FERMI-EDGE SINGULARITY IN SINGLE ELECTRON GENERATION

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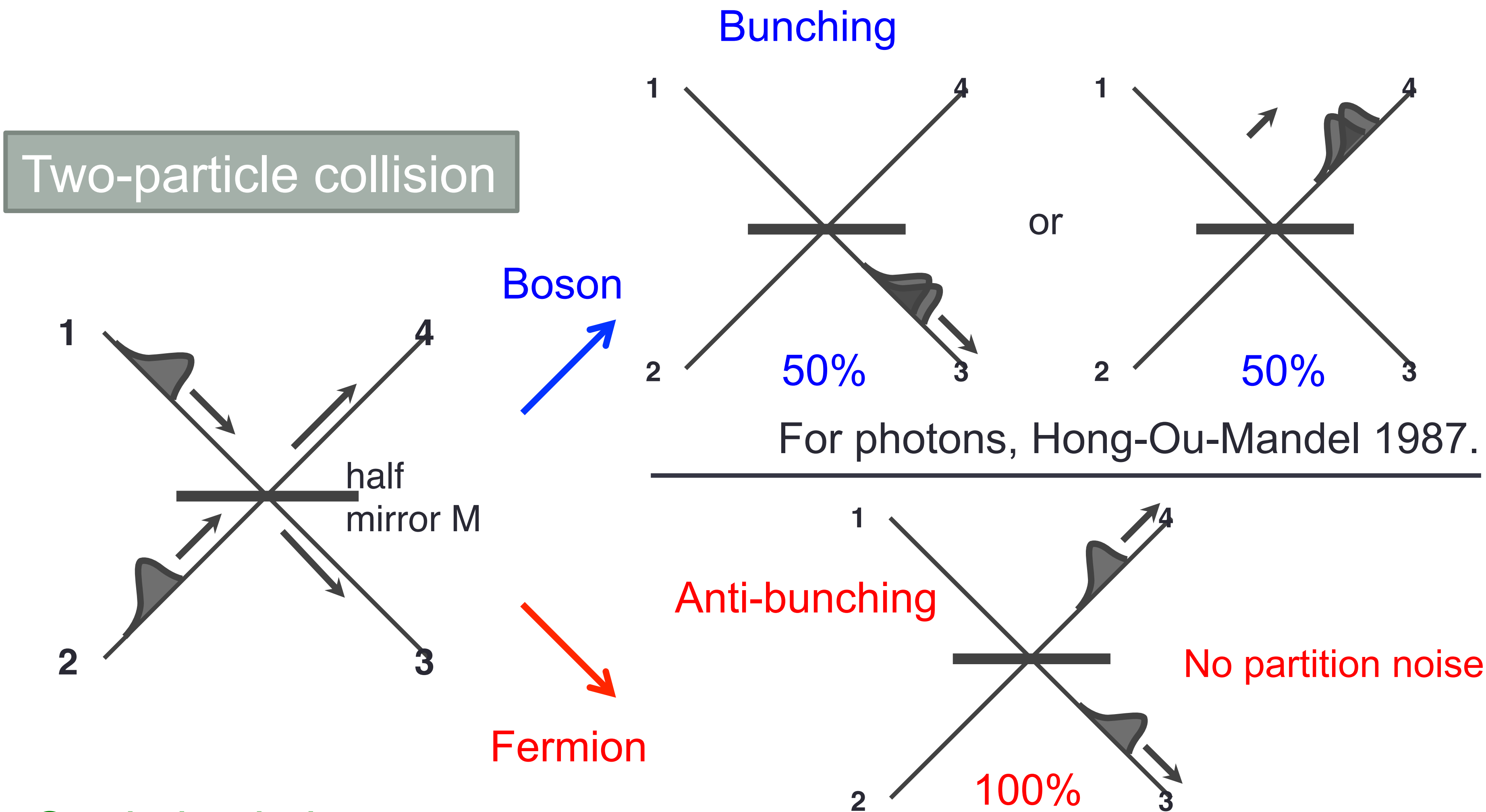
INTRODUCTION

Hong-Ou-Mandel experiment



Statistics is important.

For electrons,
Bocquillon et al. 2013 (QHE edge state).

Motivation and purpose

Theory on quality of injected electrons

Moskalets et al. ‘08, Keeling et al. ‘08, Fève et al. ‘08, Degiovanni et al. ‘09, Mahe et al, ’10, Albert et al. ’10, Grenier et al. ’11, Haack et al. ‘11, Parmentier et al. ‘12, Jonckheere et al ’12., ...

- Possible mechanism of imperfect anti-bunching
  - Small deviation of experimental parameters
  - Coulomb interaction (edge state, colliding electrons)
  - Fluctuation of environment (background charge, voltage gate etc.)
- Coulomb interaction effect at single-electron generator
- “Electronic” quantum-optical experiment.
  - Characteristic feature of electrons = “Coulomb interaction”

Model Hamiltonian

Velocity is chosen as unity.

$$H = H_0 + V$$

$$H_0 = \underline{\varepsilon_d d^\dagger d} + \int \underline{dk k a_k^\dagger a_k} + \underline{U d^\dagger d \tilde{a}_0^\dagger \tilde{a}_0}$$

quantum dot — chiral edge channel — Coulomb interaction

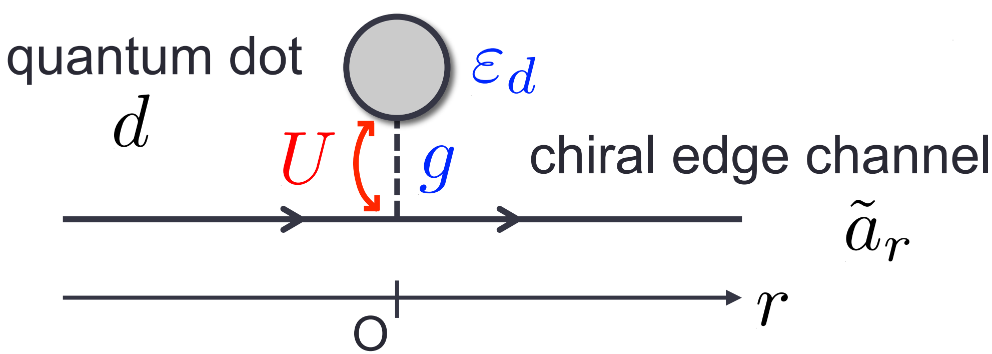
$$V = \underline{g(\tilde{a}_0^\dagger d + d^\dagger \tilde{a}_0)}$$

tunneling amplitude

$$\tilde{a}_0 \equiv \tilde{a}_{r=0}$$

$$\tilde{a}_r = \frac{1}{\sqrt{2\pi}} \int dk e^{ikr} a_k$$

Real-space representation

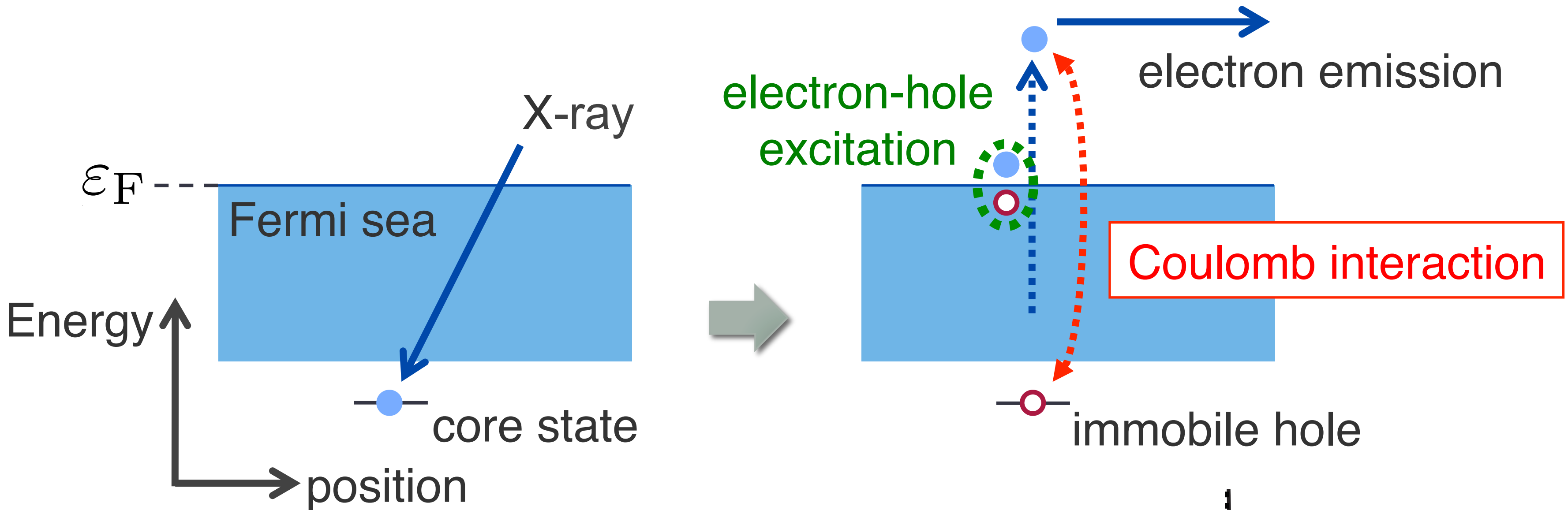


the same problem as
the X-ray absorption edge

Mahan 1967

X-ray absorption problem

Mahan 1967



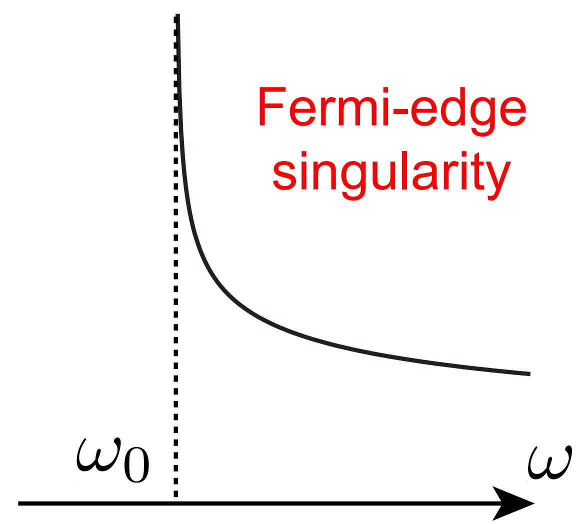
Transition probability

D : band width

$$\mathrm{Im}S(\omega) \propto \Theta(\omega - \omega_0)\left(\frac{D}{\omega - \omega_0}\right)^{\alpha}$$

$$\alpha = 2\delta/\pi - (\delta/\pi)^2$$

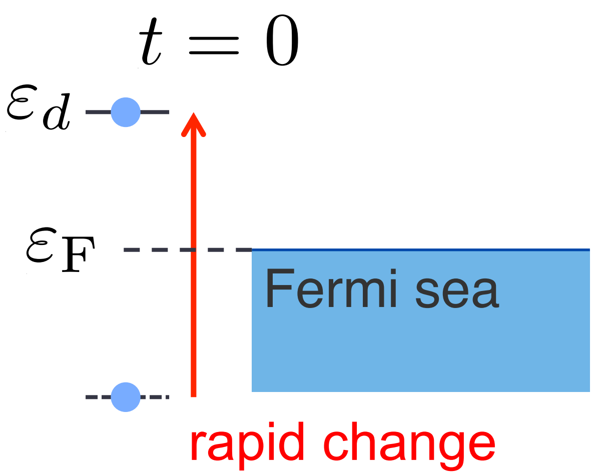
δ : phase shift



Local quantity is focused on.

METHOD

Setup



initial state

$$|\psi(0)\rangle = |n_d = 1\rangle \otimes |\mathrm{FS}\rangle$$

occupied state (quantum dot)

Fermi sea (chiral edge channel)

observables

$$\langle \hat{O}(t) \rangle = \langle \psi(0) | e^{iHt} \hat{O} e^{-iHt} | \psi(0) \rangle$$

(1) survival probability

$$P(t) = \langle d^\dagger(t) d(t) \rangle$$

(2) density matrix of emitted electrons

$$\rho(r, r', t) = \langle a^\dagger_{r'}(t) a_r(t) \rangle$$

Information of an emitted electron (including its coherence)

Diagrammatic Expansion (1)

$$H = H_0 + V$$

$$H_0 = \varepsilon_d d^\dagger d + \int dk k a_k^\dagger a_k + U d^\dagger d \tilde{a}_0^\dagger \tilde{a}_0$$

$$V = g(\tilde{a}_0^\dagger d + d^\dagger \tilde{a}_0)$$ ← perturbation

Formal expansion in V

$$\langle \hat{O}(t) \rangle = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} (-i)^m i^n \int_{0 \le t_1 \le \cdots \le t_m \le t} dt_1 \cdots dt_m \int_{0 \le t_1' \le \cdots \le t_n' \le t} dt_1' \cdots dt_n'$$

$$\times \langle \psi(0) | V_I(t_1') \cdots V_I(t_n') O_I(t) V_I(t_m) \cdots V_I(t_1) | \psi(0) \rangle$$

interaction representation

$$V_I(t) = e^{iH_0 t} V e^{-iH_0 t}$$

$$O_I(t) = e^{iH_0 t} O e^{-iH_0 t}$$

Diagrammatic Expansion (2)

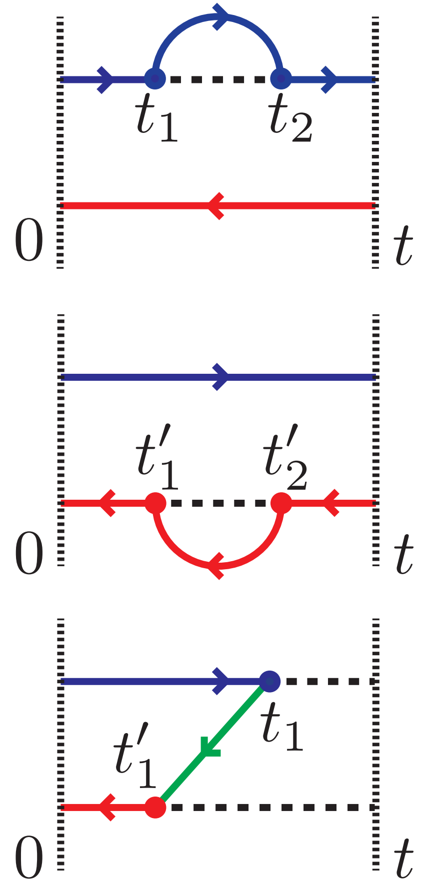
$$V = g(\tilde{a}_0^\dagger d + d^\dagger \tilde{a}_0)$$: vertex

Each term in expansion series

$$(-i)^2 \int_{0\le t_1\le t_2\le t} dt_1 dt_2 \langle O_I(t) V_I(t_2) V_I(t_1)\rangle$$

$$i^2 \int_{0\le t_1'\le t_2'\le t} dt_1' dt_2' \langle V_I(t_1') V_I(t_2') O_I(t)\rangle$$

$$(-i)i \int_{0\le t_1, t_1'\le t} dt_1 dt_1' \langle V_I(t_1') O_I(t) V_I(t_1)\rangle$$

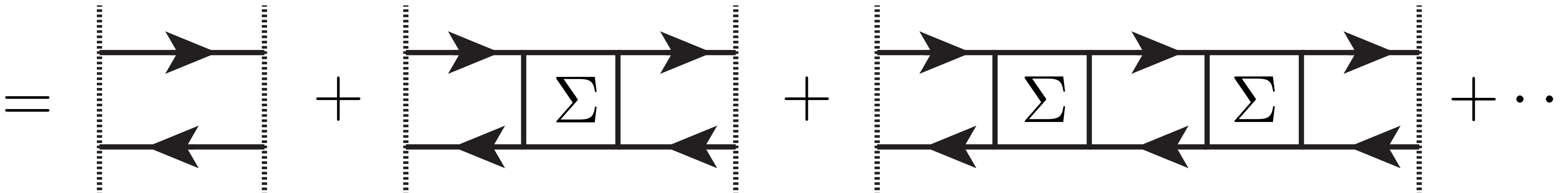


Diagrammatic Expansion (3)

Survival probability

Schoeller-Schön 1994
U. Weiss "Dissipative Quantum Systems"

$$P(t) = \langle d^\dagger(t) d(t) \rangle$$

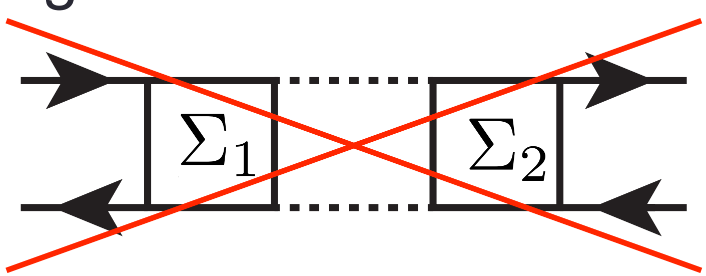


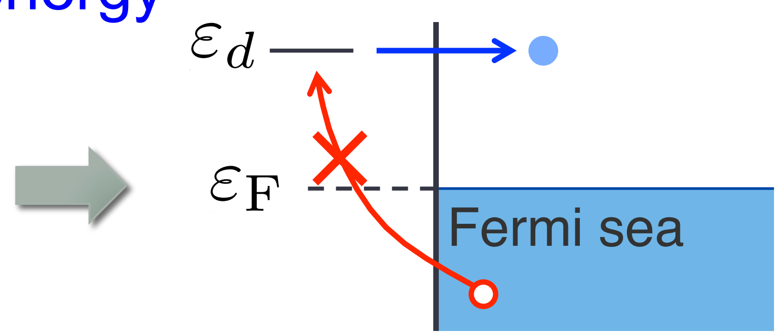
$$P(\lambda) = \int_0^\infty dt\, P(t) e^{-\lambda t} = \frac{1}{\lambda - \Sigma(\lambda)}$$

self-energy

emission only

We neglect

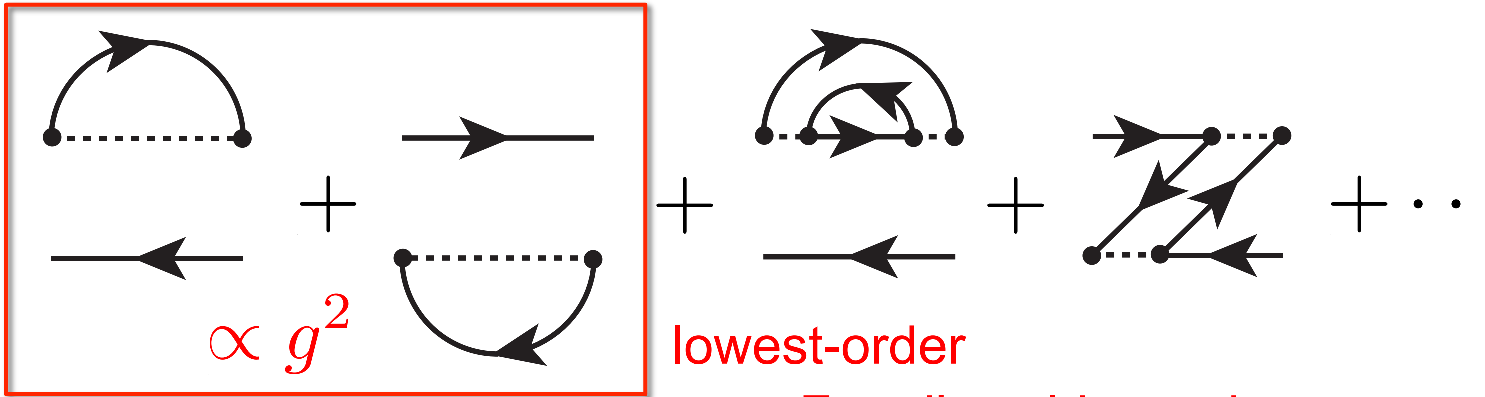




Lowest-order approximation

Self-energy

$\Sigma =$



lowest-order
= Fermi's golden rude

Example: the non-interacting case $U = 0$

$$\Sigma(t) = -g^2\delta(t) \;\rightarrow \Sigma(\lambda) = -g^2 \equiv -\Gamma$$

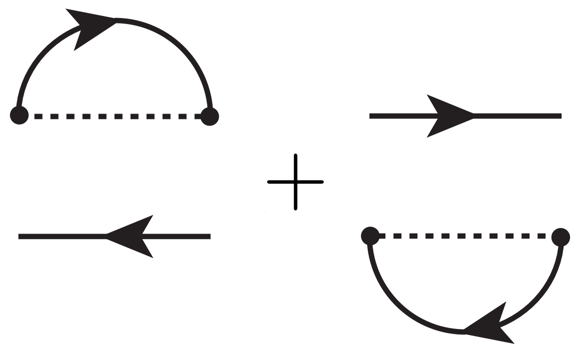
$$P(\lambda) = \frac{1}{\lambda + \Gamma} \quad \Rightarrow \quad \boxed{P(t) = e^{-\Gamma t}}$$

Γ : decay rate

exponential decay

Effect of Coulomb interaction

$\Sigma(t) =$



$= -2g^2 \mathrm{Re}[e^{i\varepsilon_d t} G(t)]$

$$G(t) = \langle \mathrm{FS}| e^{iH_{c,1}t} \tilde{a}_0 e^{-iH_{c,0}t} \tilde{a}_0 |\mathrm{FS}\rangle$$

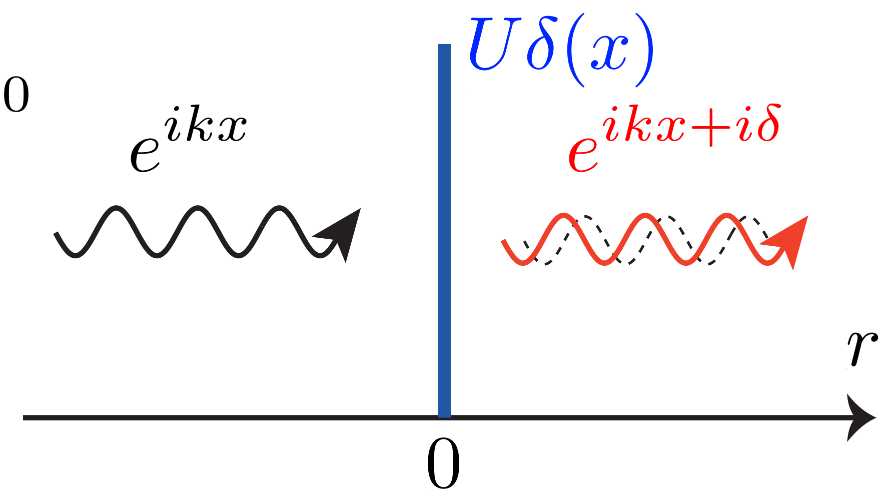
← The central quantity in Fermi-edge singularity (Nozières-de Dominicis 1969)

$\propto t^{-\alpha}$ power-law decay

$$H_{c,1} = \int dk\, k a_k^\dagger a_k + U \tilde{a}_0 \tilde{a}_0$$

$$H_{c,0} = \int dk\, k a_k^\dagger a_k$$

δ : phase shift



Analytic calculation of G(t)

- Singular integral approach (Nozières-de Dominicis 1969)
- Bosonization method (Schotte-Schotte 1969)
- Determinant formula + Riemann-Hilbert factorization (Abanin-Levitov 2004, 2005)

Determinant formula Klich 2002

$$\mathrm{tr}(e^A e^B e^C) = \det(1 + e^a e^b e^c)$$

$$O = \sum_{\alpha,\beta} o_{\alpha\beta} a^\dagger_\alpha a_\beta$$

e.g. $\langle e^{iH_{c,1}t} e^{-iH_{c,0}t} \rangle$

$$\rho = \frac{1}{Z} e^{-\sum_p \lambda_p a^\dagger_p a_p}$$

$$= Z^{-1} \mathrm{Tr}(\underline{e^{-\lambda}} e^{iH_{c,1}t} e^{-iH_{c,0}t})$$

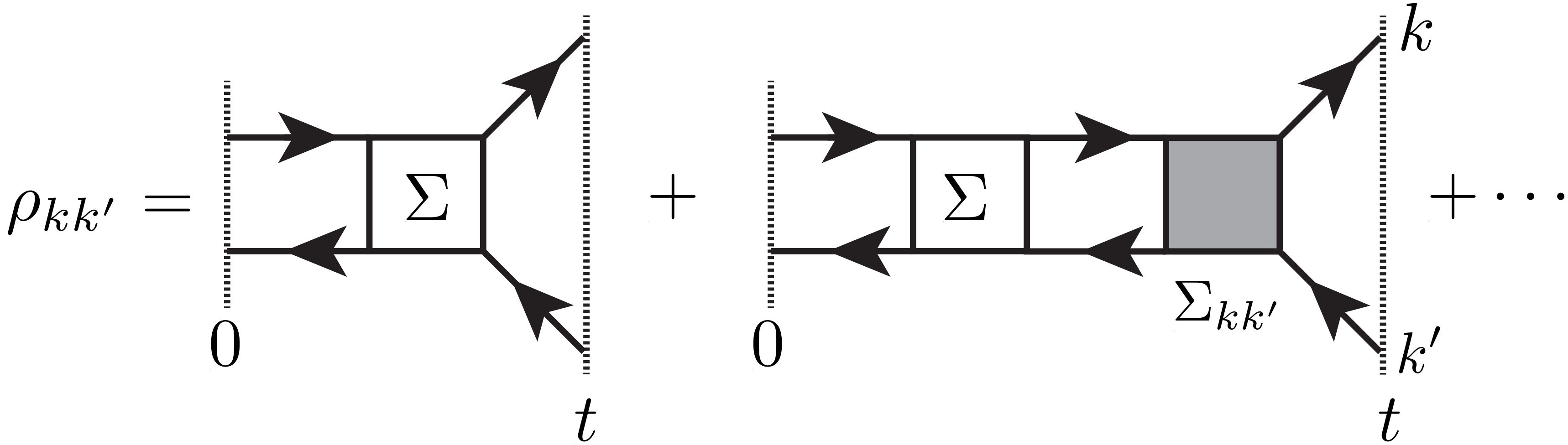
$$e^{-\lambda_p} = n(\varepsilon_p)/(1 - n(\varepsilon_p))$$

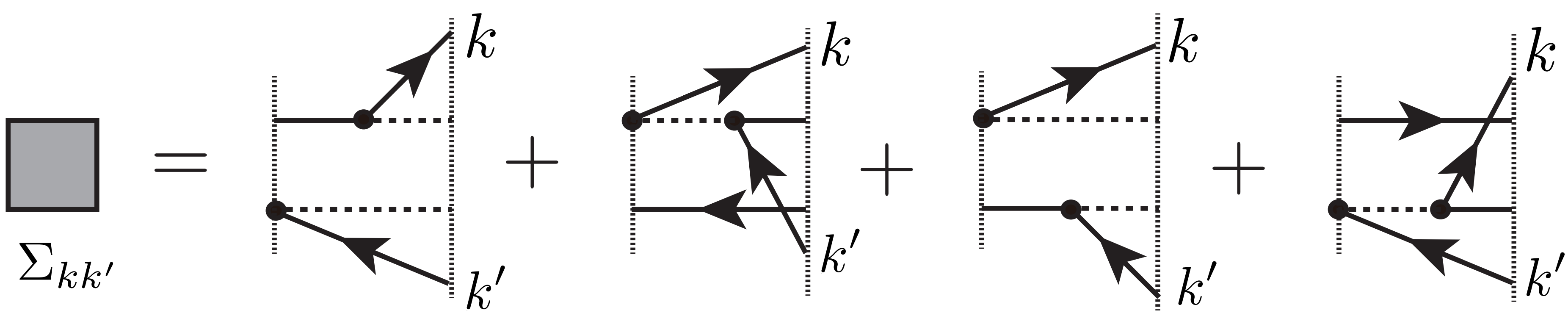
$$= \det(1 - n(\varepsilon) + e^{ih_1 t} e^{-ih_0 t} n(\varepsilon))$$

⬅ one-body problem

Density matrix

$$\rho_{kk'}(t) = \langle a^\dagger_{k'}(t) a_k(t) \rangle$$





RESULT

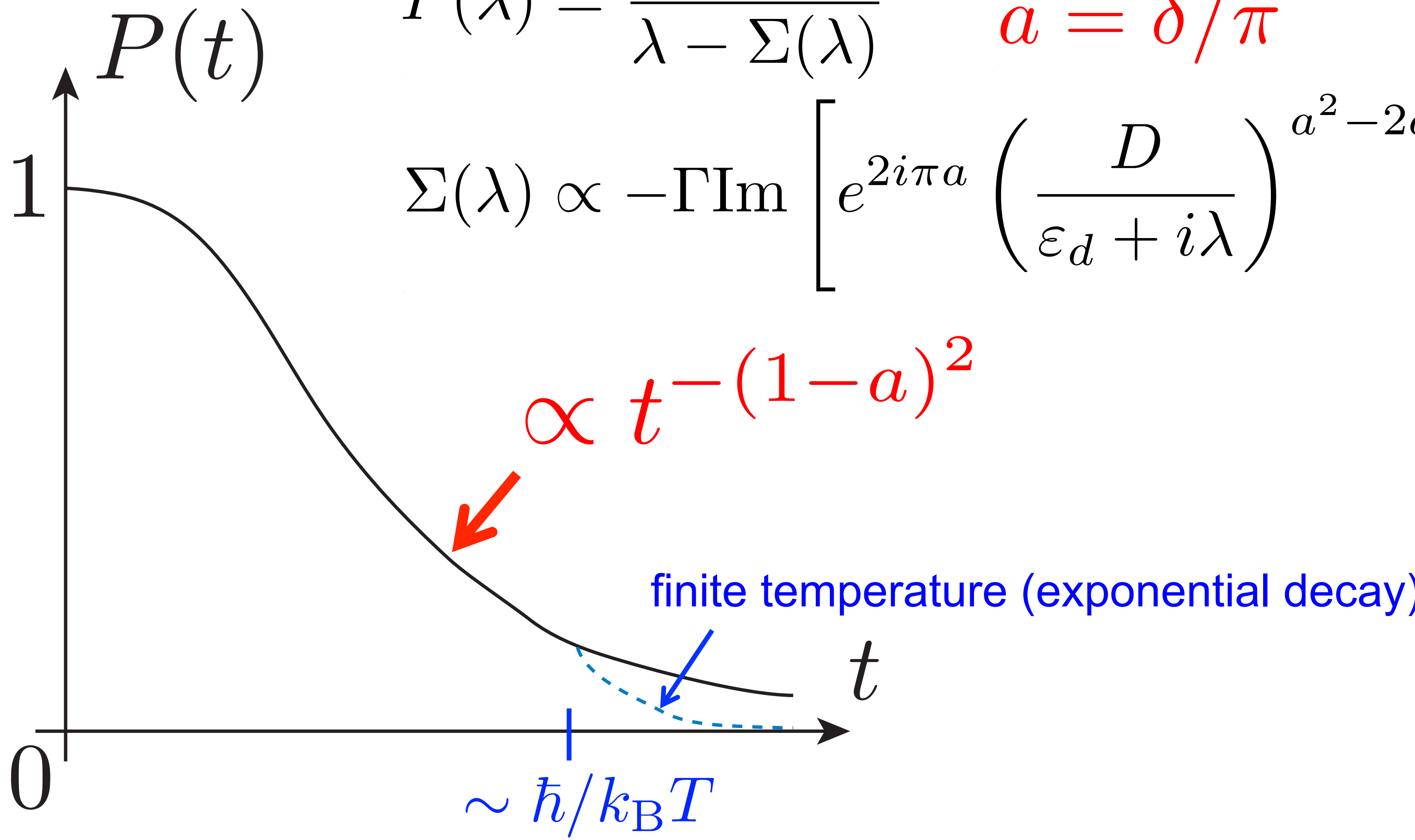
Survival probability

$$P(t) = \langle d^\dagger(t) d(t) \rangle$$

$$P(\lambda) = \frac{1}{\lambda - \Sigma(\lambda)}$$

$$a = \delta/\pi$$

$$\Sigma(\lambda) \propto -\Gamma \mathrm{Im} \left[e^{2i\pi a} \left(\frac{D}{\varepsilon_d + i\lambda} \right)^{a^2 - 2a} \right]$$

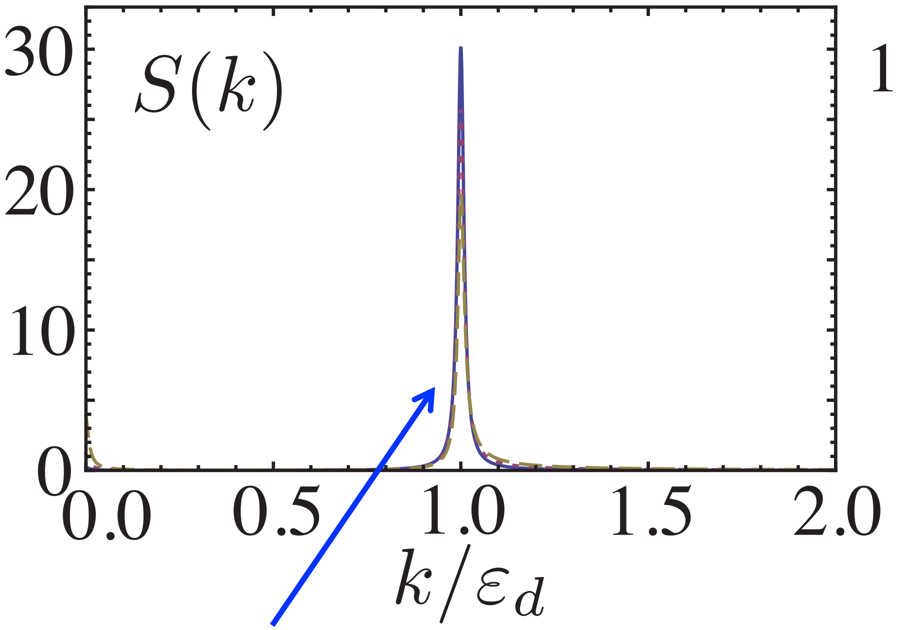


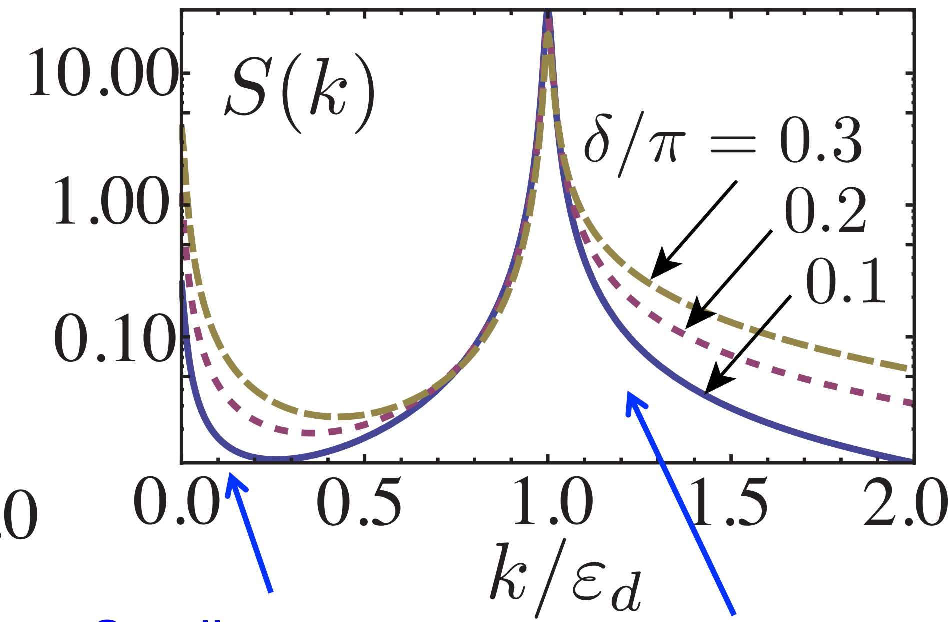
Spectrum

$$S(k) = \rho(k, k, t = \infty)$$

$E = k$: Emitted electron energy $(v = 1)$

Linear Plot → Log Plot





Emitted electron energy
$E = k = \varepsilon_d$

Small-energy electron excitation

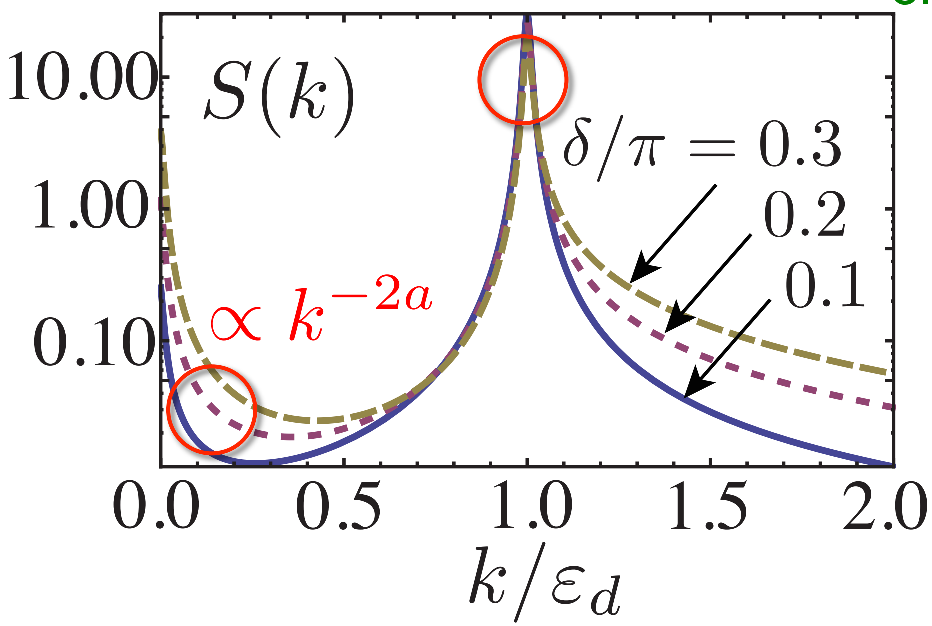
Asymmetric peak

$$a = \frac{\delta}{\pi}$$: Phase shift (Strength of interaction)

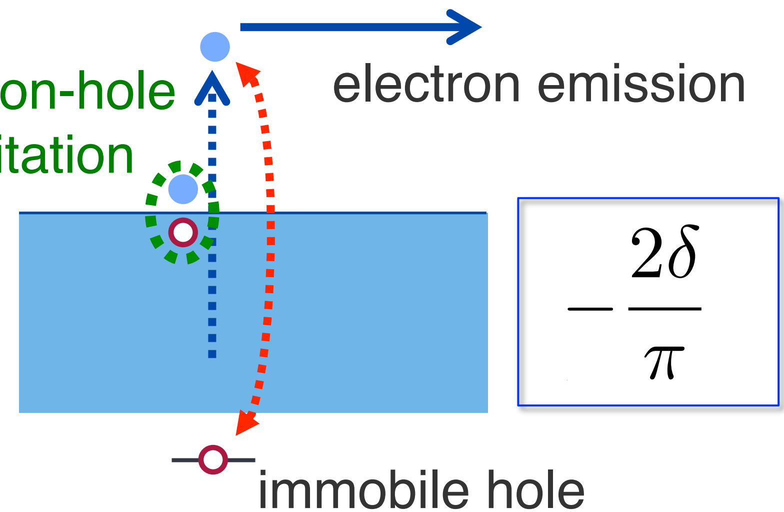
Two kinds of exponents

$a = \frac{\delta}{\pi}$

Log Plot $\propto (k - \varepsilon_d)^{2(1-a^2)}$



(1) e-h attractive interaction



$$-\frac{2\delta}{\pi}$$

c.f. absorption spectrum

$$\mathrm{Im}S(\omega) \propto \left(\frac{D}{\omega - \omega_0}\right)^{\frac{2\delta}{\pi} - \frac{\delta^2}{\pi^2}}$$

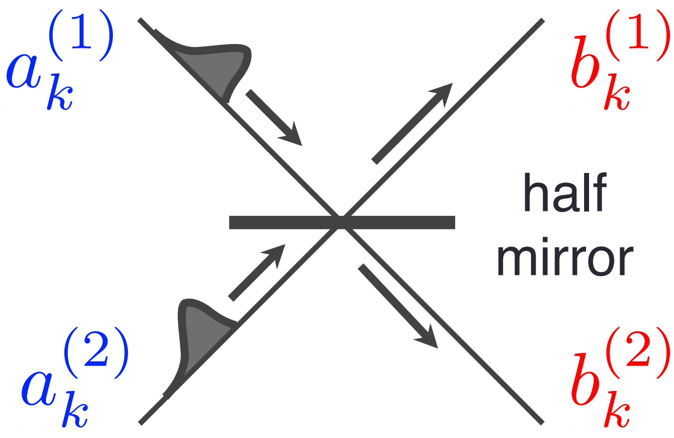
(2) overlap integral

$$|\langle \mathrm{FS}, 0 | \mathrm{FS}, 1 \rangle|^2 = N^{-(\delta/\pi)^2}$$

Anderson orthogonal theorem

$$\frac{\delta^2}{\pi^2}$$

Purity and coincidence probability



$$b_k^{(1)} = \frac{1}{\sqrt{2}}(a_k^{(1)} + a_k^{(2)})$$

$$b_k^{(2)} = \frac{1}{\sqrt{2}}(a_k^{(1)} - a_k^{(2)})$$

coincidence probability

$$P_{12} = \int dk \int dk' \langle (b_k^{(1)})^\dagger b_k^{(1)} (b_{k'}^{(2)})^\dagger b_{k'}^{(2)} \rangle = \frac{1+\mathcal{P}}{2}$$

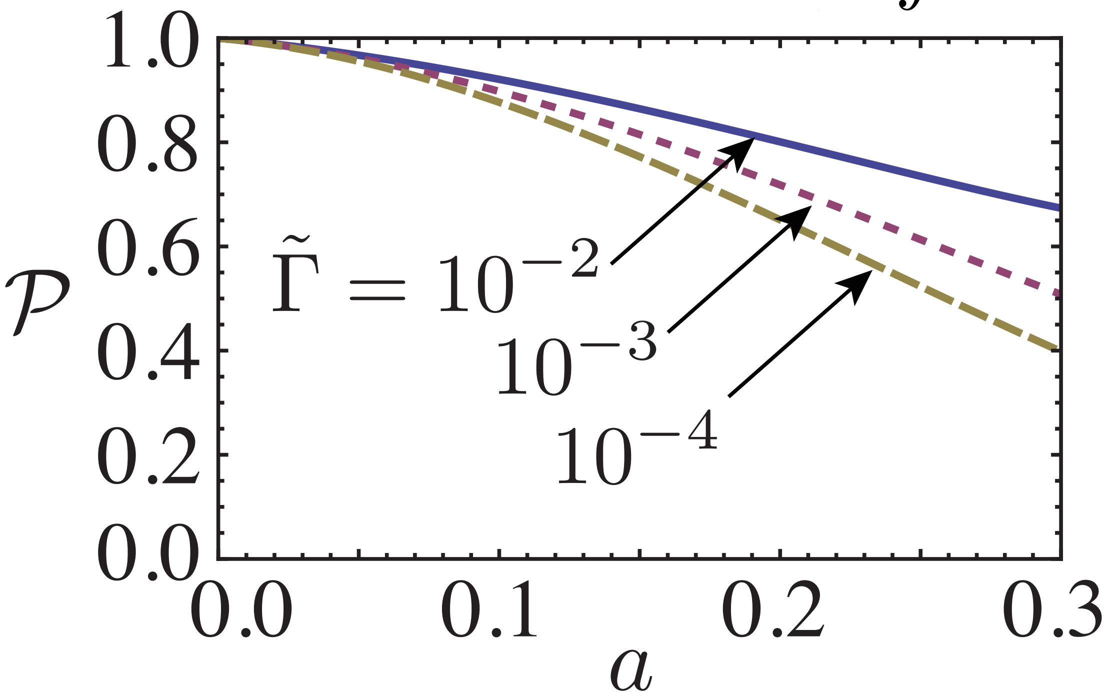
Purity $\mathcal{P} = \frac{\mathrm{Tr}(\rho^2)}{(\mathrm{Tr}\rho)^2} = \int dk \int dk' \rho(k,k')\rho(k',k)$

$\mathcal{P} \to 1 \quad P_{12} = 1$: complete anti-bunching (Fermion)

$\mathcal{P} \to 0 \quad P_{12} = 1/2$: classical

Purity (= degree of anti-bunching)

$$\mathcal{P} = \mathrm{tr}(\rho^2) = \int dk dk' \rho(k,k')\rho(k',k)$$



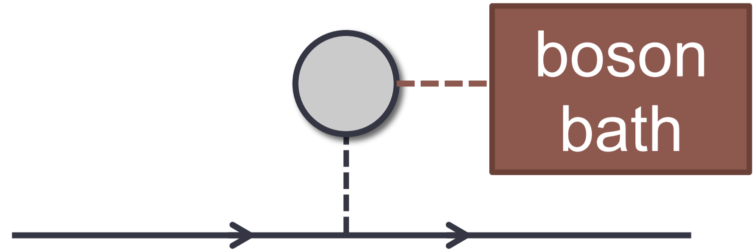
$$\tilde{\Gamma} = \frac{\Gamma}{\varepsilon_d}\left(\frac{D}{\varepsilon_d}\right)^{-2a}$$

Coincidence prob.

$$P_{12} = \frac{1+\mathcal{P}}{2}$$

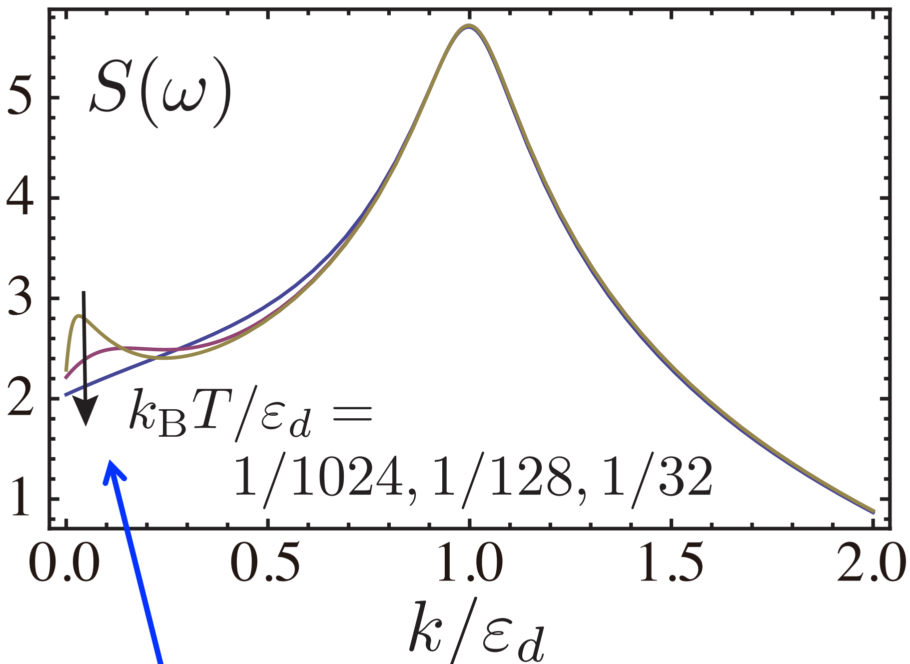
$a = \dfrac{\delta}{\pi}$: Phase shift (Strength of interaction)

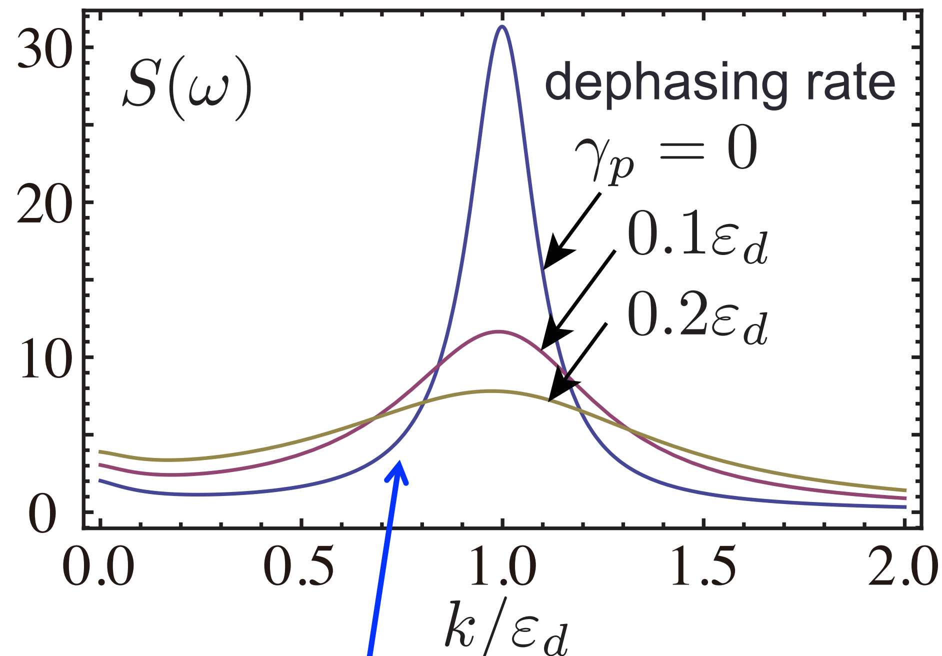
Other effect



(1) finite-temperature effect

(2) pure-dephasing effect





Suppression of small-energy excitation

Pure dephasing strongly affect quality of emitted electrons.

Summary

- Fermi-edge singularity induces singular behavior on single electron generation.
  - Survival probability
  - density matrix (including coherence of injected electrons)
- Energy spectrum of injected electrons (δ: phase shift)
  - exponent $-2\delta/\pi$ ... small-energy electron excitation
  - exponent $(\delta/\pi)^2$... decay of the main peak
- Purity (= degree of anti-bunching)
  - (modest) suppression due to Coulomb interaction
- Other effect
  - finite-temperature, pure dephasing effect
- Direct observation of many body effect through coherence of emitted electrons.