école normale
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Wigner function representation in electron quantum optics

arXiv:1308.1630, today on cond-mat!!

Dario Ferraro

Quy-Nhon, August 8, 2013

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Outline

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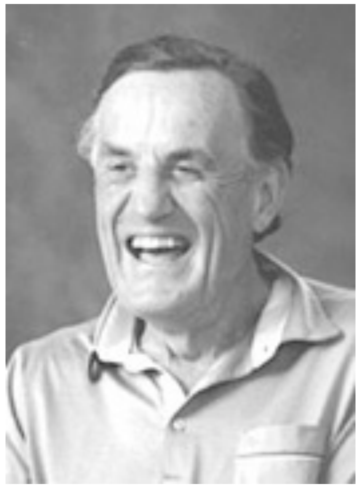
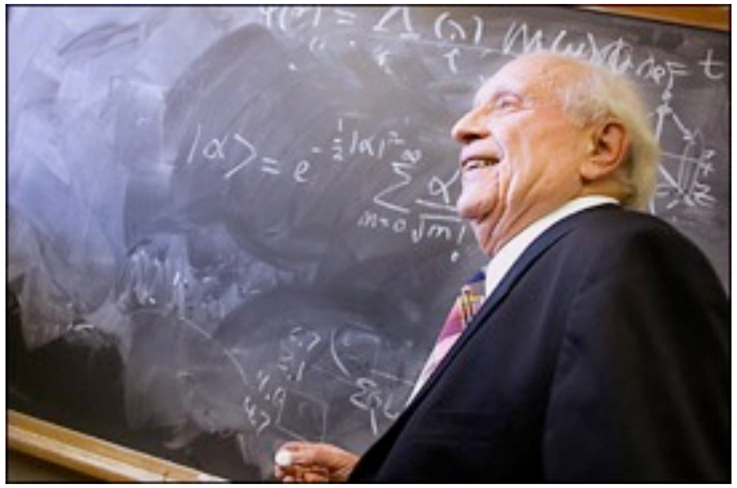
- What's electron quantum optics?
- The first order electron coherence and its representations: the Wigner function
- Unified description of single and two-electron interferometry (Mach-Zehnder, Hanbury-Brown-Twiss, Hong-Ou-Mandel)

Quantum optics: 50 years of history





The Quantum Theory of Optical Coherence*

Roy J. Glauber
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts

(Received 11 February 1963)

Phys. Rev. **130**, 2529 (1963)

Phys. Rev. Lett. **10**, 84 (1963)

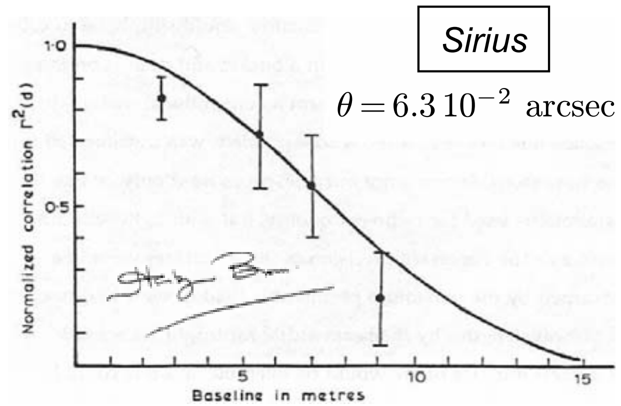
Phys. Rev. **131**, 2766 (1963)

R. Hanbury Brown

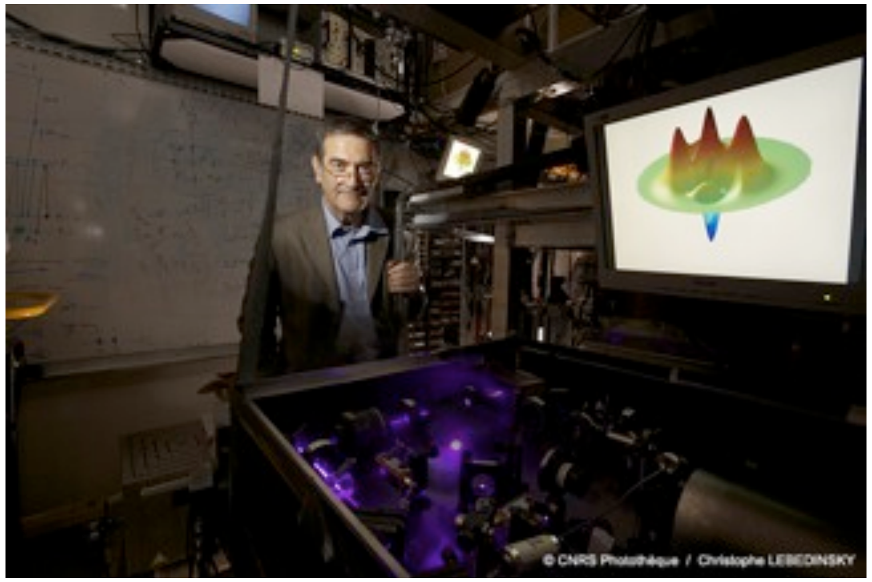
R.J. Glauber

From 1956: stellar interferometry...

...to 2012: Nobel prize *"for ground-breaking experimental methods that enable measuring and manipulation of individual quantum systems"*



$\Gamma^2(d) = [2J_1(x)/x]^2$; $x = \pi\theta d/\lambda$



R. Hanbury Brown and R. Q. Twiss, Nature **178**, 1046 (1956)

Electron quantum optics

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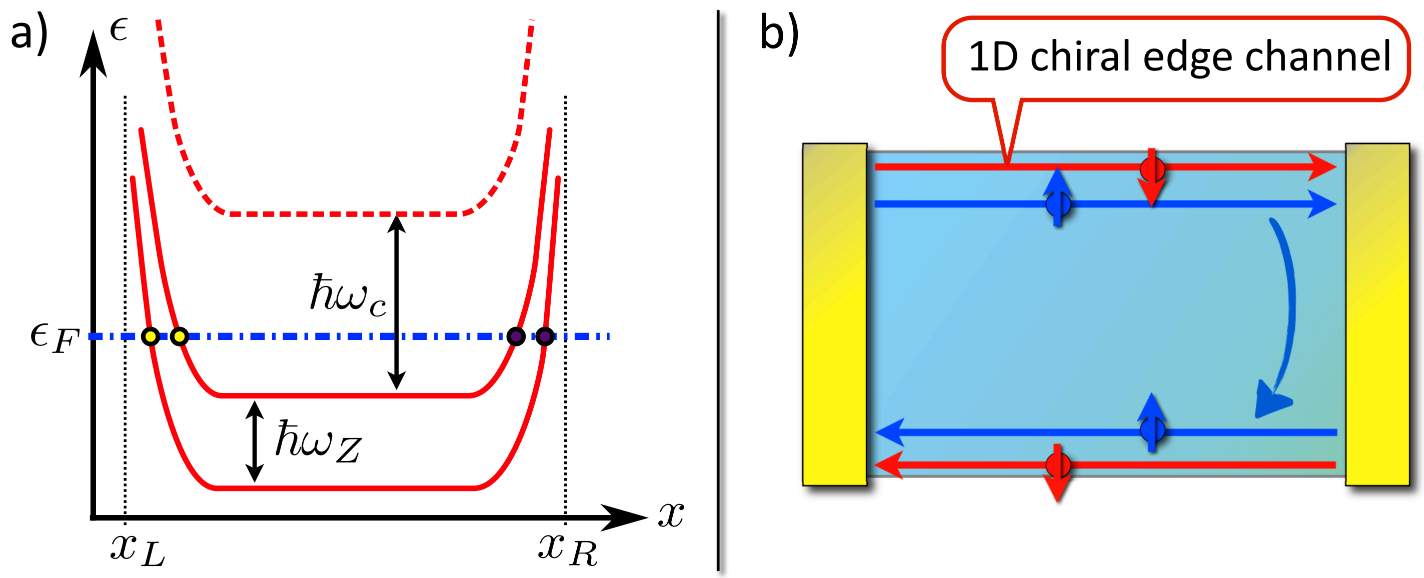
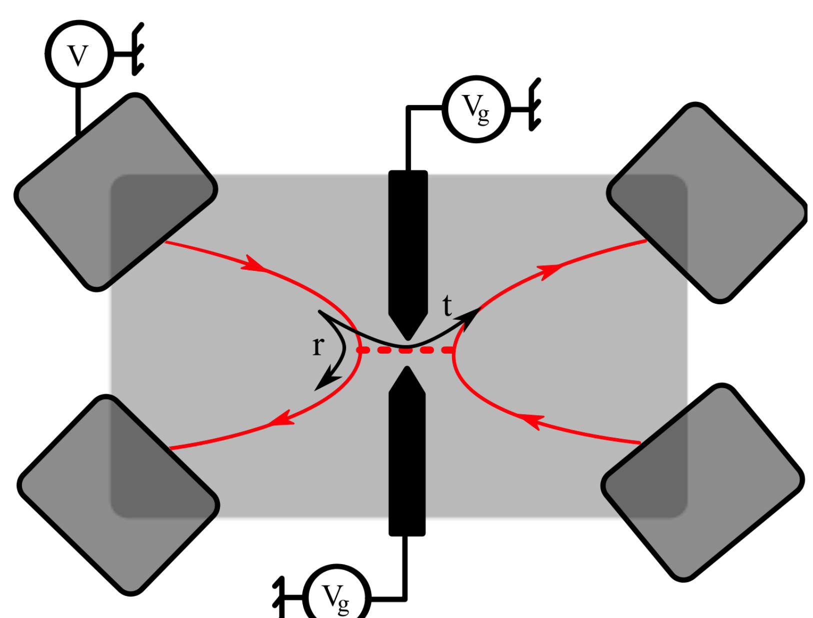
Photon *vs* electron quantum optics: *the dictionary*



Quantum Hall edge channels as wave-guides
(several micrometers of elastic mean free path)



Quantum point contact as a beam splitter

Elementary electron sources

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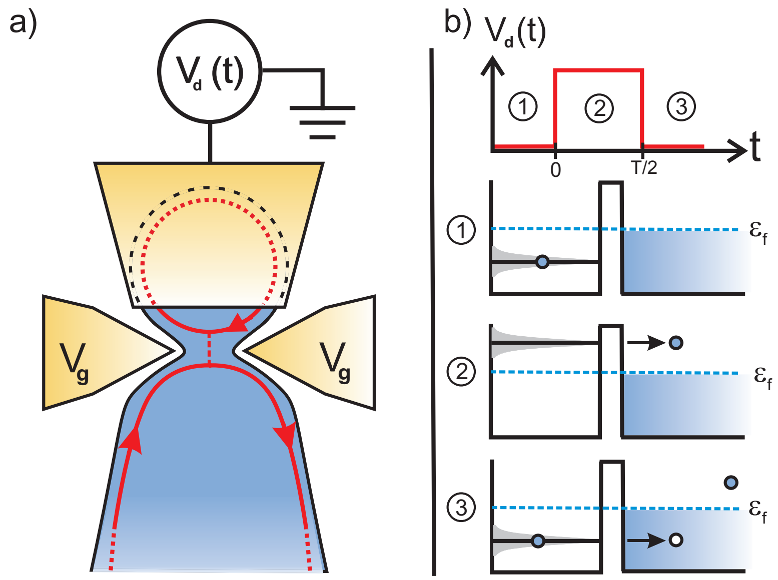
Driven mesoscopic capacitor (LPA, ENS Paris)

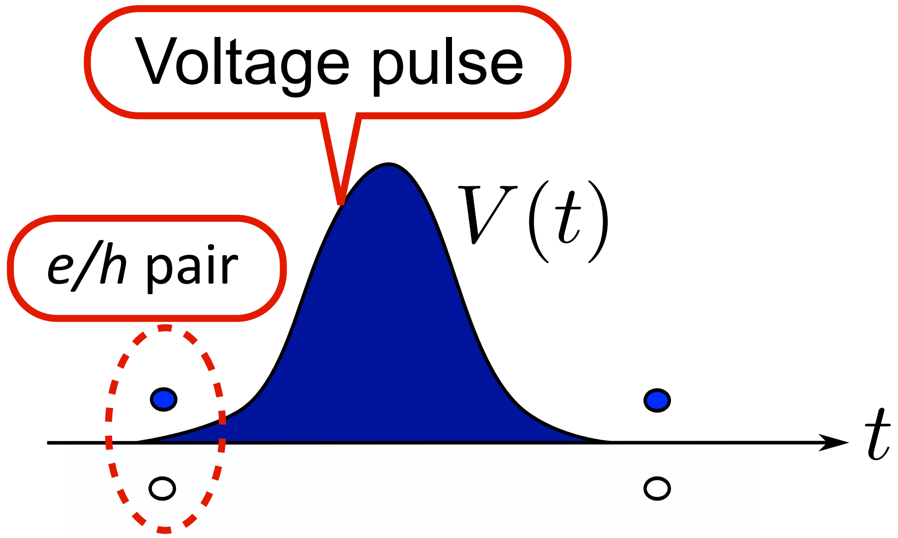
Theory: M. Buttiker *et al.*, Phys. Lett. A **180**, 364 (1993), Moskalets *et al.*, Phys. Rev. Lett. **100,** 086601 (2008) ;
Experiments (LPA, Paris): G. Fève *et al.*, Science **316**, 1169 (2007), A. Mahé *et al.*, Phys. Rev. B **82**, 201309(R) (2010)



In the optimal regime one electron and one hole emitted in each period

Lorentzian voltage pulse (CEA, Saclay)

Theory: L. S. Levitov *et al.,* J. Math. Phys. **37**, 4845 (1996), J. Keeling *et al.*, Phys. Rev. Lett. **97,** 116403 (2006);
Experimental proposals (Glattli's Group, Saclay): J. Dubois *et al.*, Phys. Rev. B **88**, 085301 (2013)



For proper lorentzian pulse in time no particle-hole contribution

Two-electrons interferometry: experiments

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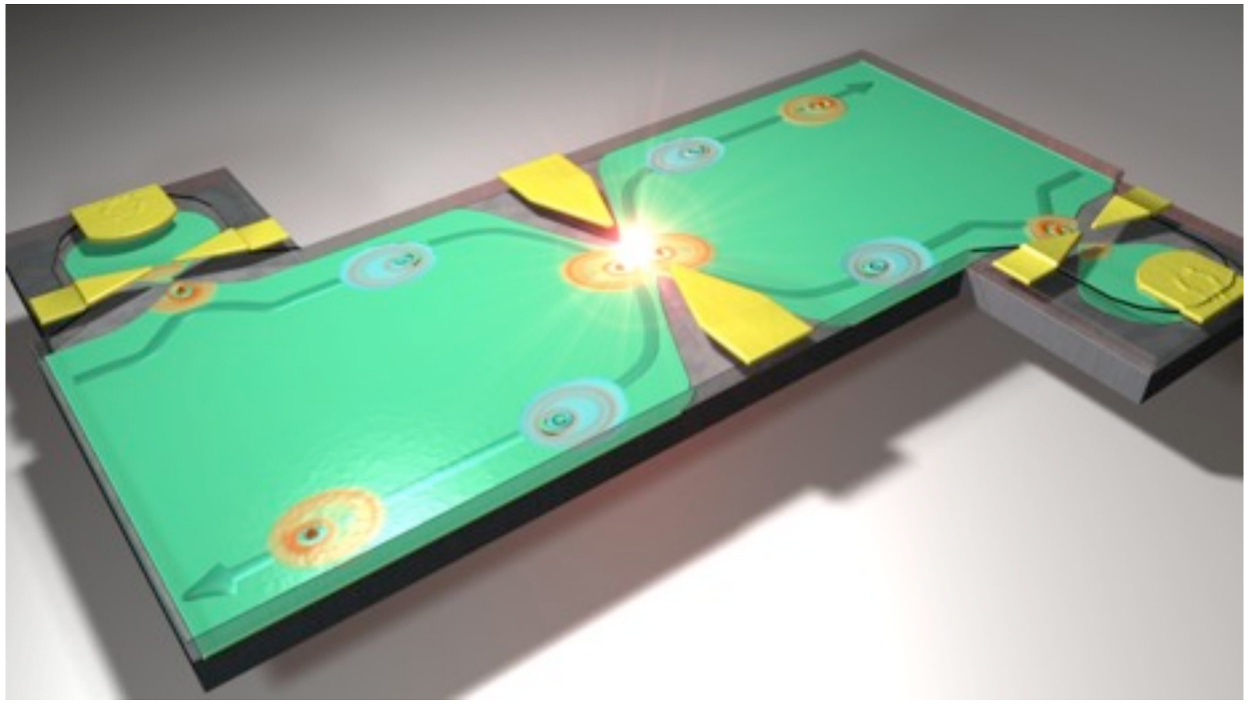
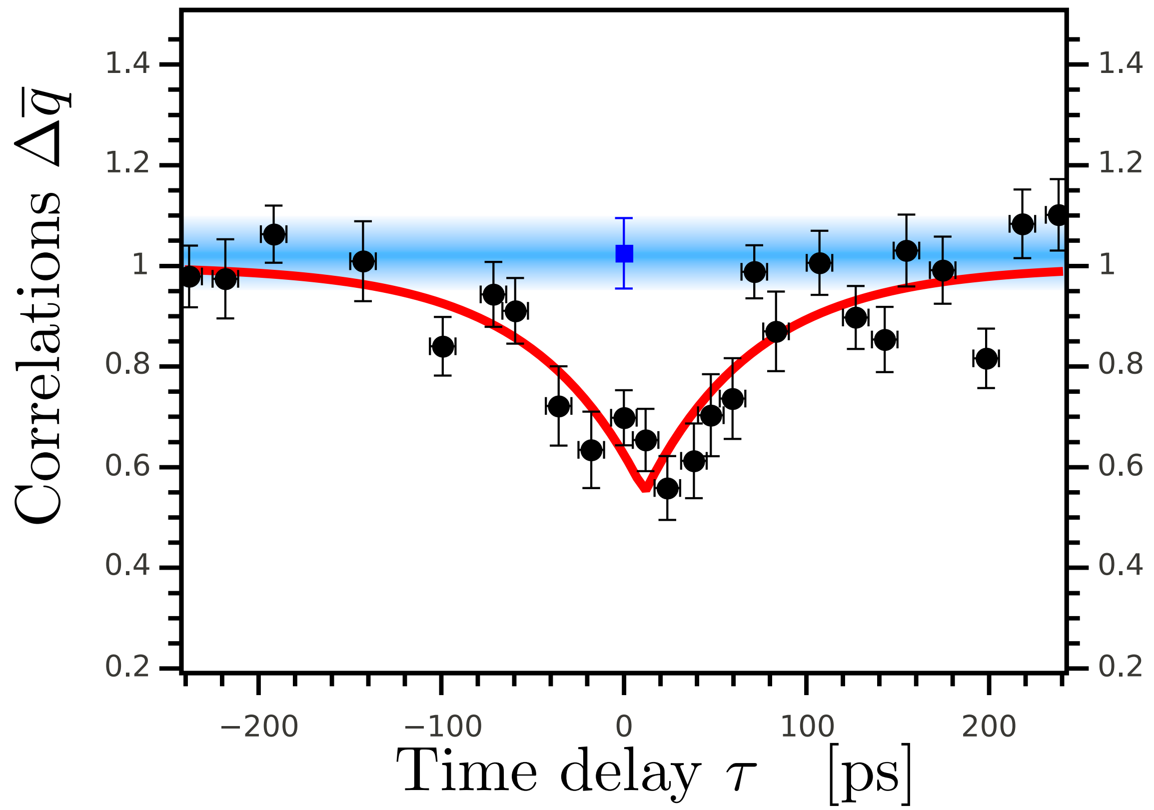
Hanbury-Brown-Twiss (HBT) and Hong-Ou-Mandel (HOM) interferometers with electrons (LPA, Paris)

Electron quantum optics: partitioning electrons one by one
E. Bocquillon *et al.* Phy. Rev. Lett. **108**, 196893 (2012)

Coherence and Indistinguishability of Single Electrons Emitted by Independent Sources
E. Bocquillon *et al.* Science **339**, 1054 (2013)





Electrons *vs* photons: differences

Differences between electron and photon quantum optics

Fermionic *vs* bosonic statistics

Fermi sea *vs* real photonic vacuum

Interacting electrons *vs* free photons

Decoherence phenomena *vs* flying photons

Theoretical framework: Glauber's coherences

First order electron coherence

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Glauber's formalism for electron quantum optics

C. Grenier *et al.*, New J. Phys. **13**, 093007 (2011); C. Grenier *et al.*, Mod. Phys. Lett. B **25**, 1053 (2011)

$$\mathcal{G}^{(e)}(t,t') = \mathrm{Tr}\left[\Psi(t)\rho\Psi^\dagger(t')\right] = \langle \Psi^\dagger(t')\Psi(t)\rangle_\rho$$

Ψ electron annihilation operator

ρ density matrix

Electron coherences as fundamental quantities

Useful to separate the Fermi sea contribution

$$\mathcal{G}^{(e)}(t,t') = \mathcal{G}_\mu(t-t') + \Delta\mathcal{G}^{(e)}(t,t')$$

G. Haack *et al.*, Phys. Rev. B **87**, 201302(R) (2013)

For an electron wave-packet above the Fermi sea

$$\mathcal{G}^{(e)}(t,t') = \mathcal{G}_\mu(t-t') + \varphi(t)\varphi^*(t')$$

Time representation of the 1st order coherence

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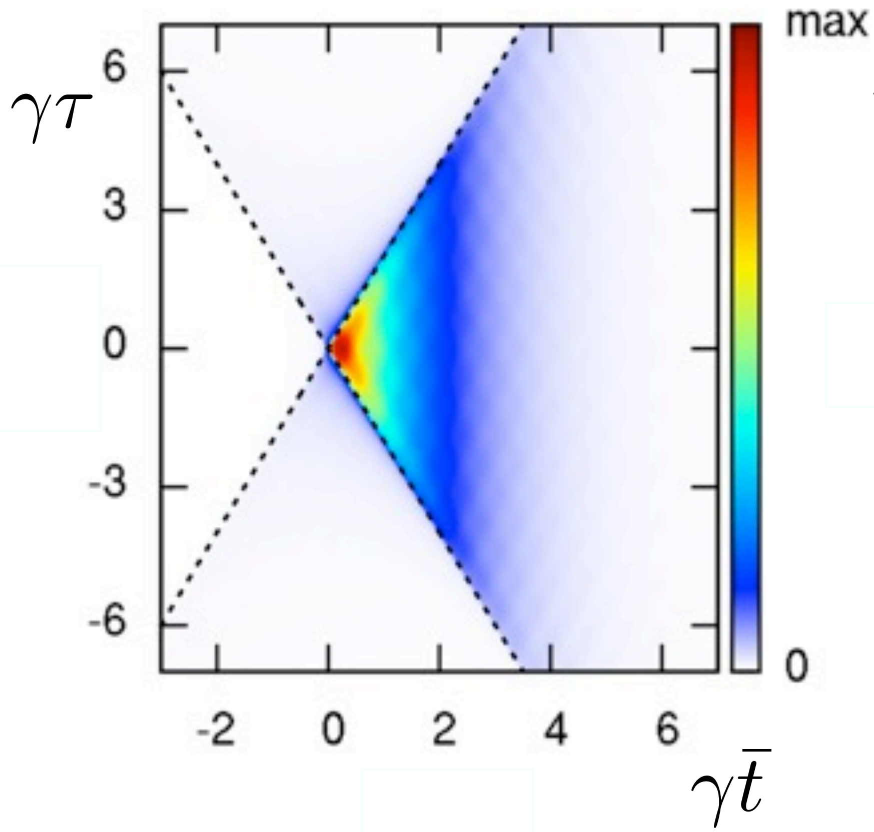
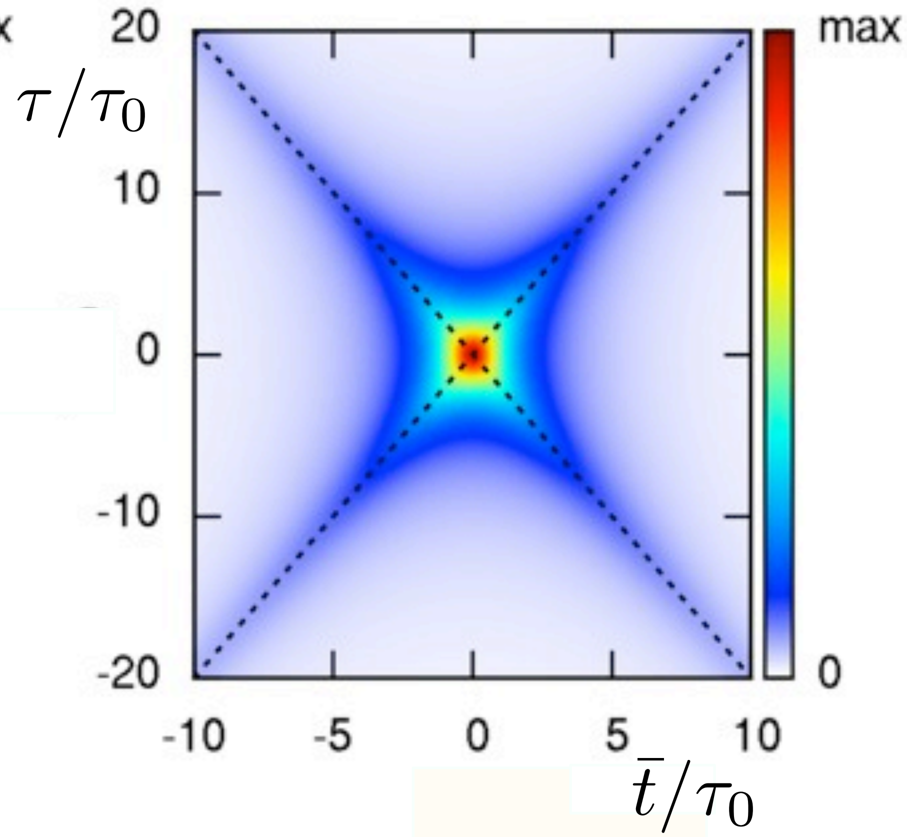
$$|\Delta\mathcal{G}^{(e)}\left(\bar{t}+\tau/2, \bar{t}-\tau/2\right)|$$

$\bar{t} = \dfrac{t+t'}{2}$ time evolution $\qquad \tau = t - t'$ coherence in time



Energy resolved wave-packet



Lorentzian voltage pulse (*n*=1)

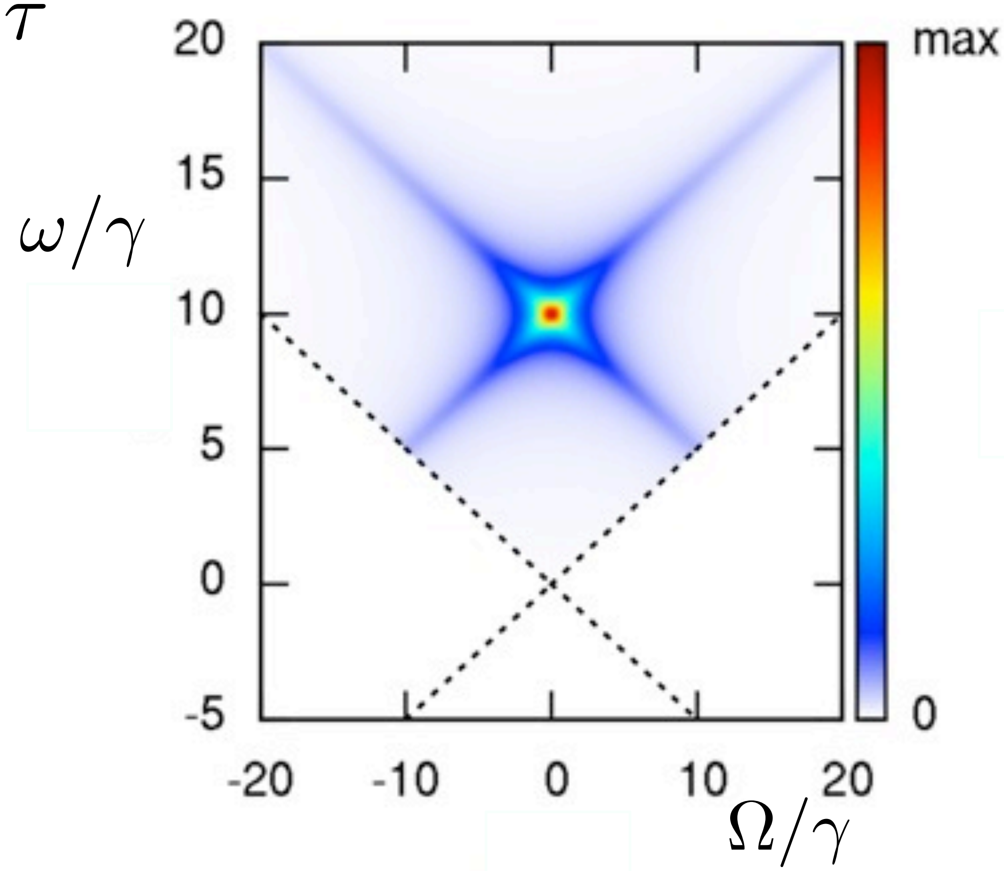
Good visualization of the time dependence

Difficult to extract information about the nature of excitations in energy

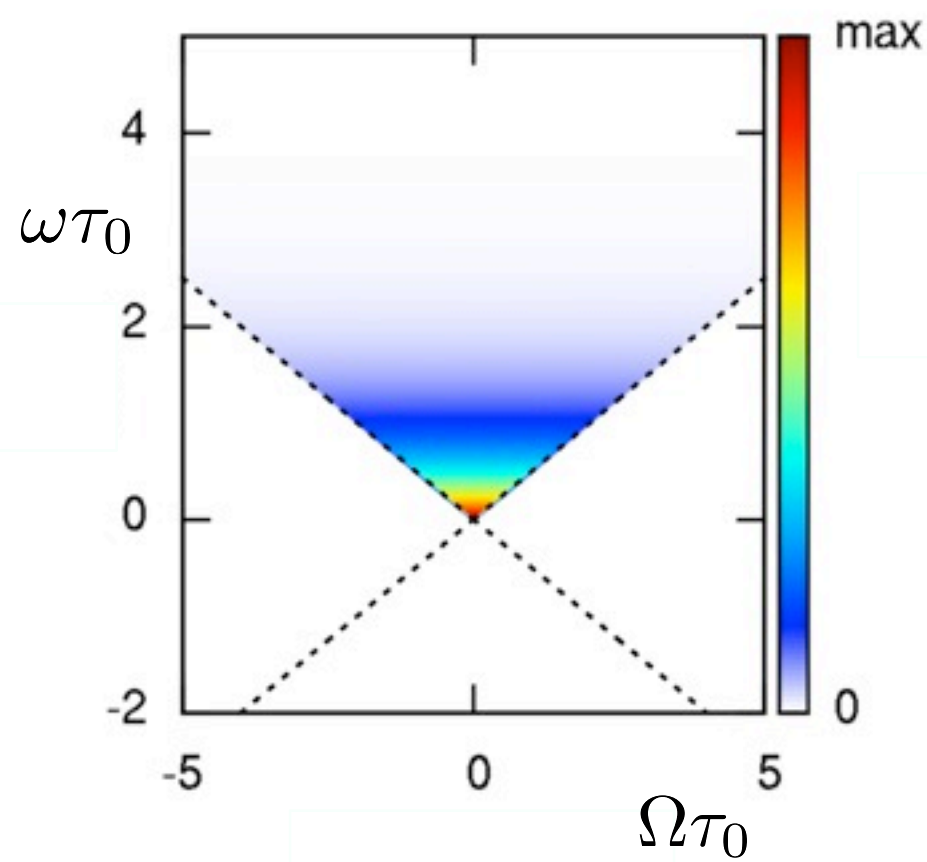
Frequency representation of the 1st order coherence

$$\left|\Delta\tilde{\mathcal{G}}^{(e)}\left(\omega+\Omega/2,\omega-\Omega/2\right)\right|$$

$\omega \leftrightarrow \tau$ $\Omega \leftrightarrow \bar{t}$



Energy resolved wave-packet



Lorentzian voltage pulse (n=1)

Good visualization of the nature of excitations in energy

Difficult to extract information about the time dependence

Tomography protocol to reconstruct the coherence in the energy domain

C. Grenier *et al.*, New J. Phys. **13,** 093007 (2011)

Wigner functions representation: definition and properties

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Quasi-probability distribution in phase space

E. Wigner, Phys. Rev. **40**, 749 (1932)

Theory of signal processing in the time-frequency domain

J. Ville, Cables and Transmission, **2A**, 61 (1948)

Time-frequency representation of the first order coherence

$$\mathcal{W}^{(e)}(\bar{t},\omega) = v_F \int_{-\infty}^{+\infty} d\tau e^{i\omega\tau} \mathcal{G}^{(e)}\left(\bar{t}+\frac{\tau}{2},\bar{t}-\frac{\tau}{2}\right)$$

Real function encoding the properties of electron coherence

Very useful to visualize both the time dependence and the nature of the excitations

For a wave-packet above the Fermi sea

$$\mathcal{W}^{(e)}(\bar{t},\omega) = f_\mu(\omega) + \Delta\mathcal{W}^{(e)}(\bar{t},\omega)$$

Few electron sources: Levitov pulses

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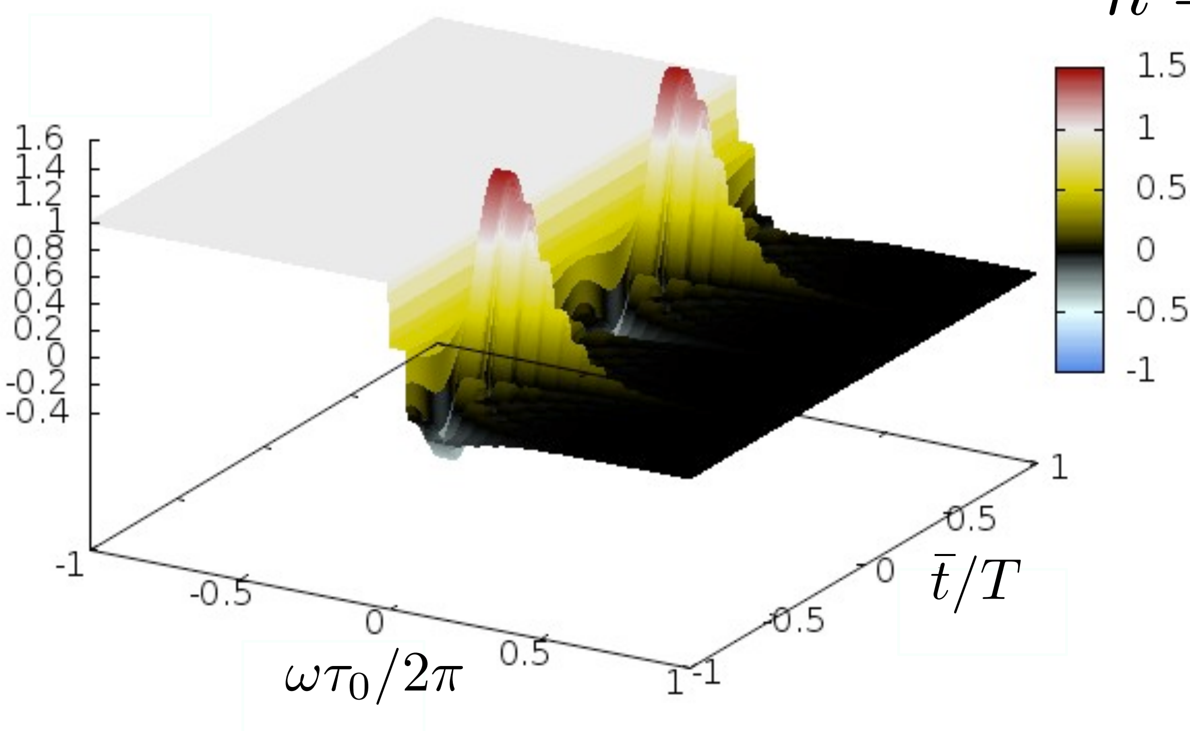
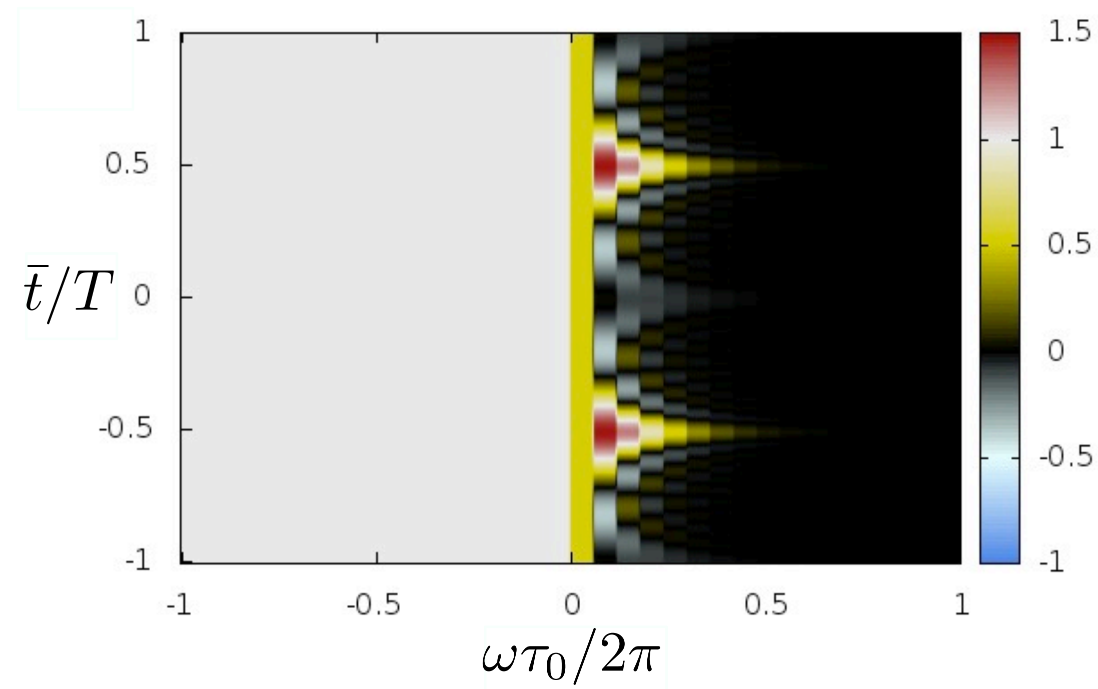
Lorentzian voltage pulse

L. S. Levitov *et al.*, J. Math. Phys. **37**, 4845 (1996); J. Keeling *et al.*, Phys. Rev. Lett. **97**, 116403 (2006)

$n = 1$





Wave-packet well defined (lorentzian) in time

For a *n* electron pulse

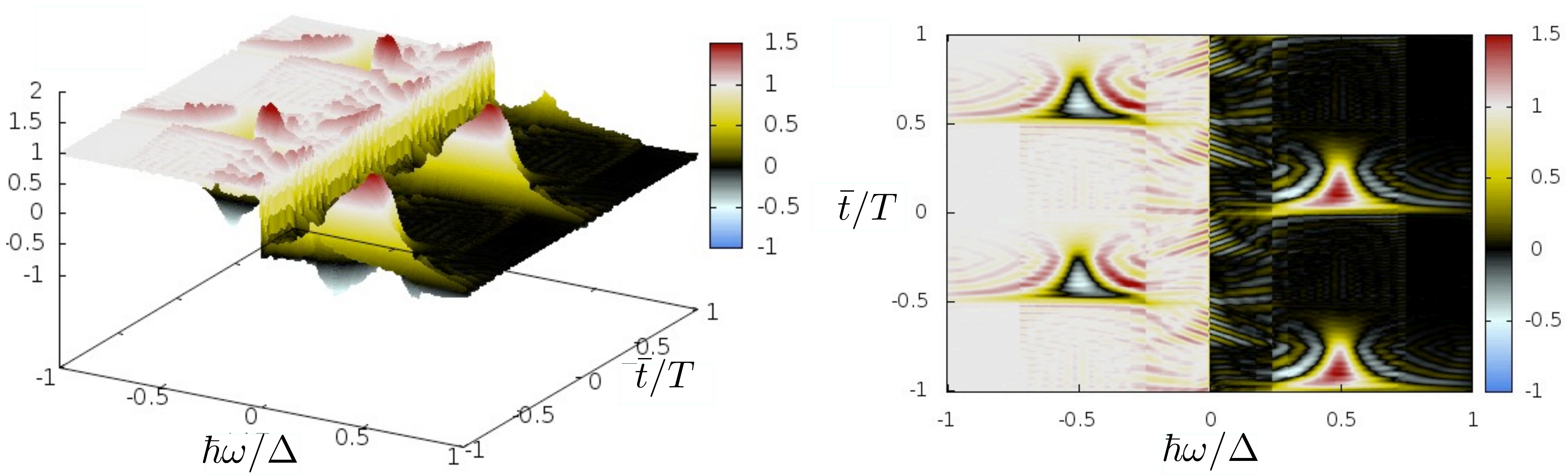
$$\Delta\mathcal{W}_n^{(e)}(\bar{t},\omega) = \sqrt{4\pi}e^{-2\omega\tau_0}\sum_{k=0}^{n-1}\sum_{l=0}^{k}\frac{1}{l!}\left(\frac{2\omega\tau_0}{\sqrt{\omega\bar{t}}}\right)^{2l+1} L_{k-l}^{(2l)}(4\omega\tau_0)J_{l+\frac{1}{2}}(2\omega\bar{t})$$

τ_0 width of the voltage pulse

Few electron sources: LPA

Single electron source of the LPA

G. Fève *et al.*, Science **316**, 1169 (2007)



Wave-packet well defined (lorentzian) in energy

For a single electron pulse with energy well above the Fermi sea

$$\Delta\mathcal{W}^{(e)}(\bar{t},\omega) = 2\gamma e^{-\gamma\bar{t}}\frac{\sin\left[2\bar{t}(\omega-\omega_0)\right]}{\omega-\omega_0}\Theta(\bar{t})$$

ω_0 emission energy of the electron (w.r.t. the Fermi sea)

γ spreading in energy

Generating non classical coherences: the Mach-Zehnder interferometer

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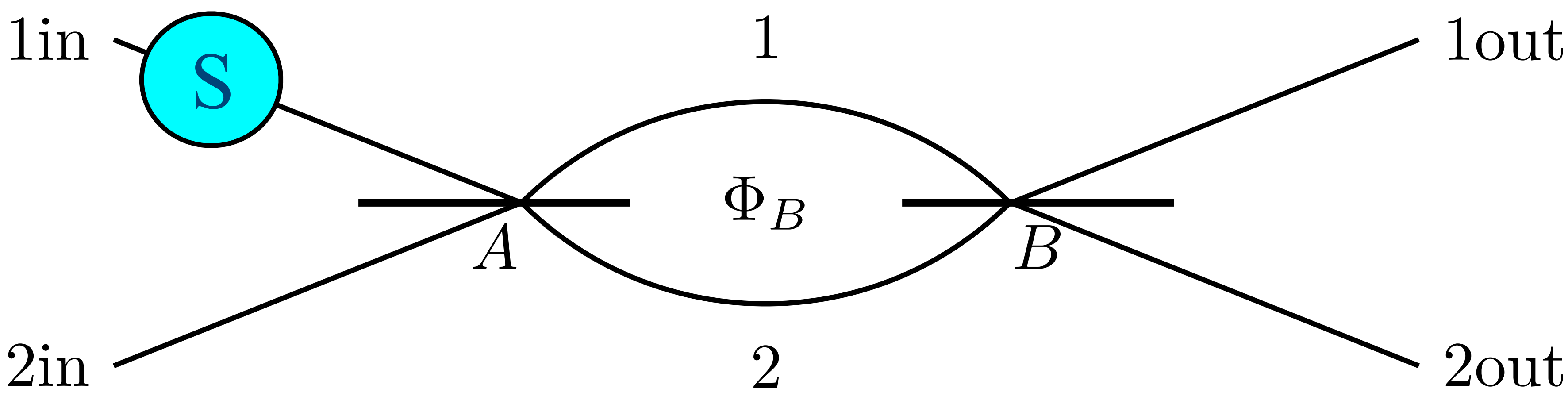


Experimentally realized with a continous current by Y. Ji *et al.*, Nature **422**, 415 (2003); Roulleau *et al.*, Phys. Rev. B **76**, 161309 (2007)

General relation for the excess coherence emitted by an electron source S

$$\Delta\mathcal{W}^{(e)}_{1,out}(\bar{t},\omega) = \frac{1}{4}\left\{\Delta\mathcal{W}^{(e)}_{1,in}(\bar{t}-\tau_1,\omega) + \Delta\mathcal{W}^{(e)}_{1,in}(\bar{t}-\tau_2,\omega) + 2\cos\left[\omega(\tau_1-\tau_2)+\phi\right]\Delta\mathcal{W}^{(e)}_{1,in}\left(\bar{t}-\frac{\tau_1+\tau_2}{2},\omega\right)\right\}$$

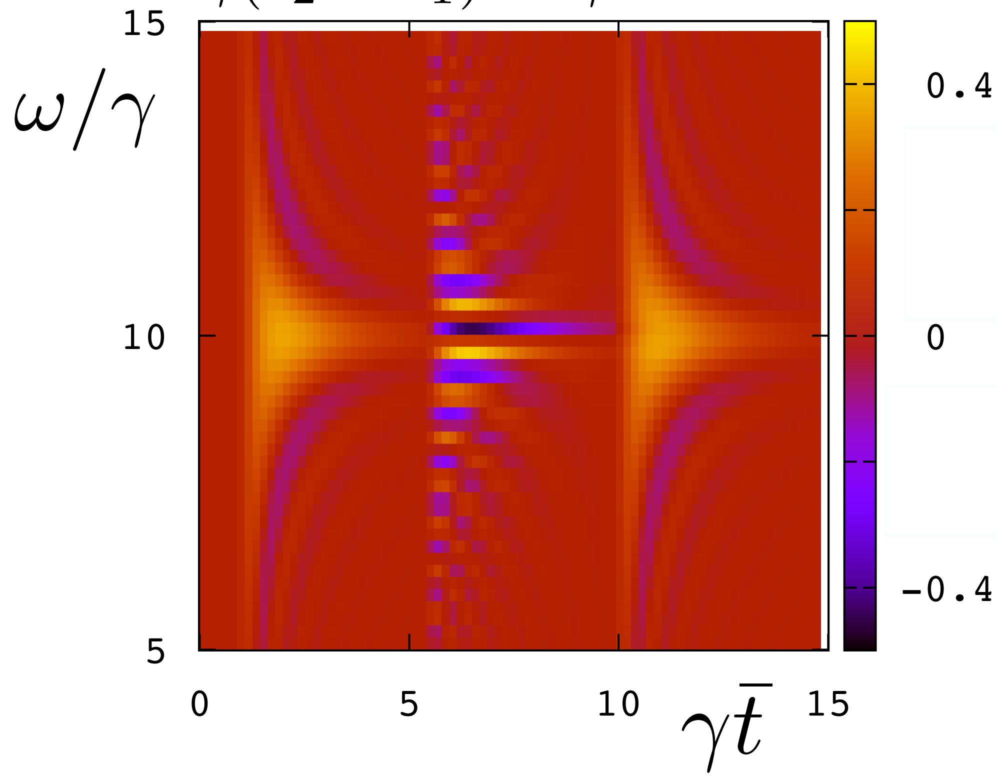
τ_1, τ_2 times of flight along the two arms of the MZI

ϕ phase related to the Aharonov-Bohm flux

Two Wigner functions delayed in time and another one in the middle modulated in frequency

Interference patterns

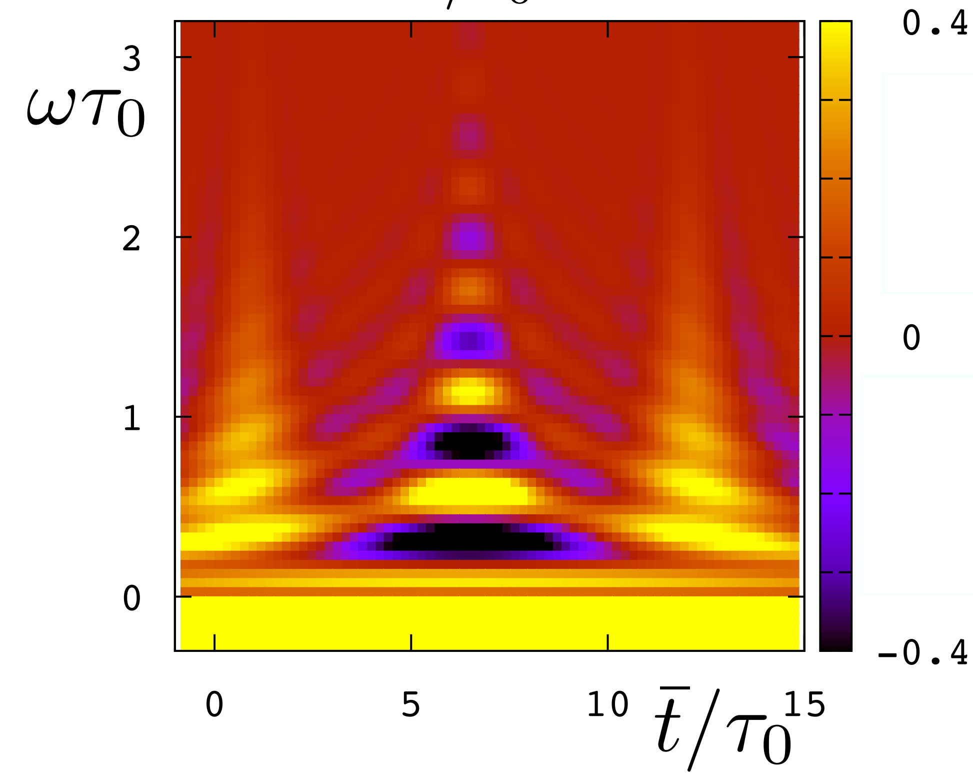
$\gamma(\tau_2 - \tau_1) = \gamma\Delta\tau = 9$



Energy resolved wave-packet

$\phi = 0$

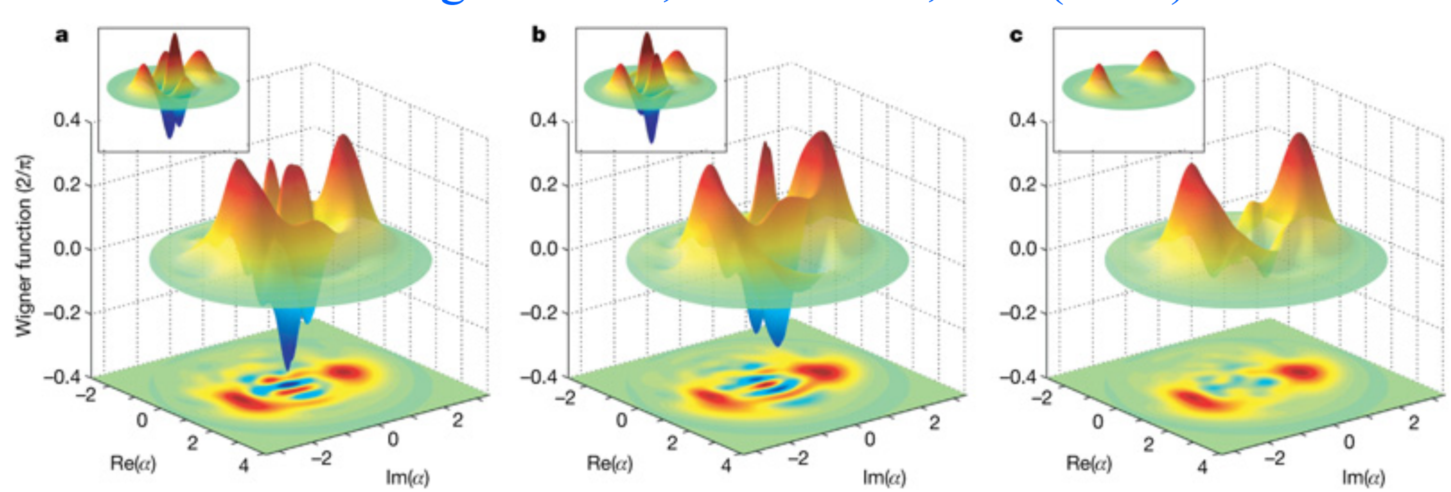
$\Delta\tau/\tau_0 = 11$



Lorentzian voltage pulse (n=1)

Analogy with "Schrödinger cats" in quantum optics

S. Deléglise *et al.*, Nature **455**, 501 (2008)

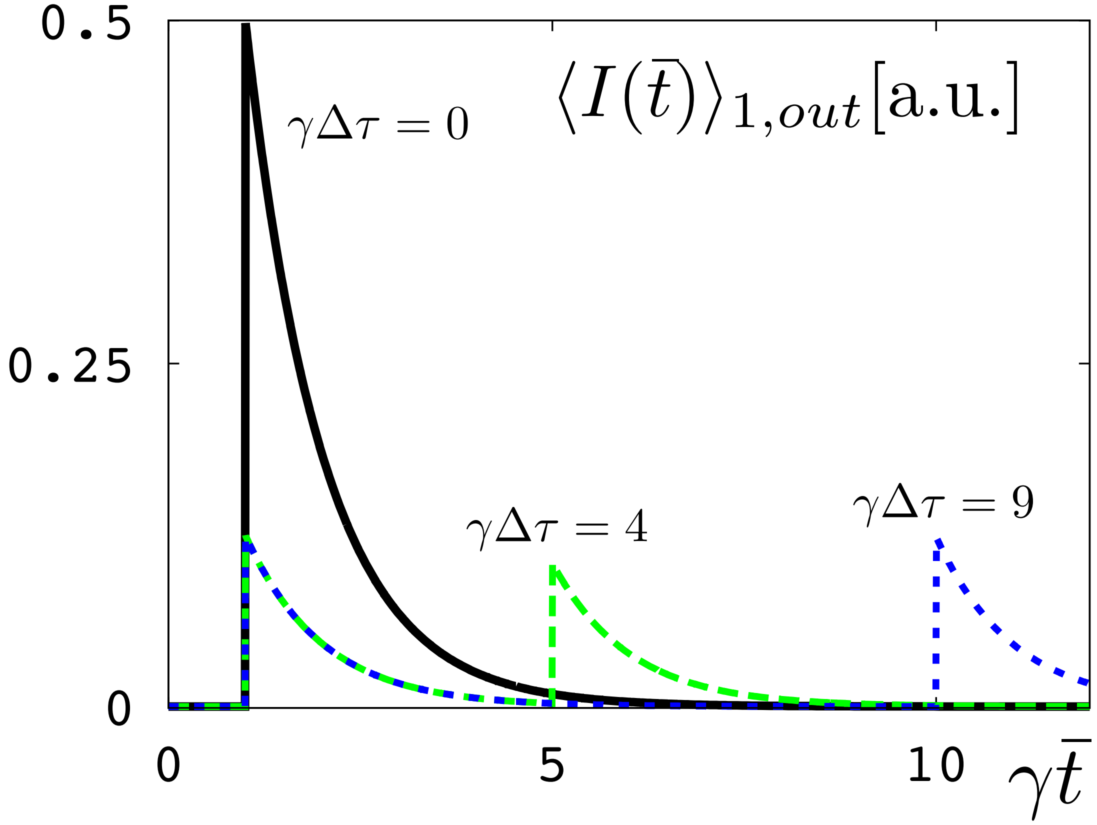


Marginal distributions: the current

$$\langle I(\bar{t})\rangle_{1,out} = -e\int_{-\infty}^{+\infty}\frac{d\omega}{2\pi}\Delta\mathcal{W}^{(e)}_{1,out}(\bar{t},\omega)$$

Single electron source LPA

$\phi = 0$

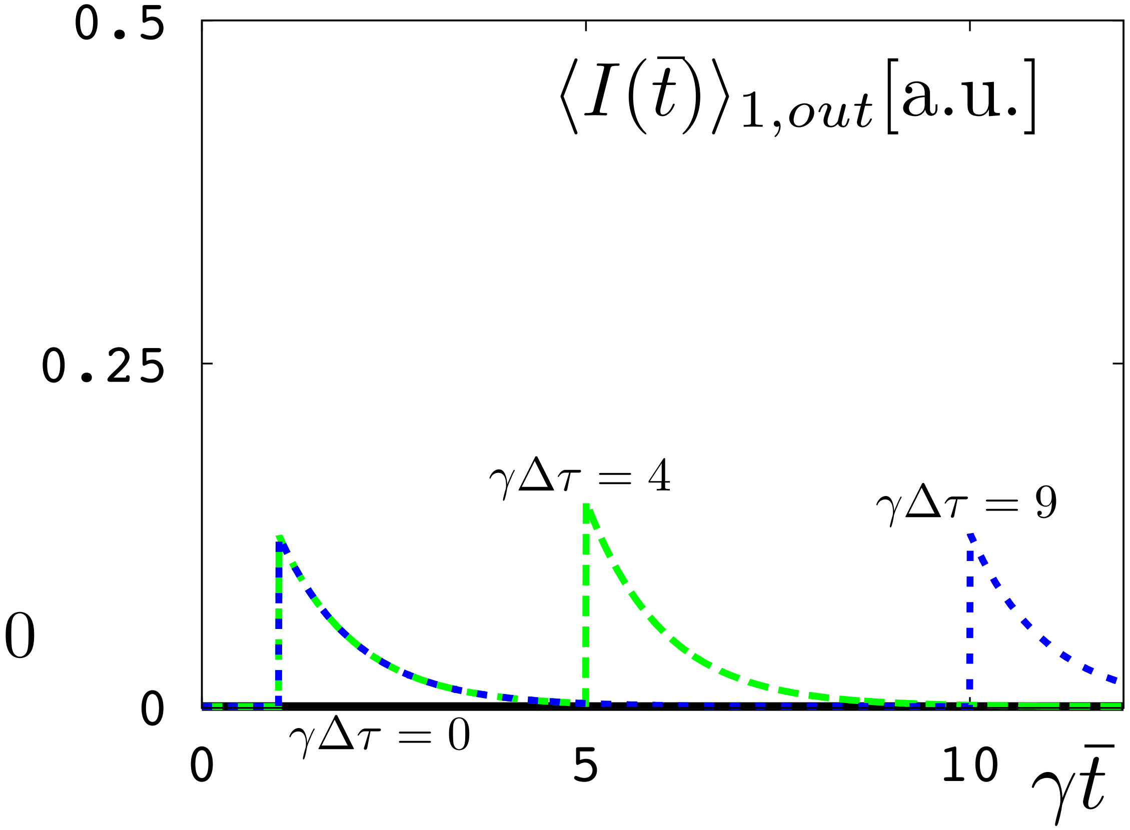


Constructive interference

$\gamma\tau_1 = 1$

$\omega_0/\gamma = 10$

$\phi = \pi$



Destructive interference

Coherence length of the wave-packet from current measurements

G. Haack *et al.* Phy. Rev. B. **84**, 081303 (2011)

No interference for $\gamma\Delta\tau \gg 1$

Marginal distributions: the channeled spectrum

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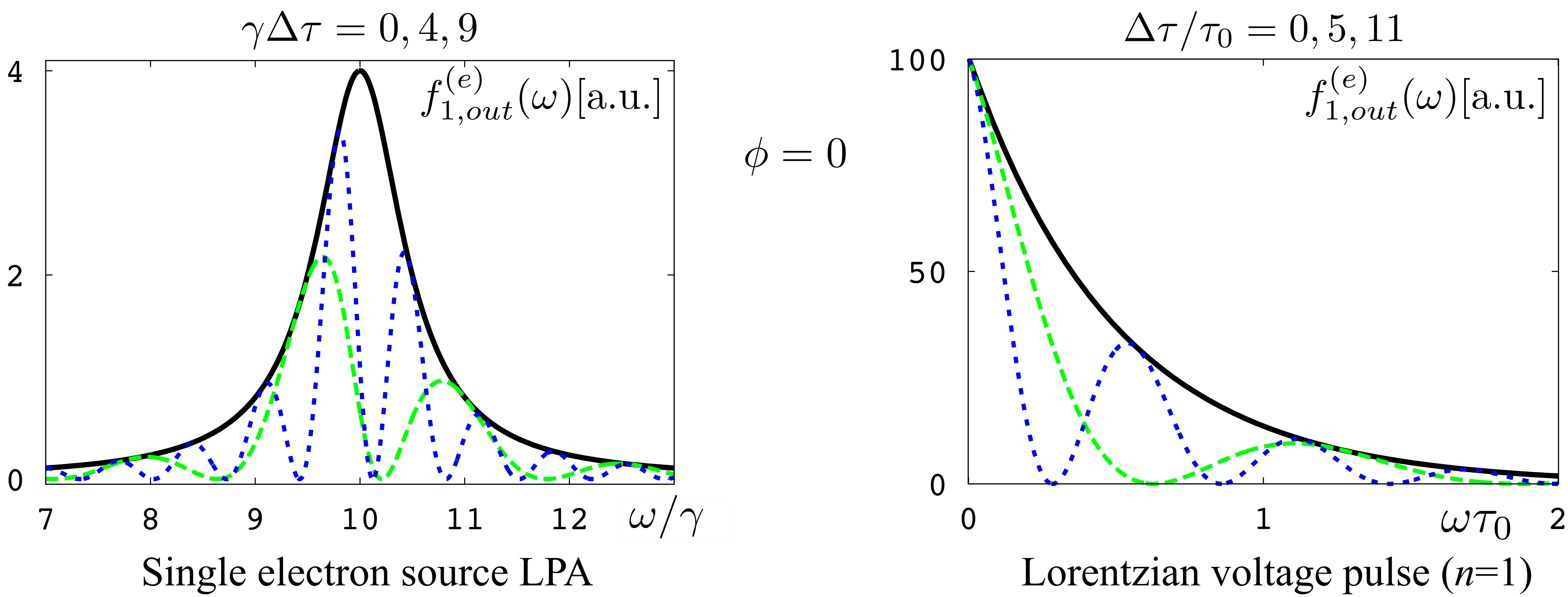
$$f^{(e)}_{1,out}(\omega) = \int_{-\infty}^{+\infty} d\bar{t}\Delta\mathcal{W}^{(e)}(\bar{t},\omega)$$



Increasing number of oscillations by increasing the difference in the times of flight

Possibility to measure this quantity by using a dot as a filter

C. Altimiras *et al.*, Nature Physics **6**, 34 (2009)

HBT in the Wigner representation

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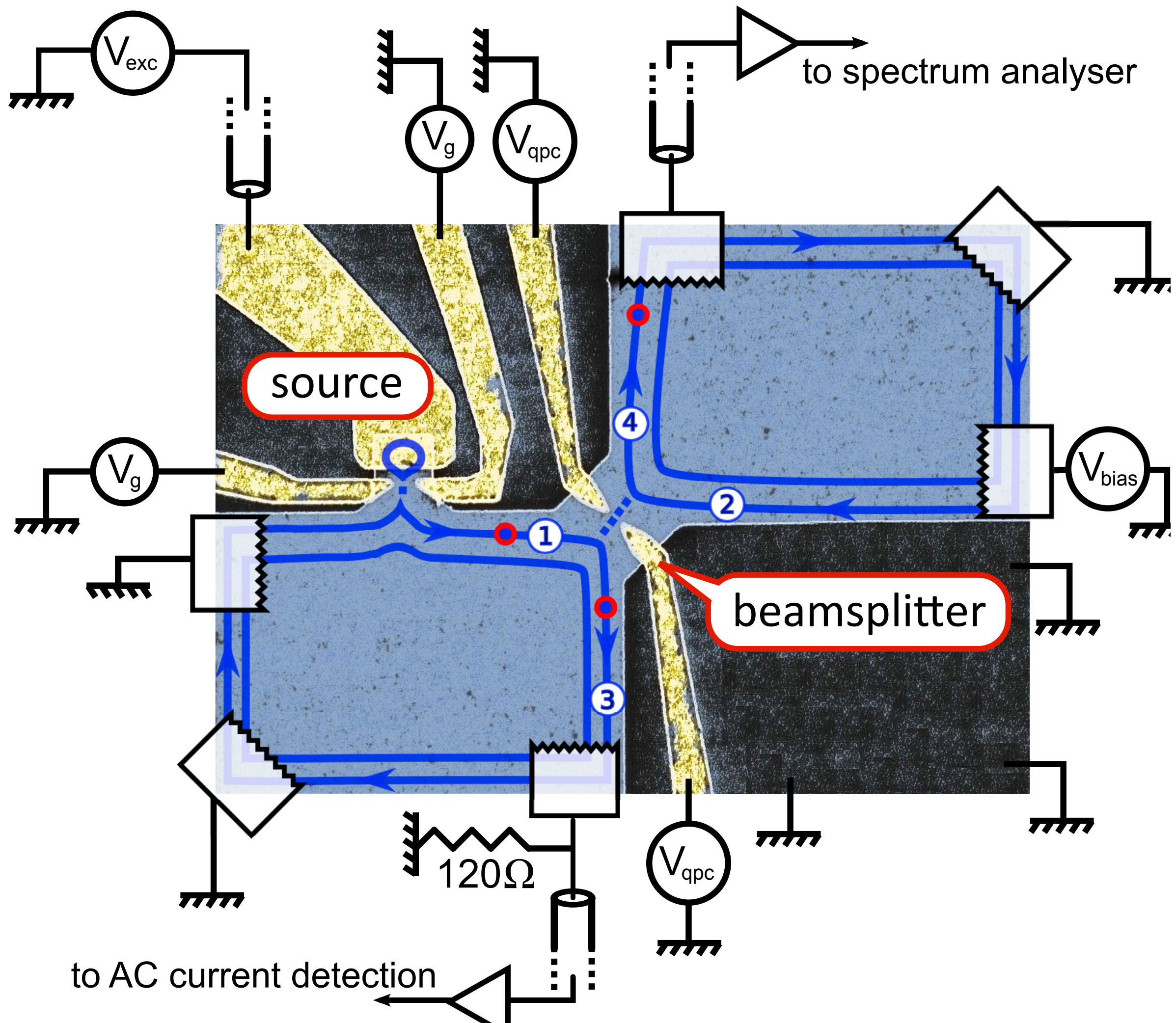

HBT contribution to the noise in the Wigner representation

$$\mathcal{S}_{\mathrm{HBT}} = \frac{e^2}{2\pi} \int d\bar{t} d\omega \Delta \mathcal{W}_1^{(e)}(\bar{t},\omega) \left[1 - 2 f_{\mu_2}(\omega)\right]$$

Anti-bunching with the electrons of the Fermi sea

E. Bocquillon *et al.* Phy. Rev. Lett. **108**, 196893 (2012)

HOM in the Wigner representation

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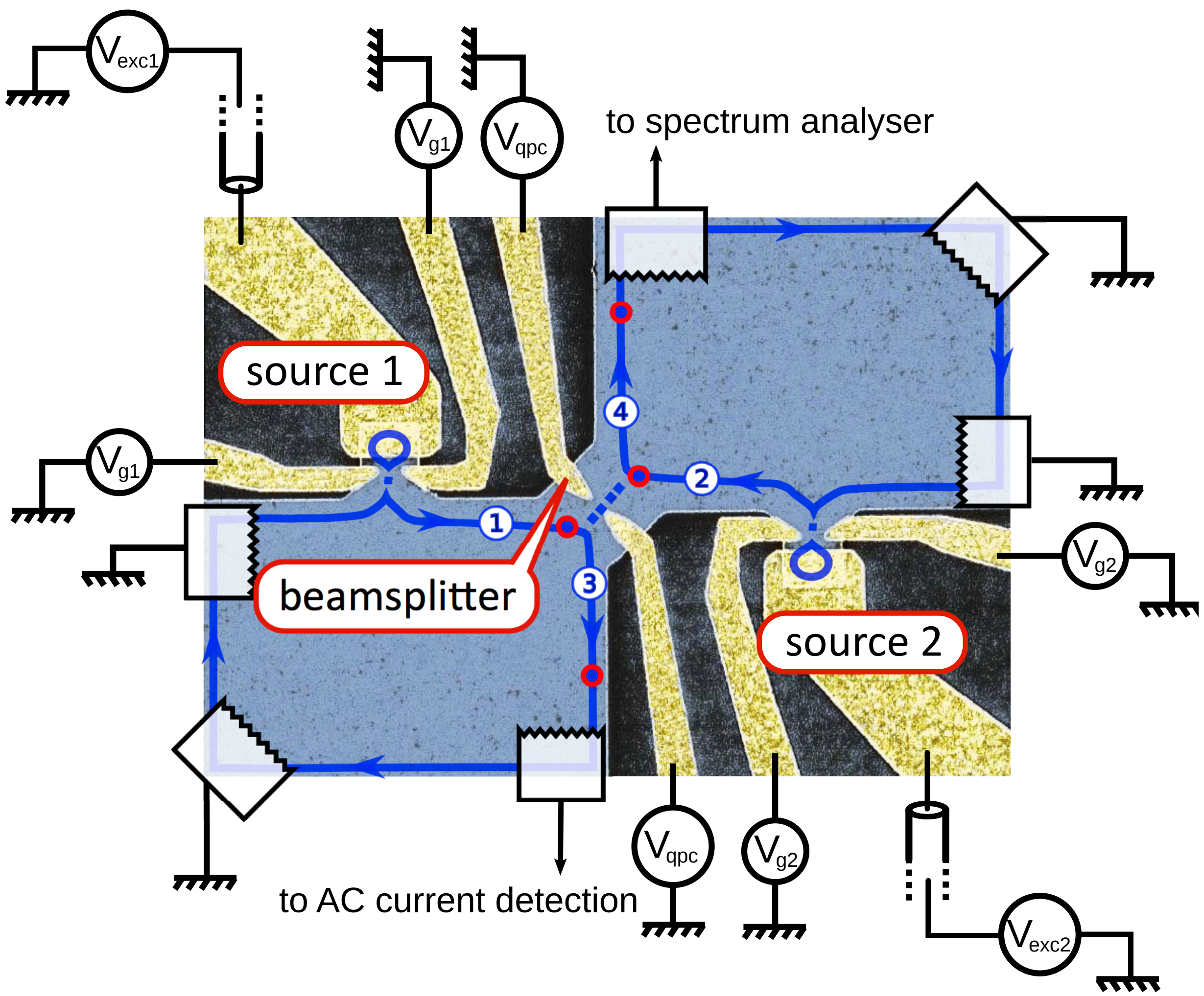


Measured noise

$$\mathcal{S} = \mathcal{S}_{\mathrm{HBT},1} + \mathcal{S}_{\mathrm{HBT},2} + \mathcal{S}_{\mathrm{HOM}}$$

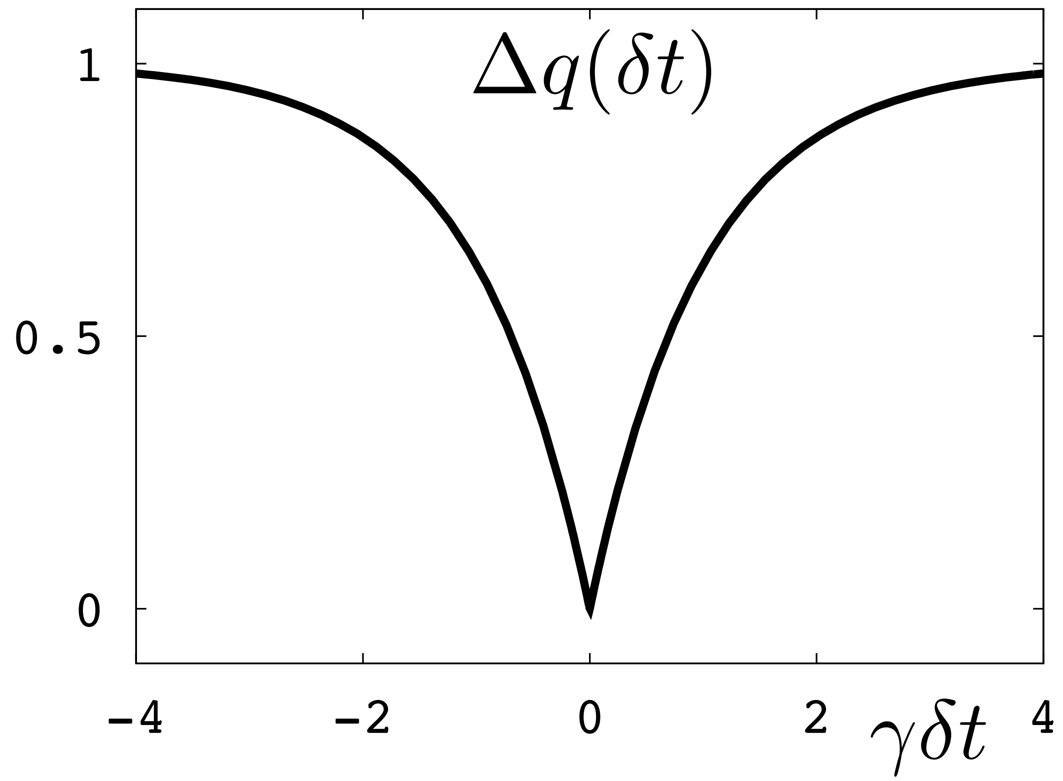
$$\mathcal{S}_{\mathrm{HOM}}(\delta t) = -\frac{e^2}{\pi} \int d\bar{t} d\omega \Delta\mathcal{W}_1^{(e)}(\bar{t},\omega)\Delta\mathcal{W}_2^{(e)}(\bar{t}-\delta t,\omega)$$

Overlap between the Wigner functions of the two sources

HOM for identical wave-packet

$$\Delta q(\delta t) = 1 - \mathcal{S}_{\mathrm{HOM}}(\delta t)/2\mathcal{S}_{\mathrm{HBT}}$$

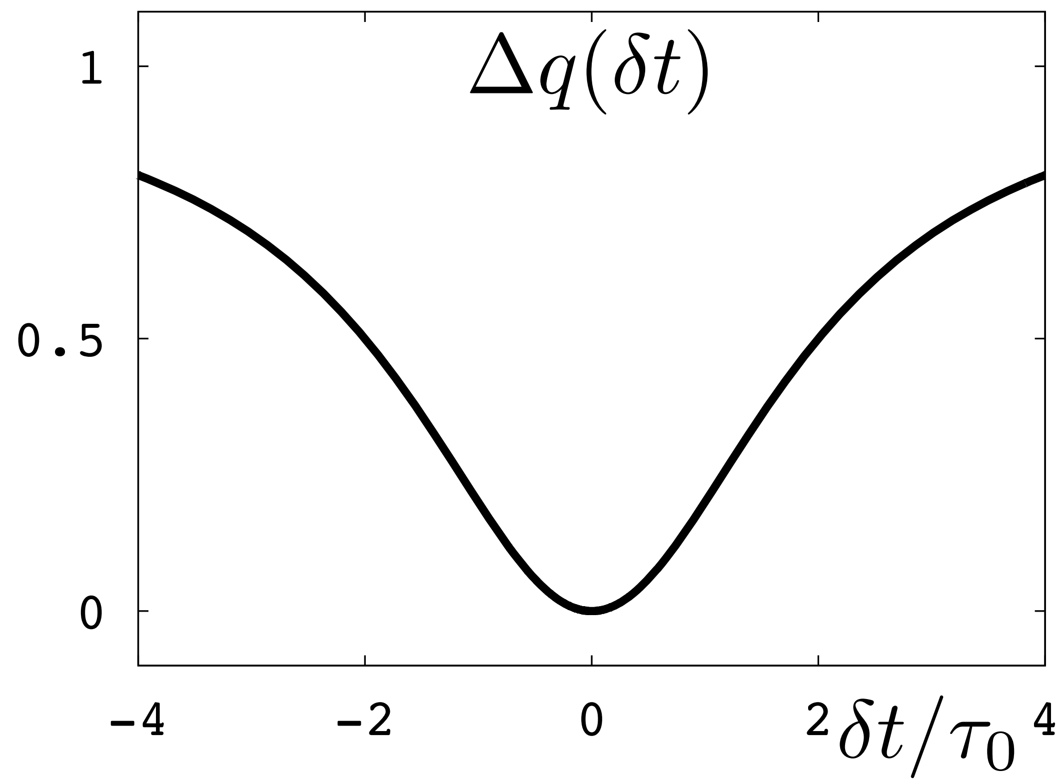
Identical electron wave-packets



Single electron source LPA

T. Jonckheere *et al.*, Phys. Rev. B **86**, 125425 (2012)

$$\Delta q(\delta t) = 1 - e^{-\gamma|\delta t|}$$



Lorentzian voltage pulse (*n*=1)

J. Dubois *et al.*, arXiv: 1212.3921

$$\Delta q(\delta t) = 1 - \frac{1}{1 + \left(\frac{\delta t}{2\tau_0}\right)^2}$$

Role of gauge invariance

Conclusions and perspective

Advantages

- Good visualization of the first order coherence

- Clear description of interference effects

- Unified view of single and two-particle interferometers

(MZI, HBT, HOM)

Perspectives

- New tomography protocol

- Take into account interaction effects