Spin-dependent thermoelectric transport in HgTe/CdTe quantum wells *

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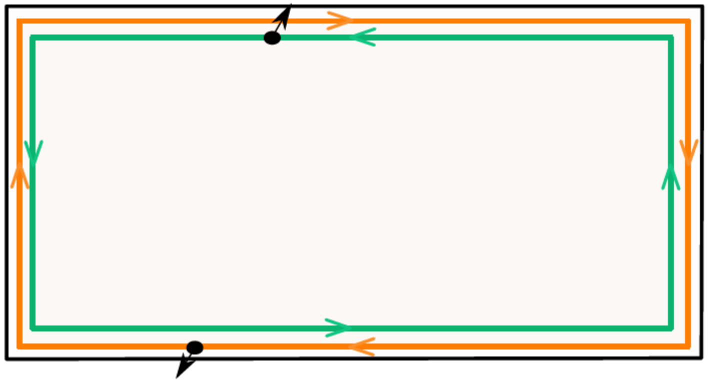
Spin-dependent thermoelectric transport in HgTe/CdTe quantum wells : outline

- Context and Motivations:
  - Quantum Spin Hall Insulator
  - Thermoelectric transport
- System :
  - HgTe quantum wells
  - Model
- Spin Nernst effect :
  - Landauer-Büttiker formalism
  - Carried by edge states (QSHE)
  - Carried by bulk states (SHE)

CONTEXT : quantum spin Hall insulators



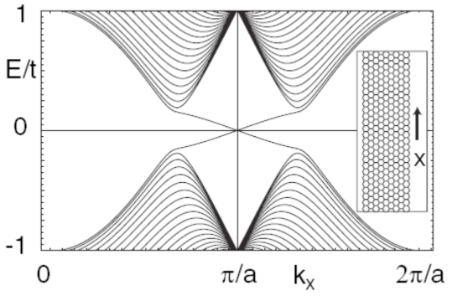
2D band insulator with **helical metal** on the edge

Absence of backscattering by non-magnetic imputities

No Anderson localization

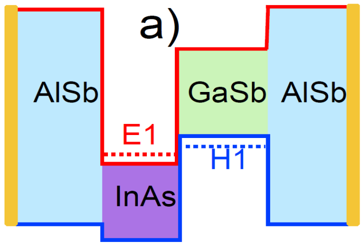
Theory:

- Graphene with spin-orbit coupling,*Kane and Mele*, PRL 2005
- HgTe/CdTe wells, *Bernevig, Hughes, Zhang*, Science 2006
- InAs/GaSb wells, *Liu, Hughes, Qi, Wang, Zhang*, PRL 2008



Experiment:

- HgTe/CdTe wells,*Molenkamp's group*, Science 2007 and 2009
- InAs/GaSb, Knez, Du,Sullivan, arXiv:1105.0137, 1106.5819

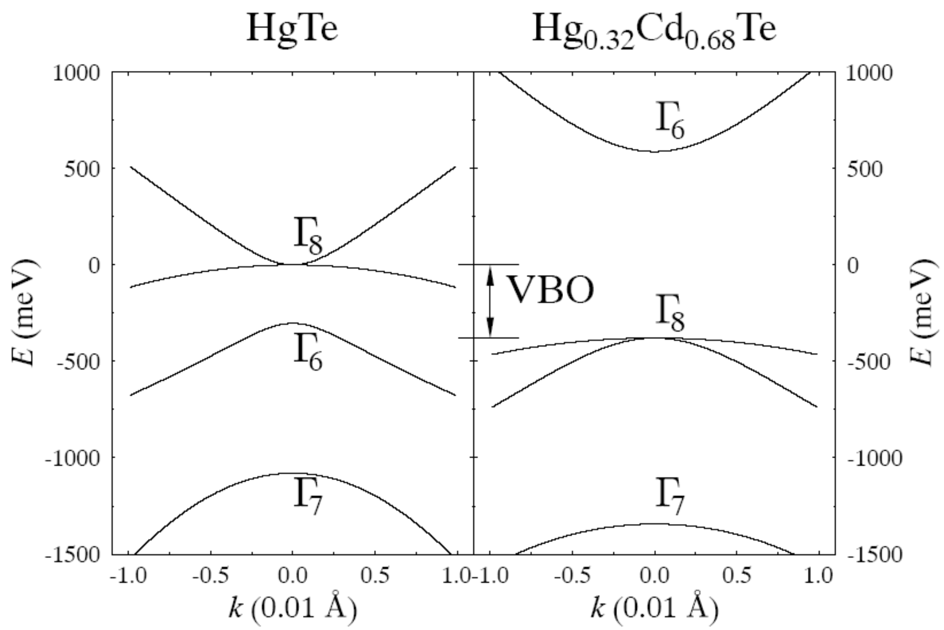


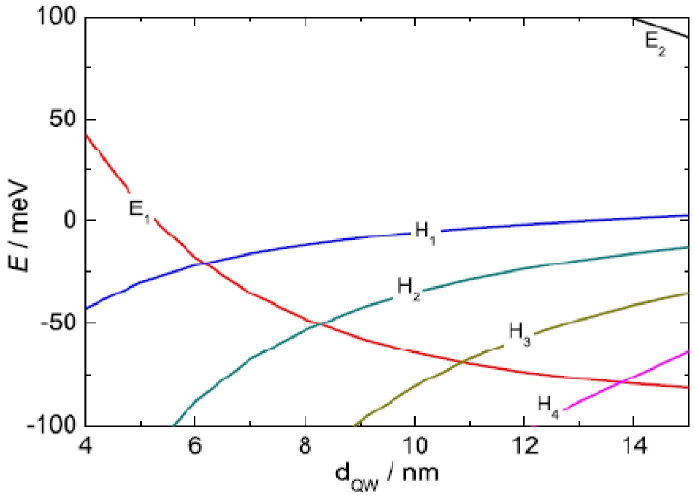
CONTEXT : band structure of HgTe QWs

Common semiconductor : CB = electrons in s orbitals
VB = electrons in p orbitals

Strong spin-orbit coupling

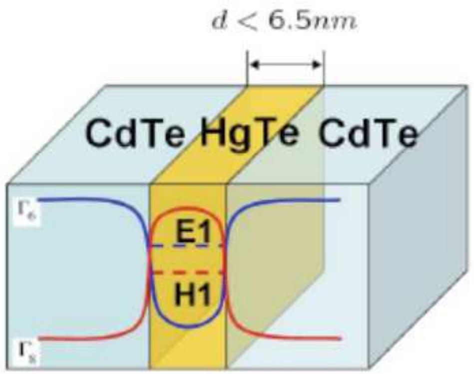
BAND INVERSION



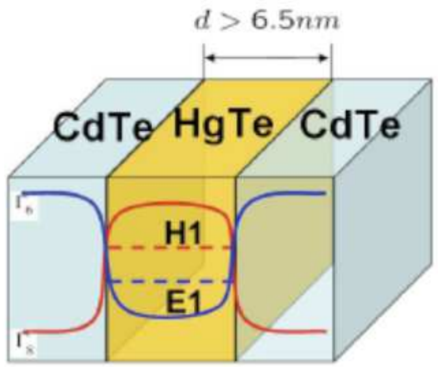


E and H = 2D modes in the 3D structure

normal

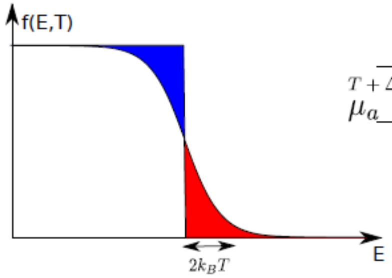


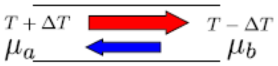
inverted



CONTEXT : thermoelectric transport

- Thermoelectric properties = efficiency of a system to convert heat into electrical power
  - Seebeck effect : longitudinal bias thermally induced
  - Nernst effect : tranverse electric current due to **B**





- Recent alliance of spintronics and thermoelectric transport,
  - spin Seebeck effect
  - anomalous Nernst effect, in the case of ferromagnetic systems
  - spin Nernst effect, in TRS systems with strong SOC

- What make thermoelectric coefficients interesting,
  - heat-voltage conversion
  - combine informations from energy and electric flows
  - more sensitve to the density of states

Bauer, arXiv:1107.4395

Dubi and Di Ventra, RMP (11)

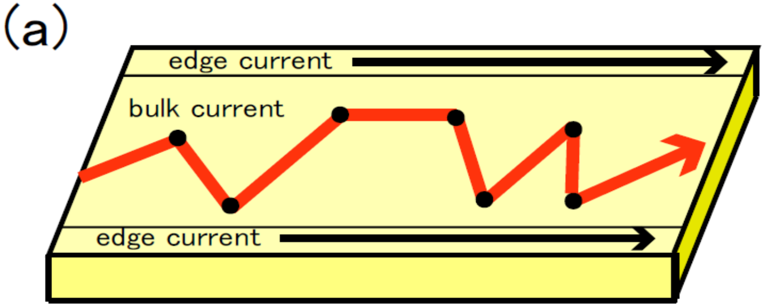
MOTIVATIONS

- Topological insulators proposed as good heat-voltage converter
- Absence of backscattering by non magnetic impurities so no reduction of electric transport in disordered systems

  → HgTe/CdTe QW in inverted regime — Murakami, J. of. Phys. (2011)

  → 3D topological insulators with line dislocations — Takahashi, PRL 2010

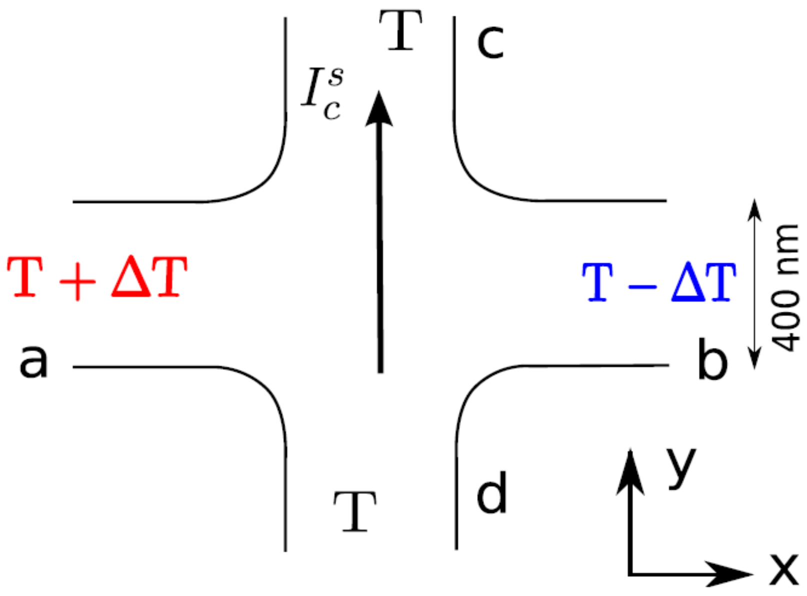
  → 3D topological insulators with holes — Tretiakov, APL 2010,2011



- HgTe/CdTe QWs = systems with strong SOC
- Host **quantum spin Hall effect**, **spin Hall effect**
- Use thermoelectric coefficients to probe the dynamics of HgTe QW and generate spin current ?

SYSTEM

- Four terminal cross-bar setup,
- Use of a quantum spin Hall insulator based on HgTe/CdTe qw,
- Thermal gradient between lateral leads :
  - Longitudinal electric bias,
  - Transverse spin current,
- Both used as probe of topological phase and finite size effects.



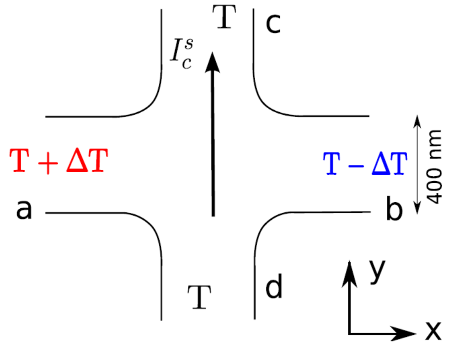
MODEL

$$Hamiltonian\ for\ the\ (E1\uparrow, HH1\uparrow, E1\downarrow, HH1\downarrow)\ basis$$

$$H = V_m(\mathbf{r})\tau_z - Dk^2 + \begin{pmatrix} h(k) & 0 \\ 0 & h^*(-k) \end{pmatrix},$$

with $h(k) = \begin{pmatrix} \mathcal{M}(k) & Ak_+ \\ Ak_- & -\mathcal{M}(k) \end{pmatrix}$, $k_\pm = k_x \pm ik_y$, and $\mathcal{M}(k) = M - Bk^2$.

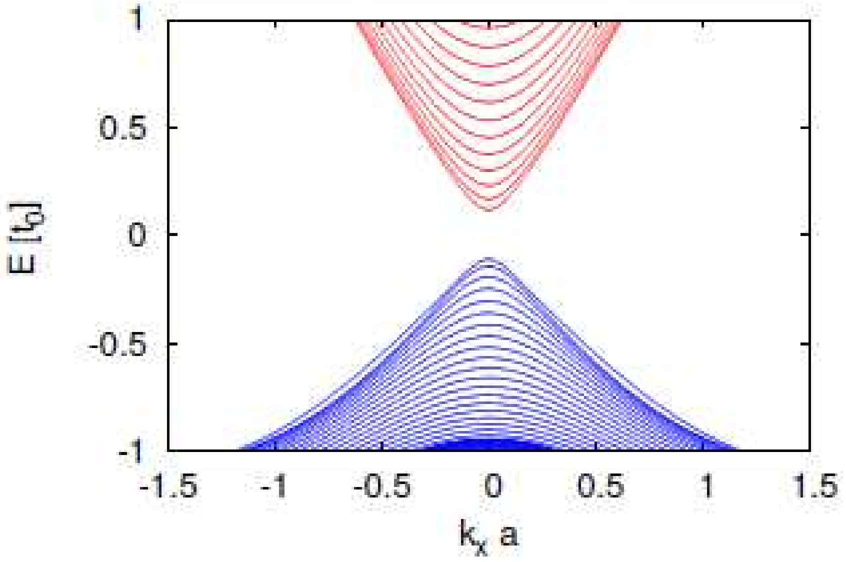
- 2+1D massive Dirac Hamiltonian
- M is the gap parameter (tuned in exp. by changing d) :
    - M>0 : trivial insulator,
    - M<0 : topological insulator
- V



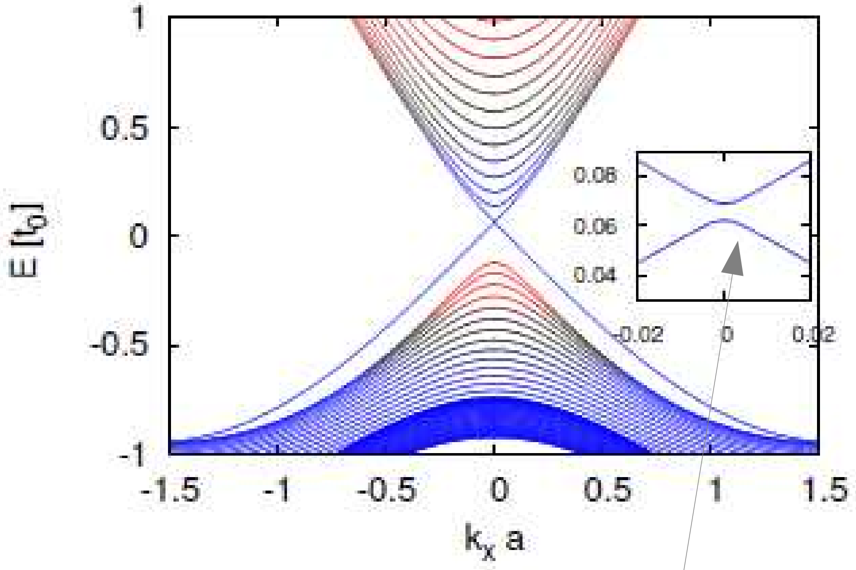
- V_m is the in-plane confinement potential
- Use of tight binding approach to model the setup and treat the thermoelectric transport

Bernevig, Nature (06) ; Rothe, NJP (10)

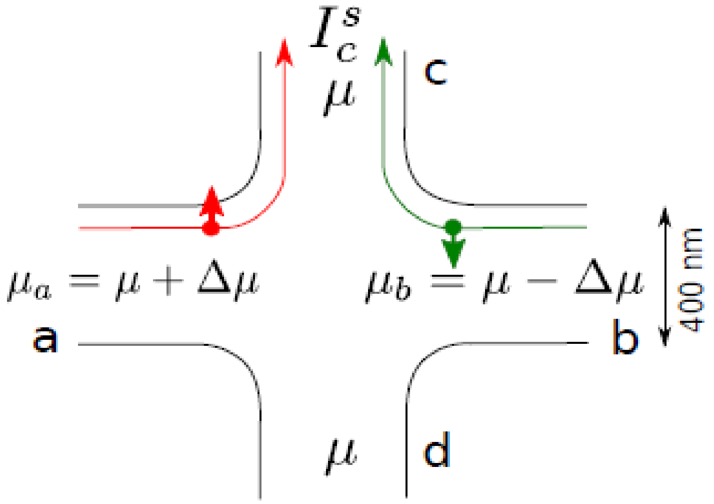
INVERTED vs. NORMAL REGIME



Normal regime



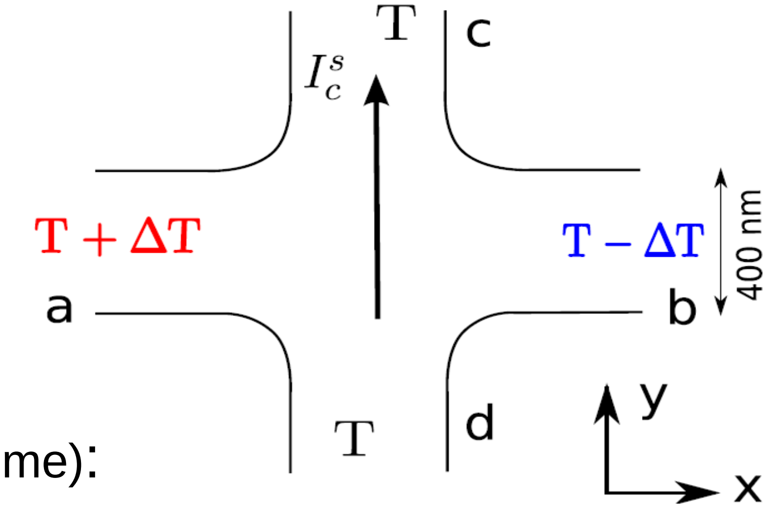
Inverted regime



Overlap of states from opposite edges
Mini-gap opens,

Zhou, PRL 2008

SPIN NERSNT COEFFICIENT Ns



Definition :

$$N_s = \frac{I_c^s}{2\Delta\mathrm{T}}\bigg|_{\mu_{\mathrm{c,d}}=\mu}$$

with $I_p^s = (\hbar/2)(I_{p\uparrow} - I_{p\downarrow})$

Landauer-Büttiker formula (linear response regime):

$$I_{p\sigma} = \frac{1}{h}\sum_{q\neq p}\int dE\, T_{p\sigma,q}(E)(f_p - f_q)$$

$$f_p = (e^{(E-\mu_p)/k_B \mathrm{T}_p} + 1)^{-1}$$

Transmission matrix

Mott-like expression (low temperature limit) :

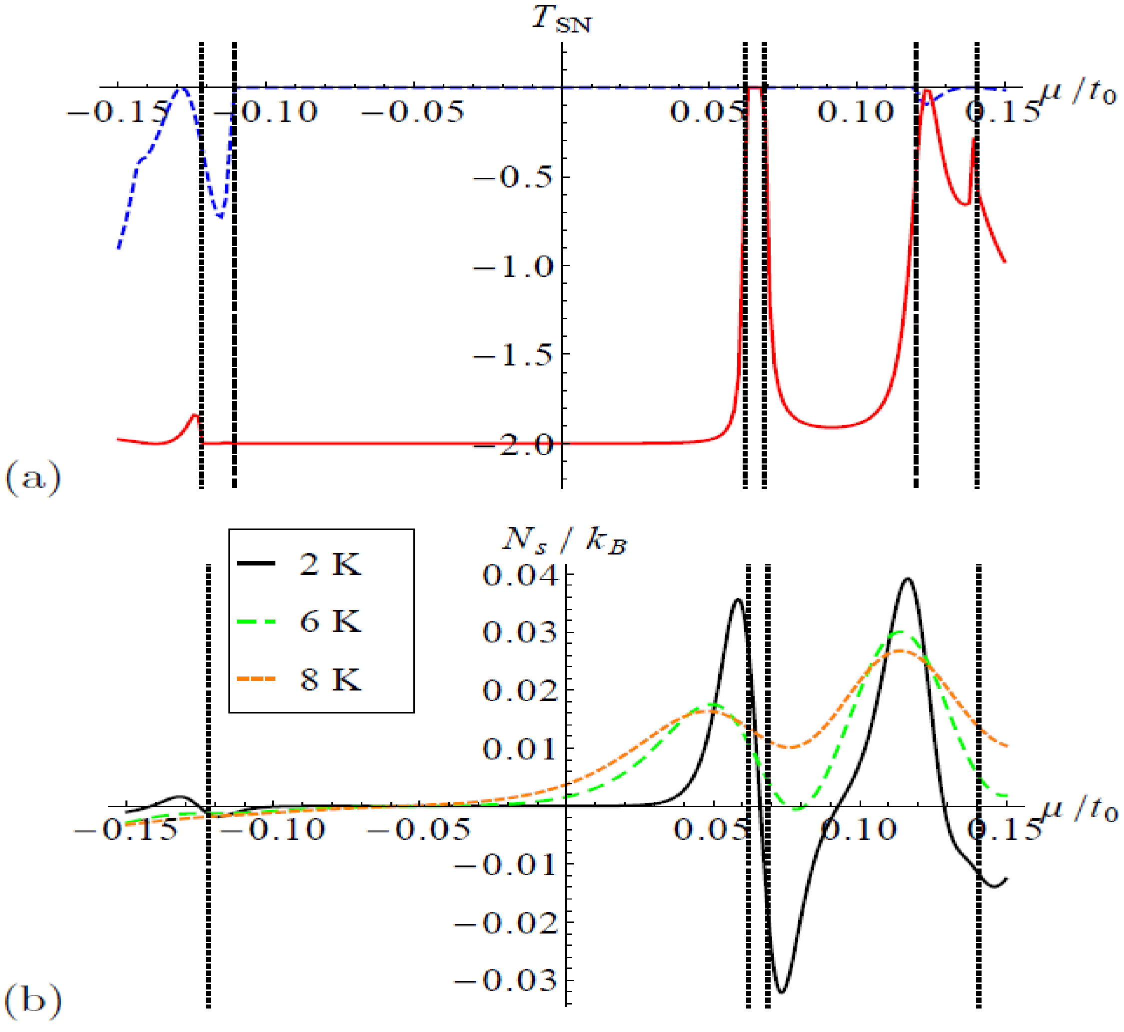
$$N_s \approx \frac{2\pi^2 k_B^2 \mathrm{T}}{3e}\,\frac{dG_{sH}(E)}{dE}\bigg|_{E=\mu(\mathrm{T}=0)}$$

With the spin Hall conductance at T= 0 :

$$G_{sH}(E,\mathrm{T}=0) = \frac{e}{8\pi}\mathcal{T}_{SN}(E)$$

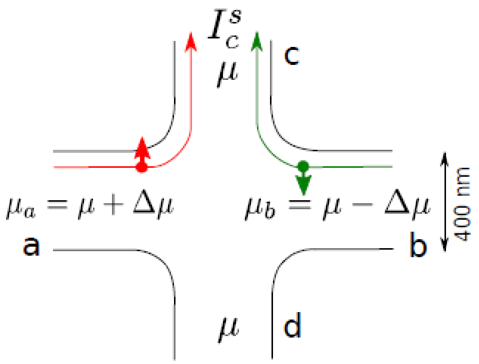
$$\mathcal{T}_{SN}(E) = \Delta T_{\mathrm{c,b}} - \Delta T_{\mathrm{c,a}}$$

THERMOELECTRIC TRANSPORT CARRIED BY EDGE STATES



· · · Bulk and mini- gaps in inverted regime

— — Bulk gap in normal regime



$$N_s \approx \frac{2\pi^2 k_B^2 \mathrm{T}}{3e}\,\frac{dG_{sH}(E)}{dE}\bigg|_{E=\mu(\mathrm{T}=0)}$$

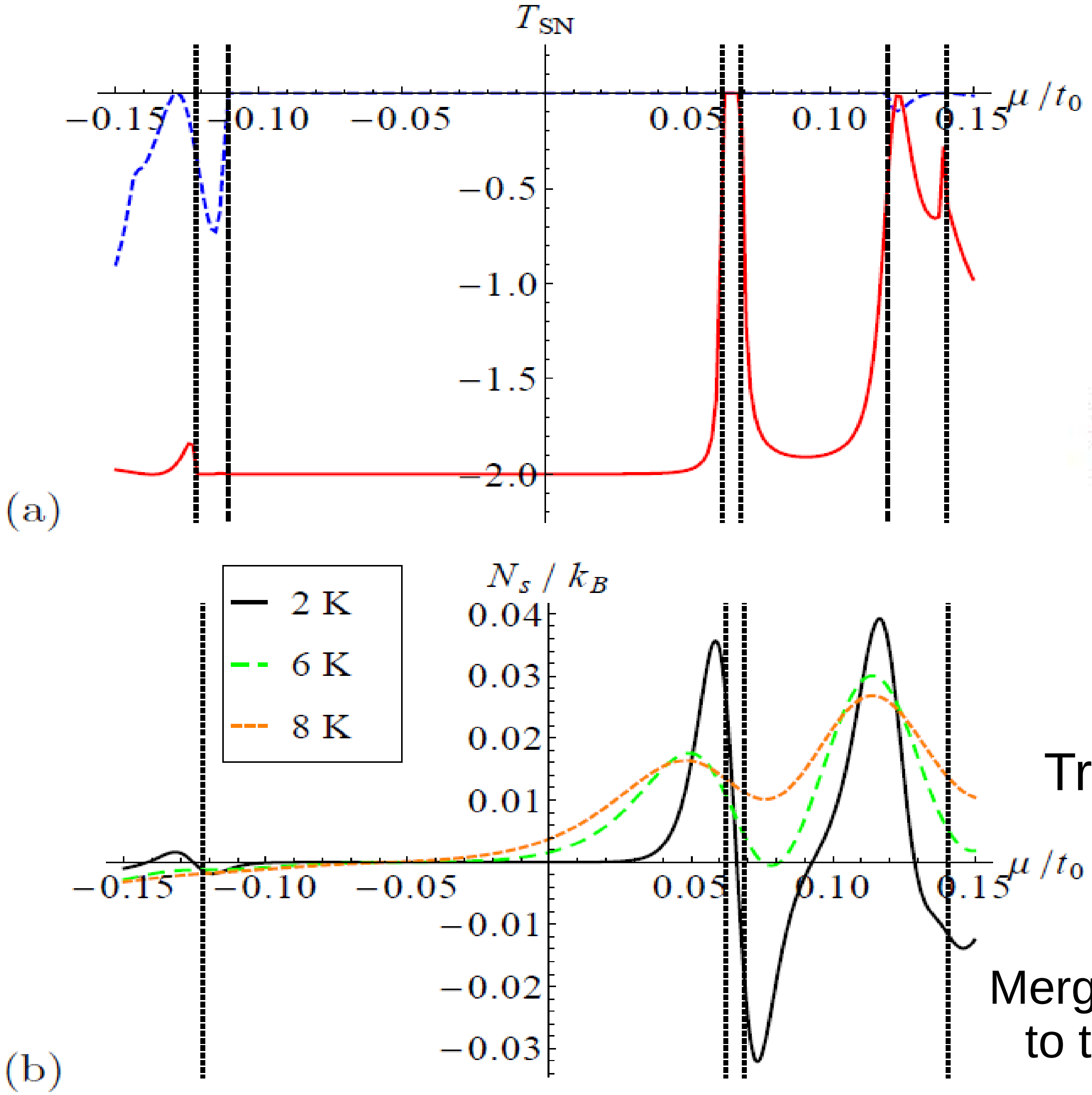
$$G_{sH}(E,\mathrm{T}=0) = \frac{e}{8\pi}\mathcal{T}_{SN}(E),$$

Symmetric mini-gap peak in transmission

→

Anti-symmetric peak in spin Nernst signal

THERMOELECTRIC TRANSPORT CARRIED BY EDGE STATES



· · · Bulk and mini- gaps in inverted regime

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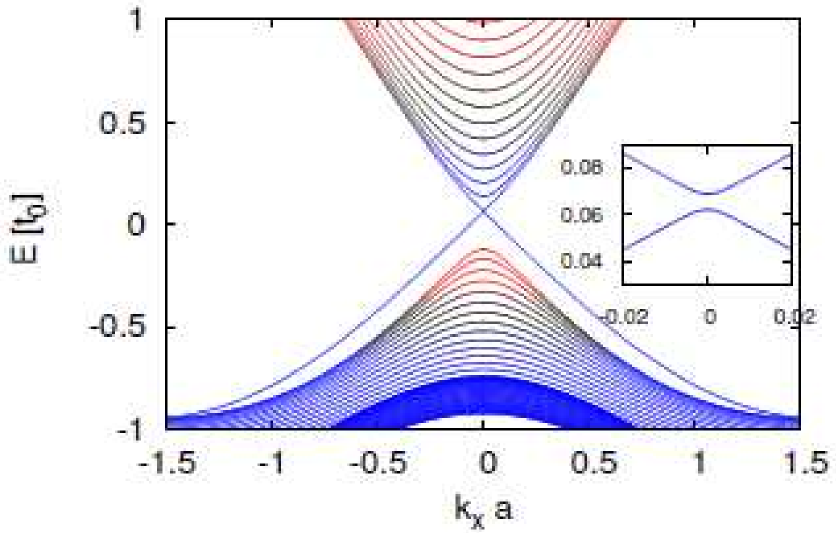


Transmission vanishes then reappears

Merging of the edge state to the conduction band

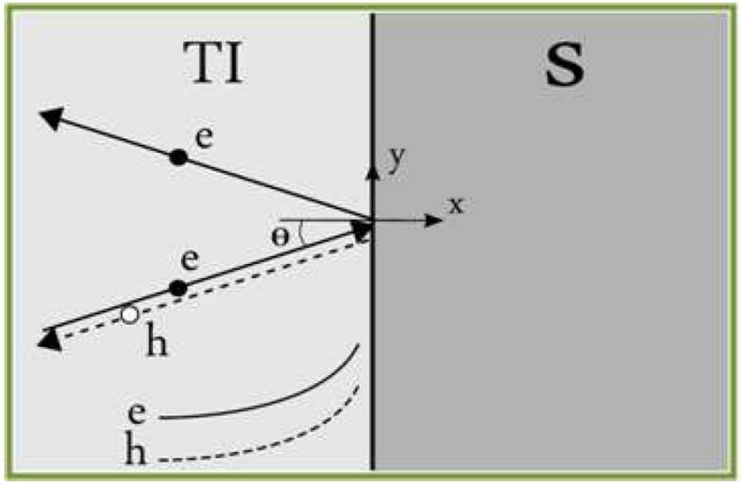
Formation of the first bulk state

➡ **Comparable magnitude of merging gap and mini gap which allows to mark positions where the edge state merges**

SPIN HALL EFFECT INDUCED BY BULK STATES

In-plane potential, mass confinement potential can generate spin Hall signal

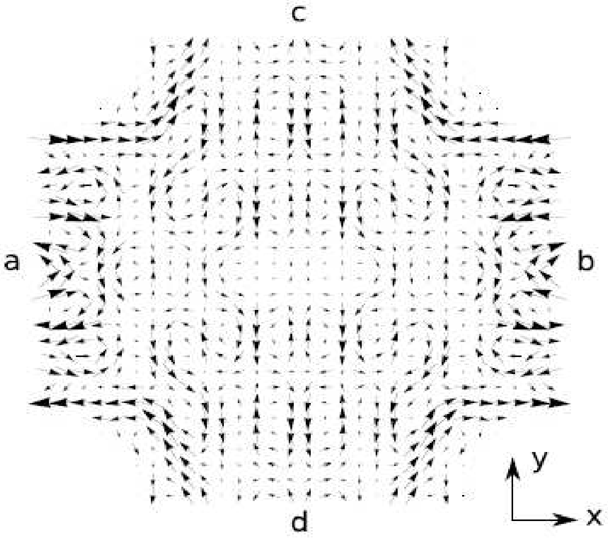
Yokoyama, PRL 2009 ; Rothe, NJP 2010 ; Guigou, PRB 2011



From Mott-like relation, spin Hall conductance gives rise to spin Nernst signal.

Local spin current at T=0 :

$$\mathbf{J}^z(\mathbf{r}) = \sum_p \mathrm{Tr}\left[\hat{\mathbf{J}}^z(\mathbf{r}) G^R \Gamma_p G^A\right] \mu_p .$$



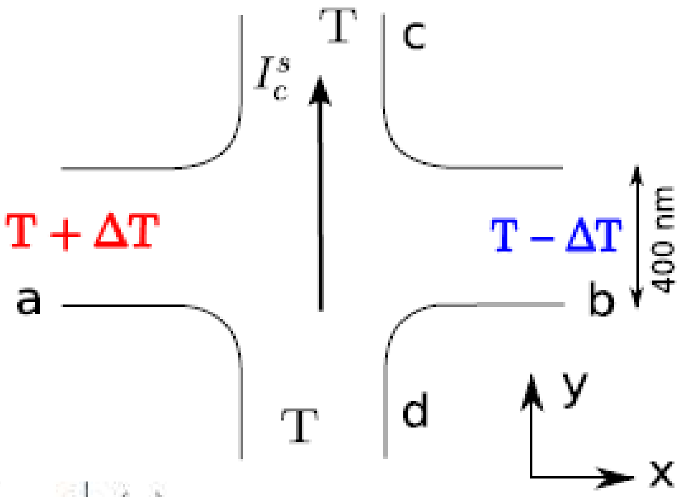
Spin Nernst effect carried by bulk modes ?

HARD WALL BOUNDARY SPIN CURRENT

We start with the following ansatz for the spin ↑ wav function

$$\psi^{\uparrow}(y) = \langle y|\psi^{\uparrow}\rangle = e^{ik_y y}u_{k_y} + re^{-ik_y y}u_{-k_y} + ce^{\lambda y}u_{i\lambda},$$

$$\psi^{\uparrow}(0) = 0$$

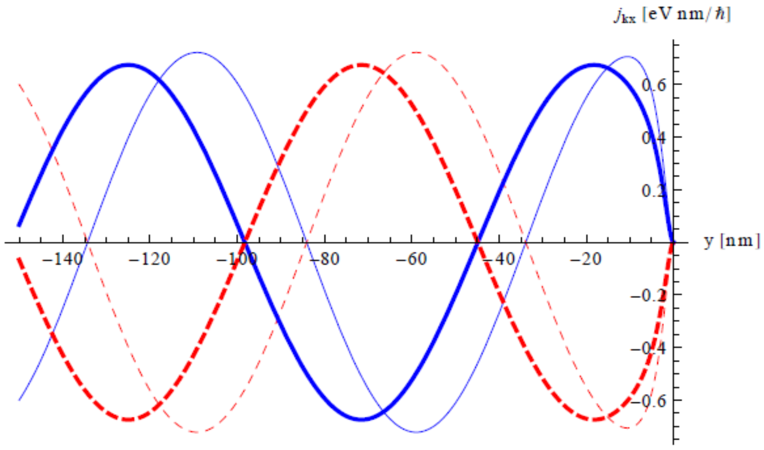


Spin current along y with tranverse velocity $j^s(y) = j^{\uparrow}_{k_x}(y) - j^{\downarrow}_{k_x}(y)$

with $j^{\uparrow}_{k_x}(y) = \langle\psi^{\uparrow}|\delta(y-\hat{y})V_x^{\uparrow}|\psi^{\uparrow}\rangle,$

Phase shift between $j^{\uparrow}$ and $j^{\downarrow}$.

Oscillatory pattern due to the superposition of incoming and reflected propagating waves.



Spin current scales as $j_s(y) \propto \dfrac{A^2 k}{|M|}.$

Ns FOR THE BULK METALLIC REGIME

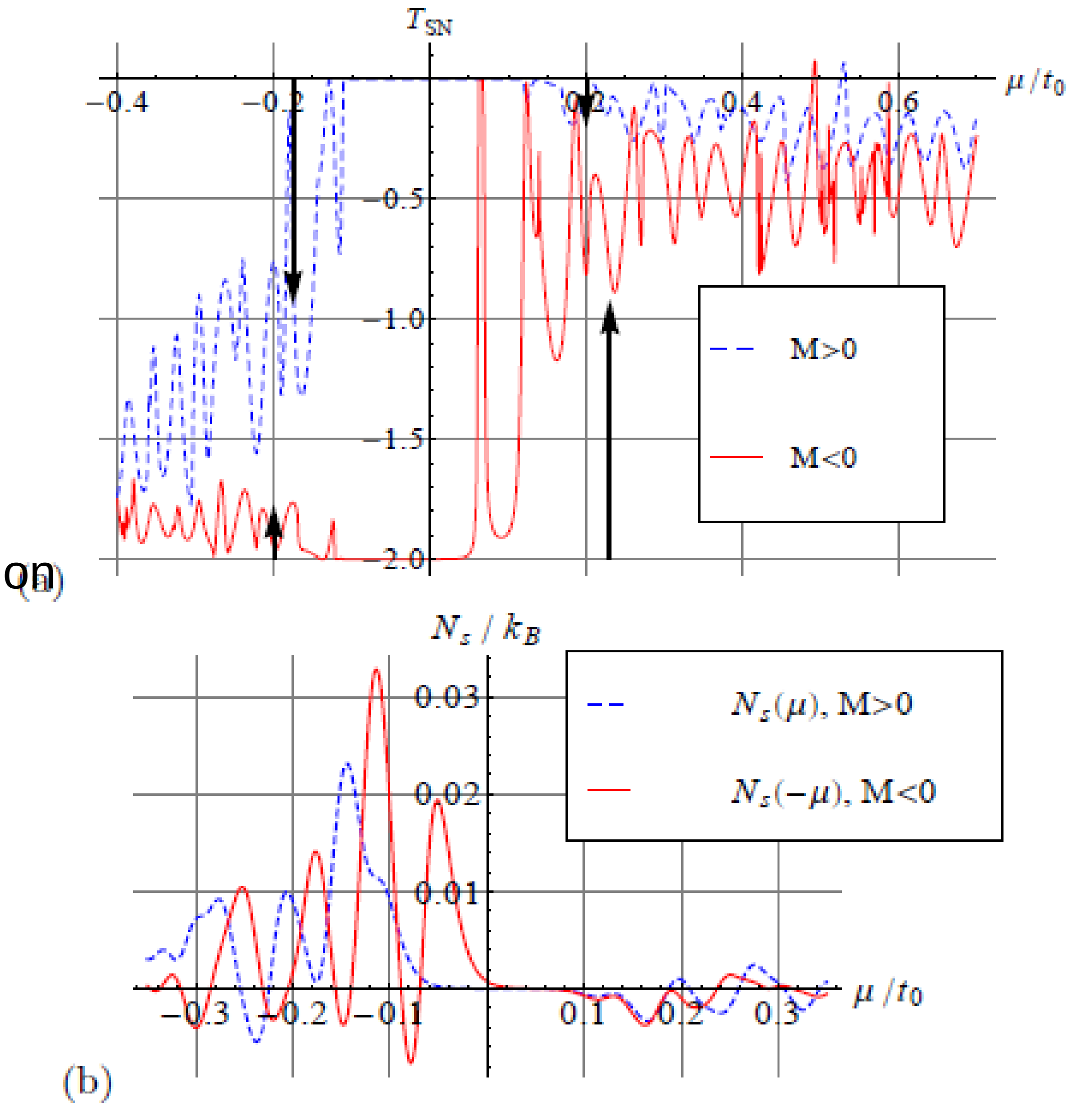
Applying voltage induces spin and charge response :

$$G_{sH} = \frac{I_s}{I} G_{xx}$$

$$\longrightarrow G_{sH} \sim m_{e/h} A^2/M$$

$$\longrightarrow \mathcal{T}_{SN} \sim m_{e/h}$$

Does the spin Nernst signal depend on the mass ?



- Black arrows compare Nernst transmission in valence and conduction bands ,
- Positions at energy for 4 propagating modes,
- Scaled by the ratio of the masses of electron/hole.

$$\boxed{N_s \propto |m_i| \;\; (i = e, h)}$$

CONCLUSION

- Use of thermoelectric coefficients as a probe of the dynamics in HgTe/CdTe quantum wells,

- Spin Nernst signals shows the topological phase and informs on mini-gap position in the spectrum,

- For metallic bulk regime, magnitude of spin Nernst signal depends on the mass of particles,

- Beyond the linear response regime ?