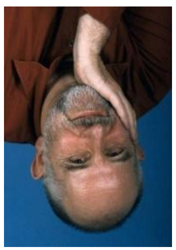
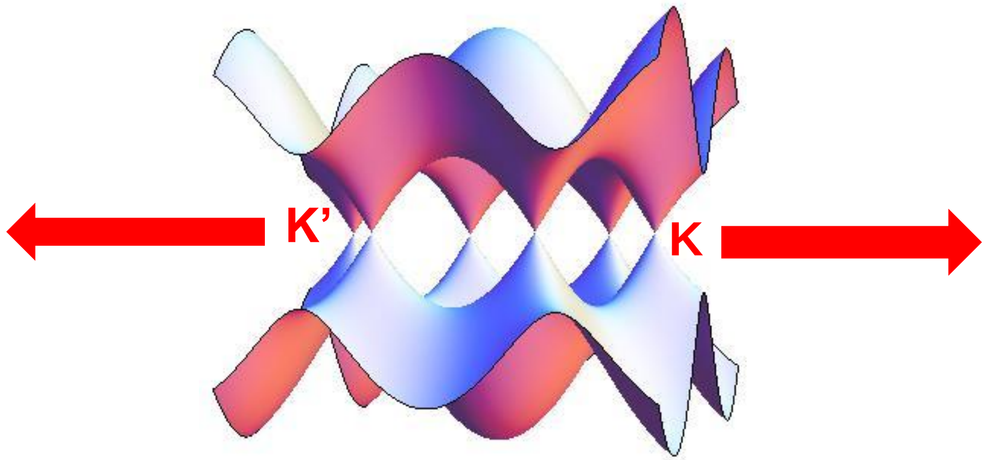
Manipulation of Dirac cones in artificial graphenes

Gilles Montambaux
Laboratoire de Physique des Solides, Orsay
CNRS, Université Paris-Sud, France

- Berry phase

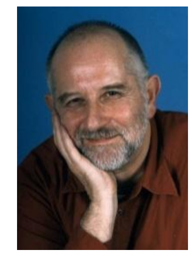


$-\pi$



Graphene electronic spectrum

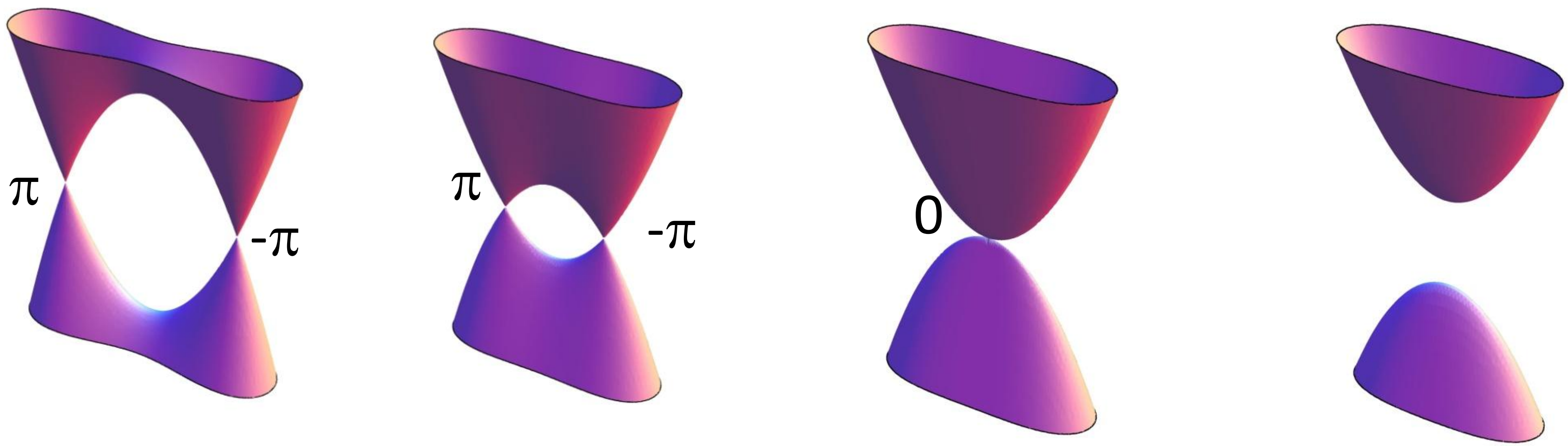
Berry phase



π

Manipulation of Dirac cones in artificial graphenes

« Life and death of Dirac points »



⇨ « Artificial » graphenes

Quy-Nhon avenues

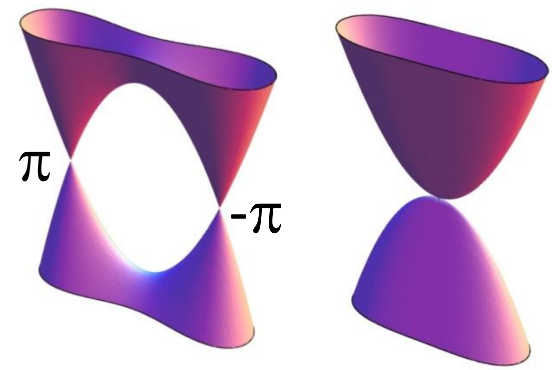
J.-N. Fuchs, M. Goerbig, F. Piéchon
P. Dietl, P. Delplace, R. De Gail (PhDs)
Lih-King Lim (post-doc)

Outline

Motion and merging of Dirac points

Modified graphene as a toy model

A universal Hamiltonian, spectrum at the merging



Physical realizations :

1) Microwaves in a honeycomb lattice of dielectric discs
Mortessagne's group, Nice (2012)

2) Graphene-like lattice of cold atoms in an optical lattice
Esslinger's group, ETH (2012)

Landau-Zener tunneling as a probe of Dirac points

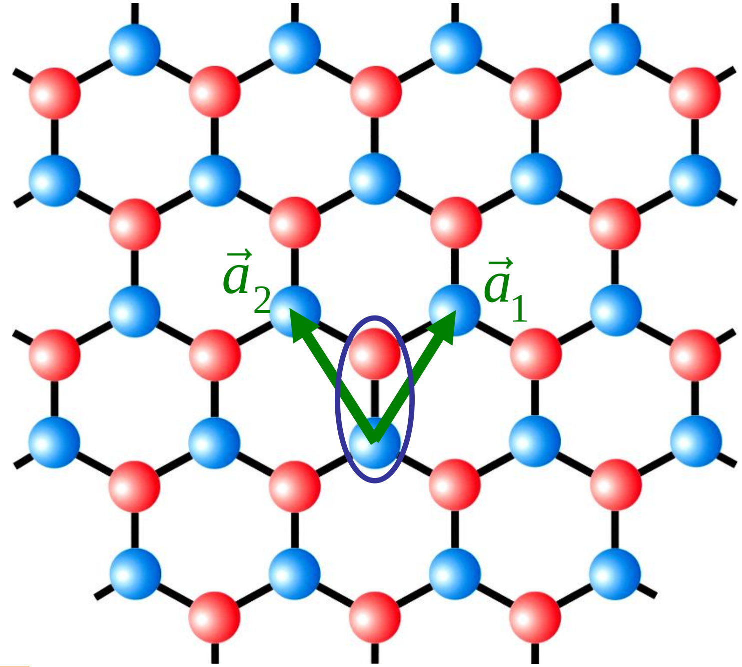
Conclusion and other artificial graphenes

Graphene

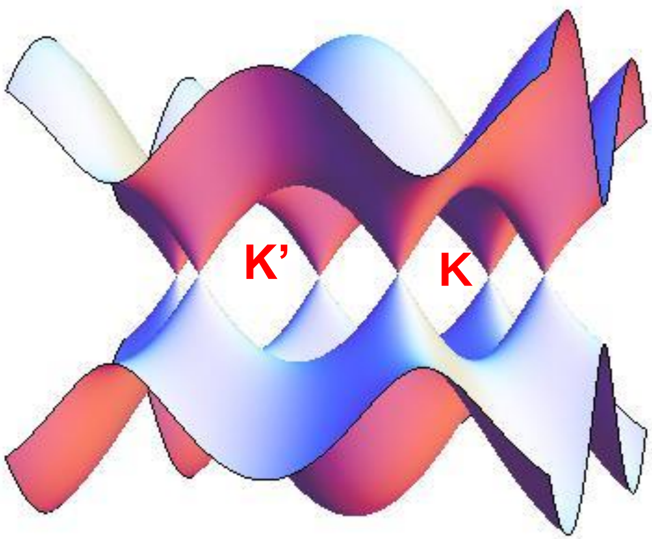
$$H = \begin{array}{cc} \color{red}{A} & \color{blue}{B} \\ \end{array}$$

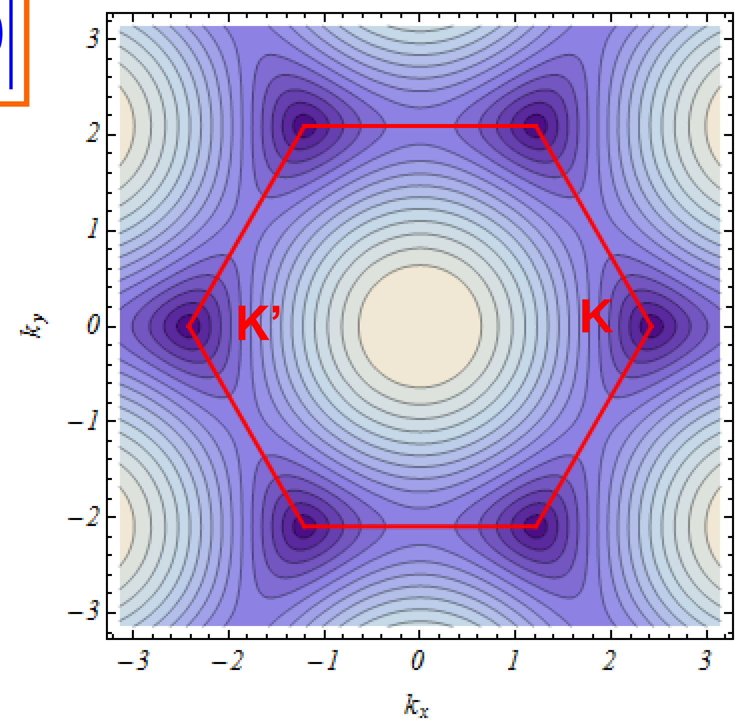
$$H = \begin{pmatrix} 0 & f(\vec{k}) \\ f^*(\vec{k}) & 0 \end{pmatrix}$$

$$f(\vec{k}) = -t\left(1 + e^{i\vec{k}\cdot\vec{a}_1} + e^{i\vec{k}\cdot\vec{a}_2}\right)$$



$$\boxed{\varepsilon(\vec{k}) = \pm\left|f(\vec{k})\right|}$$





Graphene

$$\begin{array}{cc} & \begin{array}{cc} \color{red}{A} & \color{blue}{B} \end{array} \\ H = & \begin{pmatrix} 0 & f(\vec{k}) \\ f^{*}(\vec{k}) & 0 \end{pmatrix} \end{array}$$

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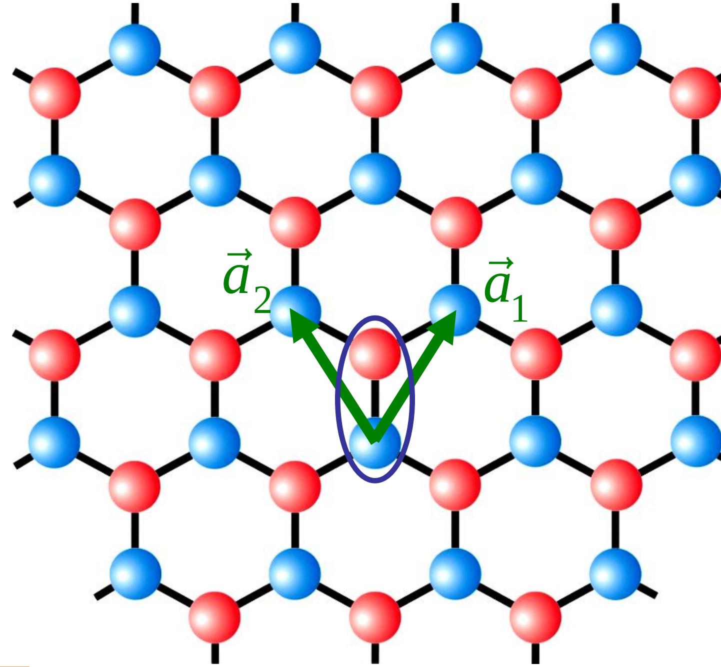
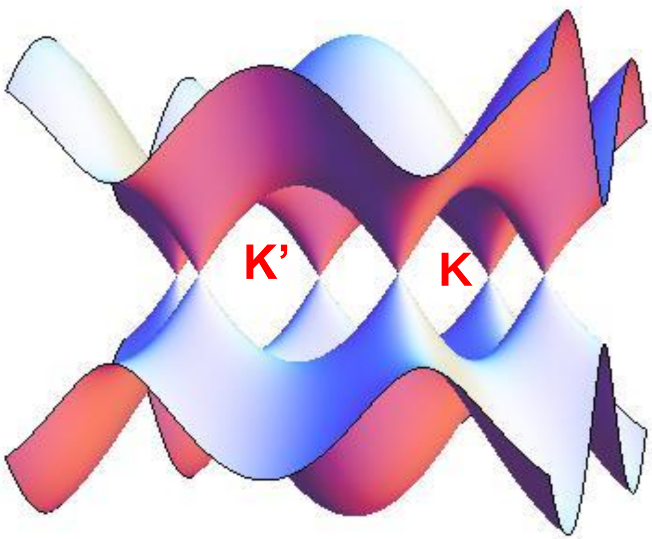
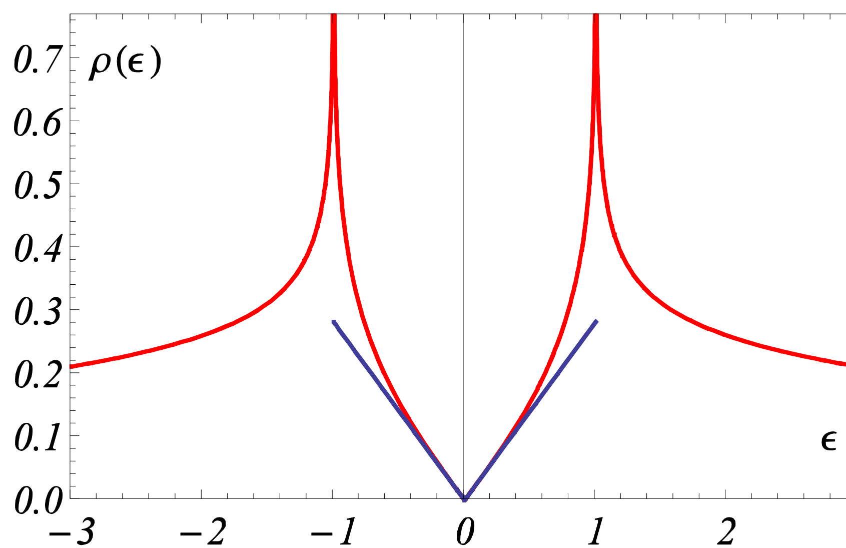




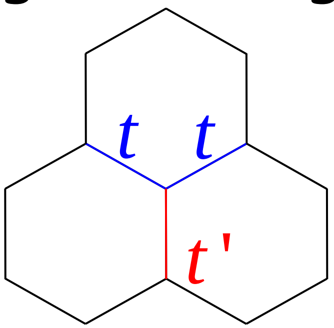
$$\boxed{\varepsilon(\vec{k}) = \pm\left| f(\vec{k})\right|}$$

DOS

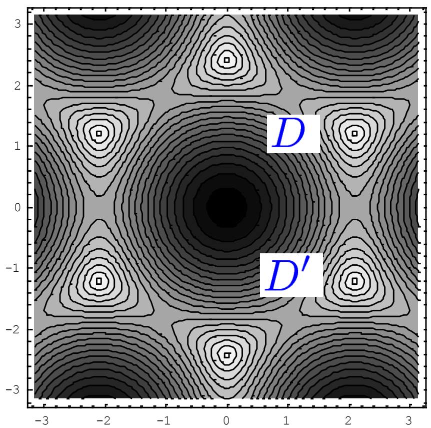


Tight-binding problem on honeycomb lattice

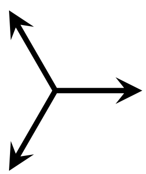
Y. Hasegawa et al., 2006

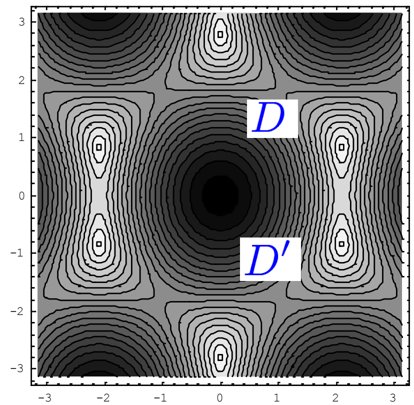


$$E = \pm \left| t' + t e^{i\vec{k}\cdot\vec{a}_1} + t e^{i\vec{k}\cdot\vec{a}_2} \right| = 0$$

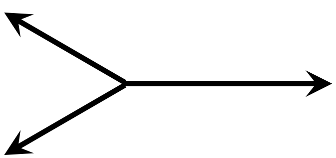


$t' = t$

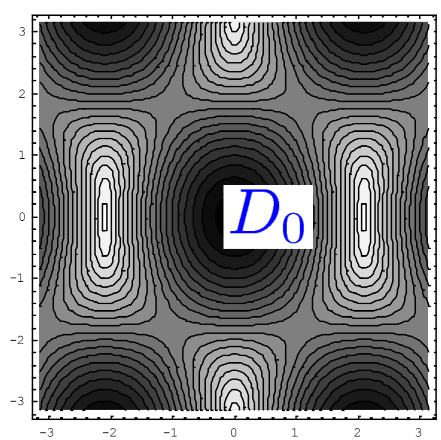




$t' = 1.5t$



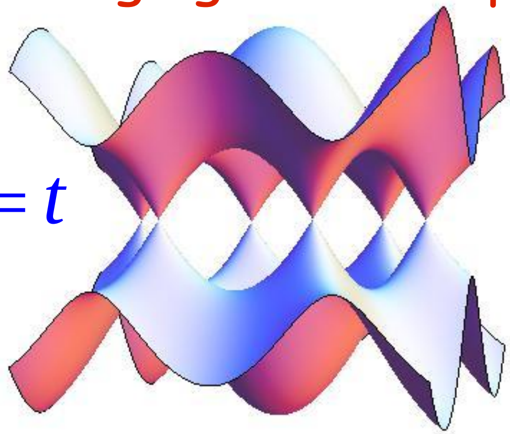
$$\vec{K}\cdot\vec{a}_1 = -\vec{K}\cdot\vec{a}_2 = \pm\frac{2\pi}{3}$$

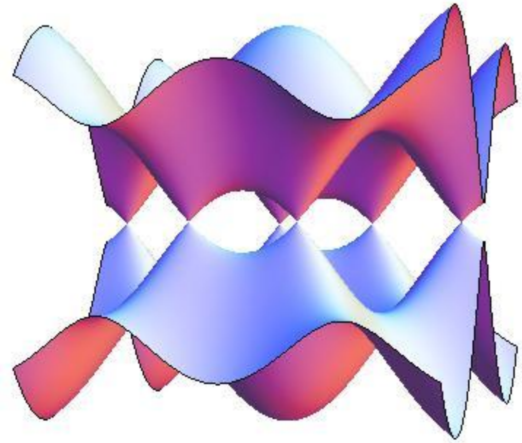


$t' = 2t$

Motion and merging of Dirac points

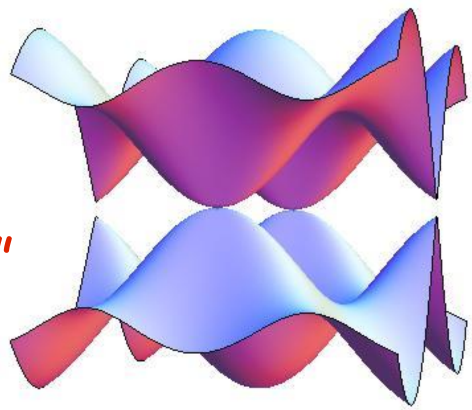
$t' = t$



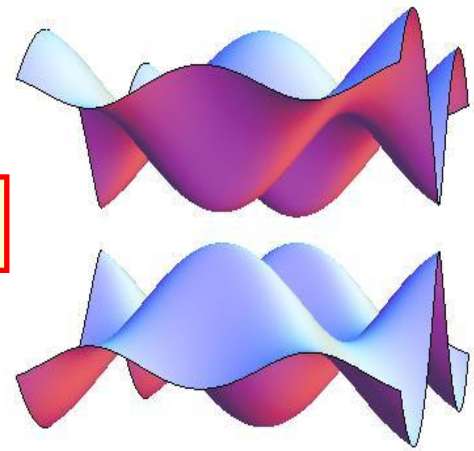


$t' = 1.5t$

"hybrid"
"semi-Dirac"



$\boxed{t' = 2t}$



$t' = 2.3t$

$$2q_D = \frac{4}{3}\arctan\sqrt{\frac{4t^2}{t'^2} - 1} \qquad c_y = \sqrt{3\left(t^2 - t'^2/4\right)}$$

$$c_x = \frac{3}{2}t' \qquad c_y \to 0 \quad \text{for} \quad t' = 2t \qquad m^* = \frac{2}{3t}$$

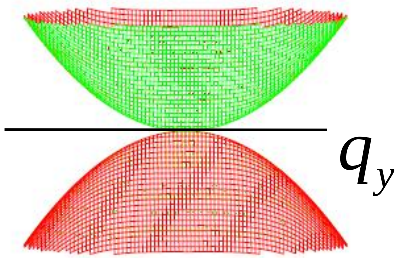
Hybrid 2D electron gas : a new dispersion relation

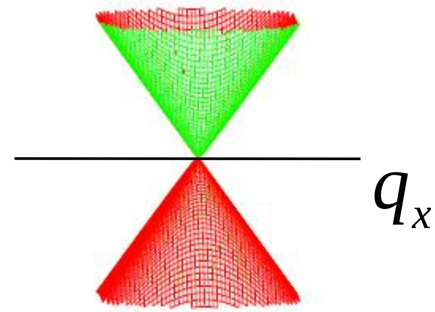
$$\boxed{t' = 2t}$$

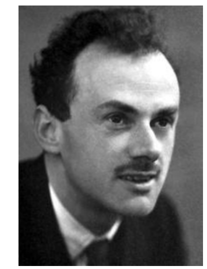
Schrödinger

Semi-Dirac

Dirac







$$\varepsilon = \pm\sqrt{\frac{q_y^4}{4m^2} + c^2 q_x^2}$$

$$\varepsilon \propto \pm\left[\left(n + 1/2\right) eB\right]^{2/3}$$

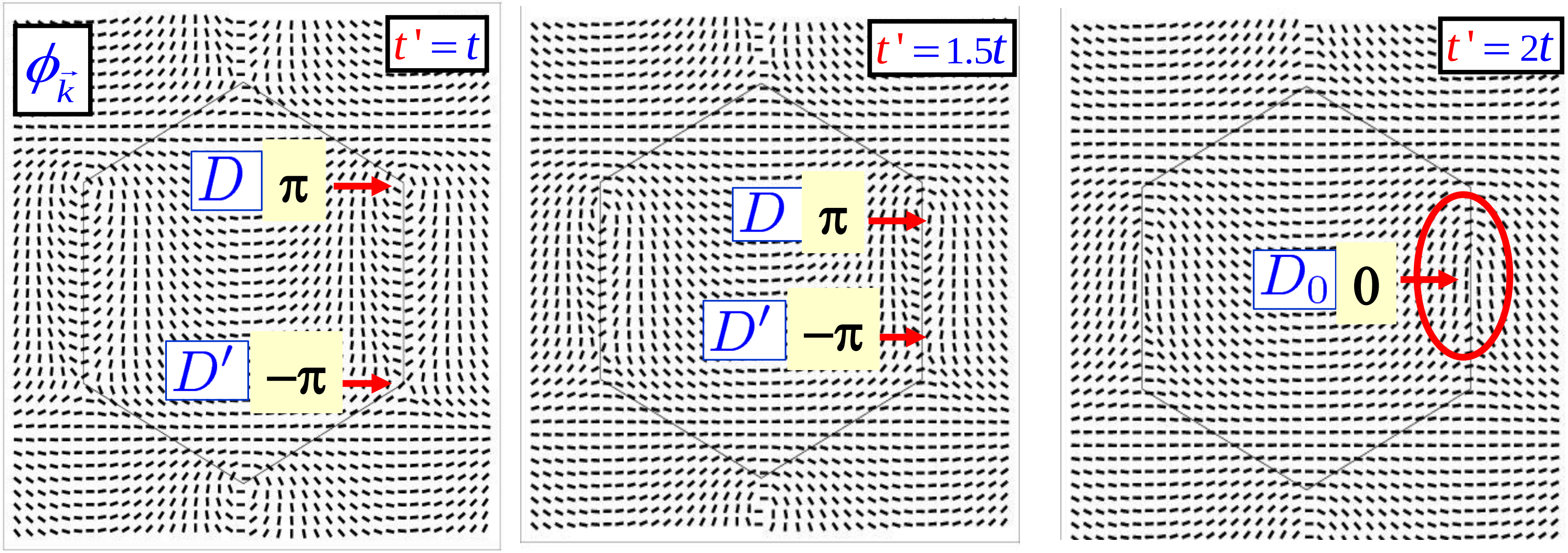
*P. Dietl, F. Piéchon, G.M., PRL **100**, 236405 (2008)*
*G. M., F. Piéchon, J.N. Fuchs, M.O. Goerbig, Eur. Phys. J. B **72,** 509 (2009),*
*Phys. Rev. B **80**, 153412 (2009)*

Berry phase

Two component wavefunction

$$|u_{k}\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ \pm e^{i\phi(\vec{k})} \end{pmatrix} e^{i\vec{k}.\vec{r}}$$

$$\boxed{\phi_B = i\oint_C \langle u_{\vec{k}} \,|\, \vec{\nabla}_{\vec{k}} u_{\vec{k}} \rangle . d\vec{k} = \frac{1}{2}\oint_C \nabla_{\vec{k}}\phi_{\vec{k}} . d\vec{k} = \pm\pi}$$



Topological transition

General description of the motion of Dirac points (with time reversal symmetry)

$$H = -\begin{pmatrix} 0 & f(\vec{k}) \\ f^*(\vec{k}) & 0 \end{pmatrix} \qquad f(\vec{k}) = \sum_{m,n} t_{mn} e^{-i\vec{k}.\vec{R}_{mn}}$$

When t_{mn} changes, $\vec{D}$ and $-\vec{D}$ move

Where is the merging point?

$$\vec{D} = -\vec{D} \Longrightarrow \boxed{\vec{D}_0 = \frac{\vec{G}}{2}}$$

4 possible positions in $\vec{k}$ space

$(0,0)$	$(0,1)$	$(1,0)$	$(1,1)$
Γ	X	Y	M

Expansion near $\vec{D}_0$

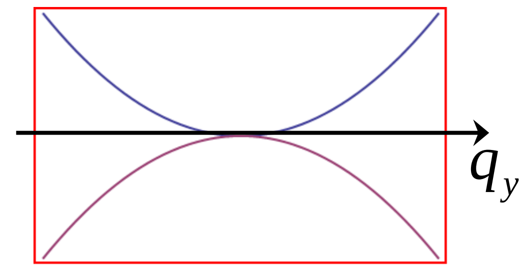
$$\boxed{f(\vec{D}_0 + \vec{q}) = -icq_x + \frac{q_y^2}{2m^*}}$$

At the merging transition :

$$\hat{cx} = \sum_{m,n} t_{mn} \vec{R}_{mn} (-1)^{\beta_{mn}}$$

$$\frac{1}{m^*} = \sum_{m,n} t_{mn} R_{mn}^2 (-1)^{1+\beta_{mn}}$$

$$H(\vec{q}) = \begin{pmatrix} 0 & -icq_x + \frac{q_y^2}{2m^*} \\ icq_x + \frac{q_y^2}{2m^*} & 0 \end{pmatrix}$$

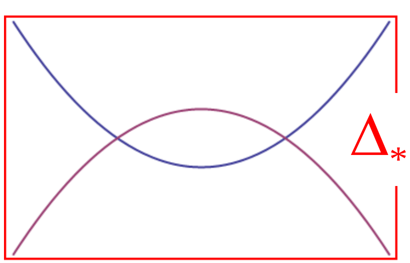


Near the transition :

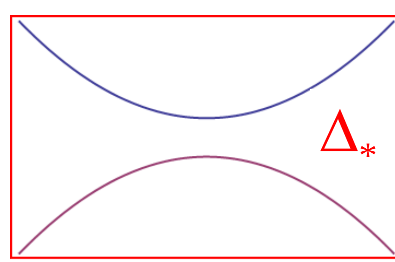
$$H(\vec{q}) = \begin{pmatrix} 0 & \Delta_* - icq_x + \frac{q_y^2}{2m^*} \\ \Delta_* + icq_x + \frac{q_y^2}{2m^*} & 0 \end{pmatrix}$$

$$\Delta_* = \sum_{m,n} t_{mn} (-1)^{\beta_{mn}}$$

$$(\Delta_* = t' - 2t)$$



$\Delta_* < 0$

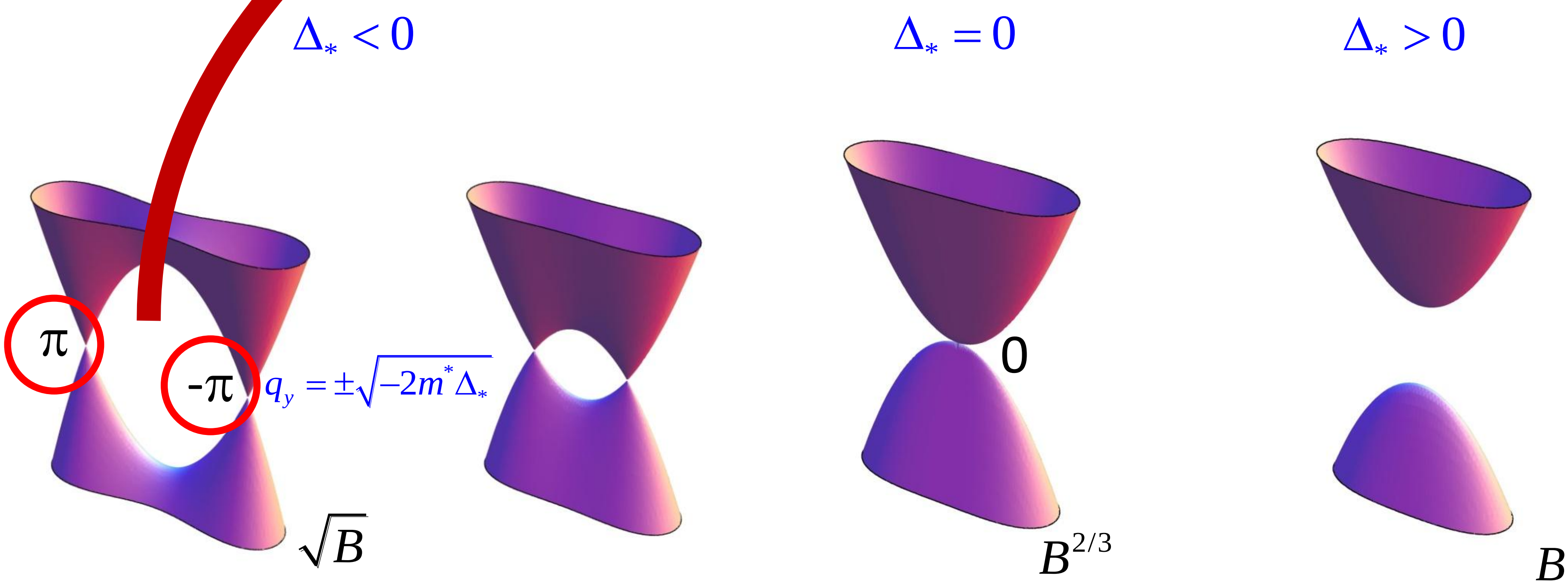


$\Delta_* > 0$

« universal Hamiltonian »

$$H(\vec{q}) = \begin{pmatrix} 0 & \Delta_* - icq_x + \dfrac{q_y^2}{2m^*} \\ \Delta_* + icq_x + \dfrac{q_y^2}{2m^*} & 0 \end{pmatrix}$$

The parameter Δ_* drives the topological transition



This Hamiltonian describes the topological transition, the coupling between valleys and the merging of the Dirac points

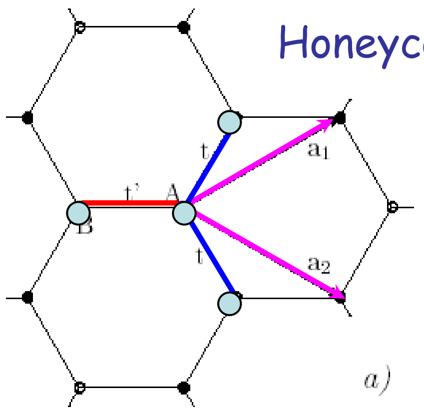
First summary: Manipulation of Dirac points and merging

By varying band parameters, it is possible to manipulate the Dirac points. They can move in k-space and they can even merge.

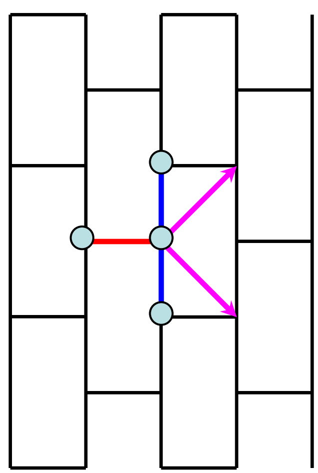
The merging transition is a topological transition: 2 Dirac points evolve into a single hybrid « semi-Dirac » point and eventually a gap opens and the Fermi surface disappears.

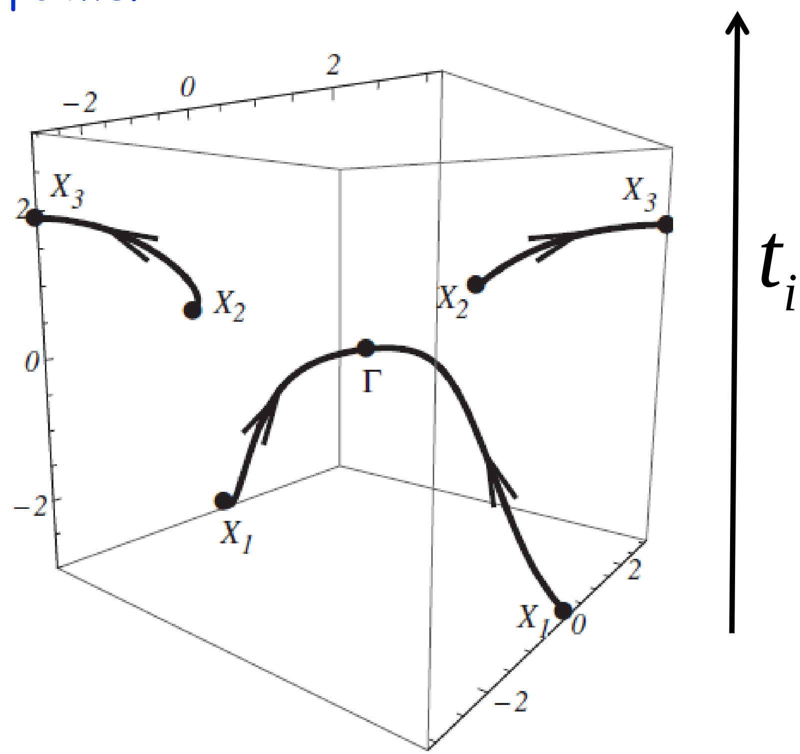
Universal description of motion and merging of Dirac points.

Honeycomb



Brick wall

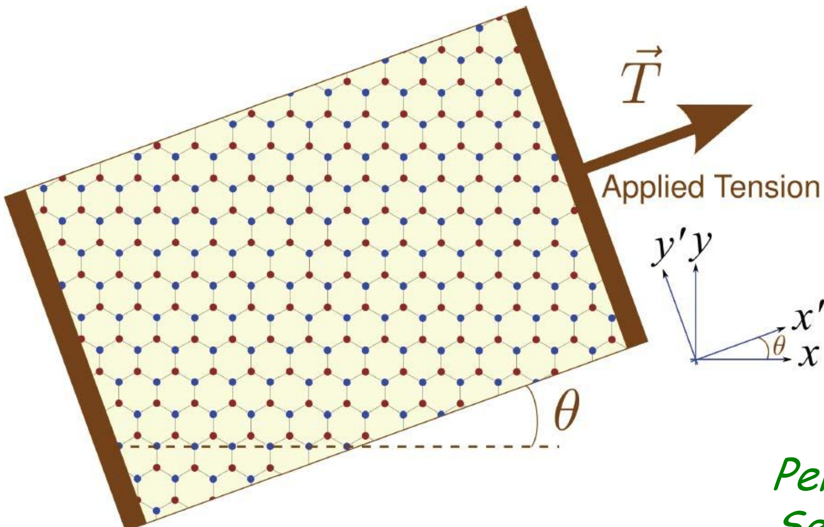


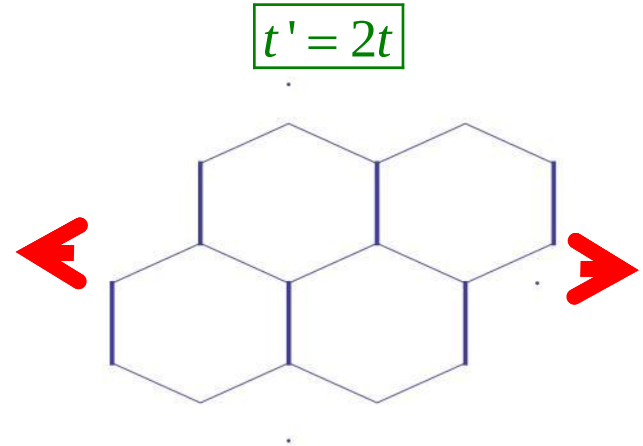


G. M., F. Piéchon, J.N. Fuchs, M.O. Goerbig, Phys. Rev. B **80**, 153412 (2009)

Physical realizations of the merging transition

* Strained graphene





strain ~ 23%

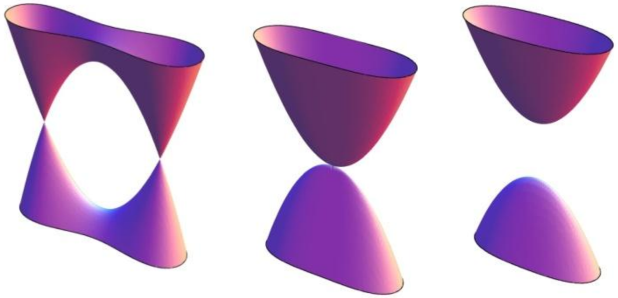
merging is unreachable

Pereira, Castro Neto, Peres, PRB 2009
See also Goerbig, Fuchs, Piéchon, G.M., PRB 2008

* Microwaves

* Ultracold atoms in optical lattices

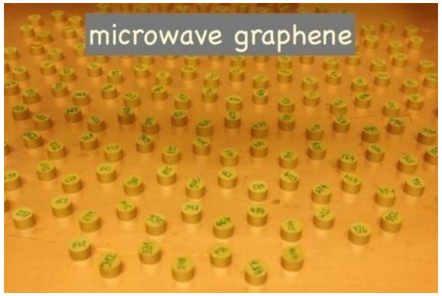
Merging of Dirac points in a 2D crystal
G. M., F. Piéchon, J.N. Fuchs, M.O. Goerbig, Phys. Rev. B **80**, 153412 (2009)



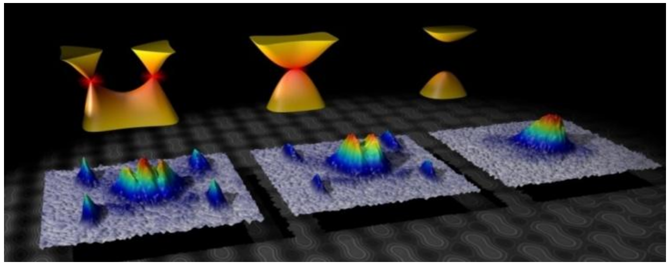
Referee A
The paper is well written and accessible to a broad audience.
The results might be applicable to optical lattice systems.
I would recommend publishing the paper in PRL as it is.

Referee B
It is a nice simple toy model...
The authors propose that merging of Dirac points might be possible with cold atoms in optical lattices.
I think that it is a very long shot, given that ... the systems are yet to be realized experimentally.
I do not recommend publication of this paper in PRL.

Referee C
While the physical system is certainly interesting, its relevance to current experiments is rather tenuous...



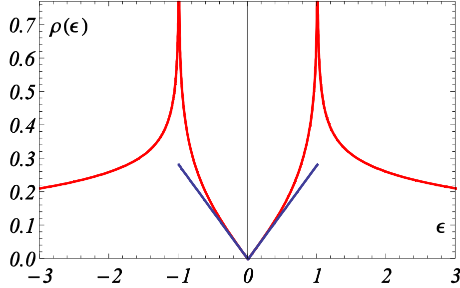
Topological transition of Dirac points
in a microwave experiment
M. Bellec et al. (2012)



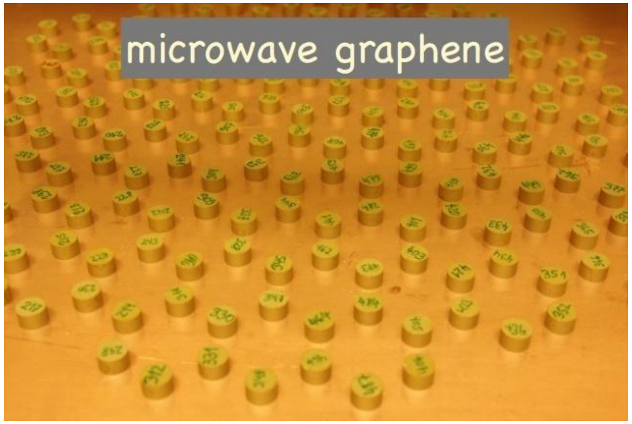
Creating moving and merging Dirac points with
a Fermi gas in a tunable honeycomb lattice,
L.Tarruell et al. Nature (2012)

Graphene physics with microwaves !

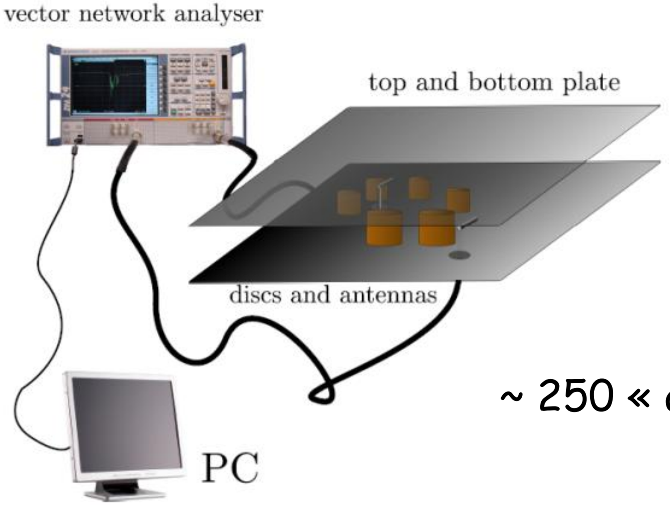
M. Bellec, U. Kuhl, F. Mortessagne (NICE)



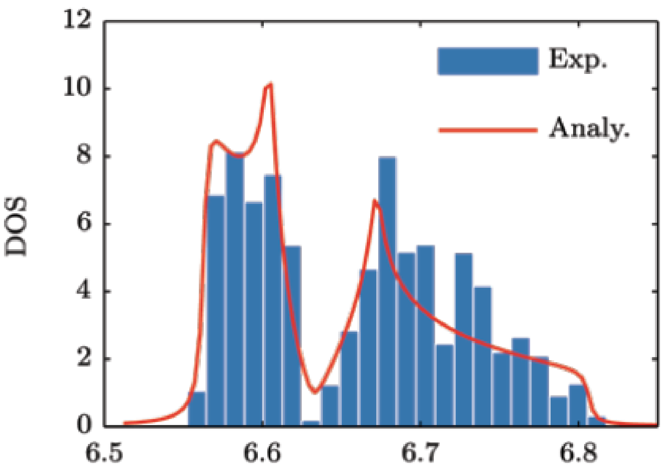
Honeycomb lattice of dielectric resonators
Evanescent propagation between the dots -> Tight-binding description
Measure of the reflexion coefficient → LDOS



~ 50cm



~ 250 « atoms »



High flexibility
1) Look for the merging transition
2) Probe the edge states

(2nd and 3rd nearest neighbor couplings not negligeable)

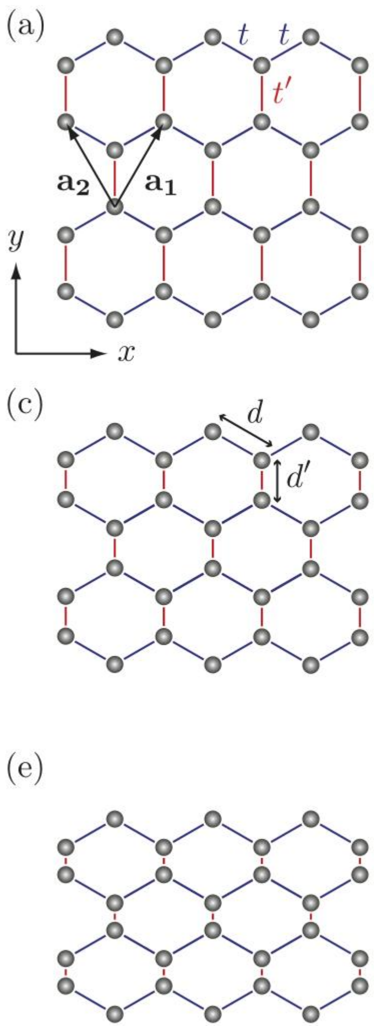
$$t_2 / t = 0.091 \qquad t_3 / t = 0.071$$

The merging transition seen in the microwave experiment

PRL 110, 033902 (2013) PHYSICAL REVIEW LETTERS week ending 18 JANUARY 2013

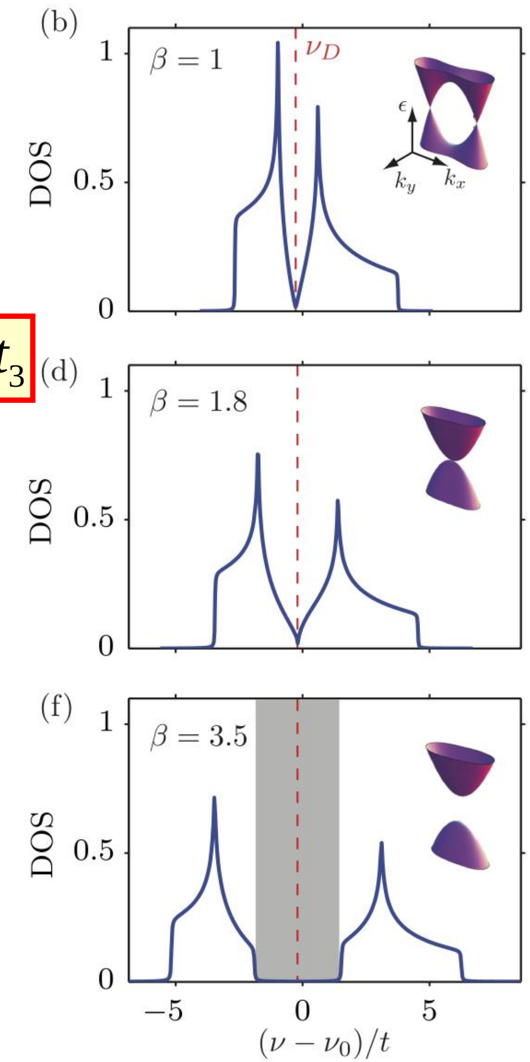
Topological Transition of Dirac Points in a Microwave Experiment

Matthieu Bellec,[1] Ulrich Kuhl,[1] Gilles Montambaux,[2] and Fabrice Mortessagne[1,*]



$$t'_{crit.} = 2t - 3t_3$$

Uniaxial strain → increase t'



The merging transition seen in the microwave experiment

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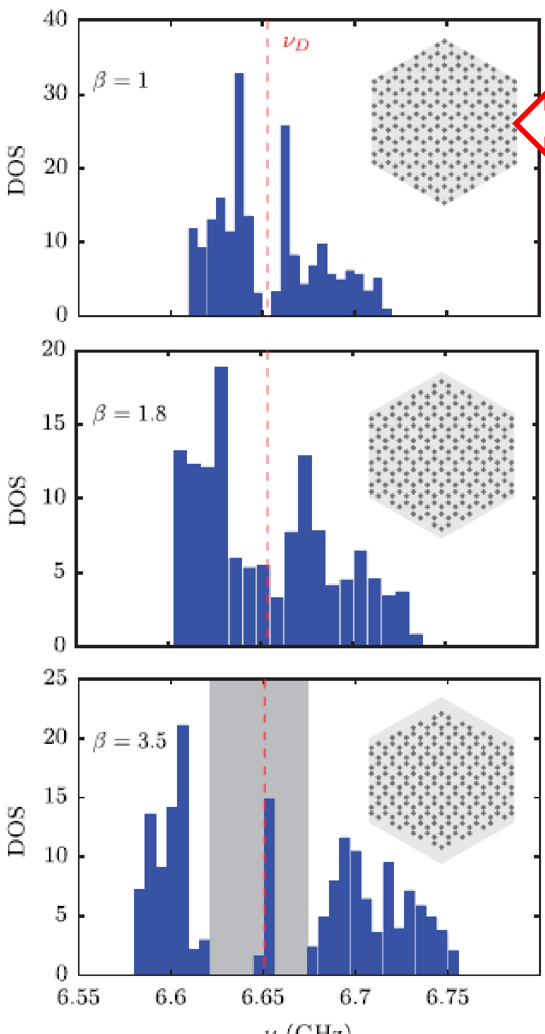


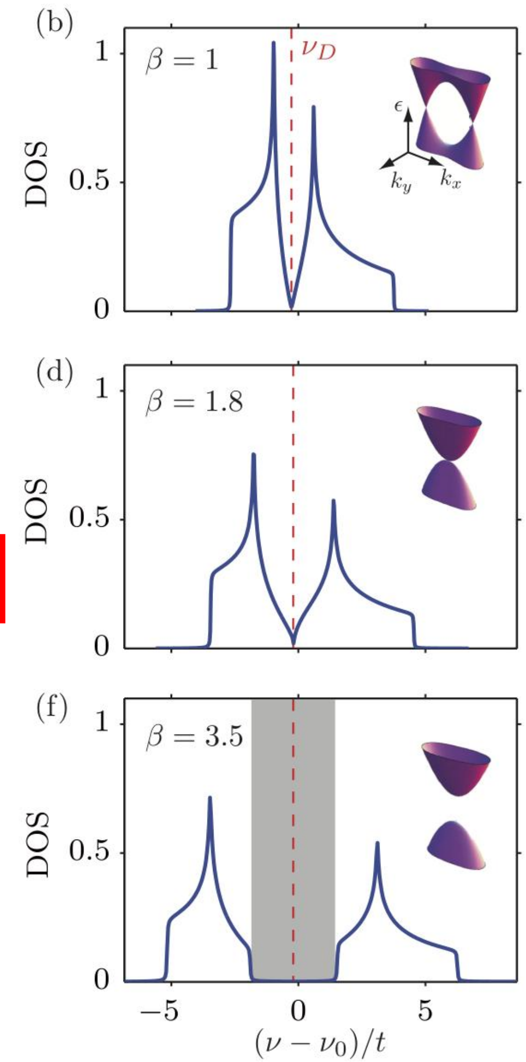
$t' = t$

Armchair edges

$t' = 1.8\,t$

$t' = 3.5\,t$

$$\boxed{t'_{crit.} = 2t - 3t_3}$$



The merging transition seen in the microwave experiment

PRL 110, 033902 (2013) PHYSICAL REVIEW LETTERS week ending 18 JANUARY 2013

Topological Transition of Dirac Points in a Microwave Experiment

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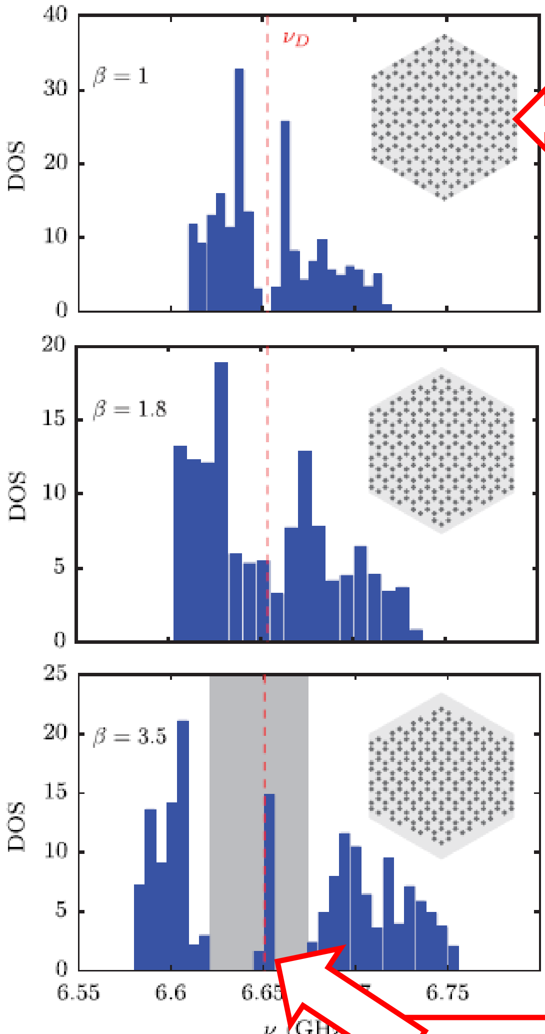


$t' = t$

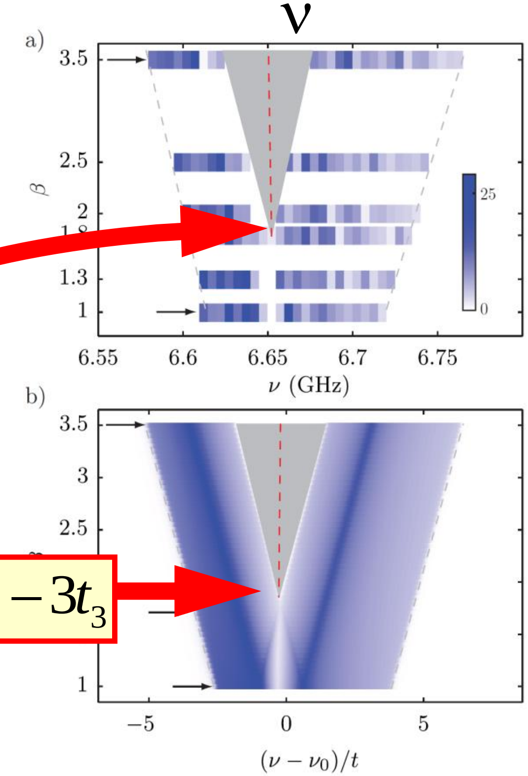
Armchair edges

$t' = 1.8\,t$

$t' = 3.5\,t$

Gapped phase + new edge states

ν t'/t



exp.

$$t'_{crit.} = 2t - 3t_3$$

th.

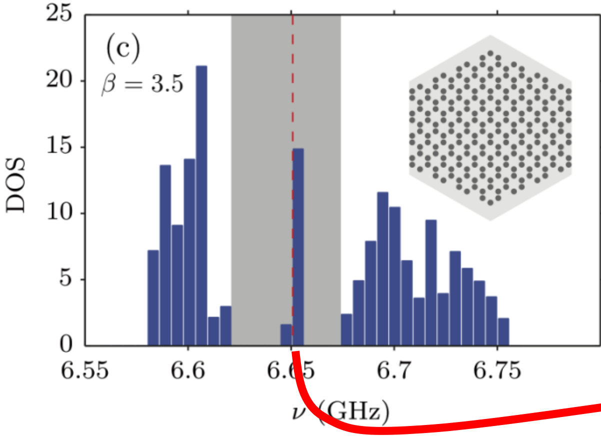
Probing the edge states…

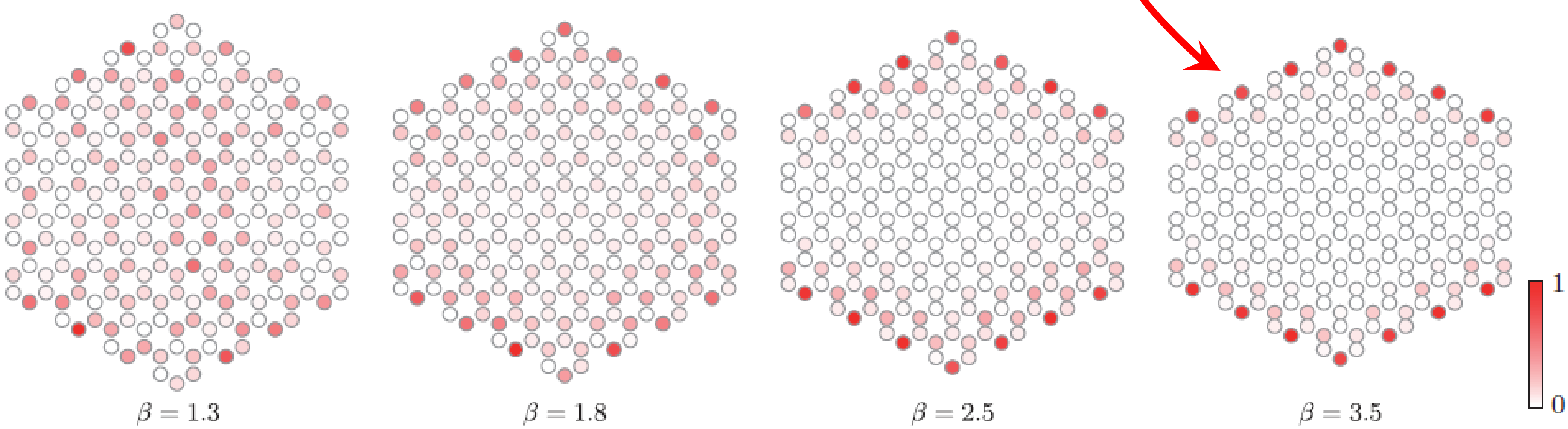
« armchair » have no edge states. But by under strain, new edge states are predicted along certain armchair edges.

P. Delplace, G.M.

PHYSICAL REVIEW B **84**, 195452 (2011)

Zak phase and the existence of edge states in graphene



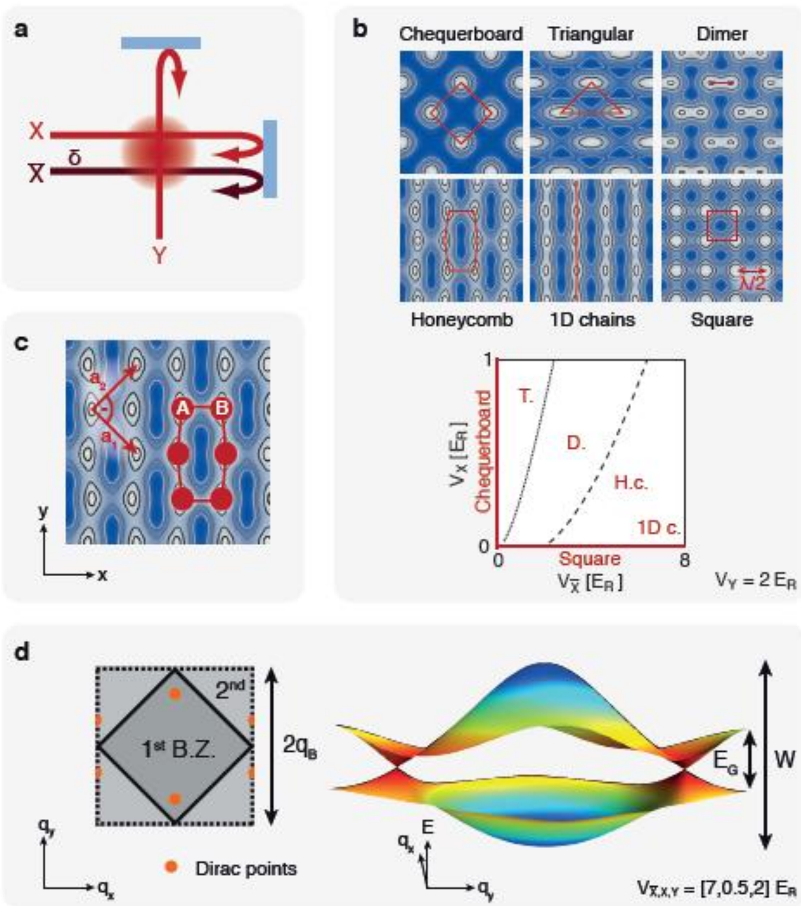


Increasing uniaxial strain

Creating, moving and merging Dirac points with a Fermi gas in a tunable honeycomb lattice

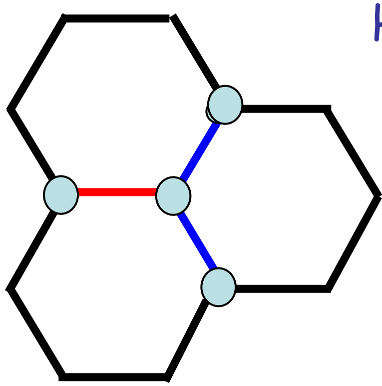
Nature 483, 302 (2012)

Leticia Tarruell, Daniel Greif, Thomas Uehlinger, Gregor Jotzu and Tilman Esslinger
Institute for Quantum Electronics, ETH Zurich, 8093 Zurich, Switzerland

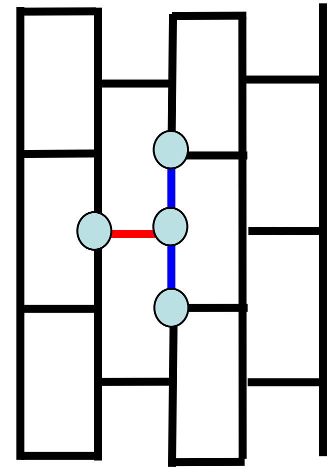


Atoms are trapped in an optical lattice potential and form an artifical crystal

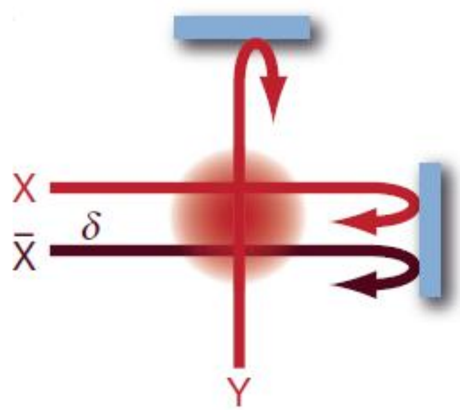
$$V(x,y) = -V_{\overline{X}}\cos^2(kx+\theta/2) - V_X\cos^2(kx)$$
$$-V_Y\cos^2(ky) - 2\alpha\sqrt{V_XV_Y}\cos(kx)\cos(ky)\cos\varphi$$

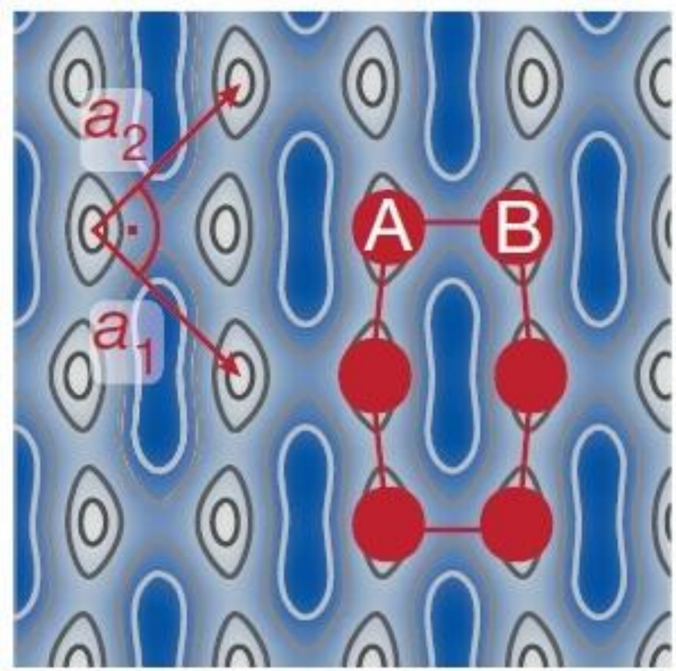


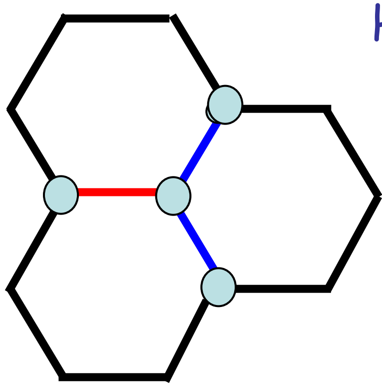
Honeycomb



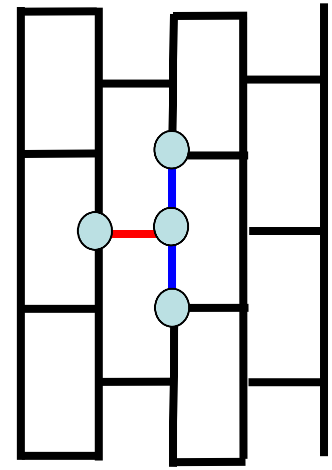
Brick wall





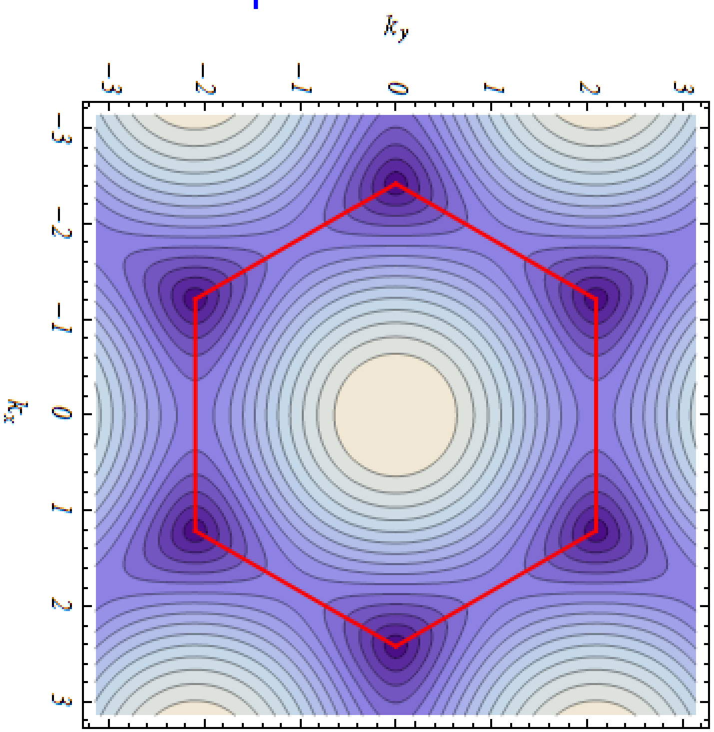


Honeycomb

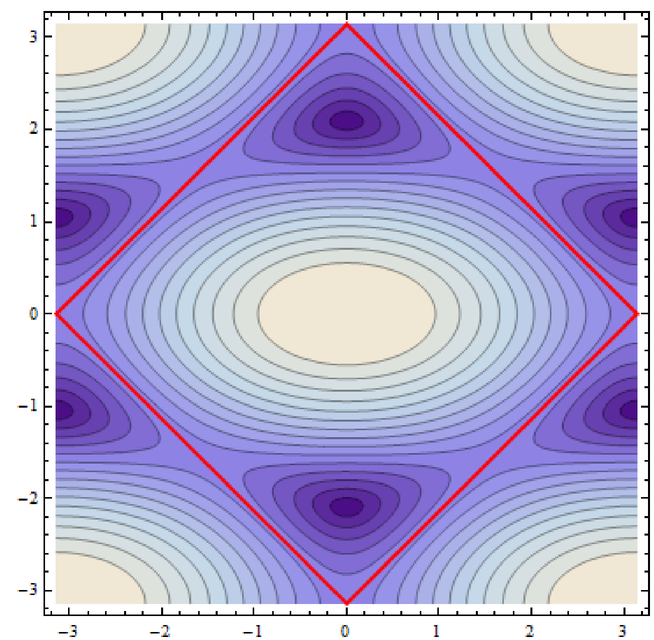


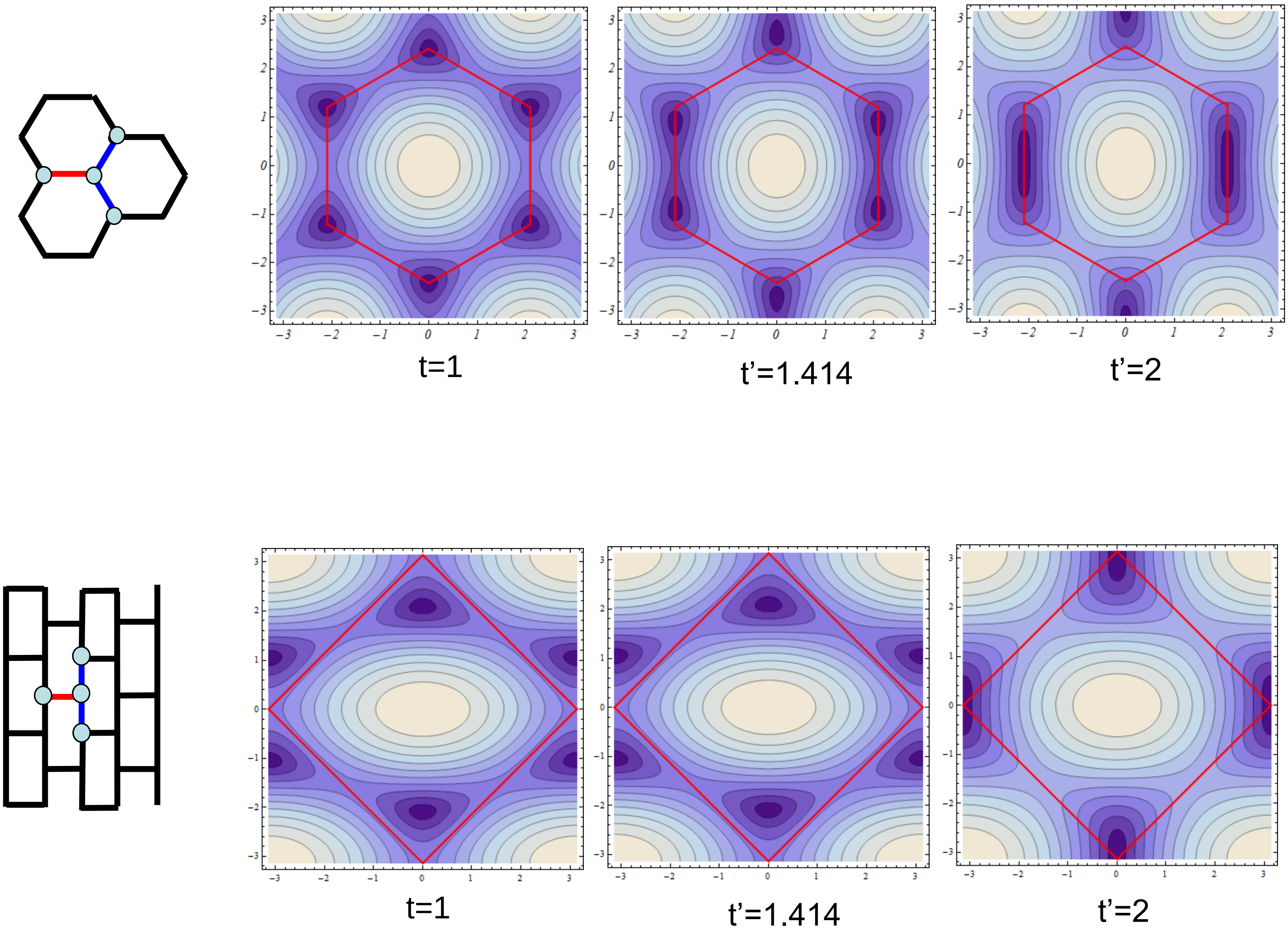
Brick wall

$$E = \pm \left| t e^{i\mathbf{k}.\mathbf{a}_1} + t e^{i\mathbf{k}.\mathbf{a}_2} + t' \right| \qquad E = \pm \left| t e^{i(k_x + k_y)a} + t e^{i(k_x - k_y)a} + t' \right|$$



$$t' = t$$

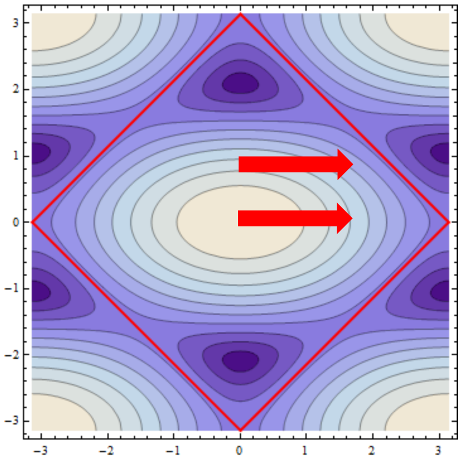


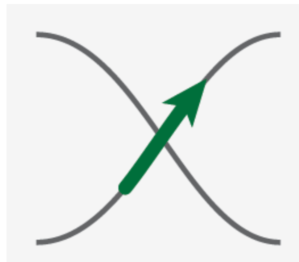

t=1
t'=1.414
t'=2
t=1
t'=1.414
t'=2

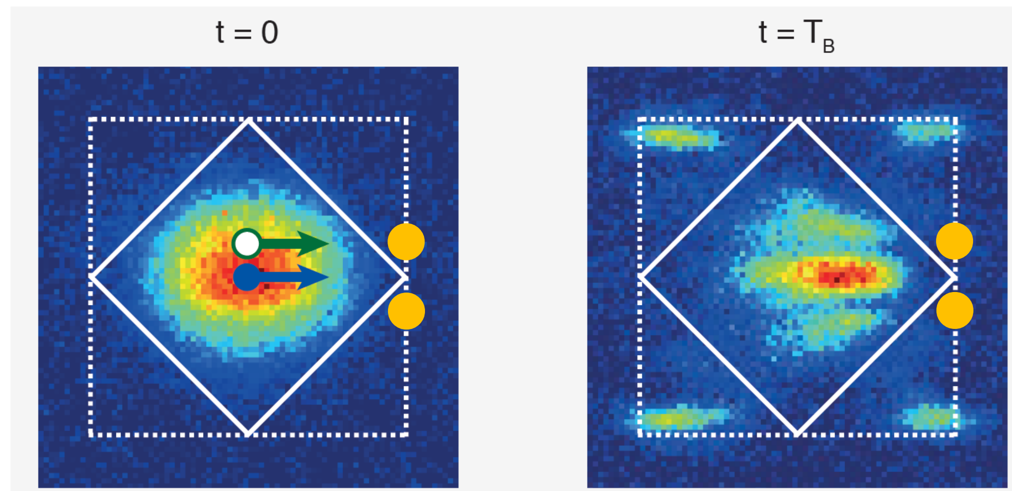
Creating, moving and merging Dirac points with a Fermi gas in a tunable honeycomb lattice

Nature 483, 302 (2012)

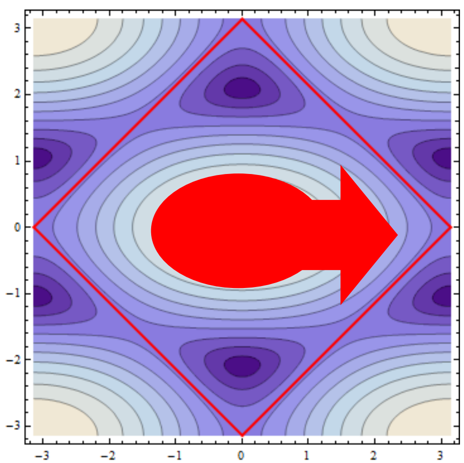
Leticia Tarruell, Daniel Greif, Thomas Uehlinger, Gregor Jotzu and Tilman Esslinger
Institute for Quantum Electronics, ETH Zurich, 8093 Zurich, Switzerland







^{40}K



Bloch oscillations

$$\hbar \frac{d\vec{k}}{dt} = \vec{F}$$

How to manipulate and merge Dirac points ?

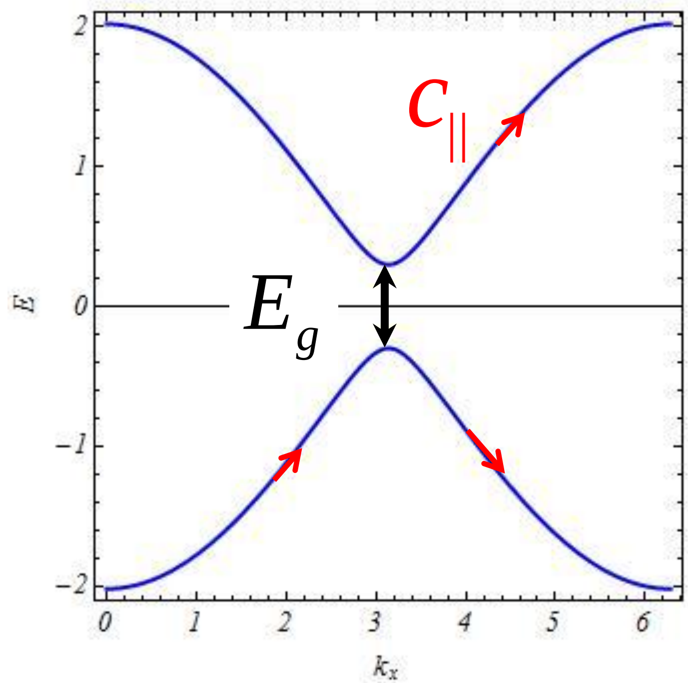
Anisotropy of the optical potential

How to detect and localize Dirac points ?

Bloch oscillations + Landau-Zener Tunneling

Measurement of the proportion of atoms in the upper band

Landau-Zener transition

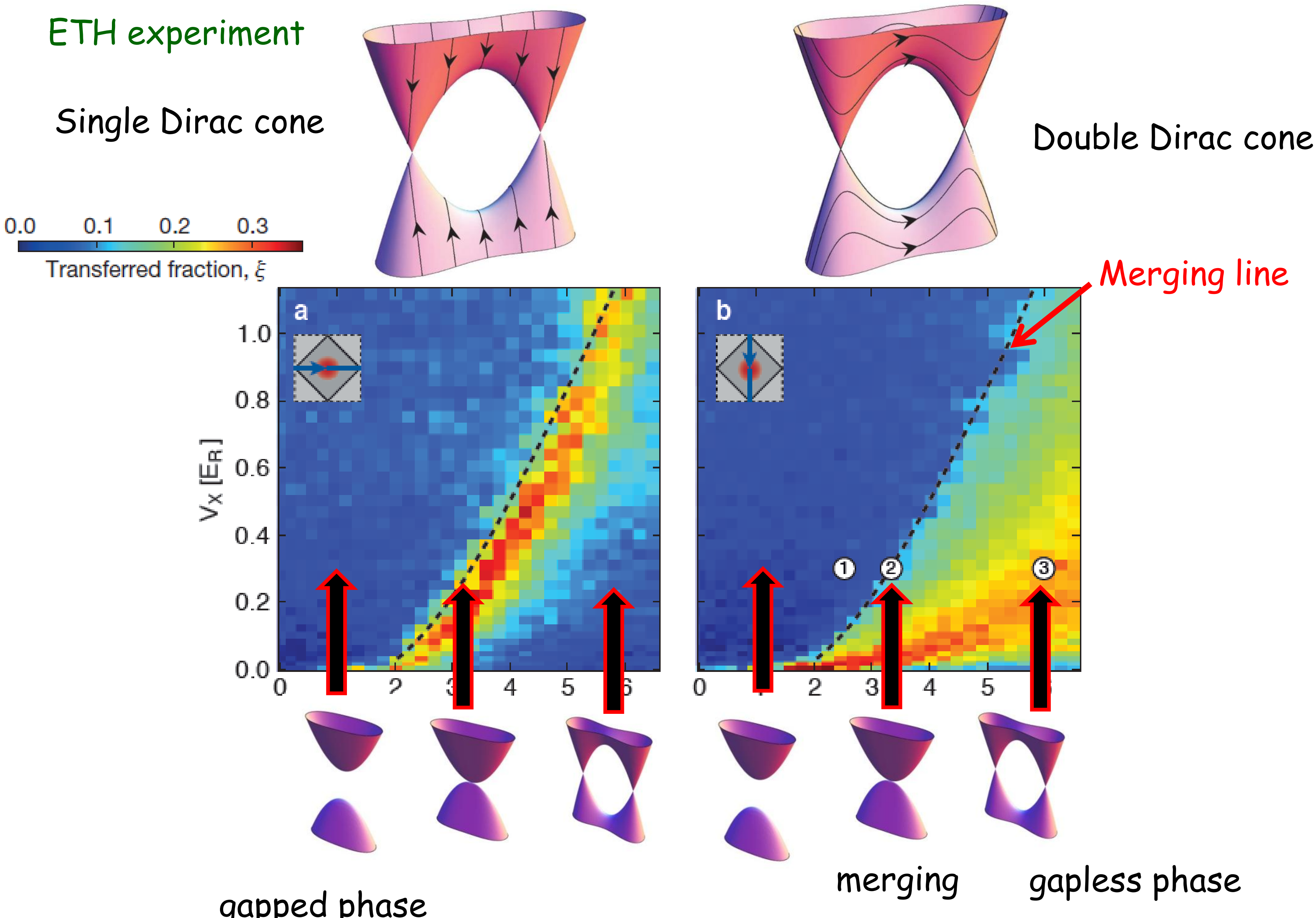


$$P_Z = e^{-\frac{\pi}{4}\frac{E_g^2}{c_{||}F}}$$

$$1 - P_Z$$

$$\boxed{\vec{k} = \frac{\overrightarrow{F}\, t}{\hbar}}$$

Measured transfered fraction of atoms: directions of motion



Explain the experimental data using Universal Hamiltonian

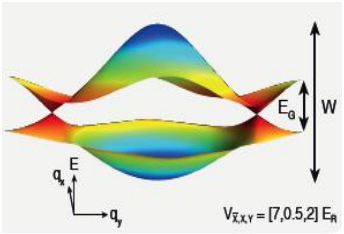
Bloch-Zener oscillations across a merging transition of Dirac points

Lih King Lim, Jean-Noel Fuchs, G. M., PRL **108**, 175303 (2012)

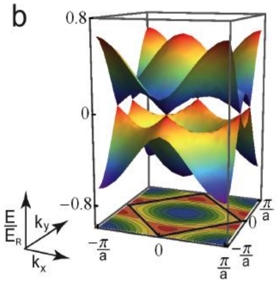
Explain the experimental data using Universal Hamiltonian

1) Relate the parameters of the optical lattice to the parameters of the Universal Hamiltonian

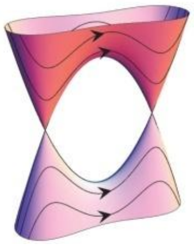
1) Ab-initio band structure from optical lattice potential: V_X, V_{Xb}, V_y (laser intensities).

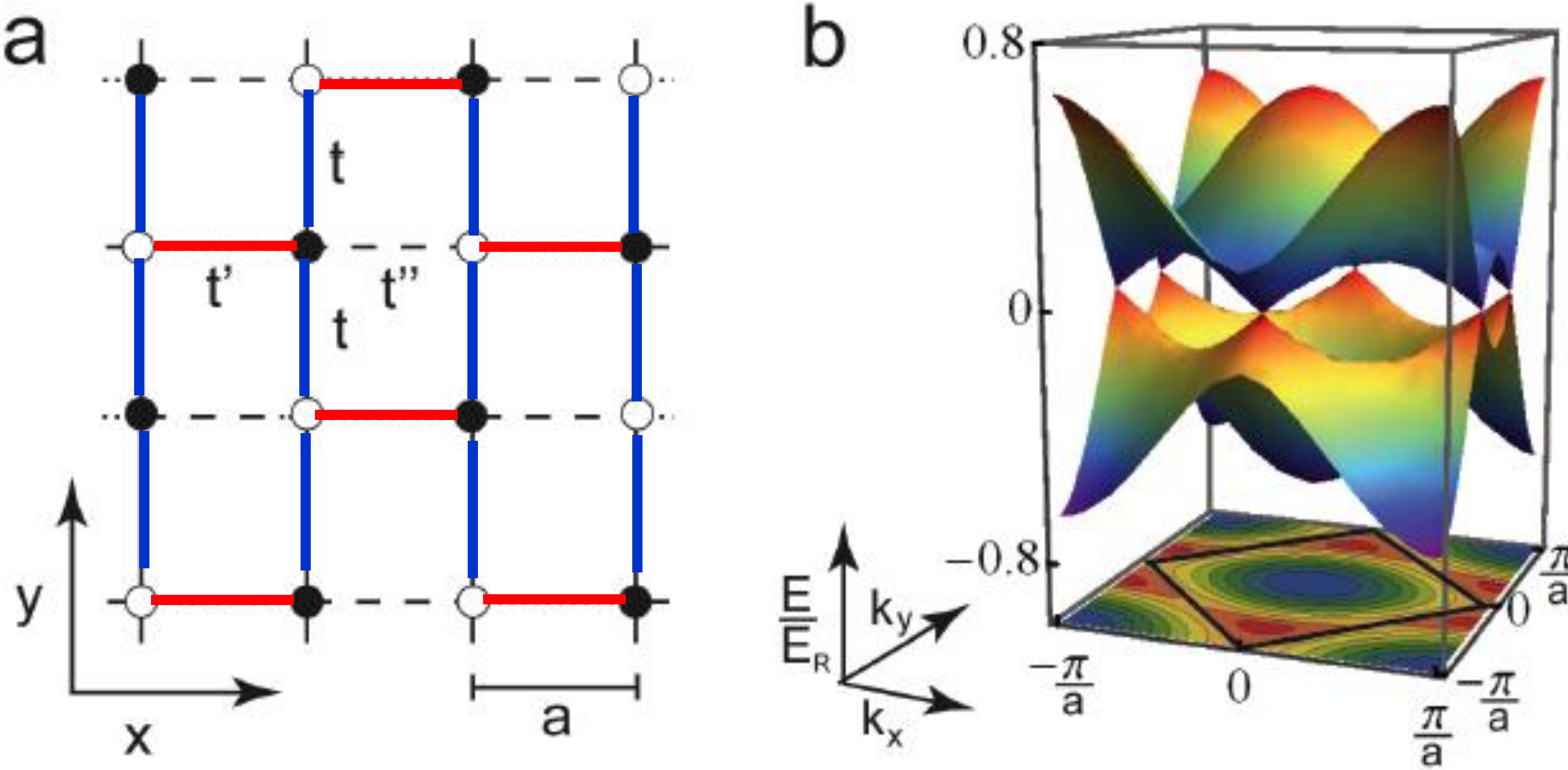


2) Tight-binding model on an anisotropic square lattice: t,t',t'' (hopping amplitudes)

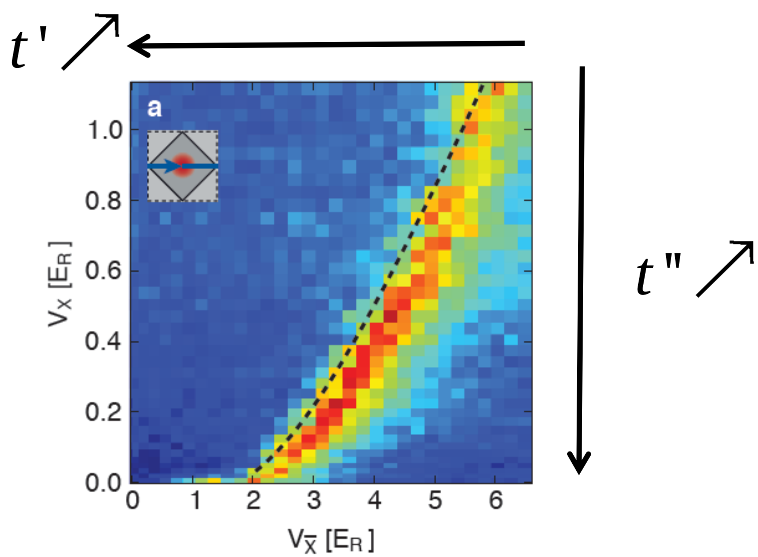


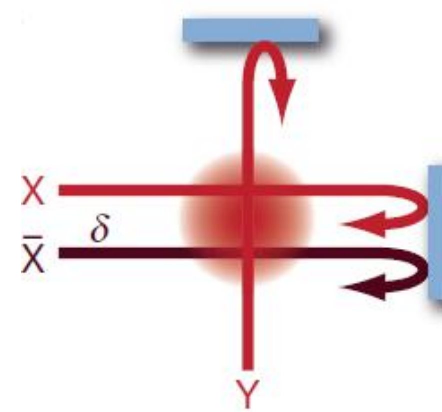
3) Universal hamiltonian describing the merging transition: Δ_*, c_x, m^*



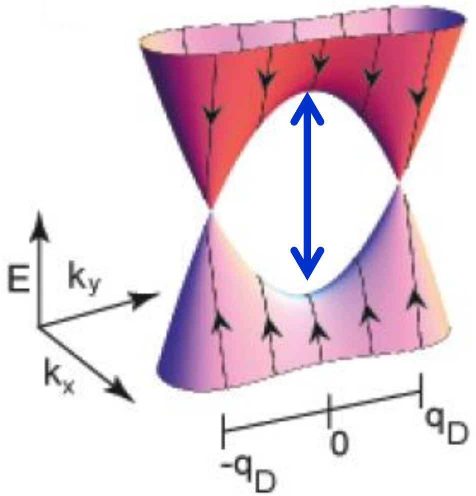


Anisotropic square lattice t-t'-t'': Dirac points, merging, gapped phase,

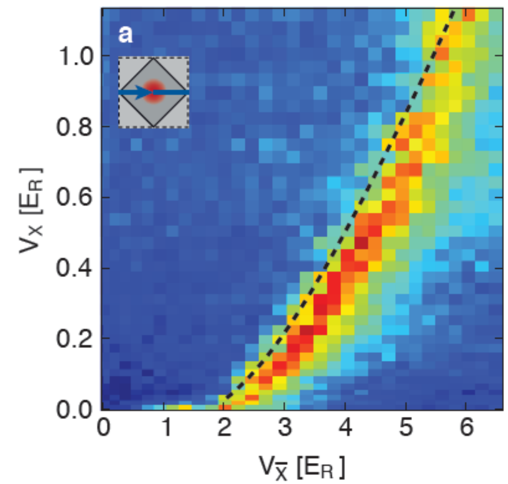




Mapping tight-binding model on universal hamiltonian



$\Delta_* \nearrow$



$c_x \searrow$

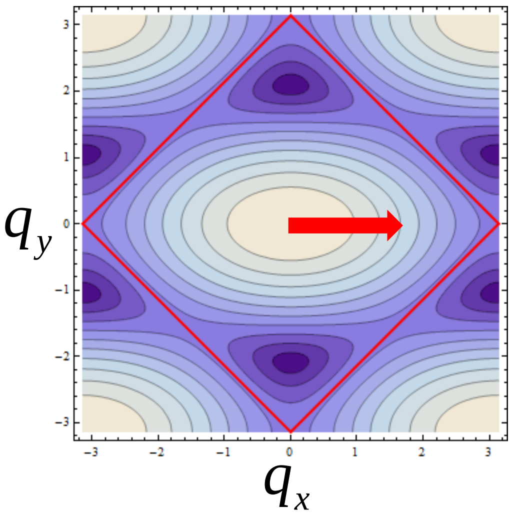
$$\Delta_* = t' - 2t + t'' \qquad c_x = t' - t''$$

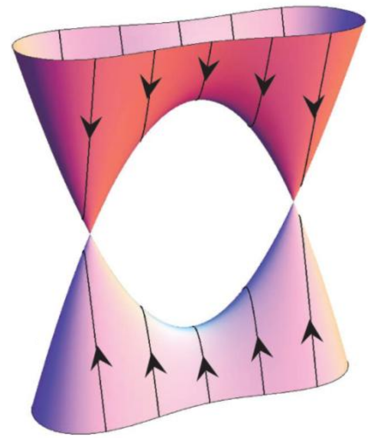
$$m^* = \frac{2}{2t + t' + t''}$$

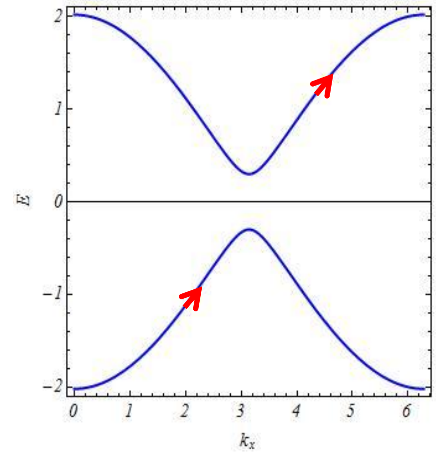
$$q_D = \sqrt{-2m^*\Delta_*} = 2\sqrt{\frac{2t - t' - t''}{2t + t' + t''}}$$

2) Compute the inter-band tunneling probability within the Universal Hamiltoninan

Single Zener tunneling

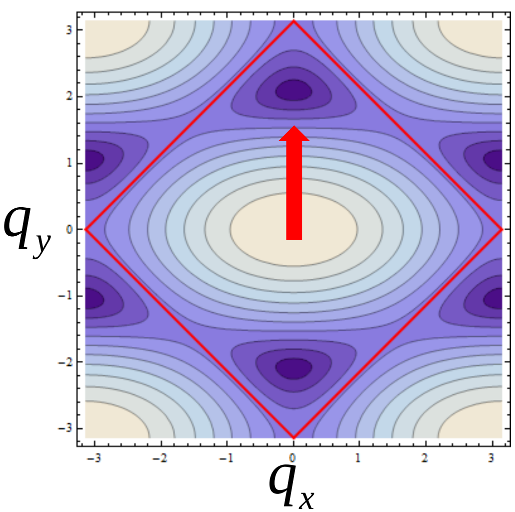


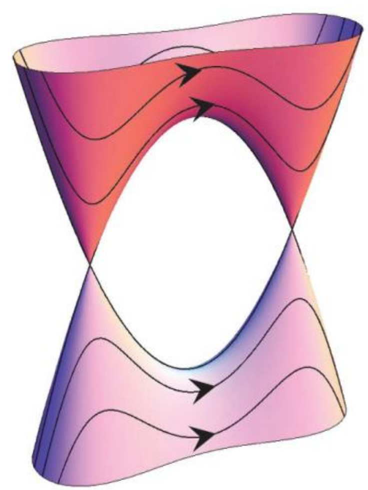




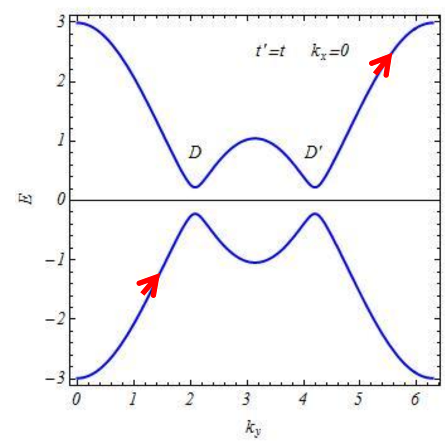
P_Z

Double Zener tunneling



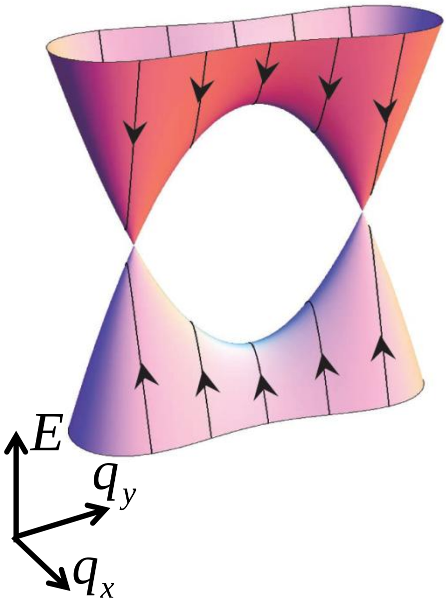


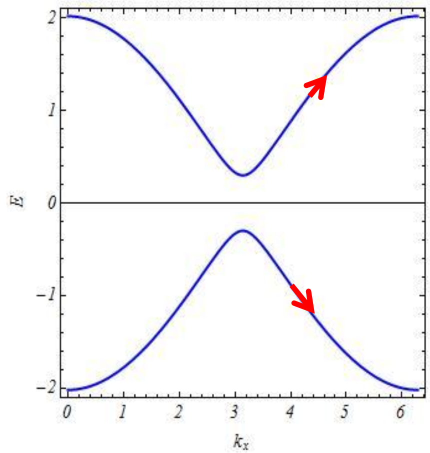
$$P_t = 2P_Z(1 - P_Z)$$

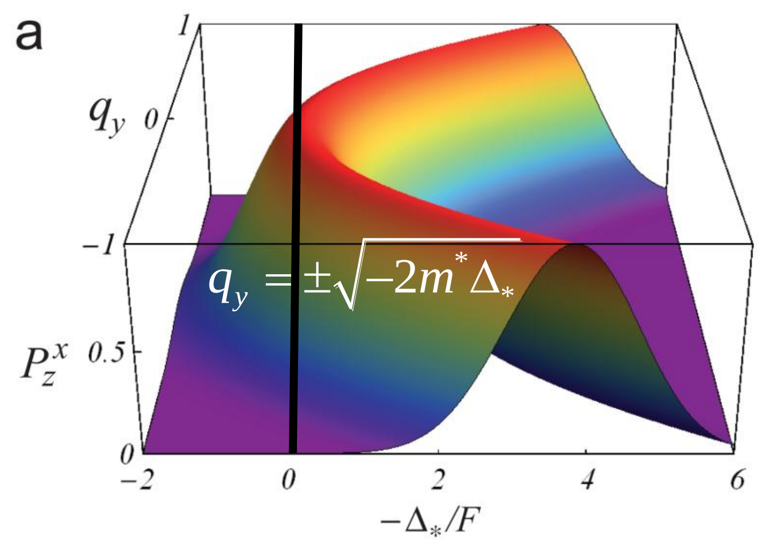


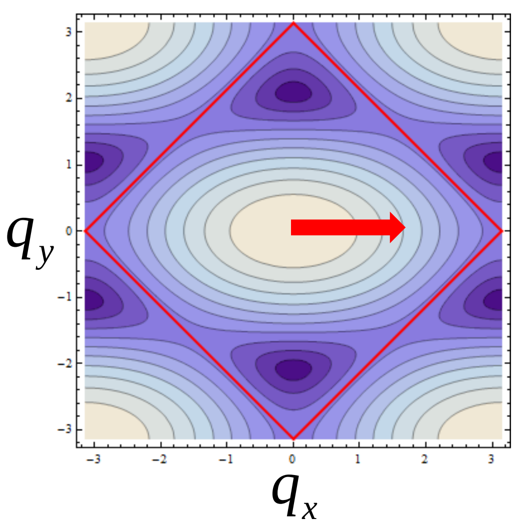
Single Dirac cone: single atom tunneling

Transfer probability as a function of q_y and Δ^*





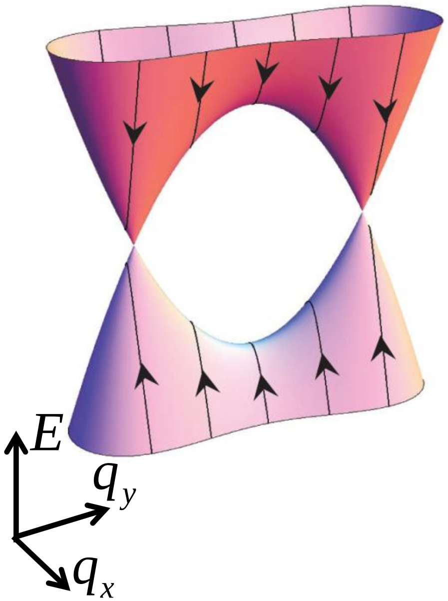


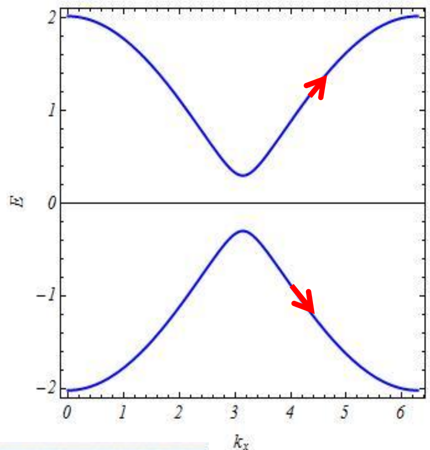


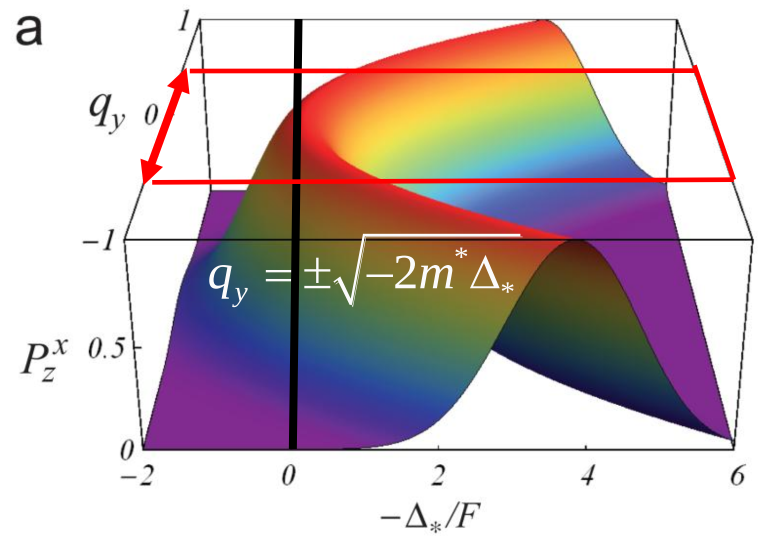
$$P_Z^x = e^{-\pi \frac{\left(\frac{q_y^2}{2m^*} + \Delta_*\right)^2}{c_x F}}$$

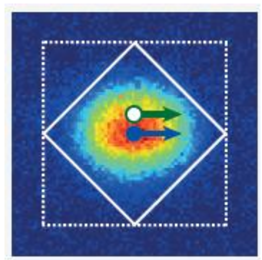
Single Dirac cone: Fermi sea tunneling

Transfer probability
as a function of q_y and Δ^*

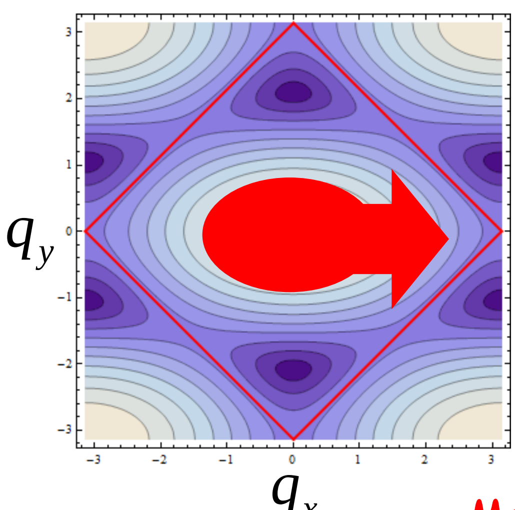




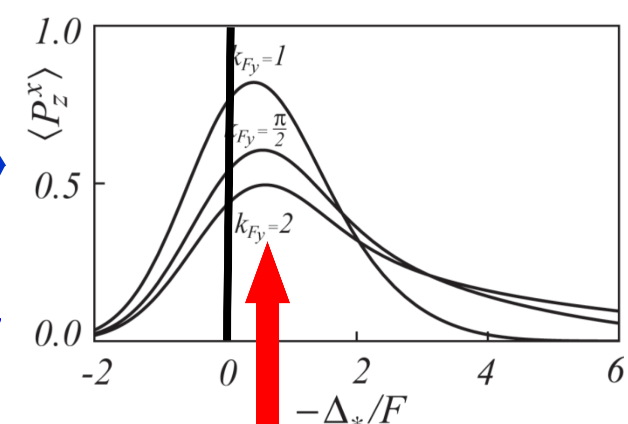




Transfer probability
for a cloud of size k_{Fy}

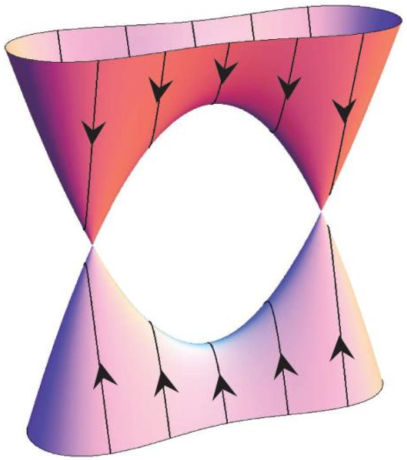


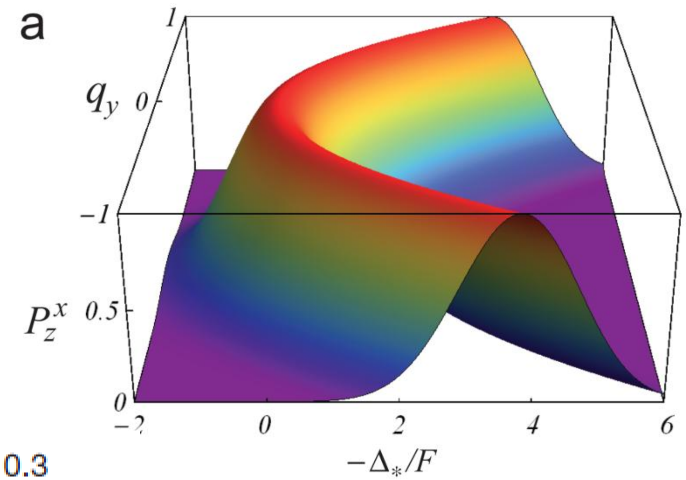
$$\left\langle P_Z^x = e^{-\pi \frac{\left(\frac{q_y^2}{2m^*}+\Delta_*\right)^2}{c_x F}} \right\rangle_{q_y}$$



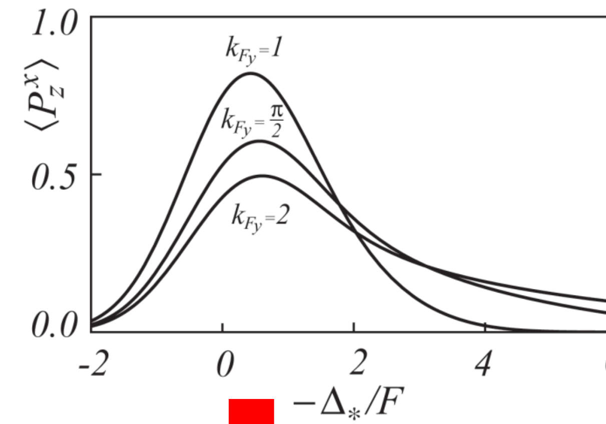
Maximum slightly inside the Dirac phase

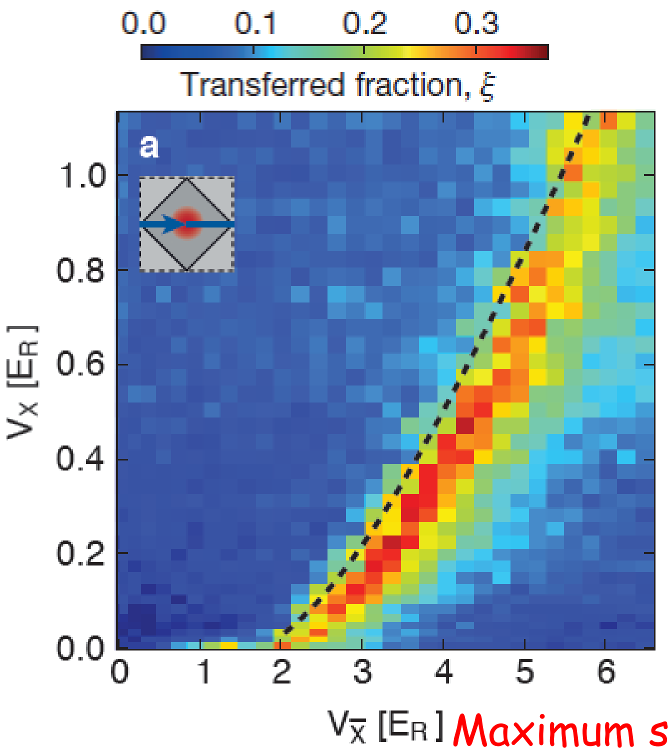
Single Dirac cone: Fermi sea tunneling

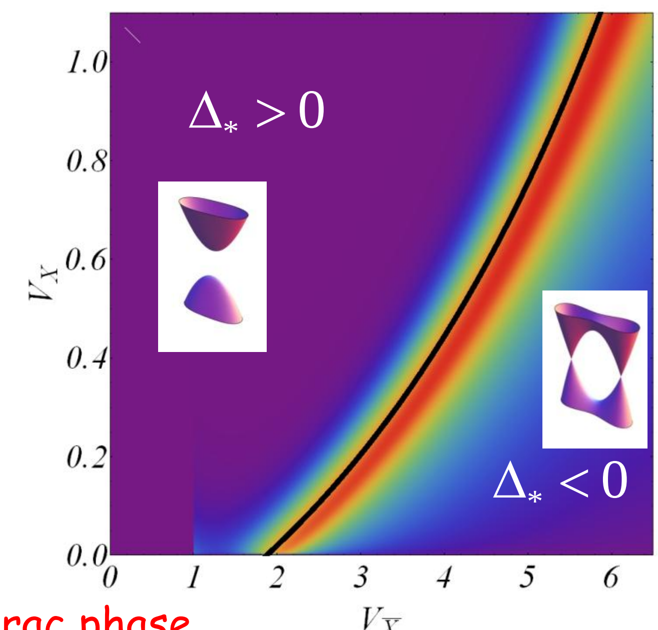




Transfer probability for a cloud of size k_{Fy}



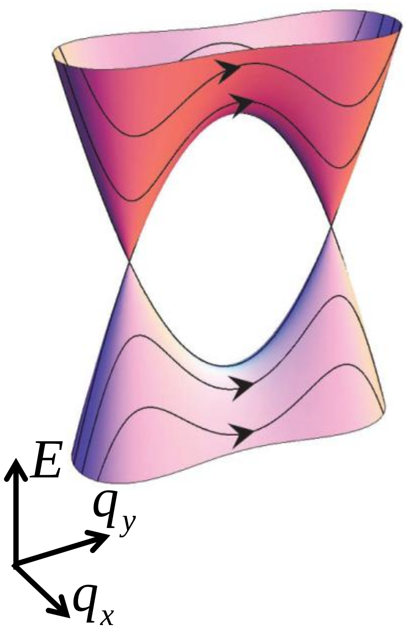


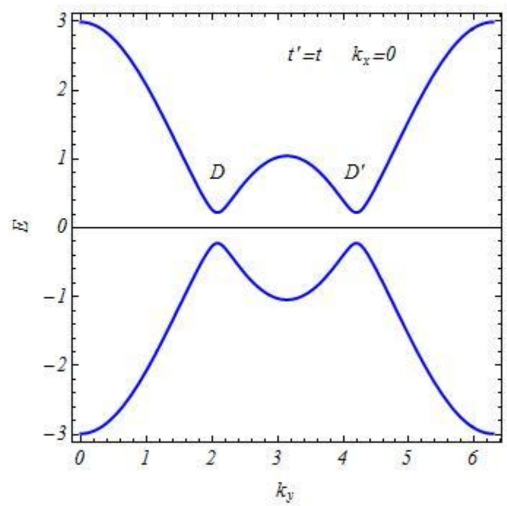


Maximum slightly inside the Dirac phase

Double Dirac cone: Fermi sea tunneling

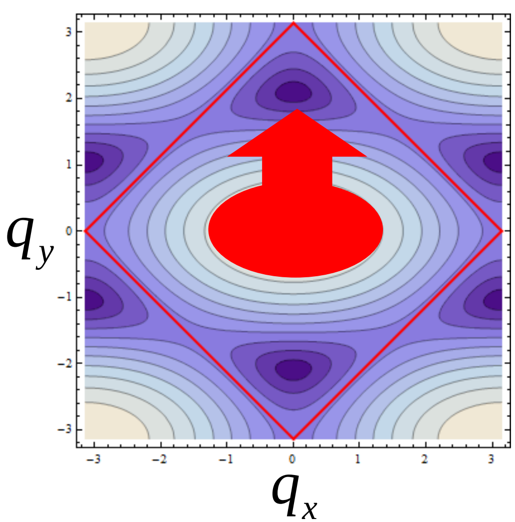
$$P_Z^y = e^{-\pi \frac{c_x^2 q_x^2}{c_y F}}$$





$$\left\langle P_t^y = 2P_Z^y(1 - P_Z^y) \right\rangle_{q_x}$$

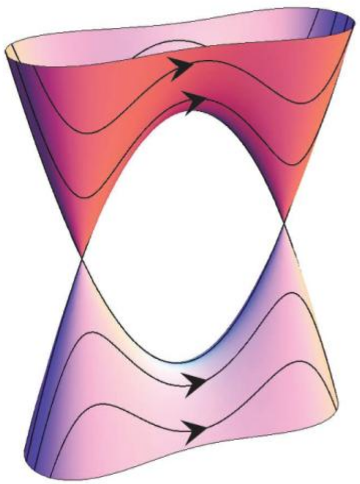
Non-monotonous function of P_Z. Maximum for P_z=1/2

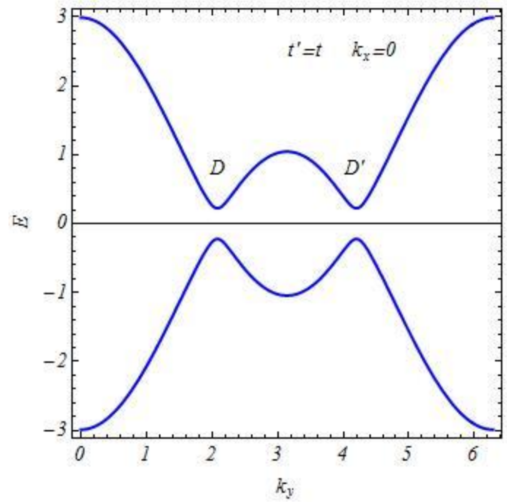


Double Dirac cone: Fermi sea tunneling

$$P_Z^y = e^{-\pi \frac{c_x^2 q_x^2}{c_y F}}$$

$$\left\langle P_t^y = 2P_Z^y(1 - P_Z^y) \right\rangle_{q_x}$$

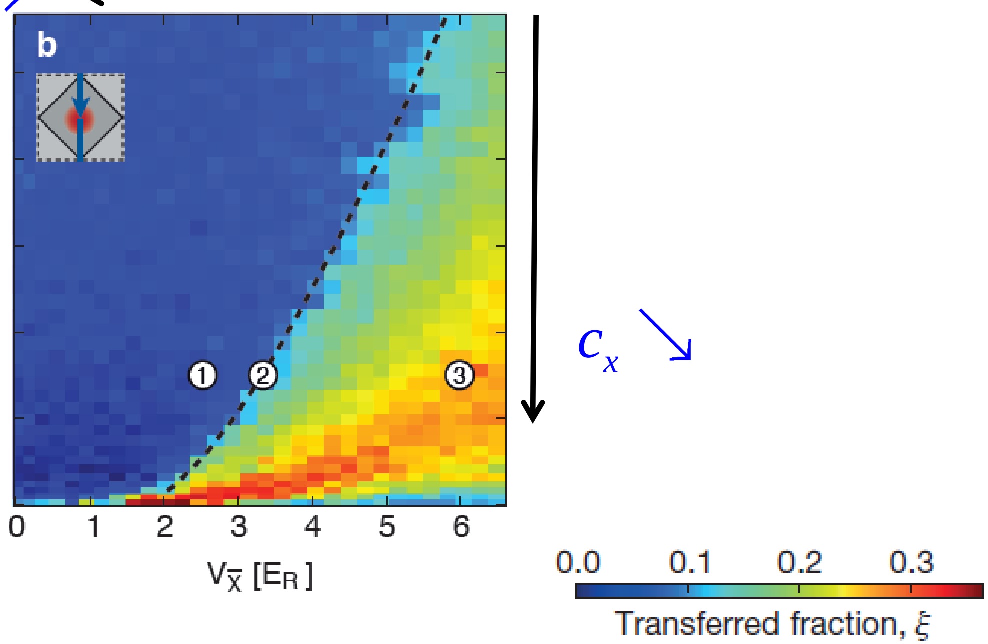


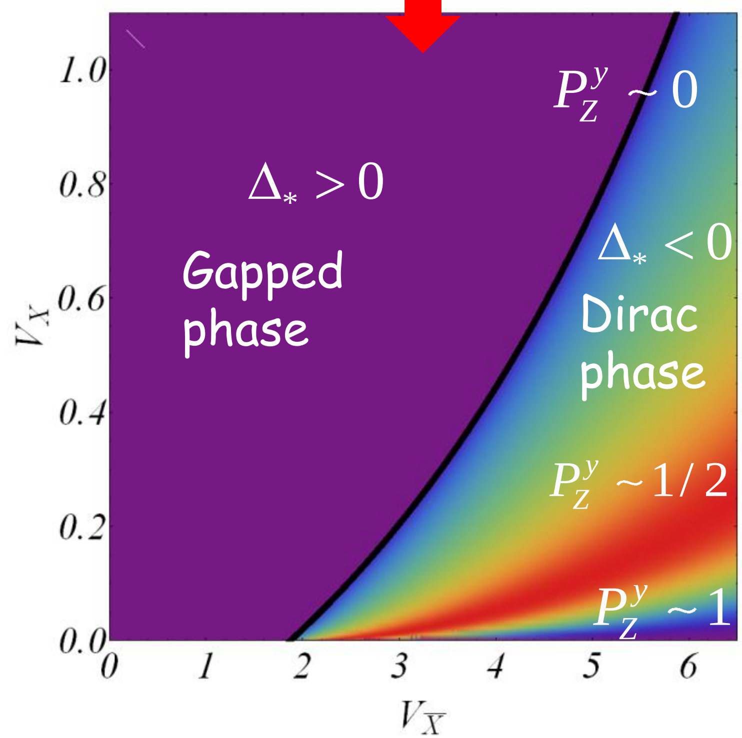


Non-monotonous function of P_Z. Maximum for P_z=1/2

$\Delta_* \nearrow$

$c_x \searrow$



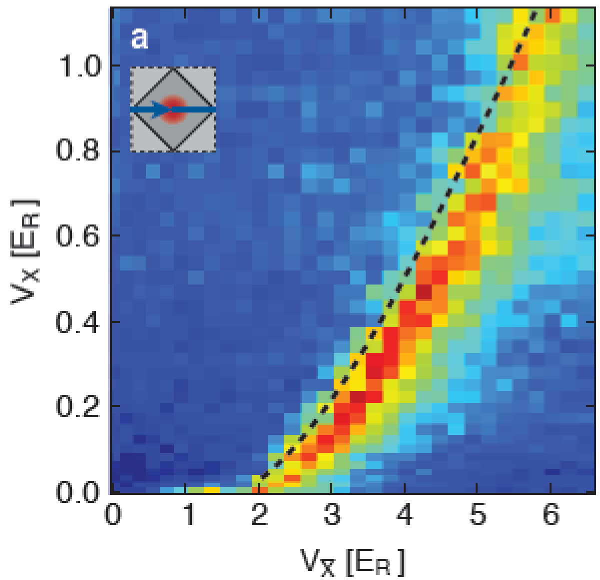


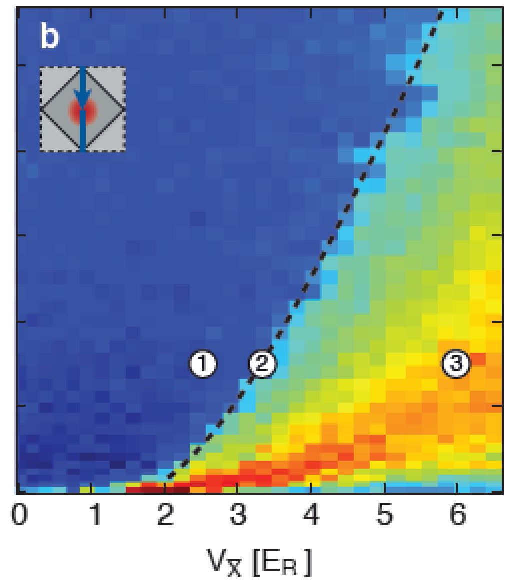
Summary

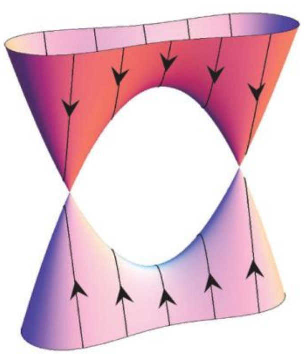
Single Dirac cone

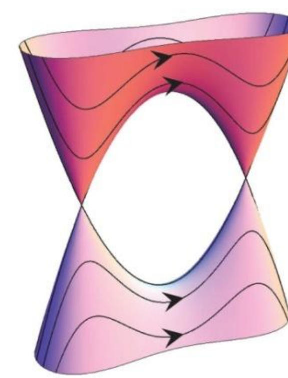
Double Dirac cone

Experiment

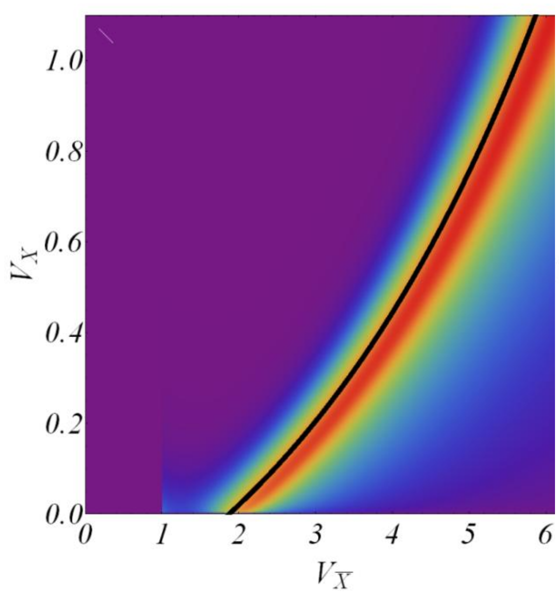


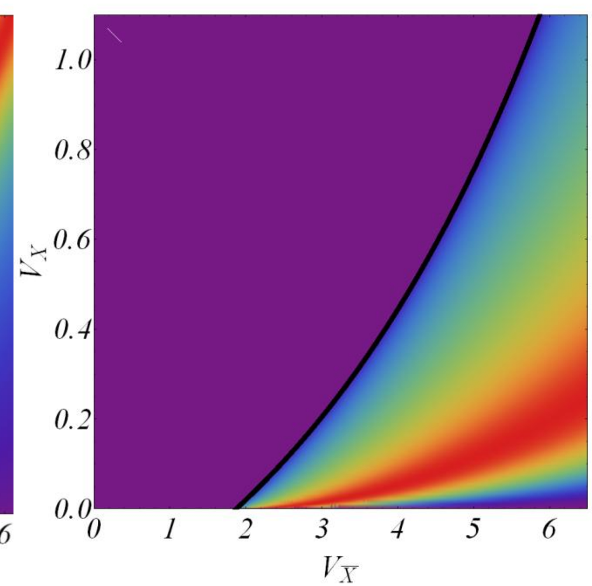






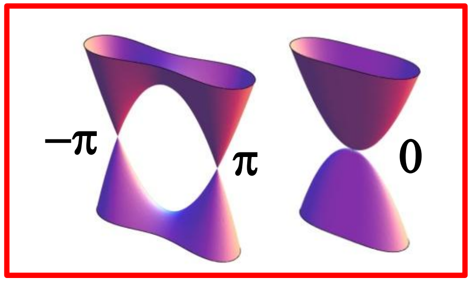
Theory

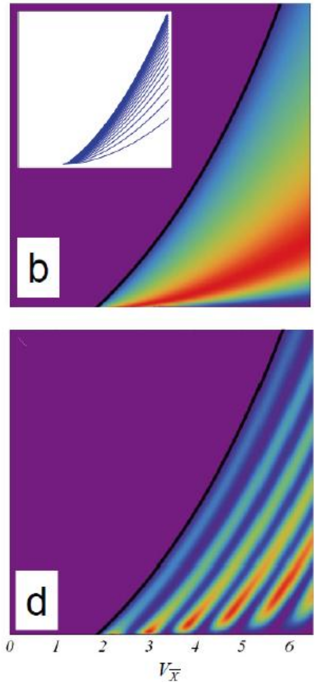




Conclusions and perspectives

Universal description of motion and merging of Dirac points in 2D crystals





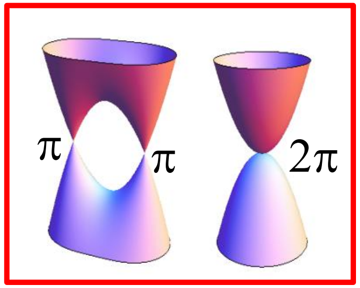
($-\pi$,π) merging : hybrid semi-Dirac spectrum

Cold atoms : Landau-Zener probe of the Dirac points

Interference effects

Condensed matter : New thermodynamic and transport properties

Interaction effects : from Dirac to Schrödinger



Other universality class : (π,π) merging