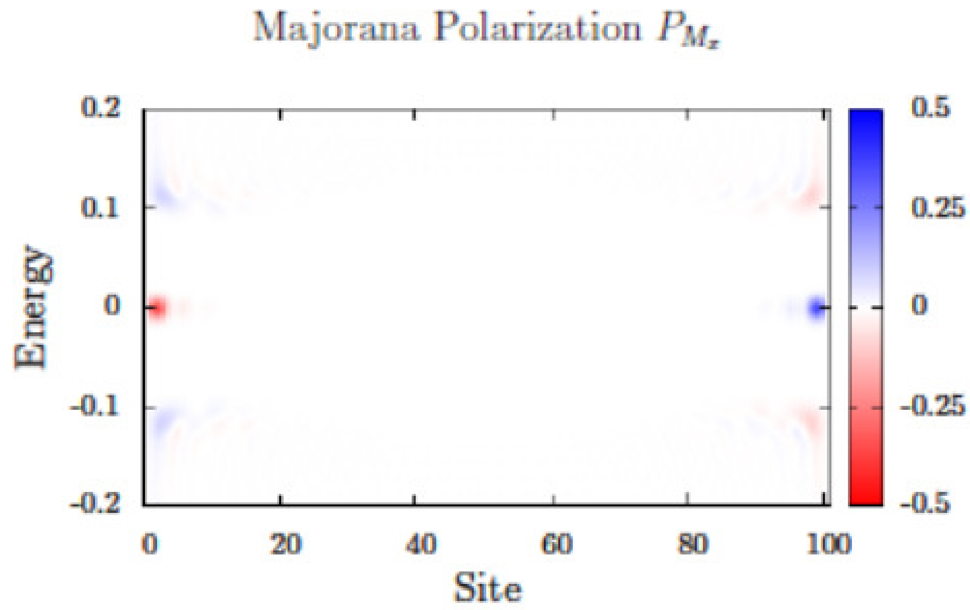
Majorana polarization
- $=0$ if $u=0$ or $v=0$ (pure fermionic states)
- is maximal for $u=v$ and $u=-v$ (purely Majorana states)

Application to 1D quantum wires



InAs nanowire on SC

Condition for topological phase : $V_z^2 > \Delta^2 + \mu^2$



$\Delta = 0.3$, $V_z = 0.4$, $\alpha = 0.2$, $\beta = 0$ and $\mu = 0$

Two Majorana bound states at the ends of the nanowire

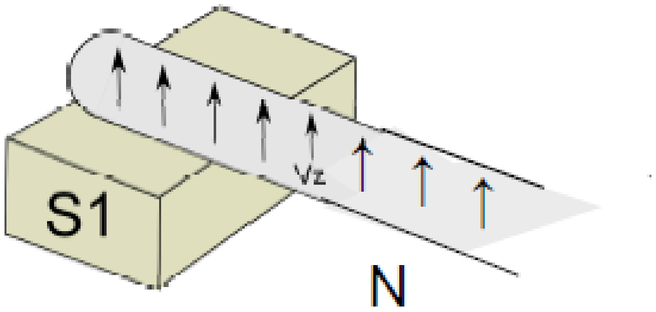
Majorana and Andreev bound states in NS and SNS junctions

*Chevallier et al, PRB **85**, 235307 (2012)*

Majorana fermions in NS junctions

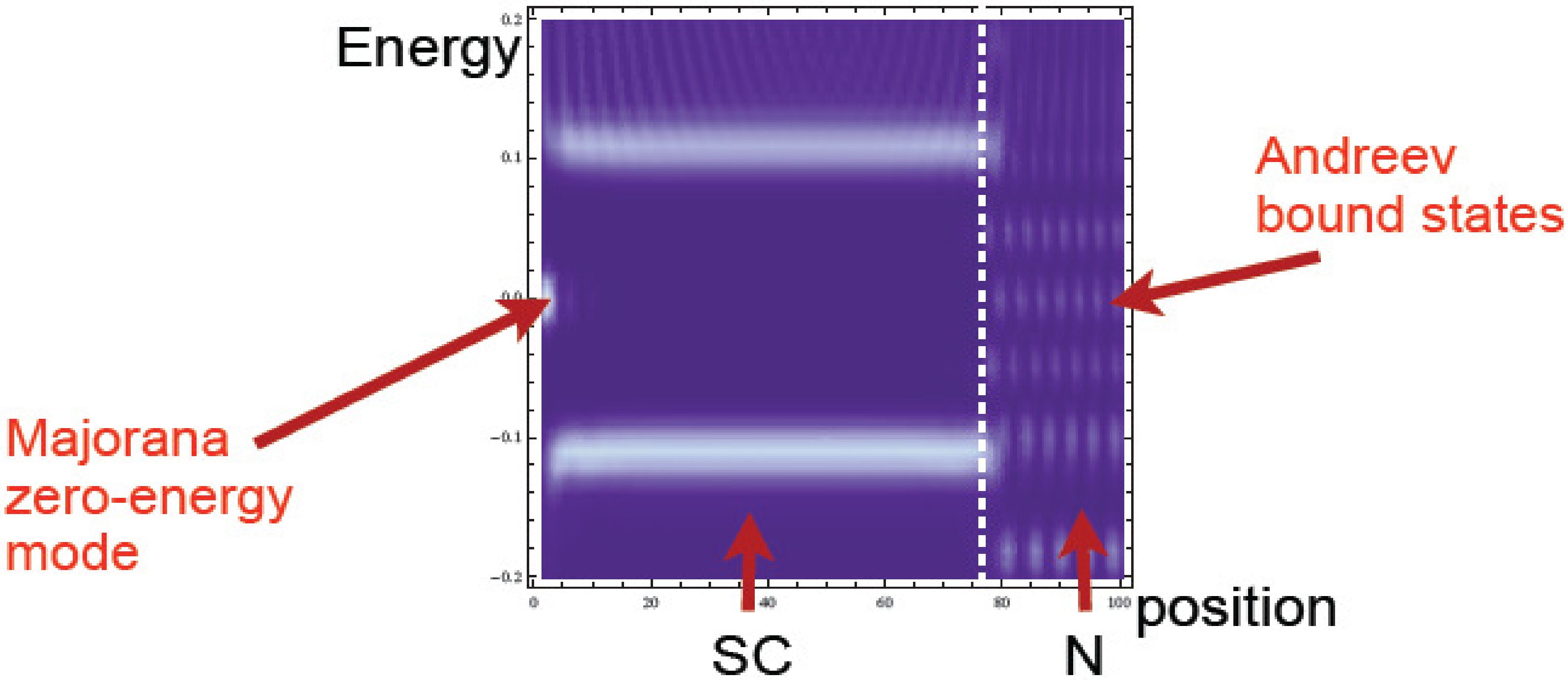
Inhomogenous tight-binding Hamiltonian

$$H = \sum_j \Psi_j^\dagger [(\mu - t)\tau_z + V_z \sigma_3 - \Delta \tau_1] \Psi_j$$
$$- \frac{1}{2} \left[\Psi_j^\dagger (t + i\alpha\sigma_y + i\beta\sigma_x) \tau_z \Psi_{j+1} + \text{h.c.} \right]$$



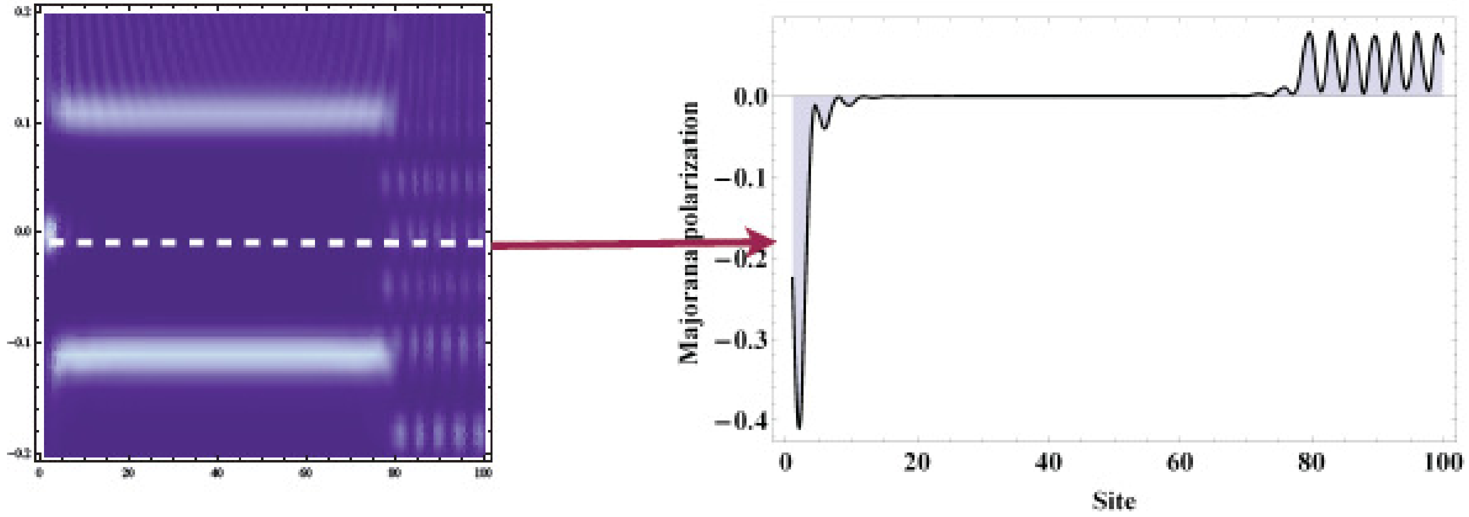
- Diagonalize numerically
- Study the LDOS and the Majorana polarization as a function of energy and position

Local Density of states of NS junctions



- No Majorana bound state at the interface !
- Majorana fermions come in pairs (conservation)
- Where is the second Majorana ?

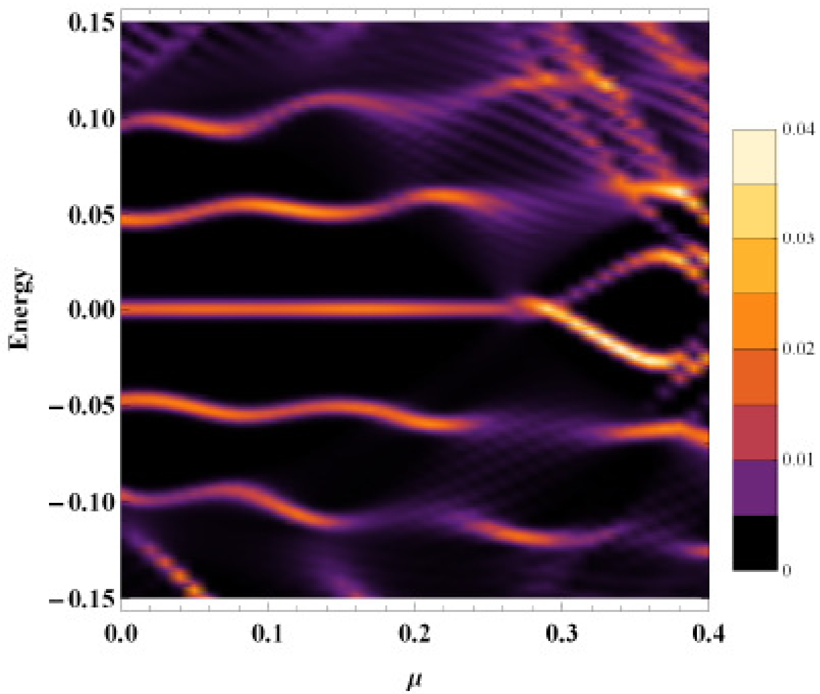
Zero-energy Majorana polarization

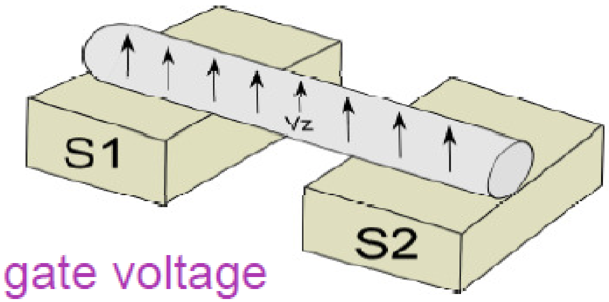


- Extended Andreev bound state at zero energy has non-zero Majorana polarization
- Integral of the MP over the normal wire = 1
- Integral of the LHS peak = -1 $\rightarrow$ MP is conserved
- Extended Majorana state in the normal state !

Spectroscopy of Andreev Bound States

- Robustness:
  - No displacement with the chemical potential, Zeeman magnetic field, etc.
- Easy to detect in spectroscopy
- Easy to manipulate: SNS junctions ?





- Easy to investigate via spectroscopy
- Easy to manipulate by the phase difference
- Majorana character - invariance with respect to gate voltage
- Andreev character - dependence on phase difference

Conclusion

- InAs/InSb nanowire carry Majorana states in topological phase
- Majorana Polarization= good local order parameter
- Interplay between Majorana and Andreev bound states in NS and SNS junctions
- Experimental detection with SNS junctions

Discussions: - ***R. Aguado*** *(ICCM, Madrid)*

Thank you for your attention!!

ENGINEERING P-WAVE SUPERCONDUCTORS

p-wave Bogoliubov qps: $$\gamma_n^+ = \sum_i \left(u_{ni} a_{i\uparrow}^+ + v_{ni} a_{i\uparrow} \right)$$

zero energy: $$\gamma^+ = \sum_i u_{0i} a_{i\uparrow}^+ + u_{0i}^* a_{i\uparrow} = \gamma .$$

$\gamma = \gamma^+ \Rightarrow$ its own antiparticle

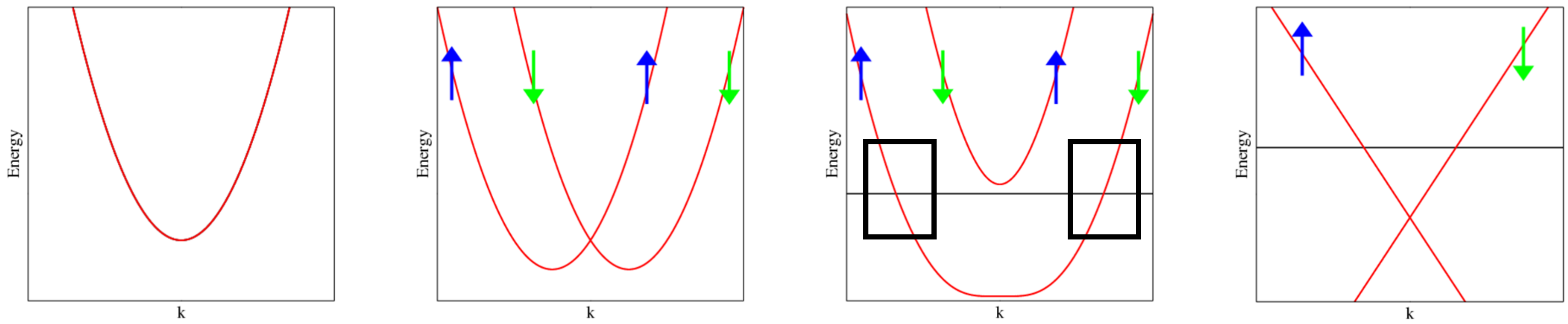
PROBLEM : p-wave superconductors are rare in nature

Few candidates : Sr_2RuO_4 or He-3 superfluid phase

s-wave Bogoliubov qps: $$\gamma_n^+ = \sum \left(u_{ni} a_{i\uparrow}^+ + v_{ni} a_{i\downarrow} \right)$$

+ Strong spin orbit coupling

Mapping between InAs nanowire and topological insulator



Semiconductor

InAs

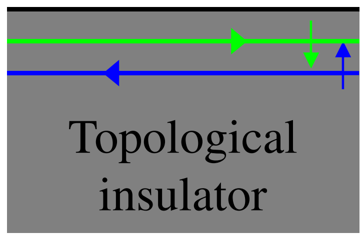
Semicond. with strong spin-orbit coupling

⇒ Rashba spin-splitting

InAs

M ↑ Magnetic insulator

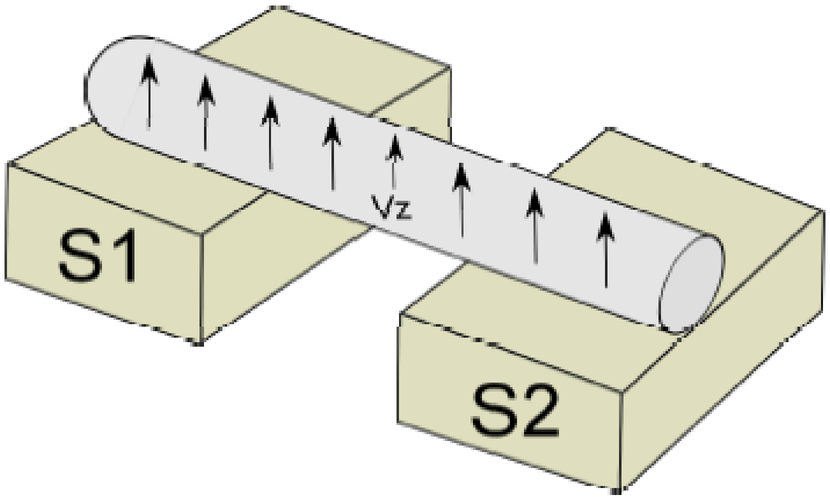
Zeeman spin-splitting



Condition for topological phase : $V_z^2 > \Delta^2 + \mu^2$

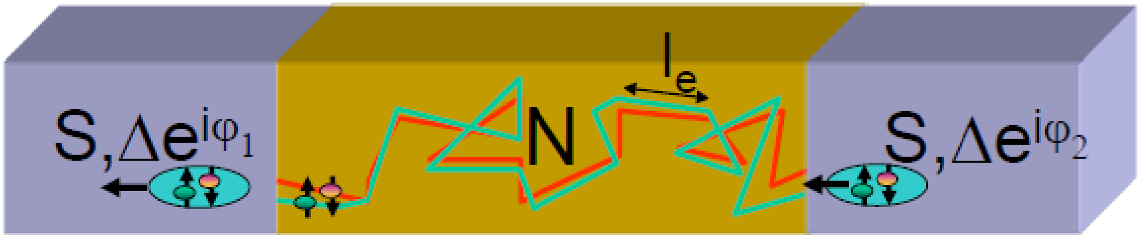
G. Fleury GDR Aussois

Majorana States in SNS junctions

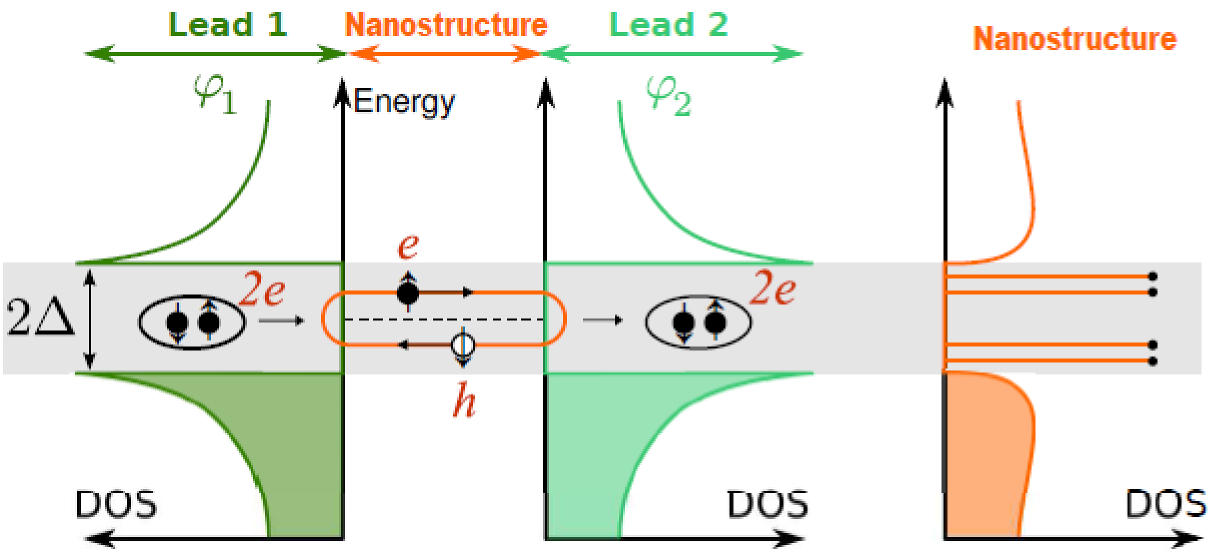


- Semiconducting wire + SO + Zeeman
- Proximity with two SCs with phase difference
- Control the chemical potential by back gate
- Search for the Majorana states using the LDOS and the MP

Andreev Bound States

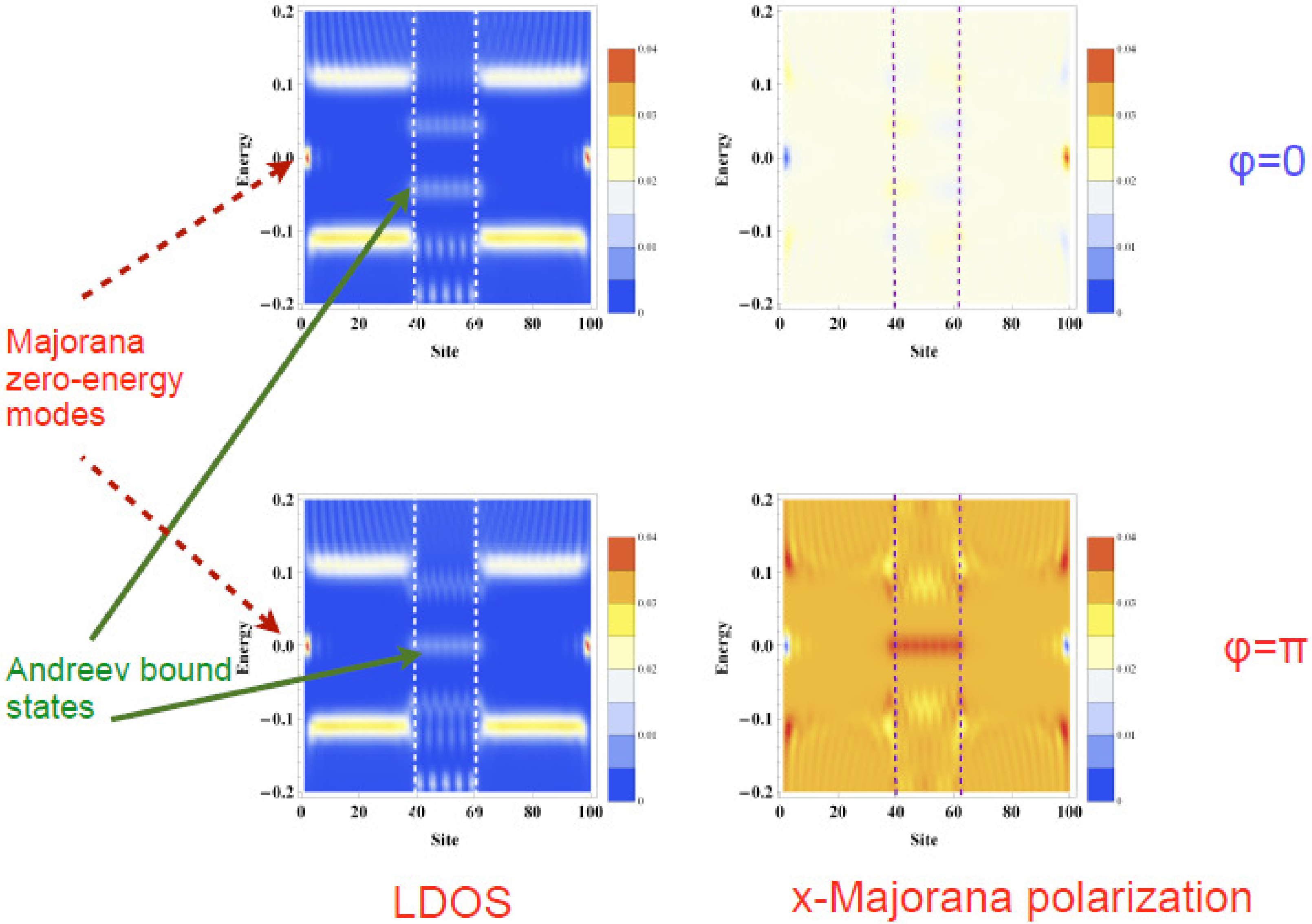


S. Guéron HDR



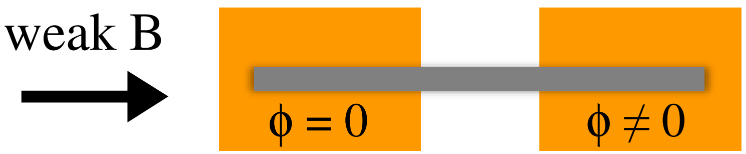
Pillet, Nat. Phys. 11 (2010)

LDOS and Majorana polarization

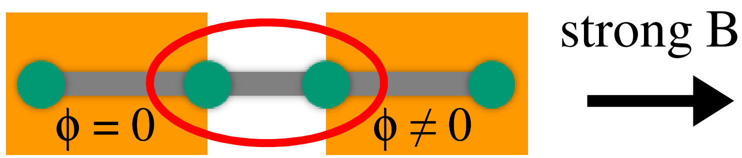


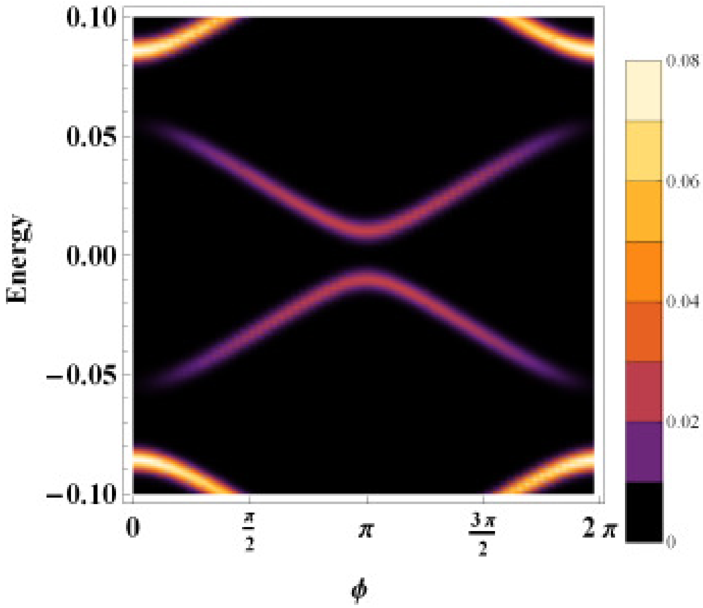
Detection by using Anomalous Josephson effect

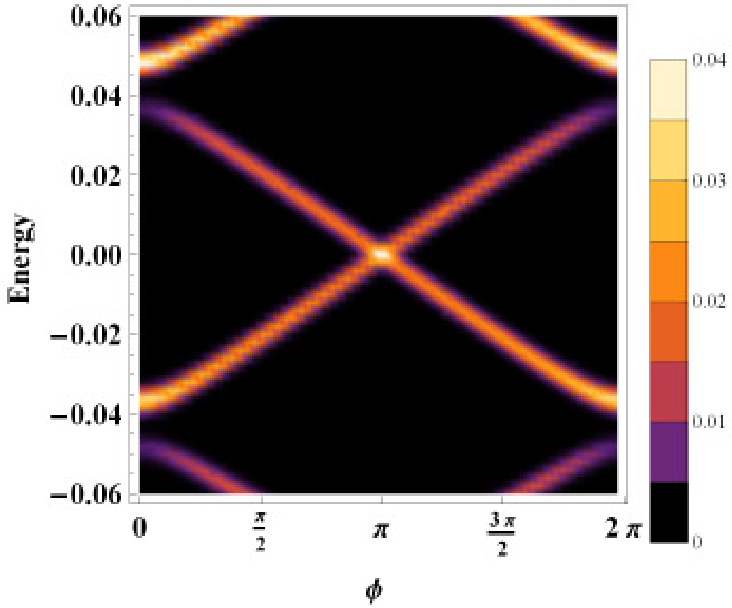
Trivial phase



Topological phase







Das Sarma PRL 2010, Meyer PRL 2011, etc