Spin-dependent thermoelectric transport in HgTe/CdTe quantum wells

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Spin-dependent thermoelectric transport in HgTe/CdTe quantum wells: outline

- **Context and Motivations:**
  - Quantum Spin Hall Insulator
  - Thermoelectric transport

- **System:**
  - HgTe quantum wells
  - Model

- **Spin Nernst effect:**
  - Landauer-Büttiker formalism
  - Carried by edge states (QSHE)
  - Carried by bulk states (SHE)
**CONTEXT**: quantum spin Hall insulators

2D band insulator with **helical metal** on the edge

Absence of backscattering by non-magnetic impurities

No Anderson localization

**Theory:**
- HgTe/CdTe wells, *Bernevig, Hughes, Zhang*, Science 2006
- InAs/GaSb wells, *Liu, Hughes, Qi, Wang, Zhang*, PRL 2008

**Experiment:**
- HgTe/CdTe wells, *Molenkamp’s group*, Science 2007 and 2009
Common semiconductor: \( \text{CB} = \text{electrons in s orbitals} \)
\( \text{VB} = \text{electrons in p orbitals} \)

Strong spin-orbit coupling

\[ \text{BAND INVERSION} \]

\[ \Gamma_8 \]

\[ \Gamma_6 \]

\[ E_{\text{meV}} \]

\[ d_{\text{QW}/\text{nm}} \]

\[ E_{\text{meV}} \]

\[ k (0.01 \text{ Å}) \]

\[ d < 6.5 \text{nm} \]

\[ d > 6.5 \text{nm} \]

\[ \text{HgTe} \]

\[ \text{Hg}_{0.32}\text{Cd}_{0.68}\text{Te} \]

\( E \) and \( H \) = 2D modes in the 3D structure
Thermoelectric properties = efficiency of a system to convert heat into electrical power

- Seebeck effect: longitudinal bias thermally induced
- Nernst effect: tranverse electric current due to $B$

Recent alliance of spintronics and thermoelectric transport,

- spin Seebeck effect
- anomalous Nernst effect, in the case of ferromagnetic systems
- spin Nernst effect, in TRS systems with strong SOC

What make thermoelectric coefficients interesting,

- heat-voltage conversion
- combine informations from energy and electric flows
- more sensitive to the density of states

Bauer, arXiv:1107.4395
Dubi and Di Ventra, RMP (11)
MOTIVATIONS

- Topological insulators proposed as good heat-voltage converter
- Absence of backscattering by non magnetic impurities so no reduction of electric transport in disordered systems
  - HgTe/CdTe QW in inverted regime
  - 3D topological insulators with line dislocations
  - 3D topological insulators with holes

- HgTe/CdTe QWs = systems with strong SOC
- Host quantum spin Hall effect, spin Hall effect
- Use thermoelectric coefficients to probe the dynamics of HgTe QW and generate spin current?
- Four terminal cross-bar setup,
- Use of a quantum spin Hall insulator based on HgTe/CdTe qw,
- Thermal gradient between lateral leads:
  - Longitudinal electric bias,
  - Transverse spin current,
- Both used as probe of topological phase and finite size effects.
MODEL

Hamiltonian for the \((E1^\uparrow, HH1^\uparrow, E1^\downarrow, HH1^\downarrow)\) basis

\[
H = V_m(r)\tau_z - Dk^2 + \begin{pmatrix}
h(k) & 0 \\
0 & h^*(-k)
\end{pmatrix},
\]

with \(h(k) = \begin{pmatrix}
M(k) & Ak_+ \\
Ak_- & -M(k)
\end{pmatrix}\), \(k_\pm = k_x \pm ik_y\), and \(M(k) = M - Bk^2\).

- 2+1D massive Dirac Hamiltonian
- \(M\) is the gap parameter (tuned in exp. by changing \(d\)):
  - \(M>0\) : trivial insulator,
  - \(M<0\) : topological insulator
- \(V_m\) is the in-plane confinement potential
- Use of tight binding approach to model the setup and treat the thermoelectric transport

Bernevig, Nature (06) ; Rothe, NJP (10)
INVERTED vs. NORMAL REGIME

Normal regime

Inverted regime

Overlap of states from opposite edges
Mini-gap opens,

\[
\mu_a = \mu + \Delta \mu \\
\mu_b = \mu - \Delta \mu
\]

Zhou, PRL 2008
Definition:

\[ N_s = \frac{I_c^s}{2\Delta T} \bigg|_{\mu_c,d=\mu} \]

with \[ I_p^s = \left( \frac{\hbar}{2} \right) (I_{p\uparrow} - I_{p\downarrow}) \]

Landauer-Büttiker formula (linear response regime):

\[ I_{p\sigma} = \frac{1}{\hbar} \sum_{q \neq p} \int dE T_{p\sigma,q}(E) (f_p - f_q) \]

Mott-like expression (low temperature limit):

\[ N_s \approx \frac{2\pi^2 k_B^2 T}{3e} \frac{dG_{SH}(E)}{dE} \bigg|_{E=\mu(T=0)} \]

With the spin Hall conductance at \( T = 0 \):

\[ G_{SH}(E, T = 0) = \frac{e}{8\pi} \mathcal{T}_{SN}(E) \]

\[ \mathcal{T}_{SN}(E) = \Delta T_{c,b} - \Delta T_{c,a} \]
THERMOELECTRIC TRANSPORT CARRIED BY EDGE STATES

Symmetric mini-gap peak in transmission

Anti-symmetric peak in spin Nernst signal

Bulk and mini-gaps in inverted regime
Bulk gap in normal regime

\[ N_s \approx \frac{2\pi^2 k_B^2 T}{3e} \frac{dG_{sH}(E)}{dE} \bigg|_{E=\mu(T=0)} \]

\[ G_{sH}(E; T = 0) = \frac{e}{8\pi} T_{SN}(E), \]
Merging of the edge state to the conduction band

Formation of the first bulk state

Transmission vanishes then reappears

Merging of the edge state to the conduction band

Formation of the first bulk state

Comparable magnitude of merging gap and mini gap which allows to mark positions where the edge state merges
In-plane potential, mass confinement potential can generate spin Hall signal

Yokoyama, PRL 2009; Rothe, NJP 2010; Guigou, PRB 2011

From Mott-like relation, spin Hall conductance gives rise to spin Nernst signal.

Local spin current at T=0:

\[
J^z(r) = \sum_p \text{Tr} \left[ \hat{J}^z(r) G^R \Gamma_p G^A \right] \mu_p.
\]

Spin Nernst effect carried by bulk modes?
We start with the following ansatz for the spin $\uparrow$ wave function:

$$\psi^\uparrow(y) = \langle y | \psi^\uparrow \rangle = e^{ik_y y} u_{k_y} + re^{-ik_y y} u_{-k_y} + ce^{\lambda y} u_{i\lambda},$$

$$\psi^\uparrow(0) = 0$$

Spin current along $y$ with transverse velocity $j^s(y) = j^\uparrow_k(y) - j^\downarrow_k(y)$

with

$$j^\uparrow_k(y) = \langle \psi^\uparrow | \delta(y - \hat{y}) V_x^\uparrow | \psi^\uparrow \rangle,$$

Phase shift between $\mathbf{j}^\uparrow$ and $\mathbf{j}^\downarrow$.

Oscillatory pattern due to the superposition of incoming and reflected propagating waves.

Spin current scales as

$$j_s(y) \propto \frac{A^2 k}{|M|}.$$
Applying voltage induces spin and charge response:

\[ G_{SH} = \frac{I_s}{I} G_{xx} \rightarrow G_{SH} \sim \frac{m_e}{h} A^2 / M \]

\[ \mathcal{T}_{SN} \sim \frac{m_e}{h} \]

Does the spin Nernst signal depend on the mass?

- Black arrows compare Nernst transmission in valence and conduction bands,
- Positions at energy for 4 propagating modes,
- Scaled by the ratio of the masses of electron/hole.

\[ N_s \propto |m_i| \quad (i = e, h) \]
CONCLUSION

- Use of thermoelectric coefficients as a probe of the dynamics in HgTe/CdTe quantum wells,

- Spin Nernst signals shows the topological phase and informs on mini-gap position in the spectrum,

- For metallic bulk regime, magnitude of spin Nernst signal depends on the mass of particles,

- Beyond the linear response regime?