Manipulation of Dirac cones in artificial graphenes

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- Berry phase

Berry phase

Graphene electronic spectrum

$\pi$

$-\pi$
Manipulation of Dirac cones in artificial graphenes

« Life and death of Dirac points »

J.-N. Fuchs, M. Goerbig, F. Piéchon
P. Dietl, P. Delplace, R. De Gail (PhDs)
Lih-King Lim (post-doc)

Quy-Nhon avenues
Outline

Motion and merging of Dirac points

Modified graphene as a toy model

A universal Hamiltonian, spectrum at the merging

Physical realizations:

1) Microwaves in a honeycomb lattice of dielectric discs
   Mortessagne’s group, Nice (2012)

2) Graphene-like lattice of cold atoms in an optical lattice
   Esslinger’s group, ETH (2012)

   Landau-Zener tunneling as a probe of Dirac points

Conclusion and other artificial graphenes
Graphene

\[
H = \begin{pmatrix}
A & B \\
0 & f(\vec{k}) \\
f^*(\vec{k}) & 0
\end{pmatrix}
\]

\[
f(\vec{k}) = -t \left(1 + e^{i\vec{k} \cdot \vec{a}_1} + e^{i\vec{k} \cdot \vec{a}_2}\right)
\]

\[
\varepsilon(\vec{k}) = \pm \left|f(\vec{k})\right|
\]
Graphene

\[ H = \begin{pmatrix} A & B \\ 0 & f(\vec{k}) \\ f^*(\vec{k}) & 0 \end{pmatrix} \]

\[ f(\vec{k}) = -t \left( 1 + e^{i\vec{k} \cdot \vec{a}_1} + e^{i\vec{k} \cdot \vec{a}_2} \right) \]

\[ \varepsilon(\vec{k}) = \pm |f(\vec{k})| \]

DOS
Tight-binding problem on honeycomb lattice

\[ E = \pm |t' + te^{i\vec{k} \cdot \vec{a}_1} + te^{i\vec{k} \cdot \vec{a}_2}| = 0 \]

\[ \vec{K} \cdot \vec{a}_1 = -\vec{K} \cdot \vec{a}_2 = \pm \frac{2\pi}{3} \]
Motion and merging of Dirac points

\[ t' = t \]

\[ t' = 1.5t \]

\[ t' = 2t \]

\[ t' = 2.3t \]

"hybrid" “semi-Dirac”

\[ 2q_D = \frac{4}{3} \arctan \sqrt{\frac{4t^2}{t'^2} - 1} \]

\[ c_y = \sqrt{3 \left( t^2 - \frac{t'^2}{4} \right)} \]

\[ c_x = \frac{3}{2} t' \]

\[ c_y \to 0 \quad \text{for} \quad t' = 2t \]

\[ m^* = \frac{2}{3t} \]
Hybrid 2D electron gas: a new dispersion relation

\[ t' = 2t \]

\[ \varepsilon = \pm \sqrt{\frac{q_y^4}{4m^2} + c^2 q_x^2} \]

\[ \varepsilon \propto \pm \left[ (n + 1/2) eB \right]^{2/3} \]

Berry phase

Two component wavefunction

\[ |u_k\rangle = \frac{1}{\sqrt{2}} \left( \frac{1}{\pm e^{i\phi(k)}} \right) e^{ik \cdot r} \]

\[ \phi_B = i \oint_C \langle u_k | \vec{\nabla}_{\vec{k}} u_{\vec{k}} \rangle \cdot d\vec{k} = \frac{1}{2} \oint_C \nabla_{\vec{k}} \phi_{\vec{k}} \cdot d\vec{k} = \pm \pi \]

Topological transition
General description of the motion of Dirac points (with time reversal symmetry)

\[ H = -\begin{pmatrix} 0 & f(k) \\ f^*(k) & 0 \end{pmatrix} \]

\[ f(k) = \sum_{m,n} t_{mn} e^{-i\vec{k} \cdot \vec{R}_{mn}} \]

When \( t_{mn} \) changes, \( \vec{D} \) and \(-\vec{D}\) move

Where is the merging point?

\( \vec{D} = -\vec{D} \)

4 possible positions in \( \vec{k} \) space

Expansion near \( \vec{D}_0 \)

\[ f(\vec{D}_0 + \vec{q}) = -icq_x + \frac{q_y^2}{2m^*} \]
At the merging transition:

\[
H(q) = \begin{pmatrix}
0 & -icq_x + \frac{q_y^2}{2m^*} \\
-icq_x + \frac{q_y^2}{2m^*} & 0
\end{pmatrix}
\]

Near the transition:

\[
H(q) = \begin{pmatrix}
0 & \Delta_* - icq_x + \frac{q_y^2}{2m^*} \\
\Delta_* + icq_x + \frac{q_y^2}{2m^*} & 0
\end{pmatrix}
\]

\[
\Delta_* = \sum_{m,n} t_{mn} (-1)^{\beta_{mn}}
\]

\[
c_x = \sum_{m,n} t_{mn} \vec{R}_{mn} (-1)^{\beta_{mn}}
\]

\[
\frac{1}{m^*} = \sum_{m,n} t_{mn} R_{mn}^2 (-1)^{1+\beta_{mn}}
\]

\[
(\Delta_* = t' - 2t)
\]
This Hamiltonian describes the topological transition, the coupling between valleys and the merging of the Dirac points.
First summary: Manipulation of Dirac points and merging

By varying band parameters, it is possible to manipulate the Dirac points. They can move in k-space and they can even merge.

The merging transition is a topological transition: 2 Dirac points evolve into a single hybrid « semi-Dirac » point and eventually a gap opens and the Fermi surface disappears.

Universal description of motion and merging of Dirac points.

Physical realizations of the merging transition

* Strained graphene

\[ t' = 2t \]

strain \( \approx 23\% \)
merging is unreachable

Pereira, Castro Neto, Peres, PRB 2009
See also Goerbig, Fuchs, Piéchon, G.M., PRB 2008

* Microwaves

* Ultracold atoms in optical lattices
Referee A
The paper is well written and accessible to a broad audience.
The results might be applicable to optical lattice systems.
I would recommend publishing the paper in PRL as it is.

Referee B
It is a nice simple toy model...
The authors propose that merging of Dirac points might be possible with cold atoms in optical lattices.
I think that it is a very long shot, given that ... the systems are yet to be realized experimentally.
I do not recommend publication of this paper in PRL.

Referee C
While the physical system is certainly interesting, its relevance to current experiments is rather tenuous...

Merging of Dirac points in a 2D crystal

Topological transition of Dirac points in a microwave experiment
M. Bellec et al. (2012)

Creating moving and merging Dirac points with a Fermi gas in a tunable honeycomb lattice,
Graphene physics with microwaves!

M. Bellec, U. Kuhl, F. Mortessagne (NICE)

Honeycomb lattice of dielectric resonators
Evanescent propagation between the dots -> Tight-binding description
Measure of the reflexion coefficient \( \rightarrow \) LDOS

\[ t_2 / t = 0.091 \quad t_3 / t = 0.071 \]
The merging transition seen in the microwave experiment

Topological Transition of Dirac Points in a Microwave Experiment

Matthieu Bellec, Ulrich Kuhl, Gilles Montambaux, and Fabrice Mortessagne

Uniaxial strain → increase $t'$

$t'_{\text{crit.}} = 2t - 3t_3$
The merging transition seen in the microwave experiment

Topological Transition of Dirac Points in a Microwave Experiment

Matthieu Bellec,¹ Ulrich Kuhl,¹ Gilles Montambaux,² and Fabrice Mortessagne¹,²*
The merging transition seen in the microwave experiment

Topological Transition of Dirac Points in a Microwave Experiment

Matthieu Bellec,1 Ulrich Kuhl,1 Gilles Montambaux,2 and Fabrice Mortessagne1,2

The merging transition is observed in the microwave experiment. For different values of the ratio \( t'/t \):

- \( t' = t \) (Armchair edges)
- \( t' = 1.8t \)
- \( t' = 3.5t \)

The theoretical critical ratio \( t'_{\text{crit.}} = 2t - 3t_3 \) connects the experimental (exp.) and theoretical (th.) results.
Probing the edge states...

« armchair » have no edge states. But by under strain, new edge states are predicted along certain armchair edges.

P. Delplace, G.M.

PHYSICAL REVIEW B 84, 195452 (2011)

Zak phase and the existence of edge states in graphene
Creating, moving and merging Dirac points with a Fermi gas in a tunable honeycomb lattice

Leticia Tarruell, Daniel Greif, Thomas Uehlinger, Gregor Jotzu and Tilman Esslinger
Institute for Quantum Electronics, ETH Zurich, 8093 Zurich, Switzerland

Nature 483, 302 (2012)

Atoms are trapped in an optical lattice potential and form an artificial crystal

\[ V(x, y) = -V_X \cos^2(kx + \theta/2) - V_X \cos^2(kx) - V_Y \cos^2(ky) - 2\alpha \sqrt{V_X V_Y} \cos(kx) \cos(ky) \cos \varphi \]
Honeycomb

\[ E = \pm \left| te^{ik_a_1} + te^{ik_a_2} + t' \right| \]  

Brick wall

\[ E = \pm \left| te^{i(k_x + k_y)a} + te^{i(k_x - k_y)a} + t' \right| \]  

\[ t' = t \]
Creating, moving and merging Dirac points with a Fermi gas in a tunable honeycomb lattice

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\[ \frac{d\vec{k}}{dt} = \vec{F} \]

\[ 40K \]
How to manipulate and merge Dirac points?

Anisotropy of the optical potential

How to detect and localize Dirac points?

Bloch oscillations + Landau-Zener Tunneling

Measurement of the proportion of atoms in the upper band
Landau-Zener transition

\[ P_Z = e^{-\frac{\pi E_g^2}{4 c||F}} \]

\[ 1 - P_Z \]

\[ \vec{k} = \frac{\vec{F} t}{\hbar} \]
Measured transfered fraction of atoms: directions of motion

ETH experiment

Single Dirac cone

Double Dirac cone

Merging line

Transferred fraction, $\xi$

0.0  0.1  0.2  0.3

$V_x [E_R]$  1.0  0.8  0.6  0.4  0.2  0.0

0  2  3  4  5  6

gapped phase

merging

gapless phase
Bloch-Zener oscillations across a merging transition of Dirac points

Lih King Lim, Jean-Noel Fuchs, G. M., PRL 108, 175303 (2012)
1) **Ab-initio band structure** from optical lattice potential: $V_X, V_{Xb}, V_Y$ (laser intensities).

2) **Tight-binding model** on an anisotropic square lattice: $t, t', t''$ (hopping amplitudes).

3) **Universal hamiltonian** describing the merging transition: $\Delta^*, c_x, m^*$

**Explain the experimental data using Universal Hamiltonian**

1) Relate the parameters of the optical lattice to the parameters of the Universal Hamiltonian.
Anisotropic square lattice $t-t'-t'':$ Dirac points, merging, gapped phase,
Mapping tight-binding model on universal Hamiltonian

\[ \Delta_* = t' - 2t + t'' \quad c_x = t' - t'' \]

\[ m^* = \frac{2}{2t + t' + t''} \]

\[ q_D = \sqrt{-2m^* \Delta_*} = 2\sqrt{\frac{2t - t' - t''}{2t + t' + t''}} \]
2) Compute the inter-band tunneling probability within the Universal Hamiltonian.

**Single Zener tunneling**

**Double Zener tunneling**

\[ P_t = 2P_Z (1 - P_Z) \]
Single Dirac cone: single atom tunneling

Transfer probability as a function of $q_y$ and $\Delta^*$

$$q_y = \pm \sqrt{-2m^*\Delta^*}$$

$$P_Z^x = e^{-\pi \frac{(\frac{q_y^2}{2m^*} + \Delta^*)^2}{c_x F}}$$
Single Dirac cone: Fermi sea tunneling

Transfer probability as a function of $q_y$ and $\Delta^*$

$$P_x^z = \langle P_z^x \rangle = e^{-\pi \frac{(q_y^2 + \Delta^*)^2}{2m^* c_x F}}$$

Maximum slightly inside the Dirac phase
Single Dirac cone: Fermi sea tunneling

Transfer probability for a cloud of size $k_{Fy}$

Maximum slightly inside the Dirac phase
Double Dirac cone: Fermi sea tunneling

\[ P^y_Z = e^{-\pi \frac{c_x^2 q_x^2}{c_y F}} \]

Non-monotonous function of \( P_Z \). Maximum for \( P_Z = 1/2 \)
Double Dirac cone: Fermi sea tunneling

$$P^y_Z = e^{-\pi \frac{c_x^2 q_x^2}{c_y F}}$$

$$\langle P^y_t = 2P^y_Z (1 - P^y_Z) \rangle$$

Non-monotonous function of $P_Z$. Maximum for $P_z = 1/2$

$\Delta_* > 0$

Gapped phase

$\Delta_* < 0$

Dirac phase

$P^y_Z \sim 0$

$P^y_Z \sim 1/2$

$P^y_Z \sim 1$
Summary

Single Dirac cone

Double Dirac cone

Experiment

Theory
Conclusions and perspectives

Universal description of motion and merging of Dirac points in 2D crystals

\[ (-\pi, \pi) \text{ merging : hybrid semi-Dirac spectrum} \]

Cold atoms : Landau-Zener probe of the Dirac points

Interference effects

Condensed matter : New thermodynamic and transport properties

Interaction effects : from Dirac to Schrödinger

Other universality class : \((\pi, \pi) \text{ merging}\)