Superconducting Proximity Effect in Quantum-Dot Systems

Jürgen König

Institut für Theoretische Physik
Universität Duisburg-Essen

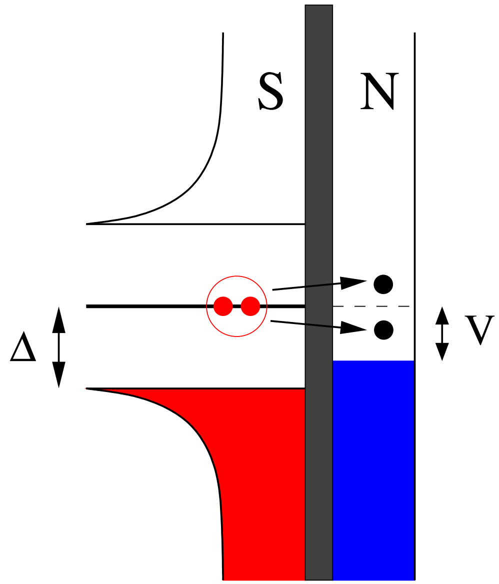
UNIVERSITÄT DUISBURG ESSEN
Open-Minded

CENIDE
CENTER FOR NANO INTEGRATION DUISBURG ESSEN

Transport through S-N and S-I-S Interfaces

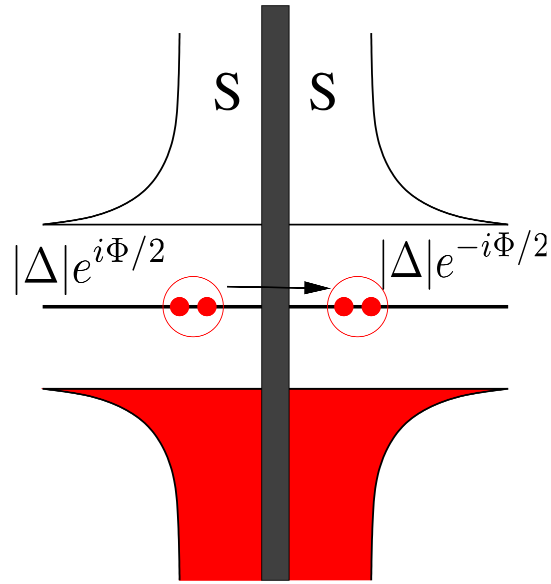
- BCS superconductivity: pair potential $\Delta = g\langle c_{-k\downarrow} c_{k\uparrow}\rangle$
  - quasiparticles with gap $|\Delta|$ in density of states
  - Cooper-pair condensate
- subgap transport

Andreev reflection



Cooper pair breaks/forms

Josephson current



transfer of Cooper pairs

$J_{\text{jos}}(\Phi) = J_c \sin\Phi$

QDs Coupled to Superconductors - Exps

- carbon nanotube quantum dot

  Buitelaar, Nussbaumer, Schönenberger, PRL '02; Cleziou, Wernsdorfer, Bouchiat, Ondarcuhu, Monthioux, Nature Nanotech. '06; Jarillo-Herrero, van Dam, Kouwenhoven, Nature '06; Jorgensen, Grove-Rasmussen, Novotny, Flensberg, Lindelof, PRL '06; Hermann, Portier, Roche, Levy Yeyati, Kontos, Strunk, PRL '10; Pillet, Quay, Morfin, Bena, Levy Yeyati, Joyez, Nature Phys. '10; . . .

- quantum dot in InAs nanowire

  van Dam, Nazarov, Bakkers, De Franceschi, Kouwenhoven, Nature '06; Hofstetter, Csonka, Nygard, Schönenberger, Nature '10; . . .

- InAs quantum dot with Al electrodes

  Buizert, Oiwa, Shibata, Hirakawa, Tarucha, PRL '07; Deacon, Tanaka, Oiwa, Sakano, Yoshida, Shibata, Hirakawa, Tarucha, PRL '10; . . .

- quantum dot in graphene

  Dirks, Hughes, Lal, Uchoa, Chen, Chialvo, Goldbart, Mason, Nature Phys. '11

QDs Coupled to Superconductors - Thy

- Andreev and multiple Andreev reflections through QDs

  Levy Yeyati, Cuevas, Lopez-Davalos, Martin-Rodero, PRB '97; Fazio, Raimondi, PRL '98, PRL '99; Kang, PRB '98; Schwab, Raimondi, PRB '99; Johansson, Bratus, Shumeiko, Wendin, PRB '99; Clerk, Ambegaokar, Hershfield, PRB '00; Cuevas, Levy Yeyati, Martin-Rodero, PRB '01; . . .

- transport in Kondo regime

  Clerk, Ambegaokar, PRB '00; Avishai, Golub, Zaikin, PRB '03; Choi, Lee, Kang, Belzig, PRB '04; Siano, Egger, PRL '04; Sellier, Kopp, Kroha, Barash, PRB '05; Nussinov, Shnirman, Arovas, Balatsky, Zhu, PRB '05; Bergeret, Levy Yeyati, Martin-Rodero, PRB '06; Lopez, Choi, Aguado, PRB '07; Karrasch, Oguri, Meden, PRB '08; Karrasch, Meden, PRB '09; . . .

- Josephson current through QD with spin-orbit coupling

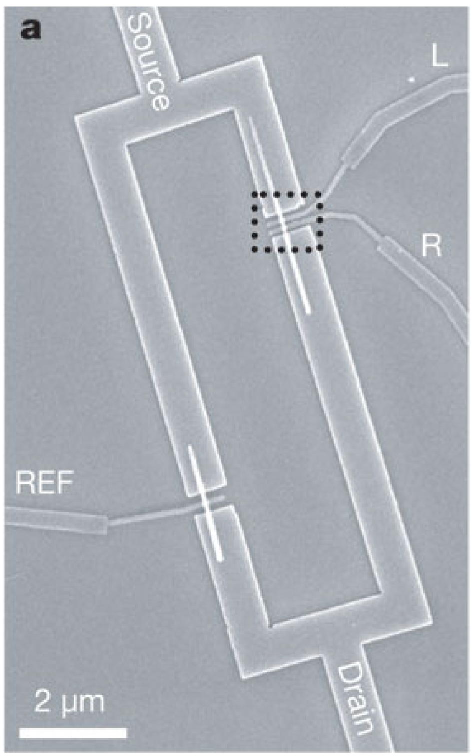
  Dell'Anna, Zazunov, Egger, Martin, PRB '07; Zazunov, Egger, Jonckheere, Martin, PRL '09; . . .

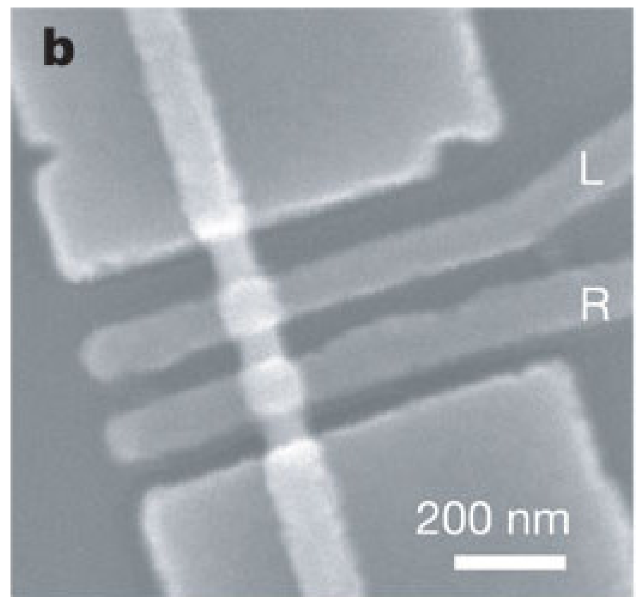
Supercurrent reversal in quantum dots

Jorden A. van Dam[1], Yuli V. Nazarov[1], Erik P. A. M. Bakkers[2], Silvano De Franceschi[1,3] & Leo P. Kouwenhoven[1]

Nature **442**, 667 (2006)

- quantum dot in InAs nanowire

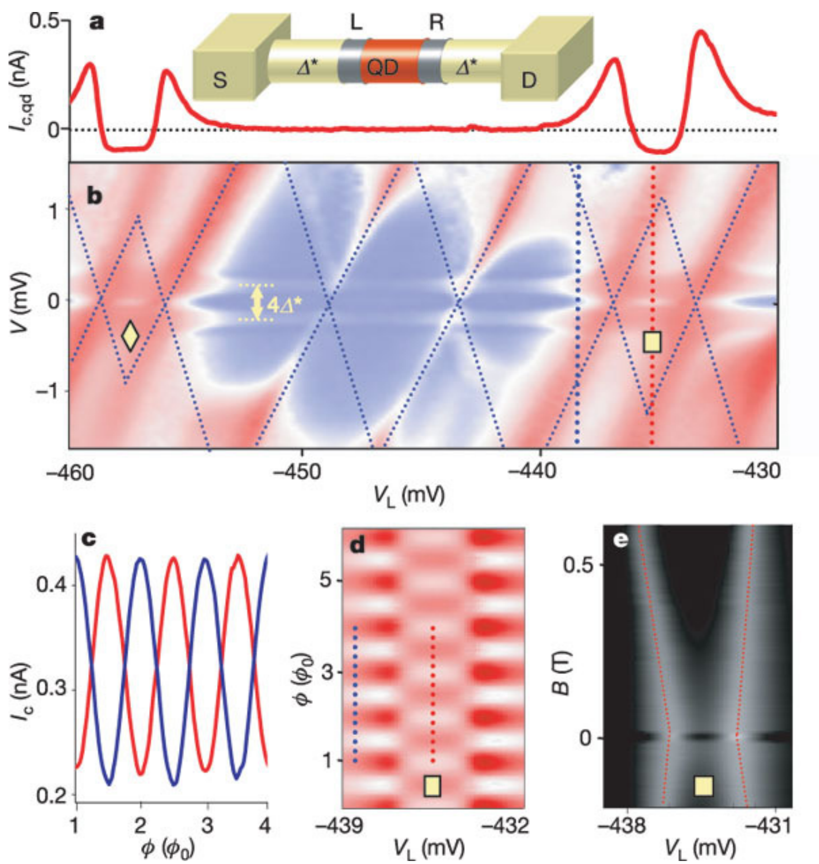




Supercurrent reversal in quantum dots

Jorden A. van Dam[1], Yuli V. Nazarov[1], Erik P. A. M. Bakkers[2], Silvano De Franceschi[1,3] & Leo P. Kouwenhoven[1]

Nature **442**, 667 (2006)

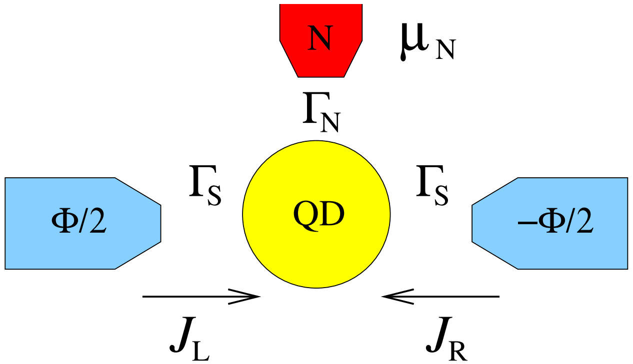


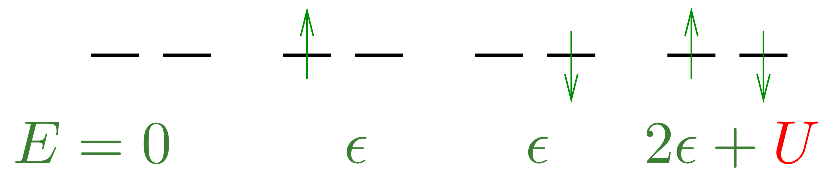
- Josephson coupling due to cotunneling
- $J_{\text{jos}} \propto \Gamma^2$
- π-state for odd occupation
- theory:
  van Dam et al., Nature '06
  Glazman, Matveev, JETP Lett. '89
  Rozhkov, Arovas, Guinea, PRB '01

How to Establish a Josephson Coupling?

- higher-order tunneling (cotunneling)
  - $J_{\text{jos}} \propto \Gamma^2$
- (equilibrium) proximity effect in single-level QD
  - finite pair amplitude: $\langle d_\downarrow d_\uparrow \rangle \neq 0$
  - requirement: $E_0 \approx E_{\uparrow\downarrow} < E_\uparrow = E_\downarrow \Rightarrow 0 < \epsilon \approx -U/2$
  - impossible for large charging energy, $U \gg k_\mathrm{B} T, \Gamma$
- nonequilibrium proximity effect in single-level QD
  - finite pair amplitude: $\langle d_\downarrow d_\uparrow \rangle \neq 0$
  - third, normal electrode drives dot out of equilibrium

Model: Anderson Hamiltonian





- quantum dot: $H_\mathrm{D} = \sum_\sigma \epsilon d^\dagger_\sigma d_\sigma + U n_\uparrow n_\downarrow$
- leads: $H_\eta = \sum_{k\sigma} \epsilon_k c^\dagger_{\eta k\sigma} c_{\eta k\sigma} - \sum_k \left(\Delta_\eta c^\dagger_{\eta k\uparrow} c^\dagger_{\eta -k\downarrow} + \mathrm{H.c.} \right)$
  - superconducting leads: phase biased with $\pm i\Phi/2$
  - normal lead: voltage biased with μ_N
- tunneling: $H_{\mathrm{tunn},\eta} = V_\eta \sum_{k\sigma} \left(c^\dagger_{\eta k\sigma} d_\sigma + \mathrm{H.c.} \right)$

$\Rightarrow$ charging energy, superconductivity, and nonequilibrium

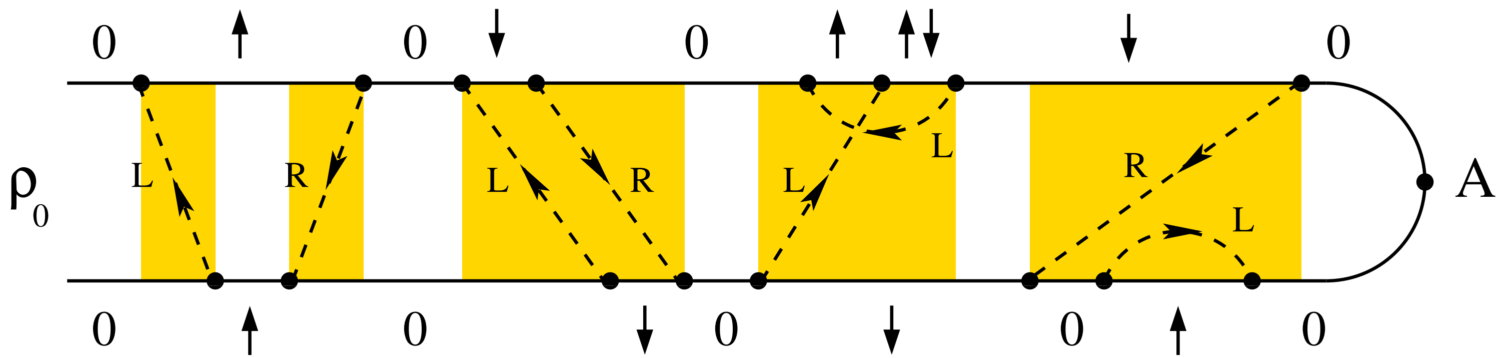
Diagrammatic Transport Theory

Pala, Governale, J.K., NJP '07; Governale, Pala, J.K., PRB '08

general idea: reduced density matrix for dot

– integrate out leads: contractions $\langle c^\dagger_{\eta k\sigma} c_{\eta k\sigma} \rangle$ and $\langle c_{\eta k\sigma} c^\dagger_{\eta k\sigma} \rangle$

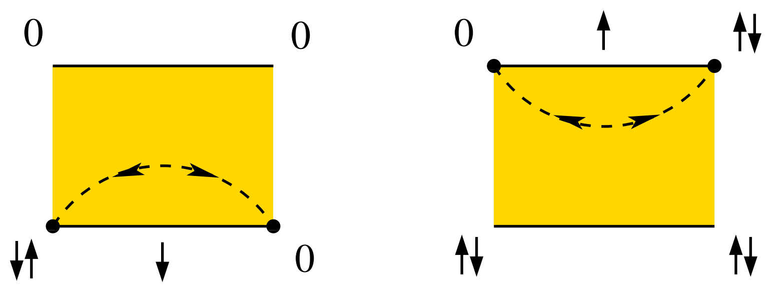
– expand in tunnel coupling, treat interaction exactly



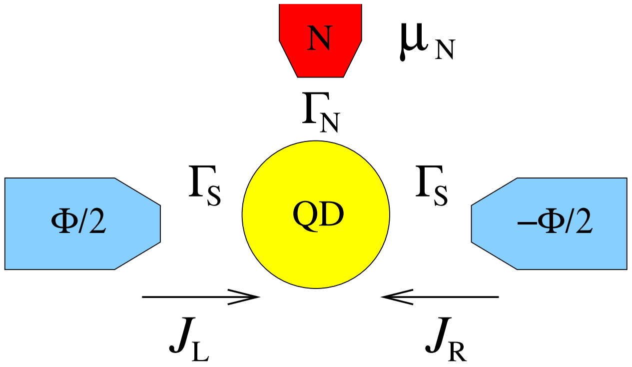
changes due to superconductivity:

– anomalous contractions: $\langle c^\dagger_{\eta k\sigma} c^\dagger_{\eta -k -\sigma} \rangle$ and $\langle c_{\eta -k -\sigma} c_{\eta k\sigma} \rangle$

– finite pair amplitude on dot: $\langle d_\downarrow d_\uparrow \rangle \neq 0$



2S-dot-N in Nonequilibrium



Governale, Pala, J.K., PRB ’08

- $|\Delta| \to \infty$: all orders in Γ_{S}
- first order in Γ_{N}

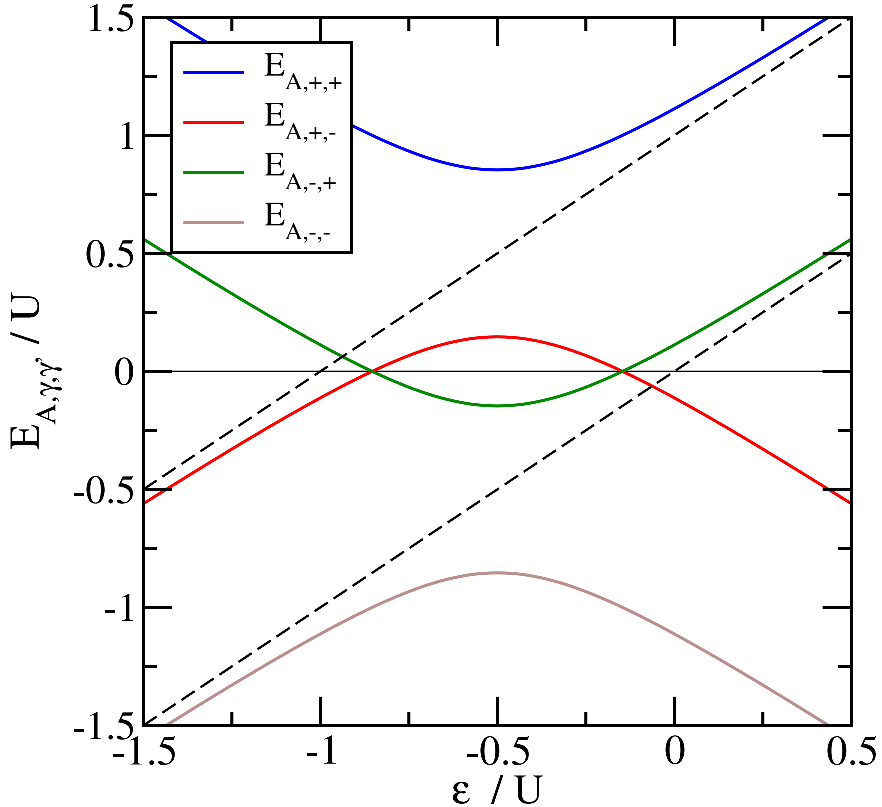
isospin representation

$$I_{\mathrm{x}} = \mathrm{Re}\langle d_\downarrow d_\uparrow \rangle; \quad I_{\mathrm{y}} = \mathrm{Im}\langle d_\downarrow d_\uparrow \rangle; \quad I_{\mathrm{z}} = \frac{\langle d_\uparrow^\dagger d_\uparrow + d_\downarrow^\dagger d_\downarrow \rangle - 1}{2}$$

- kinetic equation for isospin: $\frac{d\mathbf{I}}{dt} = \mathbf{A} - \mathbf{R} \cdot \mathbf{I} + \mathbf{I} \times \mathbf{B}$
- how to proximize the dot:
  - combined S-dot-N Andreev reflection: $\mathbf{A}^{(1)}$
  - Andreev reflection at S-dot interface: $\mathbf{I} \times \mathbf{B}^{(0)}$

Andreev bound states

- without SC: resonances at ϵ and $\epsilon + U$
- with SC: Andreev bound-state energies (for $\Delta \to \infty$):
  $E_{\mathsf{A},\gamma',\gamma} = \gamma' \frac{U}{2} + \gamma \sqrt{(\epsilon + U/2)^2 + \Gamma_{\mathsf{S}}^2 \cos^2(\Phi/2)}$ with $\gamma, \gamma' = \pm$



- four resonances
- avoided crossing
  - at $\delta = 2\epsilon + U = 0$
  - with gap $2\Gamma_{\mathsf{S}} |\cos(\Phi/2)|$

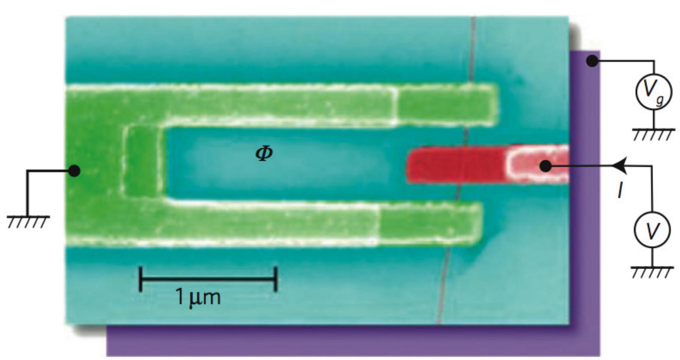
nature physics

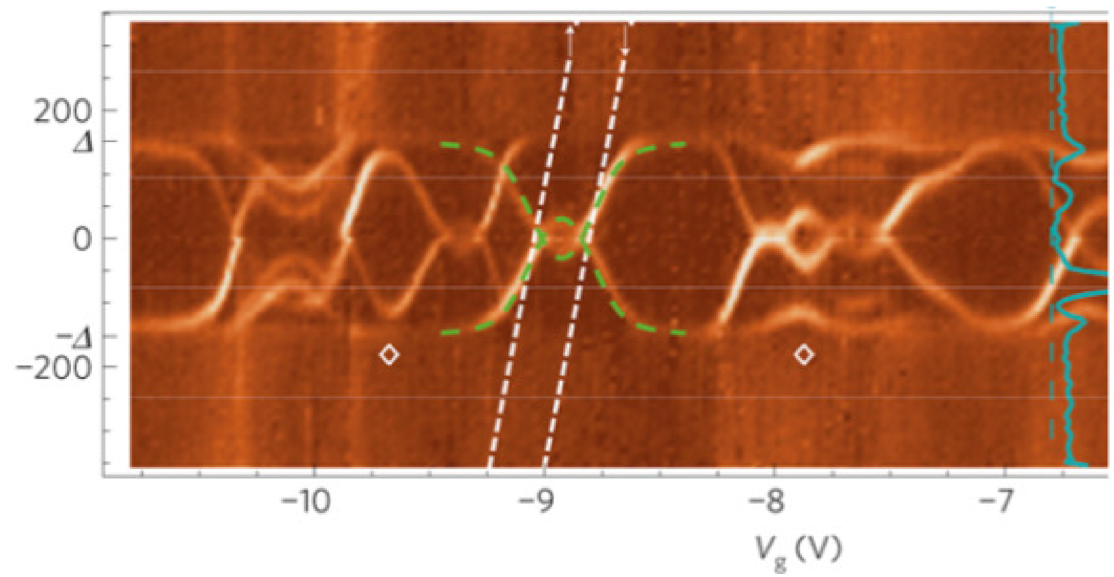
LETTERS

PUBLISHED ONLINE: 14 NOVEMBER 2010 | DOI: 10.1038/NPHYS1811

Andreev bound states in supercurrent-carrying carbon nanotubes revealed

J-D. Pillet[1], C. H. L. Quay[1†], P. Morfin[2], C. Bena[3,4], A. Levy Yeyati[5] and P. Joyez[1★]



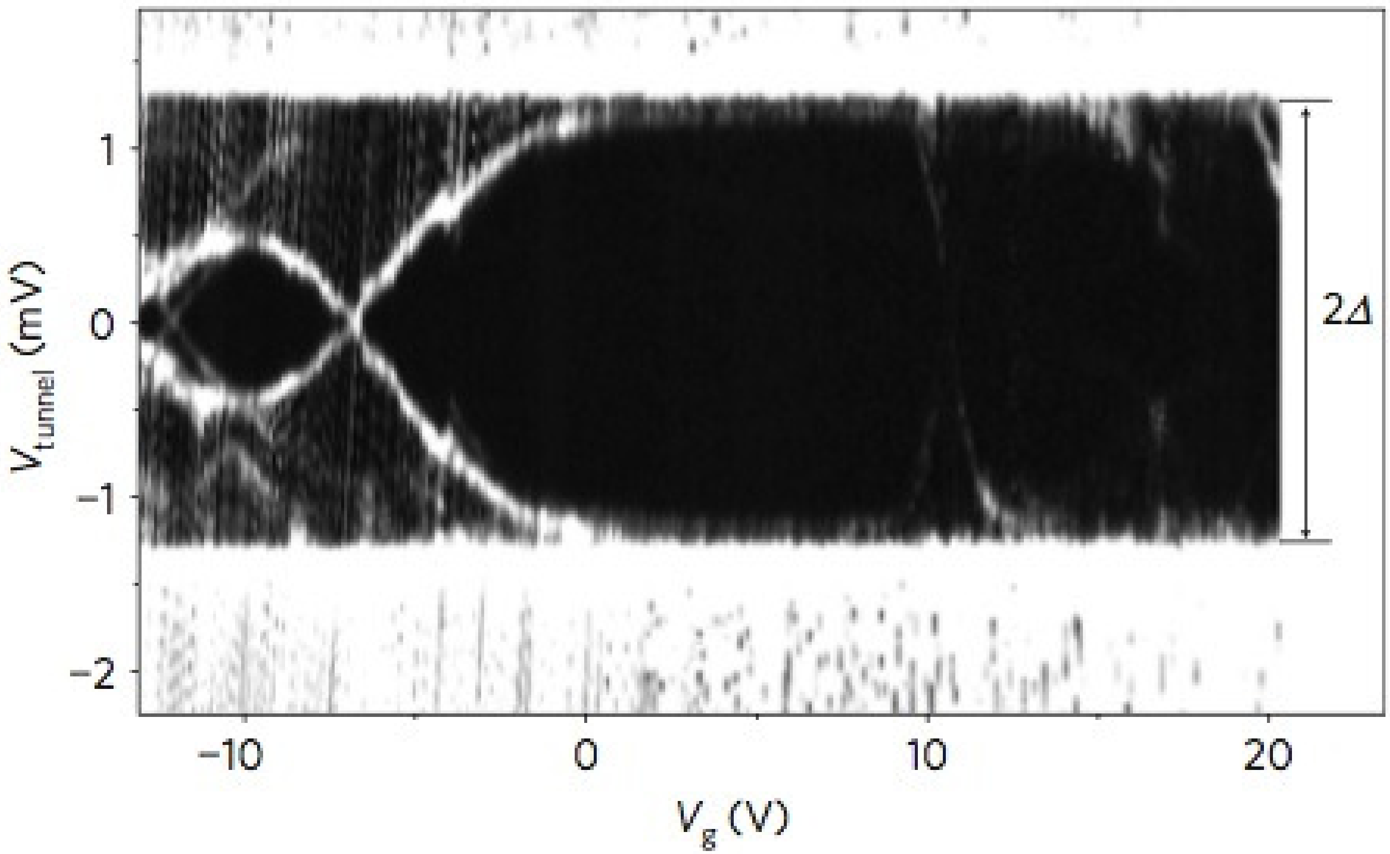


LETTERS
PUBLISHED ONLINE: 6 FEBRUARY 2011 | DOI: 10.1038/NPHYS1911

nature physics

Transport through Andreev bound states in a graphene quantum dot

Travis Dirks, Taylor L. Hughes, Siddhartha Lal†, Bruno Uchoa, Yung-Fu Chen, Cesar Chialvo, Paul M. Goldbart† and Nadya Mason*

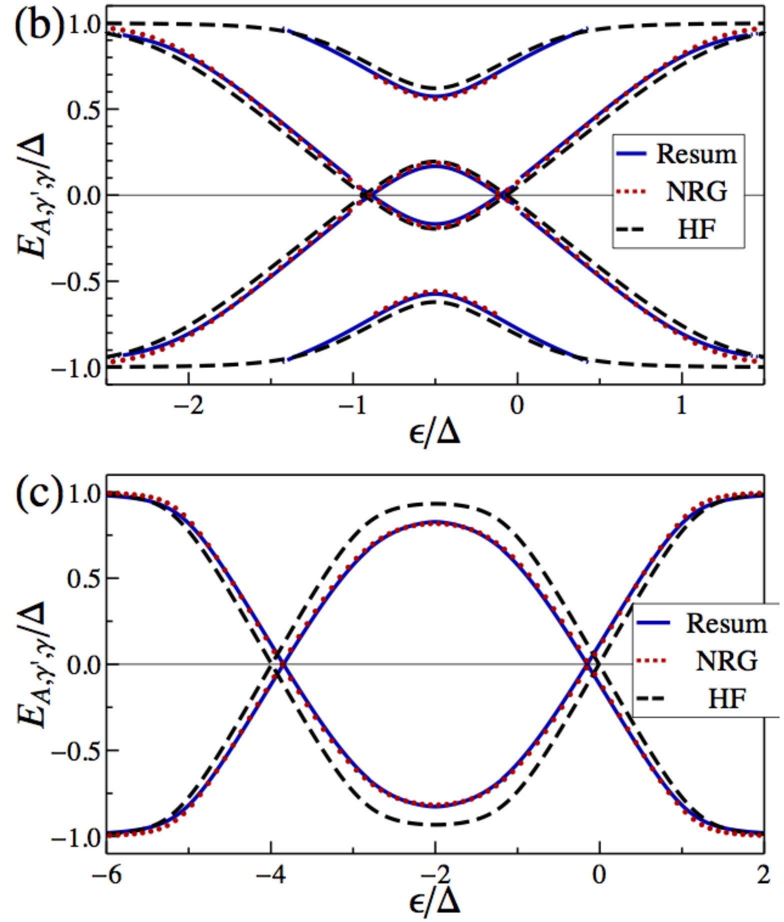


Andreev Bound States for Finite Δ

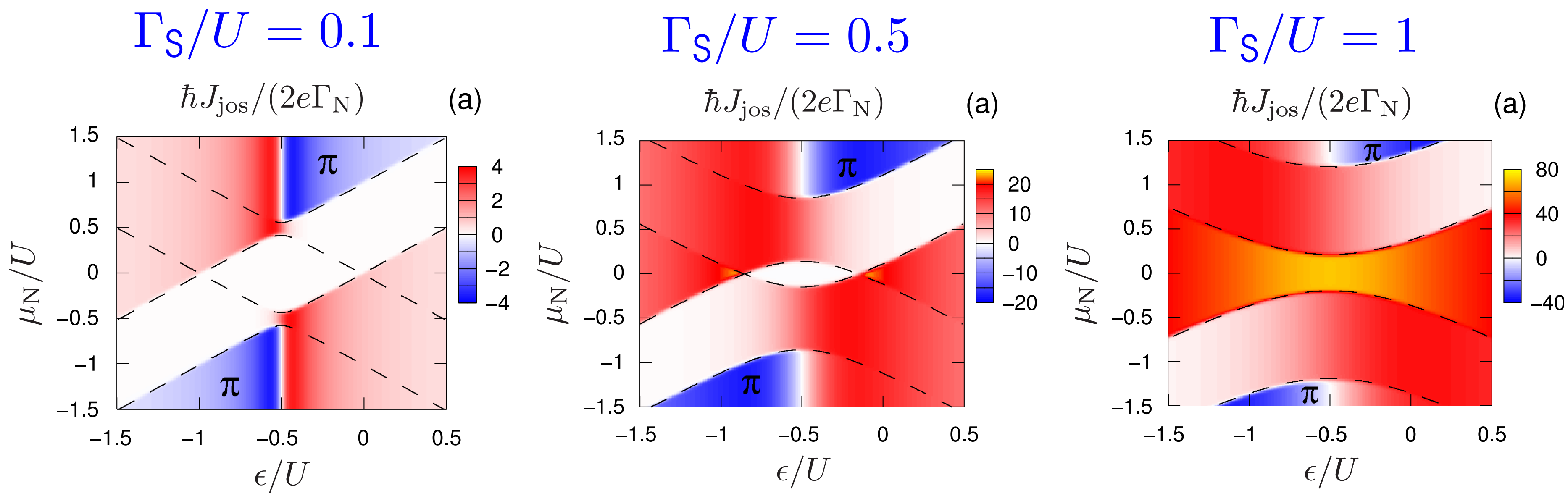
Futterer, Swiebodzinski, Governale, J.K., PRB '13

- resummation scheme for finite Δ
- exact for $\Delta \to \infty$
- very nice agreement with NRG

  Martin Rodero, Levy Yeyati, JPCM '12
- much better than Hartree Fock



Josephson Current $J_L - J_R$

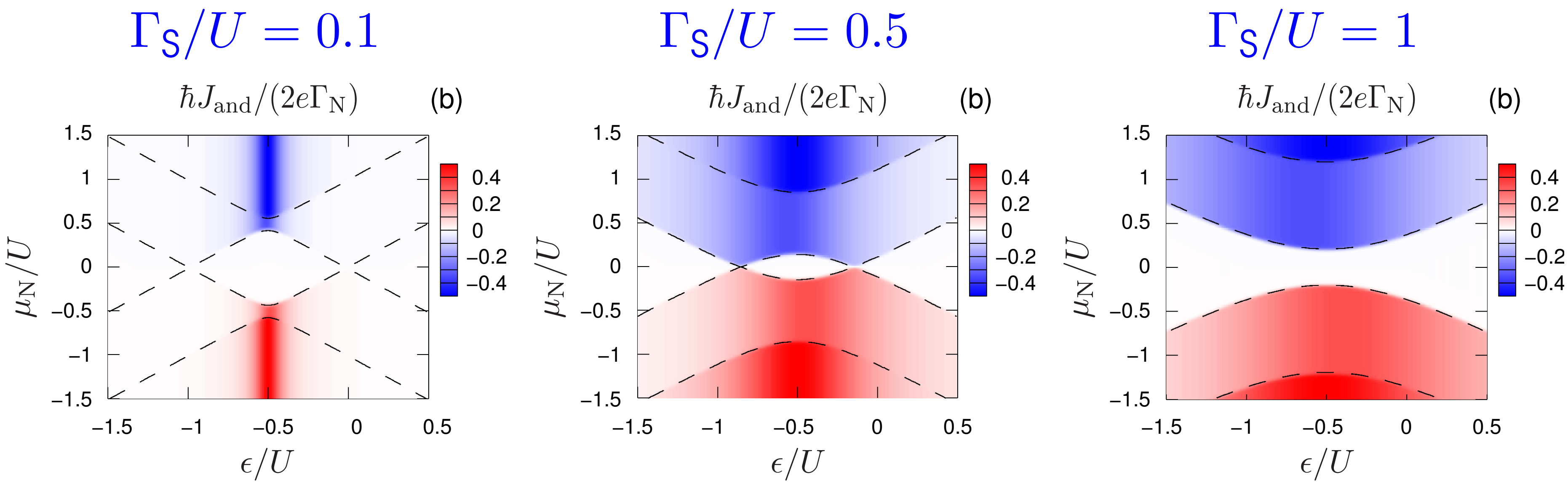


- equilibrium Josephson current suppressed by charging
- nonequilibrium Josephson current for finite $\mu_{\rm N}$
- π-transition driven by ϵ or $\mu_{\rm N}$
- Andreev bound-state energies:

$$E_{\mathrm{A},\gamma',\gamma} = \gamma' \frac{U}{2} + \gamma \sqrt{(\epsilon + U/2)^2 + \Gamma_{\mathrm{S}}^2 \cos^2(\Phi/2)} \text{ with } \gamma, \gamma' = \pm$$

Governale, Pala, J.K., PRB '08

Andreev Current $J_L + J_R$



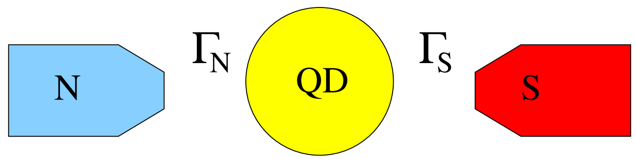
- Andreev current probes nonequilibrium proximity effect
- maximal around $\varepsilon = -U/2$
- Andreev bound-state energies:

  $E_{\mathrm{A},\gamma',\gamma} = \gamma' \frac{U}{2} + \gamma \sqrt{(\epsilon + U/2)^2 + \Gamma_{\mathrm{S}}^2 \cos^2(\Phi/2)}$ with $\gamma, \gamma' = \pm$

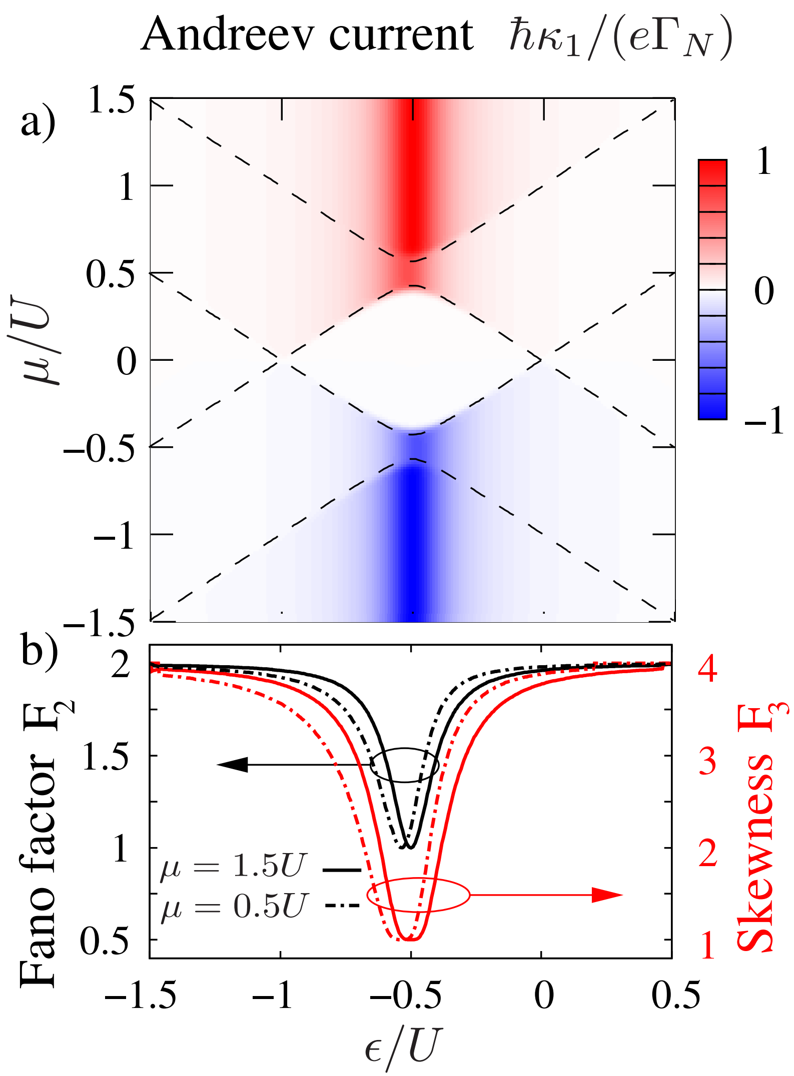
Governale, Pala, J.K., PRB '08

Shot Noise Reveals Proximity Effect

Braggio, Governale, Pala, J.K., SSC '11

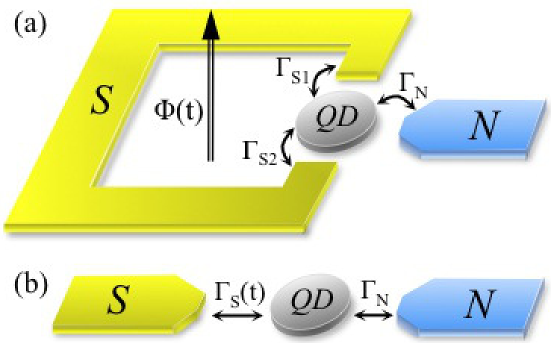


- off resonance ($|\delta| \gg \Gamma_S$)
  - Cooper pair cotunneling
  - Poissonian transfer of $2e$
  - Fano factor $F = 2$
- on resonance ($|\delta| \ll \Gamma_S$)
  - Cooper pair oscillations, interrupted by SET
  - Poissonian transfer of e
  - Fano factor $F = 1$



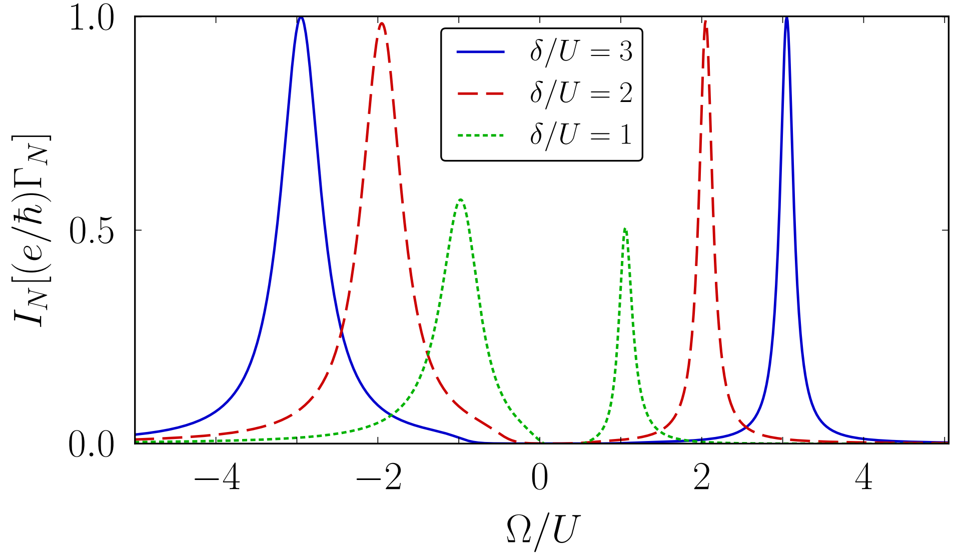
Proximity by AC Driving

Moghaddam, Governale, J.K., PRB '12



- off resonant: $|\delta| \gg \Gamma_S \Rightarrow |I_z| = \frac{1}{2}$
- no bias voltage: $\mu = 0$
- ac driving: $\Gamma_S(t) = \Gamma_S + \delta\Gamma_S \cos \Omega t$
  $\Rightarrow \mathbf{B}(t)$ acts on $\mathbf{I} \Rightarrow$ Rabi oscillations

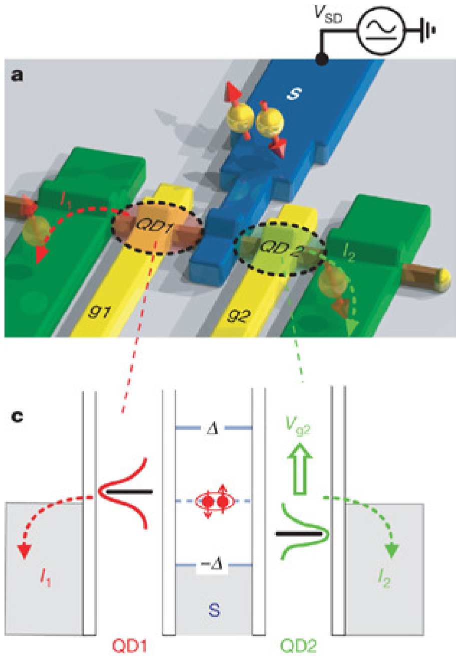
- DC current into N lead
- resonance at $\Omega = \delta$

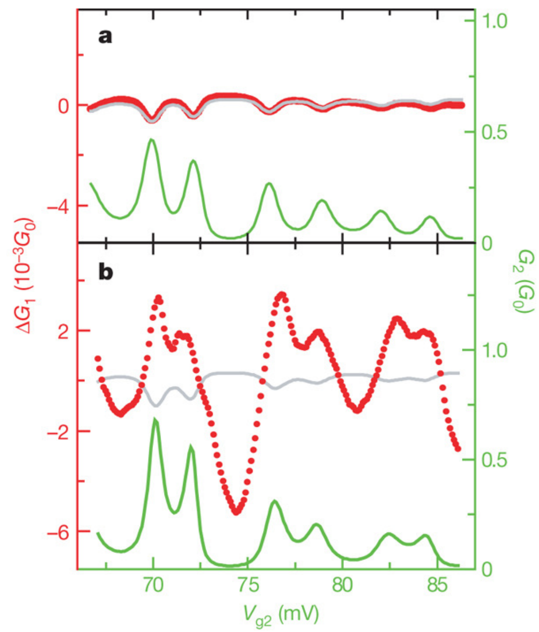


Cooper Pair Splitter with Double Dots

- double dot in InAs quantum wire

  Hofstetter, Csonka, Nygard, Schönenberger, Nature '09





  Das, Ronen, Heiblum, Mahalu, Kretinin, Shtrikman, Nat. Comm. '12

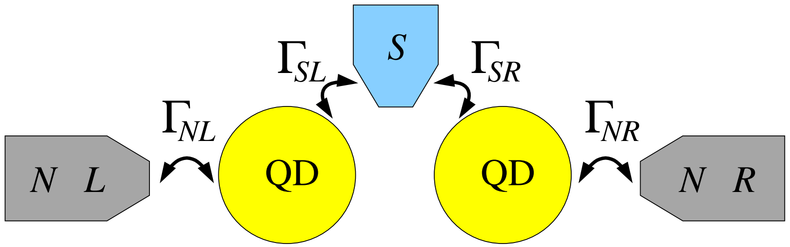
- double dot in CNT

  Hermann, Portier, Levy Yeyati, Kontos, Strunk, PRL '10

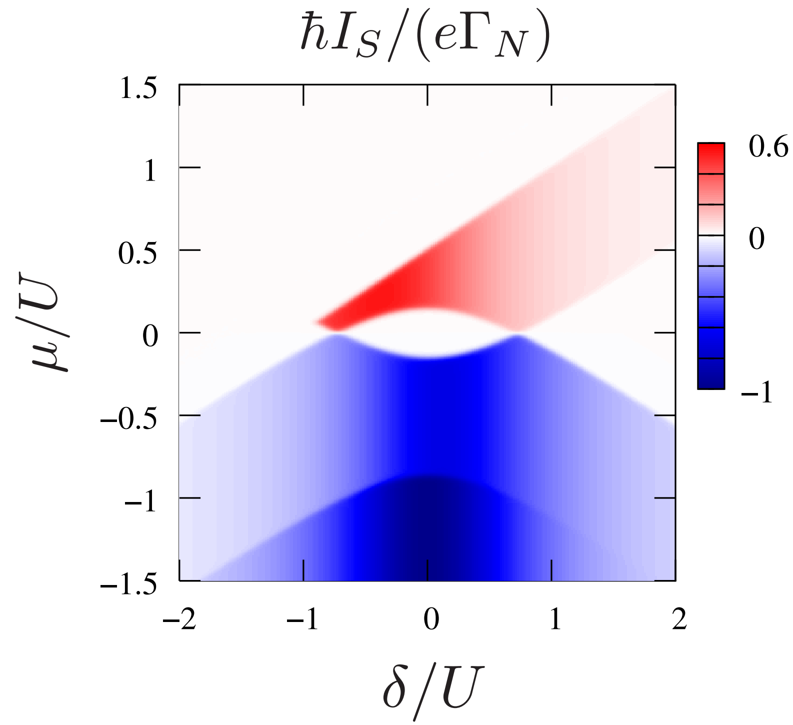
$\rightarrow$ spin-entangled electron pair in normal leads

Nonlocal Proximity Effect in Double Dots

Eldridge, Pala, Governale, J.K., PRB '10



- non-local pair amplitude:
  $\langle d_{\mathrm{L}\uparrow} d_{\mathrm{R}\downarrow} - d_{\mathrm{L}\downarrow} d_{\mathrm{R}\uparrow} \rangle \neq 0$
- negative bias $\mu_{\mathrm{S}} < 0$:
  - transport through singlet
  - Cooper pair splitter
- positive bias $\mu_{\mathrm{S}} > 0$:
  - spin triplet occupied
  - triplet blockade

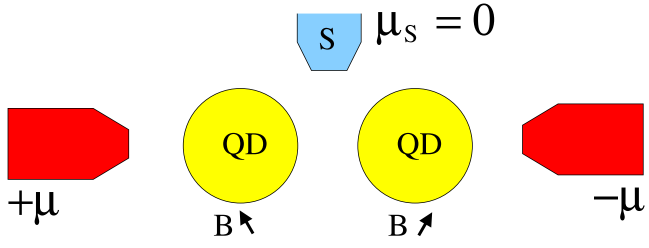


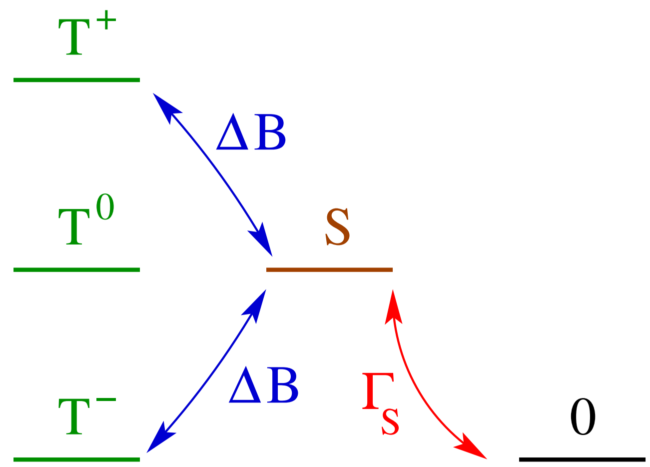
Unconventional Superconductivity

- pair amplitudes: $F_{i\sigma,i'\sigma'}(t) = \langle T c_{i\sigma}(t) c_{i'\sigma'}(0) \rangle$
- symmetry: under exchange of spin/orbital/time
  - even-singlet: odd/even/even, BCS
  - even-triplet: even/odd/even, ^{3}He, Sr_2RuO_4
  - odd-triplet: even/even/odd, FM-SC hybids
  - odd-singlet: odd/odd/odd, ?
- even/odd - singlet/triplet pairing in double dots?

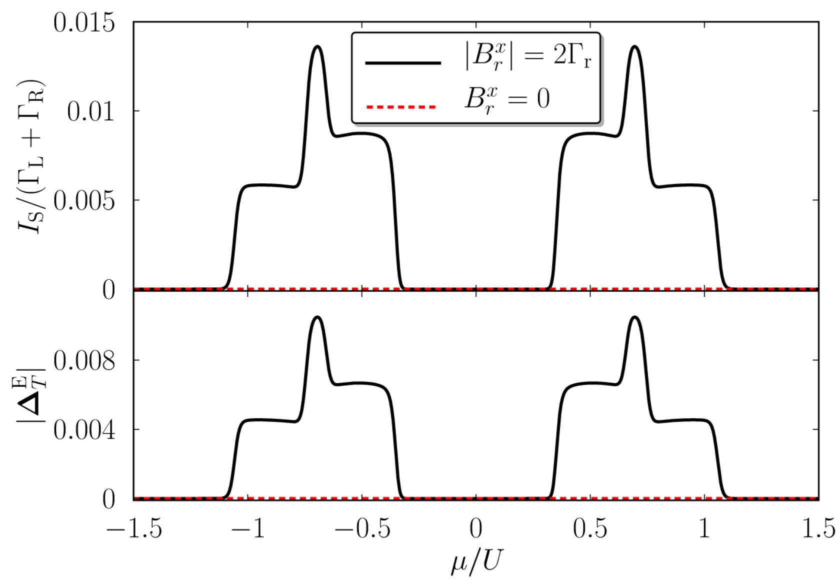
Unconventional SC in Double Dots

Weiss, Sothmann, Governale, J.K., in preparation





- large B field: T^- occupied
- supercond. lead: $0 \leftrightarrow S$
- inhom. B field: $S \leftrightarrow T^0$
- noncol. B field: $T^0 \leftrightarrow T^-$



Superconducting Proximity Effect in . . .

- quantum dot + superconductor
  - 0-π-transitions for Josephson current
  - local and non-local Andreev transport
- double dot + superconductor
  - non-local Cooper pairs
  - triplet blockade
- double dot + superconductor + magnetic field
  - non-local unconventional pairing

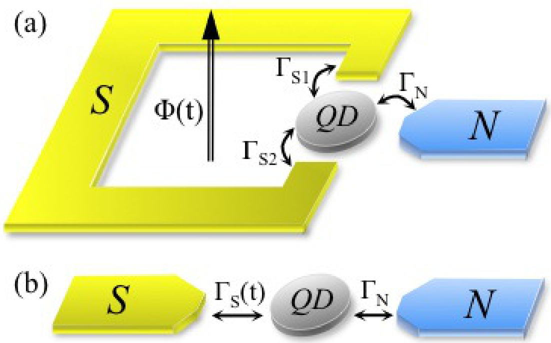
Collaborators

Alessandro Braggio	Uni Genova, Italy
James Eldridge	VU Wellington, New Zealand
Rosario Fazio	SNS Pisa, Italy
David Futterer	Uni Duisburg-Essen, Germany
Michele Governale	VU Wellington, New Zealand
Bastian Hiltscher	Uni Duisburg-Essen, Germany
Ali Moghaddam	Uni Duisburg-Essen, Germany
Marco Pala	CNRS Grenoble, France
Björn Sothmann	Uni Geneva, Switzerland
Janine Splettstoesser	RWTH Aachen, Germany
Jacek Swiebodzinski	Uni Duisburg-Essen, Germany
Fabio Taddei	SNS Pisa, Italy
Stephan Weiß	Uni Duisburg-Essen, Germany

extra slides

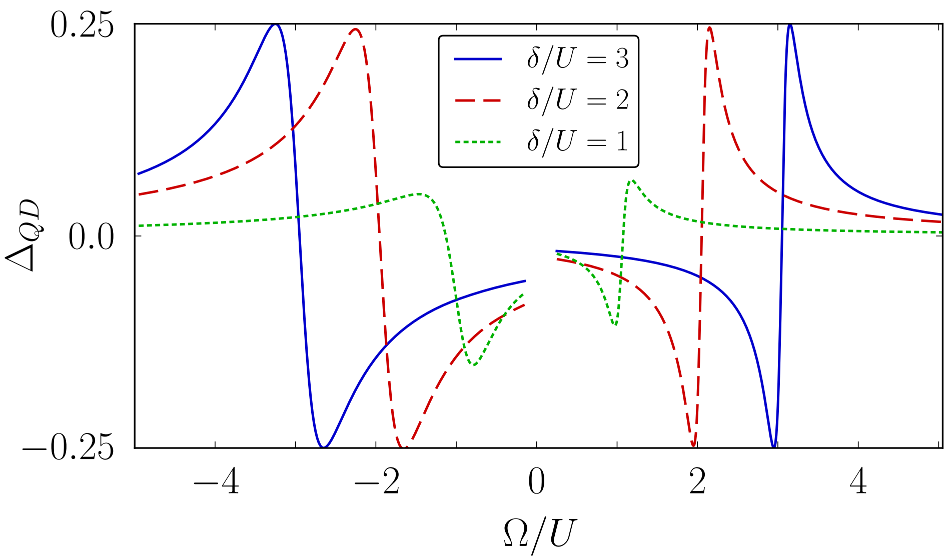
Proximity by AC Driving

Moghaddam, Governale, J.K., PRB '12



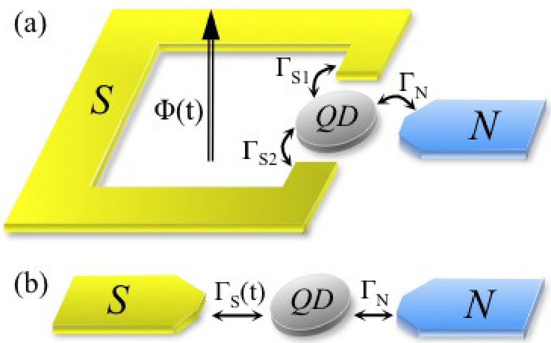
- off resonant: $|\delta| \gg \Gamma_{\rm S} \Rightarrow |I_{\rm z}| = \frac{1}{2}$
- no bias voltage: $\mu = 0$
- ac driving: $\Gamma_{\rm S}(t) = \Gamma_{\rm S} + \delta\Gamma_{\rm S} \cos \Omega t$
  $\Rightarrow \mathbf{B}(t)$ acts on $\mathbf{I} \Rightarrow$ Rabi oscillations

- ac-induced proximity
- resonances at $\Omega = \delta \pm \Gamma_{\rm S}$



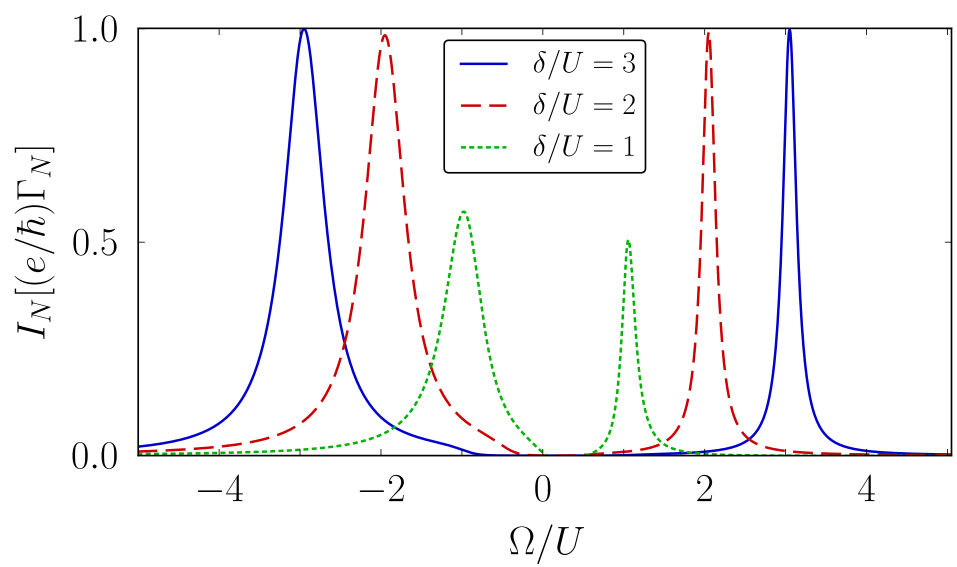
Proximity by AC Driving

Moghaddam, Governale, J.K., PRB '12



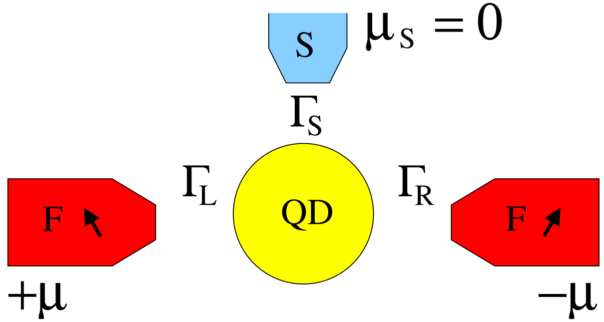
- off resonant: $|\delta| \gg \Gamma_S \Rightarrow |I_z| = \frac{1}{2}$
- no bias voltage: $\mu = 0$
- ac driving: $\Gamma_S(t) = \Gamma_S + \delta\Gamma_S \cos\Omega t$
  $\Rightarrow \mathbf{B}(t)$ acts on $\mathbf{I} \Rightarrow$ Rabi oscillations

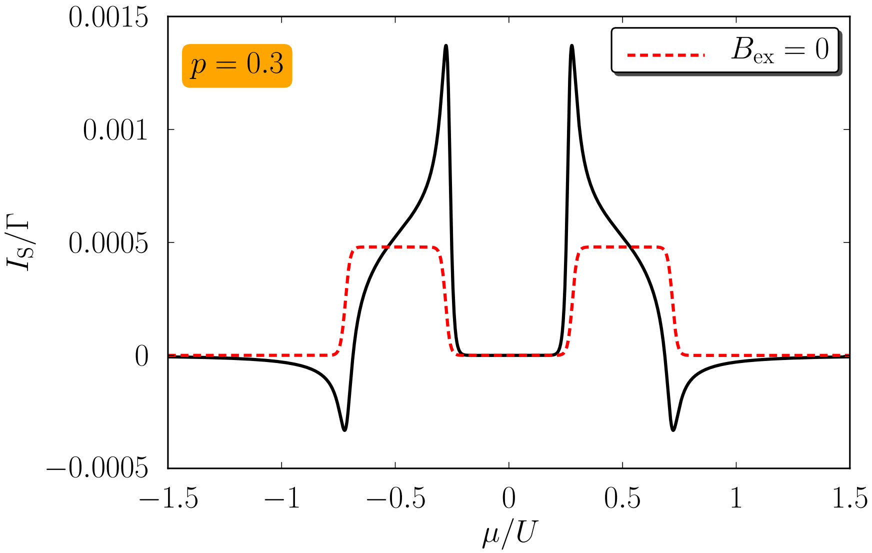
- DC current into N lead
- resonance at $\Omega = \delta$



Quantum Dot + FM + SC

Sothmann, Futterer, Governale, J.K., PRB '10





- non-collinear fm leads
- spin accumulation on QD
- fm leads $\rightarrow$ exchange field
- current into superconductor (symmetric in μ)
  - very sensitive to exchange field
  - indication of unconventional pairing