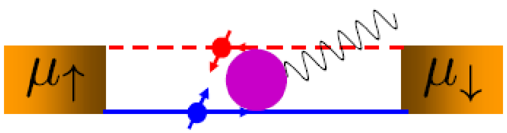
Transport properties of helical Tomonaga Luttinger liquids

IXth Rencontres du Vietnam
Nanophysics: from fundamentals to applications



August 4-10, 2013
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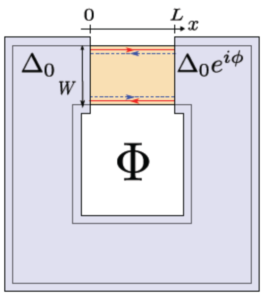

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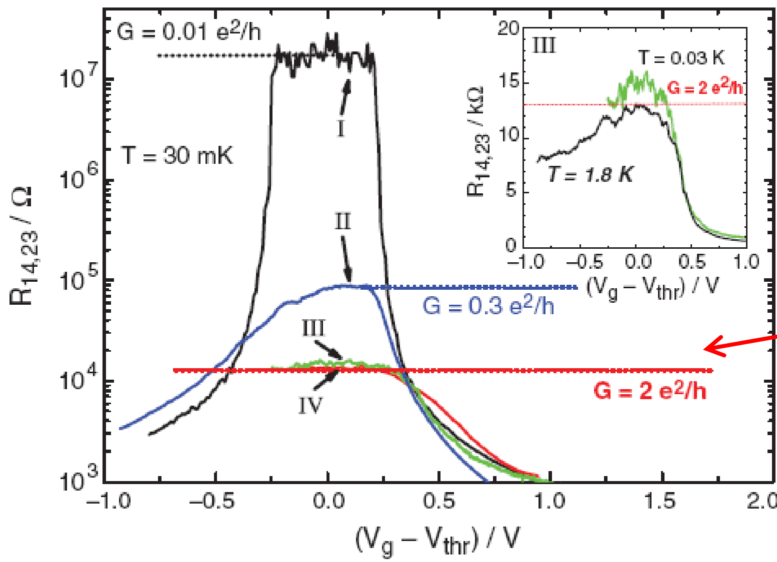
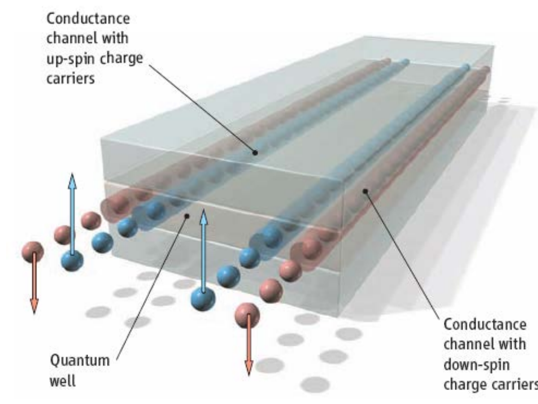
Source of backscattering?

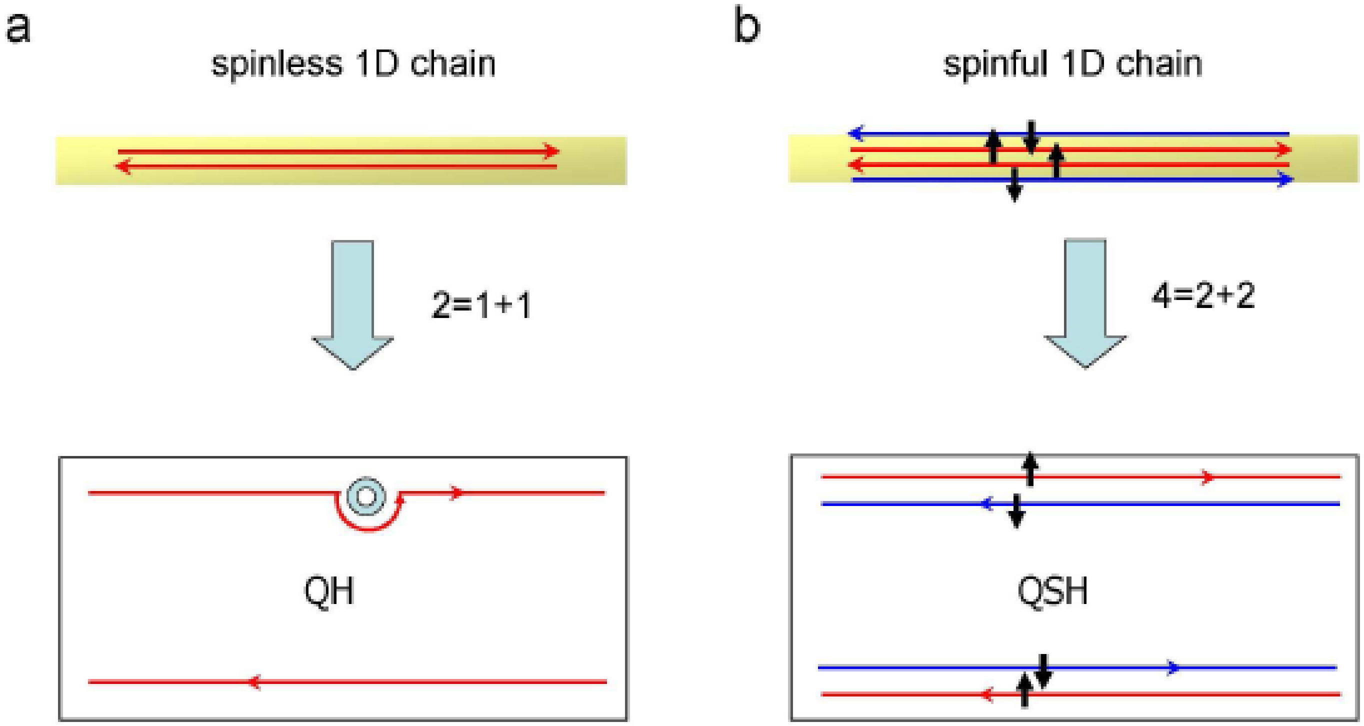


König et al. Science 2007

ultimate sources of backscattering that can alter G?

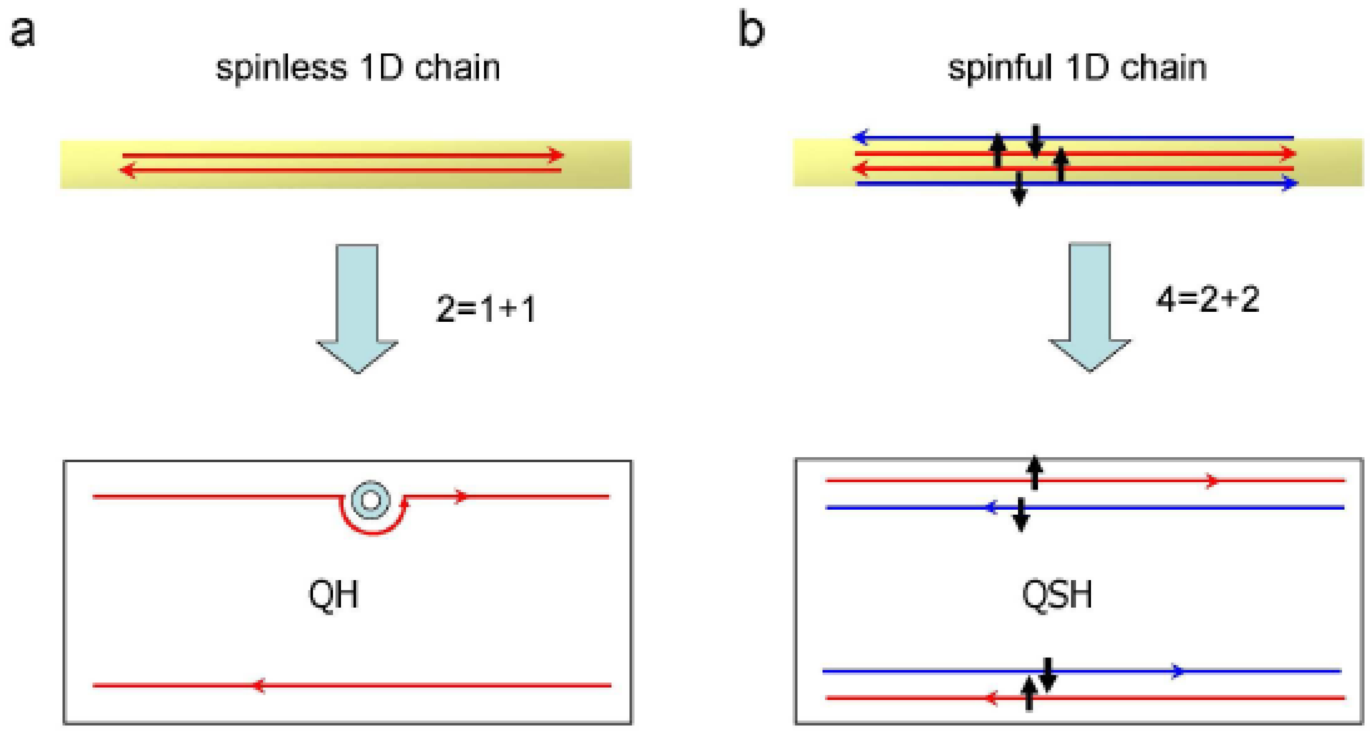


Spin detector



Qi & Zhang Phys Today 2010

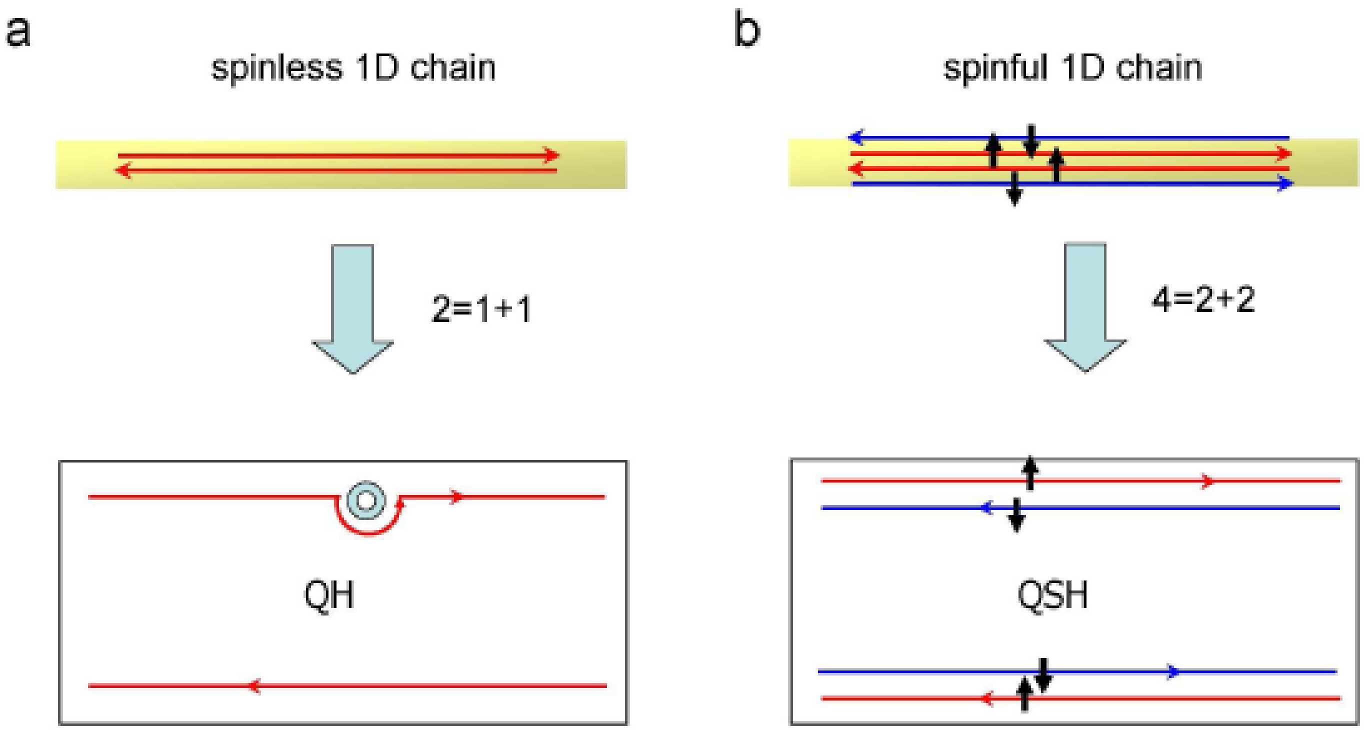
Spin detector



has been used as a perfect charge detector!

Qi & Zhang Phys Today 2010

Spin detector



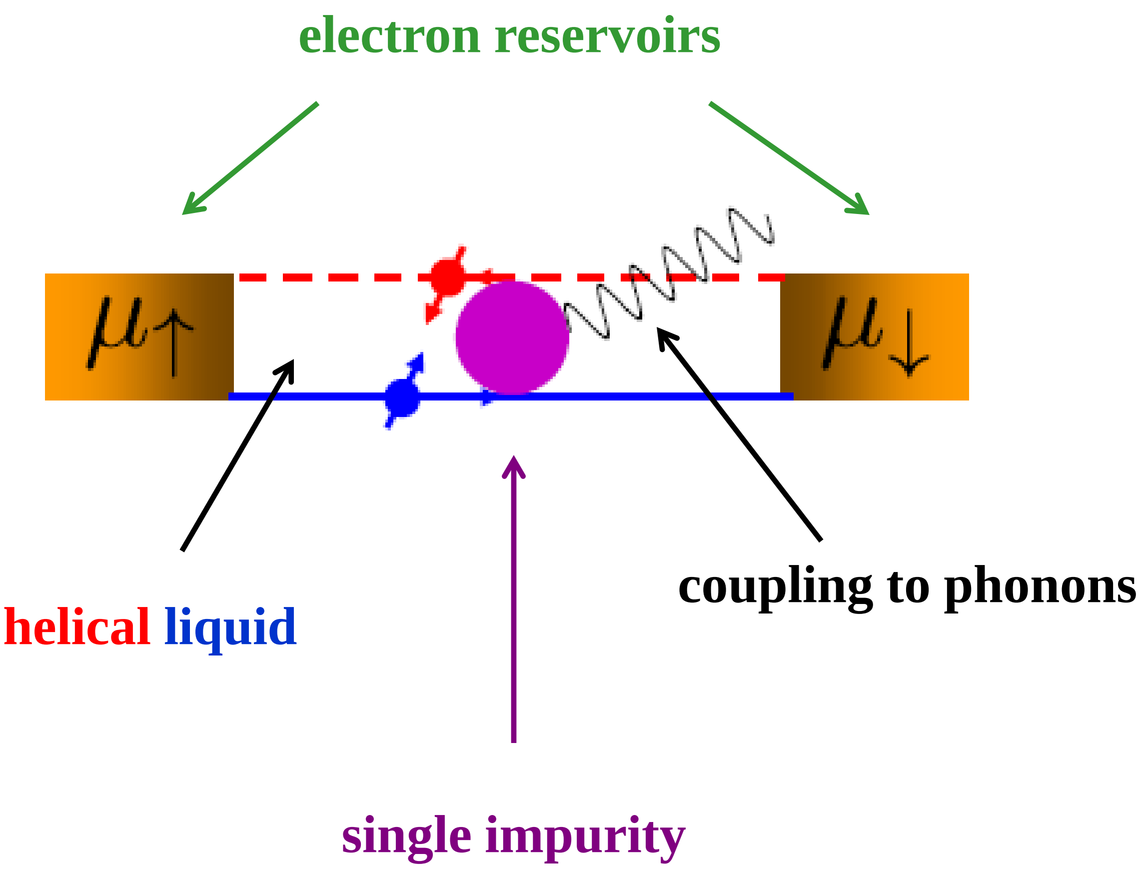
use it as the perfect spin detector!

Qi & Zhang Phys Today 2010

Outline

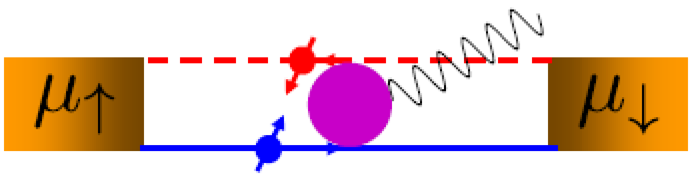
- Phonon-induced backscattering in helical edge states **(2T)**
- RG analysis for scattering off a Rashba impurity in a helical liquid **(2T)**
- Kondo screening cloud of two helical liquids **(4T)**
- Parity measurement in topological Josephson junctions **(0T)**

System



Budich, Dolcini, Recher & BT PRL 2012

System & Hamiltonian



$$\Psi = \left(\Psi_{R\uparrow}, \Psi_{L\downarrow}\right) = \left(\Psi_{+}, \Psi_{-}\right)$$

$$H = H_{hl} + H_{p} + H_{ep} + H_{R}$$

$$H_{hl} = \int dx \Psi^{\dagger}(x)\, p \sigma_{z} \Psi(x)$$

$$H_{p} = \frac{1}{2}\int dx \left[\left(\Pi_{d}(x)\right)^{2} + c^{2}\left(\partial_{x} d(x)\right)^{2}\right]$$

$$H_{ep} = \lambda \int dx \Psi^{\dagger}(x)\, \sigma_{0} \Psi(x)\, \partial_{x} d(x)$$

$$H_{R} = \frac{1}{2}\int dx \Psi^{\dagger}(x)\left(\left\{\alpha_{1}(x), p\right\} + \left\{\alpha_{3}(x), p^{3}\right\}\right)\sigma_{y} \Psi(x)$$

Budich, Dolcini, Recher & BT PRL 2012

No backscattering: □$_1$ term

left mover → p_f^-

right mover → p_i^+

$$M_{if} = \left\langle p_f^-, q_{ph} \middle| H_I G_0 H_I \middle| p_i^+ \right\rangle$$

$$H_I = H_{ep} + H_R$$

Budich, Dolcini, Recher & BT PRL 2012

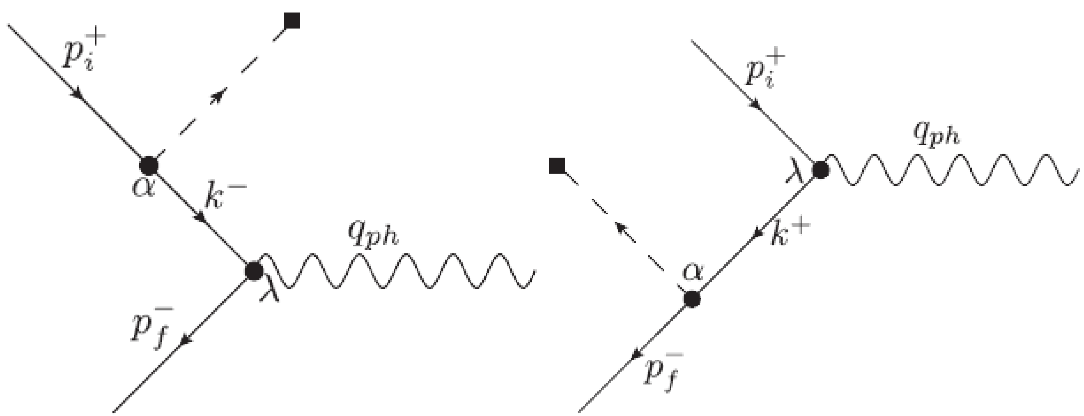
No backscattering: □$_1$ term

left mover — right mover

$$M_{if} = \left\langle p_f^-, q_{ph} \left| H_I G_0 H_I \right| p_i^+ \right\rangle$$

$$H_I = H_{ep} + H_R$$

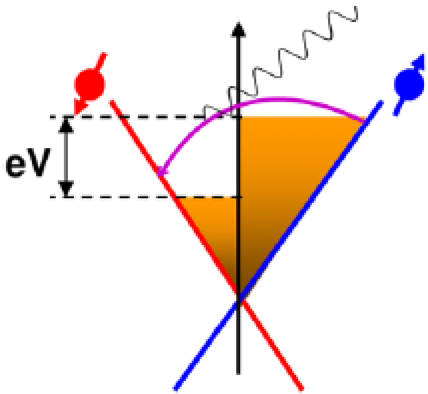
destructive interference of these diagrams:



$$\Rightarrow M_{if} = 0$$

Budich, Dolcini, Recher & BT PRL 2012

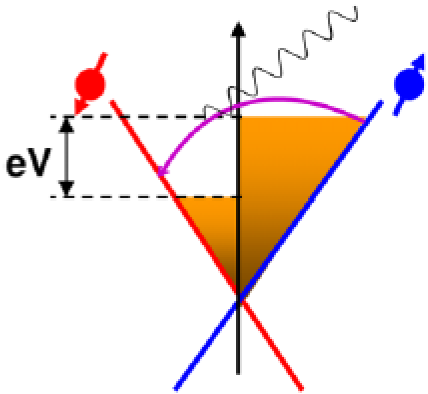
How about the □$_3$-term?



$$M_{if} = \frac{\lambda^2 c}{16\pi}\tilde{\alpha}_3^2 \left(q_{ph} + p_f^- - p_i^+\right)\left|q_{ph}\right|^5$$

Budich, Dolcini, Recher & BT PRL 2012

How about the □$_3$-term?



$$M_{if} = \frac{\lambda^2 c}{16\pi} \tilde{\alpha}_3^2 \left(q_{ph} + p_f^- - p_i^+ \right) \left| q_{ph} \right|^5$$

Fermi‘s golden rule calculation □

$$I_{BS} = 2\pi e \int dp_i^+ \int dp_f^- \int dq_{ph} \left[f^+\left(\varepsilon_i^+\right)\left(1 - f^-\left(\varepsilon_f^-\right)\right) \left| M_{if} \right|^2 \delta\left(\varepsilon_i^+ - \varepsilon_f^- - \omega_{ph}\right) \right]$$

$$I_{BS} = \frac{\alpha_3^2 \lambda^2 e}{672 \pi^2 c^5} V^7$$

$$\left. \frac{dI_{BS}}{dV} \right|_{V=0} \propto T^6$$

Budich, Dolcini, Recher & BT PRL 2012

Presence of ee interactions

calculation of
average current:

$$I\left(x,t\right)=\frac{e}{\sqrt{\pi}}\partial_t\left\langle\varphi\left(x,t\right)\right\rangle$$

on basis of generating functional:

$$Z\left[J\right]=\int D\left(\varphi,\theta,d\right)e^{iS_0-i\int_C H_R+i\frac{e}{\sqrt{\pi}}E^T\sigma_3\varphi+\frac{i}{\sqrt{2}}J^T\varphi}$$

along Keldysh contour

 $$I=\frac{e^2}{h}V$$ up to second order in $_1$

Budich, Dolcini, Recher & BT PRL 2012

Take home messages

- $\square_1$-Rashba impurity does not lead to inelastic backscattering (in the presence of a simple phonon bath) at quadratic order

- holds in the presence and the absence of ee interactions in the helical liquid

- $\square_3$-Rashba impurity term can lead to inelastic backscattering at finite voltage or temperature.

Outline

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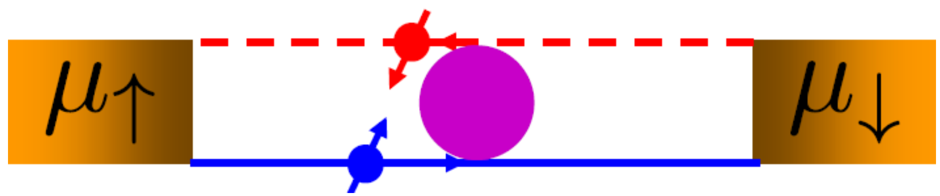
“Simpler” Hamiltonian



$$H = H_0 + H_I + H_R$$

$$H_0 = \int dx \sum_{\eta=\pm} \Psi_\eta^\dagger(x)\left(-i\eta v_F \partial_x - E_F\right)\Psi_\eta(x)$$

$$H_I = \int dx \int dx' \Psi_+^\dagger(x)\Psi_-^\dagger(x') g_2(x-x')\Psi_-(x')\Psi_+(x)$$

$$H_R = \int dx {\color{red}\alpha(x)}\left[\left(\partial_x \Psi_+^\dagger\right)\Psi_- - \Psi_+^\dagger\left(\partial_x \Psi_-\right)\right](x) + \text{H.c.}$$

motivation: *Ström, Johannesson & Japaridze* PRL 2010 *Crépin, Budich, Dolcini, Recher & BT* PRB 2012

Generation of 2p backscattering?

$$H = H_0 + H_I + H_R$$
$$H_0 = \int dx \sum_{\eta=\pm} \Psi_\eta^\dagger(x)\left(-i\eta v_F \partial_x - E_F\right)\Psi_\eta(x)$$
$$H_I = \int dx \int dx' \Psi_+^\dagger(x)\Psi_-^\dagger(x') g_2(x-x')\Psi_-(x')\Psi_+(x)$$
$$H_R = \int dx \alpha(x)\left[\left(\partial_x \Psi_+^\dagger\right)\Psi_- - \Psi_+^\dagger\left(\partial_x \Psi_-\right)\right](x) + \text{H.c.}$$

Question: How is (inelastic) two-particle backscattering generated by this Hamiltonian?

$$H_{2p}^{in} = \gamma_{2p} \int dx \left[\left(\partial_x \Psi_+^\dagger\right)\Psi_+^\dagger\left(\partial_x \Psi_-\right)\Psi_-\right](x_0) + \text{H.c.}$$

Crépin, Budich, Dolcini, Recher & BT PRB 2012

RG for interacting fermions

Partition function: $$Z = \int D\Psi_{\pm}^{*} D\Psi_{\pm} e^{-S}$$

integrate out fields on the momentum shell: $$\Lambda / \left(1 + dl\right) < v_F \left|\eta k - k_F\right| < \Lambda$$

Crépin, Budich, Dolcini, Recher & BT PRB 2012

RG for interacting fermions

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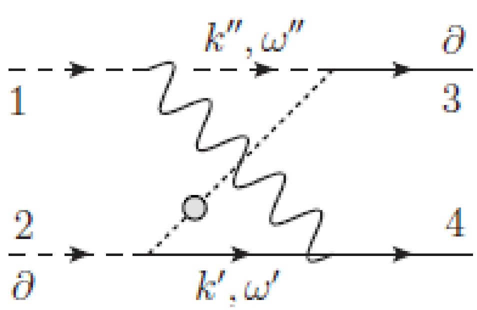
integrate out fields on the momentum shell: $$\Lambda / \left(1 + dl\right) < v_F \left|\eta k - k_F\right| < \Lambda$$

flow equations:



$$\frac{d\gamma_{2p}}{dl} = -3\gamma_{2p}\left(l\right) + \frac{\alpha\left(l\right)^2}{v_F \Lambda} \frac{g_2}{2\pi v_F}$$

$$\frac{d\alpha}{dl} = -\alpha\left(l\right)$$

$$\gamma_{2p}\left(l = 0\right) = 0$$

Crépin, Budich, Dolcini, Recher & BT PRB 2012

Next: Bosonization

$$H_{hl} = H_0 + H_I = \frac{v}{2\pi}\int dx\left[K\left(\partial_x\theta\right)^2 + \frac{1}{K}\left(\partial_x\phi\right)^2\right]$$

with bosonization identity:

$$\Psi_\pm\left(x\right) = \frac{\kappa_\pm}{\sqrt{2\pi a}} e^{\pm ik_F x} e^{-i\left(\pm\phi-\theta\right)}$$

and bosonized Rashba Hamiltonian:

$$H_R = i\kappa_+\kappa_-\int dx \frac{\alpha\left(x\right)}{\pi a}\left(\frac{2\pi a}{L}\right)^K :\partial_x\theta\left(x\right)\left(e^{-i2\phi\left(x\right)}e^{i2k_F x} + e^{i2\phi\left(x\right)}e^{-i2k_F x}\right):$$

Crépin, Budich, Dolcini, Recher & BT PRB 2012

Real space RG

RG transformation in real space:

$$a \rightarrow a' = \left(1 + dl\right) a$$

$$Z = \mathrm{Tr}\left[e^{-\beta H_0} \hat{U}\left(\beta, 0\right)\right]$$

$$\frac{d\tilde{\gamma}_{2p}^{in}}{dl} = \left(1 - 4K\right) \tilde{\gamma}_{2p}^{in}\left(l\right) + \left(1 - \frac{1}{K}\right)\left(1 - 2K\right) \tilde{\alpha}\left(l\right)^2$$

$$\frac{d\tilde{\alpha}}{dl} = -K \tilde{\alpha}\left(l\right)$$

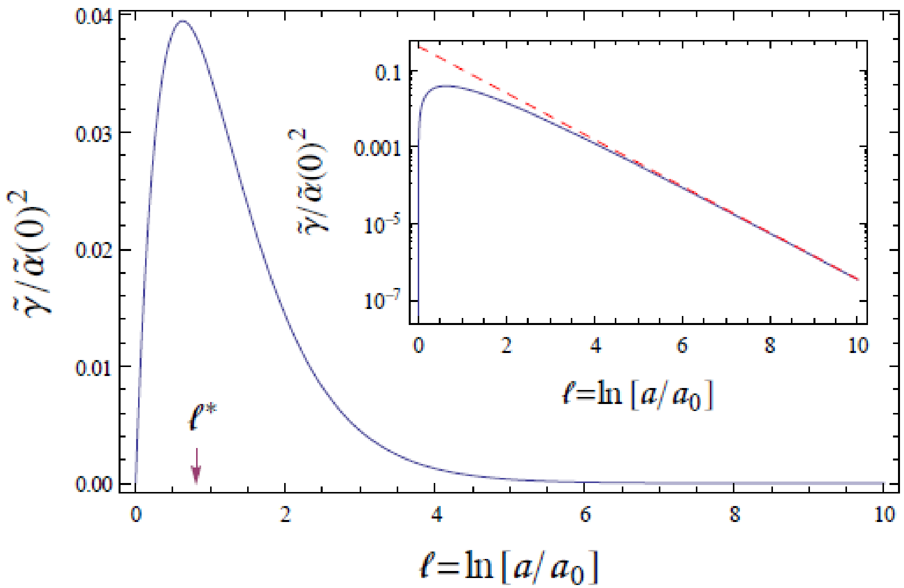
with $\tilde{\gamma}_{2p}^{in}\left(l = 0\right) = 0$

Important consequence in the absence of interactions:

$\tilde{\gamma}_{2p}^{in}\left(l\right) = 0$ at any scale l

Crépin, Budich, Dolcini, Recher & BT PRB 2012

Surprising T-dependence



$$\Rightarrow \frac{\delta G}{G_0} \sim \begin{cases} \left(a_0 T / v\right)^{4K} & \text{if } \frac{1}{2} < K < 1 \\ \left(a_0 T / v\right)^{8K-2} & \text{if } \frac{1}{4} < K < \frac{1}{2} \end{cases}$$

For weak interactions, we obtain a **T^4 correction**!

related work: *Schmidt, Rachel, von Oppen & Glazman* PRL 2012**;** *Lezmy, Oreg & Berkooz* PRB 2012

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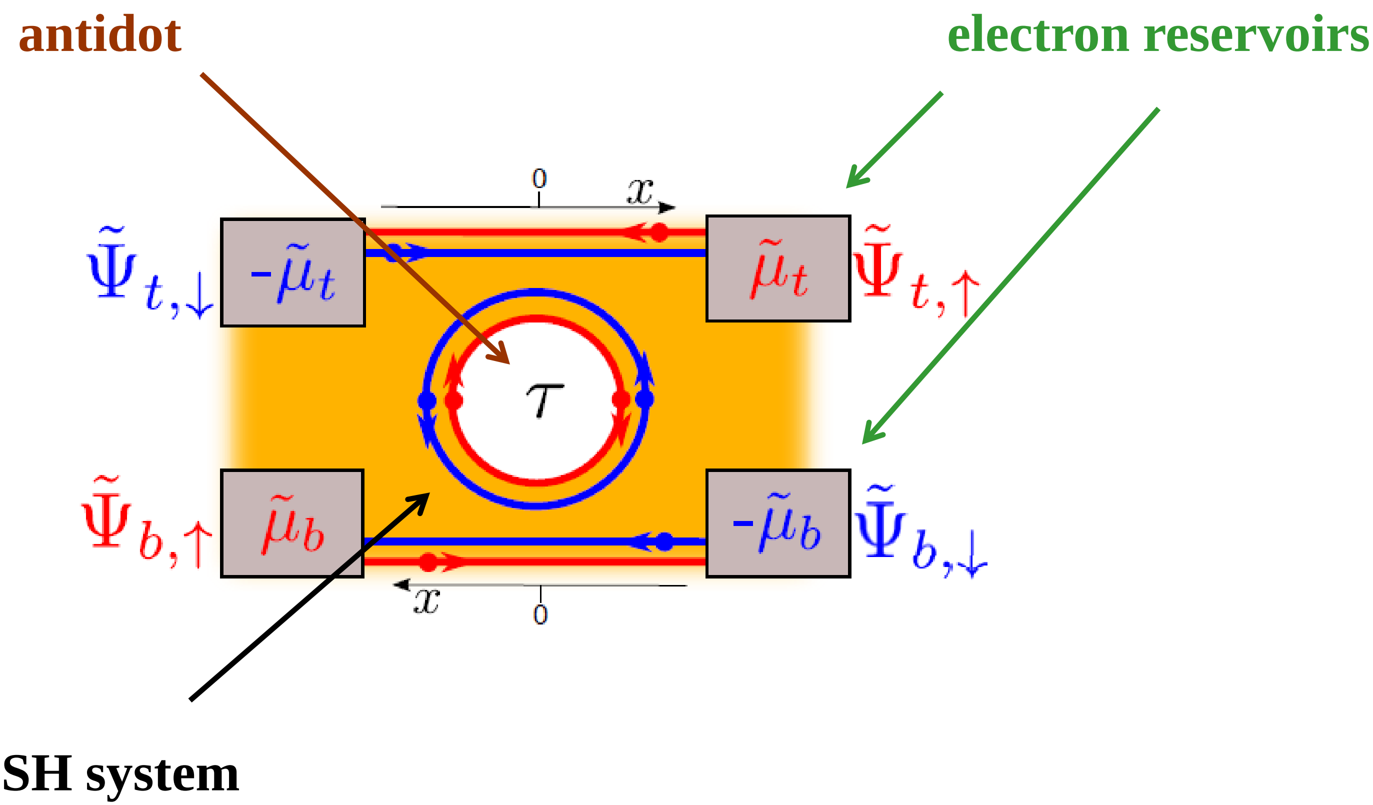
Setup



Posske, Liu, Budich & BT PRL 2013

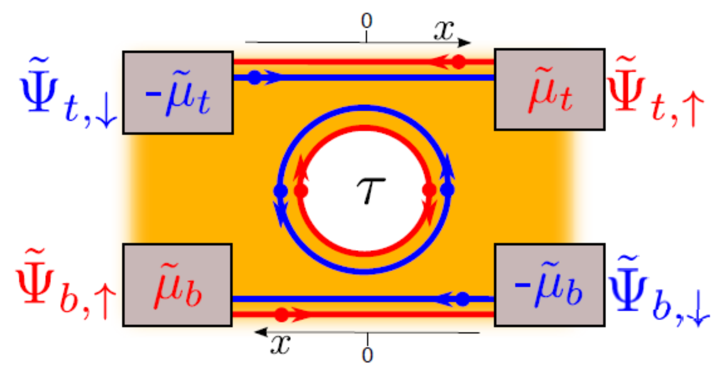
Setup & Hamiltonian

$$H = \sum_{a=t,b} H_a + H_{KXY,a} + H_{KZ,a}$$

$$H_a = \frac{v_a}{2}\int dx \sum_{\sigma=\pm}\left(\partial_x \varphi_{a,\sigma}(x)\right)^2$$

$$H_{KXY,a} = \frac{1}{4\pi a_0} J_a^{\perp} \kappa_{a\uparrow}^{\dagger} \kappa_{a\downarrow} e^{i\sqrt{2K_a}\varphi_{a,+}} \tau^- + H.c.$$

$$H_{KZ,a} = \frac{J_a^z}{8\pi}\sqrt{\frac{2}{K_a}}\partial_x \varphi_{a,+}(0)\,\tau^z$$



with non-equilibrium operator:

$$Y = \sum_{a=t,b} \tilde{\mu}_a \left(\tilde{N}_{a,\uparrow} - \tilde{N}_{a,\downarrow}\right)$$

Posske, Liu, Budich & BT PRL 2013

Toulouse points

- Emery-Kivelson rotation

$$U = \exp\left(i \sum_a \lambda_{a,+} \varphi_{a,+}(0) \tau^z \right)$$

- Transform bosonic fields

$$\phi_j = \sum_{a,\sigma} M_{j,a,\sigma} \varphi_{a,\sigma}$$

- Refermionize the Hamiltonian

⇒ two sets of Toulouse points

$$\text{type A: } K_t + K_b = 2$$
$$\text{type B: } K_t + K_b = 1$$

Posske, Liu, Budich & BT PRL 2013

Two (new) Toulouse points

Type A:

$$\tilde{H}_A = -\mu_x N_2 - \mu_s N_4 + H_0 + \frac{1}{\sqrt{8\pi a_0}}\left(J_L^\perp \Psi_4^\dagger\left(0\right)c + J_R^\perp \Psi_4\left(0\right)c + H.c.\right)$$

$$c = \kappa_2^\dagger \tau^-$$

Type B:

$$\tilde{H}_B = -\left(\mu_x - \mu_s\right)N_2 - \left(\mu_x + \mu_s\right)N_4 + H_0 + \frac{1}{\sqrt{8\pi a_0}}\left(J_L^\perp \Psi_4^\dagger\left(0\right)c + J_R^\perp \Psi_2^\dagger\left(0\right)c + H.c.\right)$$

$$c = \tau^-$$

$$\mu_x = \tilde{\mu}_t + \tilde{\mu}_b \qquad \mu_s = \tilde{\mu}_t - \tilde{\mu}_b$$

Posske, Liu, Budich & BT PRL 2013

Analytical result: Kondo cloud

$$\chi_a^z\left(x, K_{t,b}\right) = \left\langle \delta\tilde{\rho}_{spin,a}\left(x\right)\delta\tau^z\right\rangle$$

Nagaoka Phys. Rev. 1965
Müller-Hartmann Z. Physik 1969

Type A:

$$\chi_a^z\left(x, K_{t,b}\right) = \chi_a^z\left(x, 1\right)$$

$$\chi_t^z\left(x, K_{t,b}\right) = -\chi_b^z\left(x, K_{t,b}\right)$$

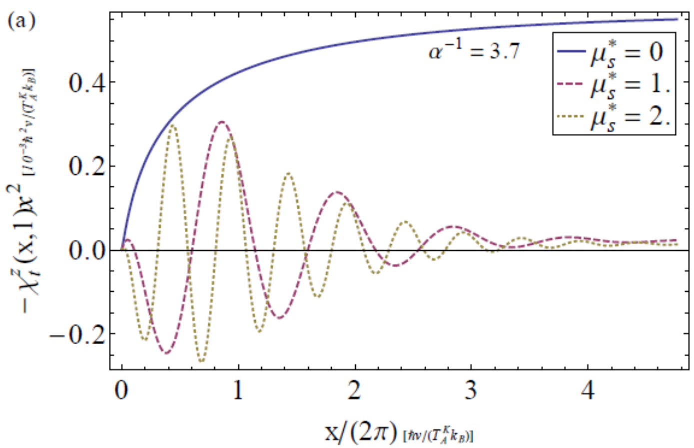
Type B:

$$\chi_b^z\left(x, K_{t,b}\right) = \frac{1}{\sqrt{K_b}}\left[\left(-\sqrt{K_b} + \sqrt{K_t}\right)\chi_b^z\left(x, \frac{1}{2}\right) + \left(\sqrt{K_b} + \sqrt{K_t}\right)\chi_t^z\left(x, \frac{1}{2}\right)\right]$$

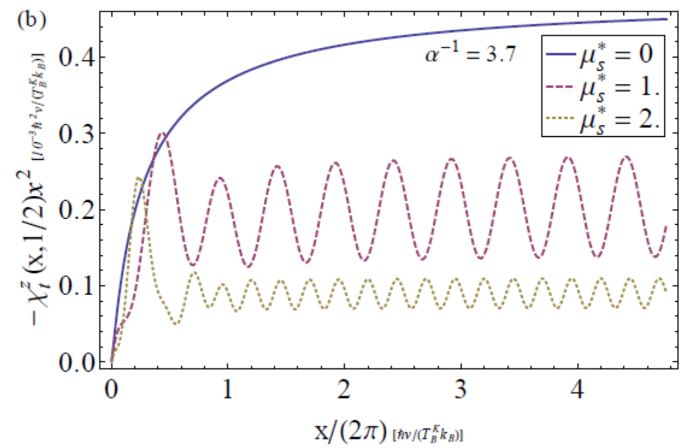
Posske, Liu, Budich & BT PRL 2013

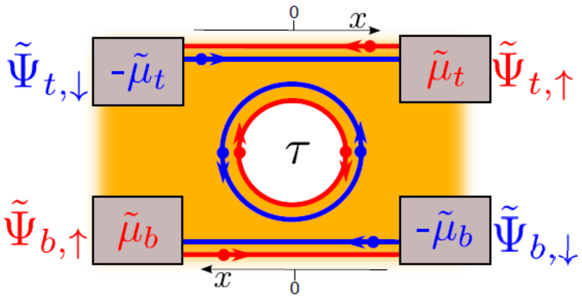
Non-equilibrium Kondo cloud

Type A:



Type B:





Posske, Liu, Budich & BT PRL 2013

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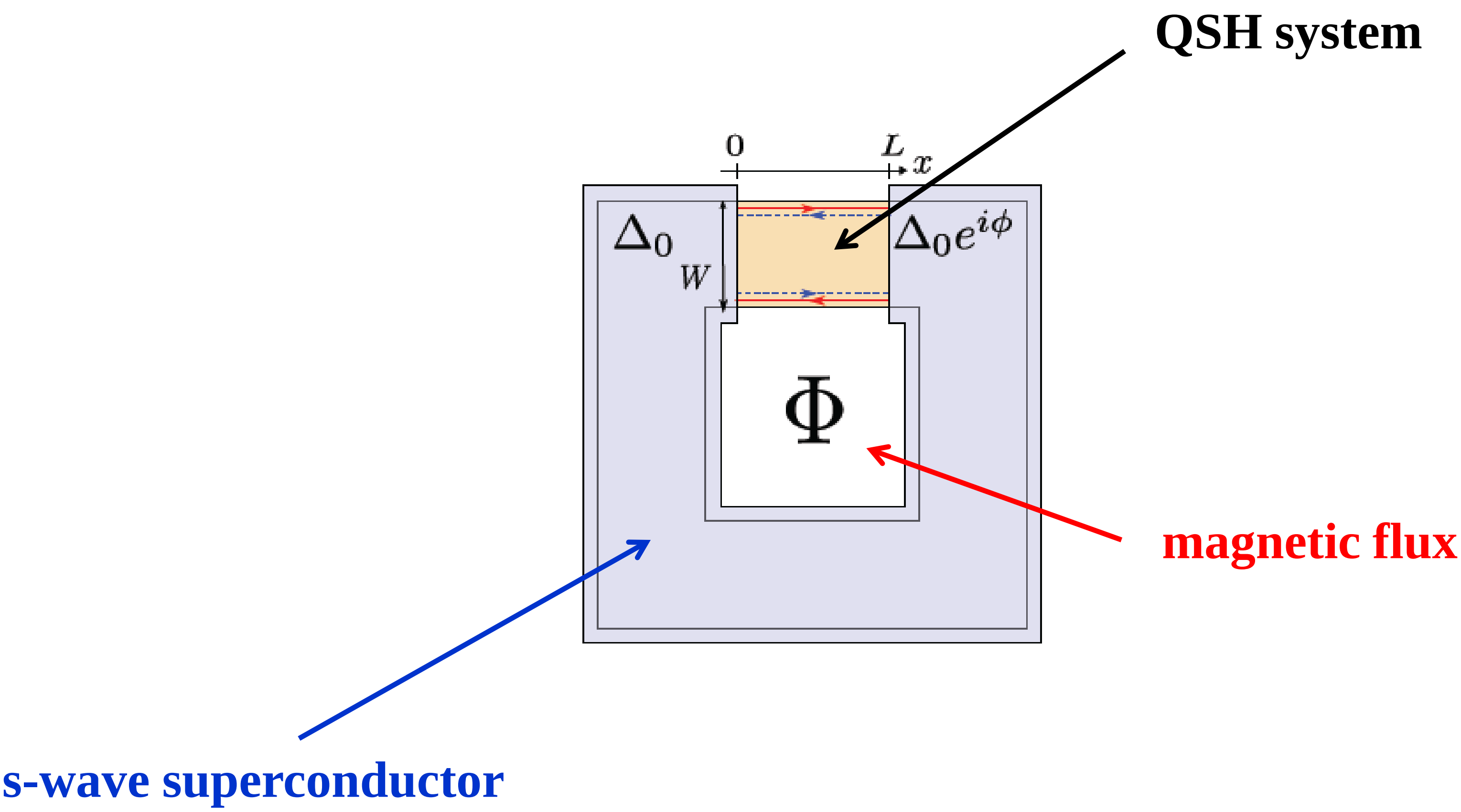
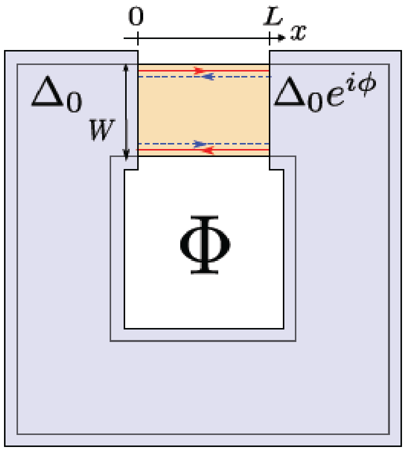


inspired by: *Fu & Kane* PRB 2009

Crépin & BT arXiv 2013

JJ current vs. parity pumping

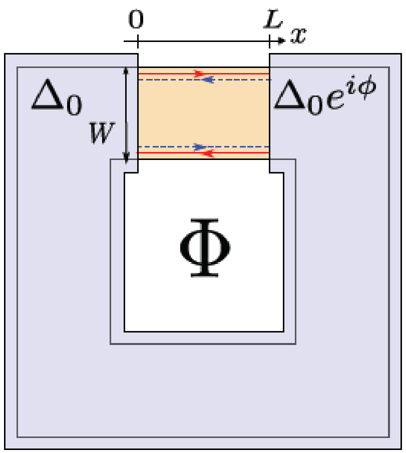


$$J_{\sigma,\sigma'}\left[\phi\right] = I_{up,\sigma}\left[\phi\right] + I_{down,\sigma'}\left[\phi\right]$$

total fermion parity number: $\Sigma = \sigma \cdot \sigma'$

Crépin & BT arXiv 2013

JJ current vs. parity pumping

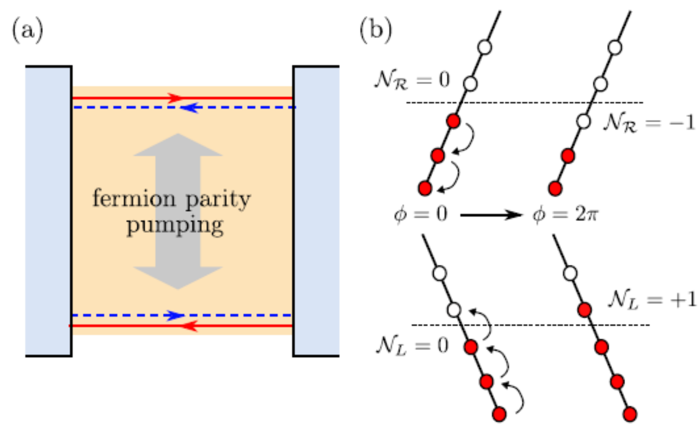


$$J_{\sigma,\sigma'}\left[\phi\right] = I_{up,\sigma}\left[\phi\right] + I_{down,\sigma'}\left[\phi\right]$$

total fermion parity number: $\Sigma = \sigma \cdot \sigma'$

$$\phi \to \phi + 2\pi$$

$$\phi / 2\pi = \Phi / \Phi_0 \quad \text{with } \Phi_0 = h / 2e$$



Fu & Kane PRB 2006

Crépin & BT arXiv 2013

Short junction limit

$$L \ll \xi = {}^{v_F}\!/_{\Delta_0}$$

$$I_{up,\pm}[\phi] = I_{down,\pm}[\phi] = \pm I_c \sin\left({}^{\phi}\!/_{2}\right)$$

$$I_c = {}^{\Delta_0}\!/_{2}$$

Crépin & BT arXiv 2013

Short junction limit

$$L \ll \xi = v_F / \Delta_0$$

$$I_{up,\pm}[\phi] = I_{down,\pm}[\phi] = \pm I_c \sin\left(\phi/2\right)$$

$$I_c = \Delta_0 / 2$$

odd parity:

$$J_{+,-}[\phi] = J_{-,+}[\phi] = 0$$

even parity:

$$J_{+,+}[\phi] = -J_{-,-}[\phi] = 2I_c \sin\left(\phi/2\right)$$

 parity detector

Crépin & BT arXiv 2013

Long junction limit

$$L \gg \xi = v_F / \Delta_0$$

twisted boundary conditions:

$$\psi_{R,\uparrow}\left(x+2L,t\right) = -e^{-i\phi}\psi_{R,\uparrow}\left(x,t\right)$$
$$\psi_{L,\downarrow}\left(x+2L,t\right) = -e^{-i\phi}\psi_{L,\downarrow}\left(x,t\right)$$
$$\psi_{R,\uparrow}\left(x,t\right) = -i\psi^{\dagger}_{L,\downarrow}\left(-x,t\right)$$

$x \in \left[-L, L\right]$

Maslov, Stone, Goldbart & Loss PRB 1996

 ABS spectrum:

$$\varepsilon_n = \frac{\pi v_F}{L}\left(n + \frac{1}{2} \pm \frac{\phi}{2\pi}\right) \quad \text{with } n \in \mathbb{Z}$$

valid for $\varepsilon \ll \Delta_0$

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Bosonization again

number of fermions w/ respect to Fermi sea

chiral bosonic field

$$\psi_{R,\uparrow}\left(x,t\right) = F_{R}\,\frac{1}{\sqrt{2\pi a}}\,e^{i\frac{\pi}{L}\left(\hat{N}_R+\frac{1}{2}-\frac{\phi}{2\pi}\right)x}\,e^{-i\tilde{\phi}_R\left(x\right)}$$

Klein factors

Crépin & BT arXiv 2013

Bosonization again

number of fermions w/ respect to Fermi sea

chiral bosonic field

$$\psi_{R,\uparrow}\left(x,t\right) = F_R \frac{1}{\sqrt{2\pi a}} e^{i\frac{\pi}{L}\left(\hat{N}_R + \frac{1}{2} - \frac{\phi}{2\pi}\right)x} e^{-i\tilde{\phi}_R\left(x\right)}$$

Klein factors

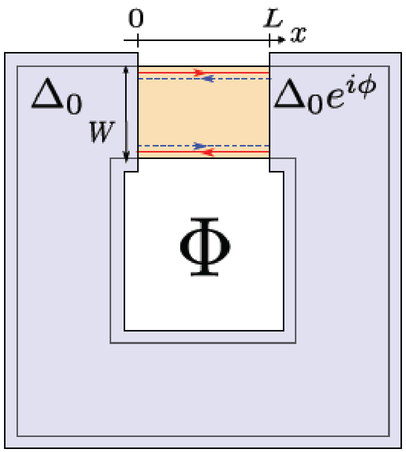
$$\tilde{\phi}_R\left(x\right) = -\sum_{q>0}\sqrt{\frac{\pi}{Lq}}\left(e^{iqx}b_{R,q} + e^{-iqx}b^{\dagger}_{R,q}\right)e^{-aq/2}$$

Crépin & BT arXiv 2013

Hamiltonian

$$H_{up} = \frac{v_F \pi}{2L}\left(\hat{N}_R - \frac{\phi}{2\pi}\right)^2 + v_F \sum_{q>0} q b^\dagger_{R,q} b_{R,q}$$

$$H_{down} = \frac{v_F \pi}{2L}\left(\hat{N}_L + \frac{\phi}{2\pi}\right)^2 + v_F \sum_{q>0} q b^\dagger_{R,q} b_{R,q}$$

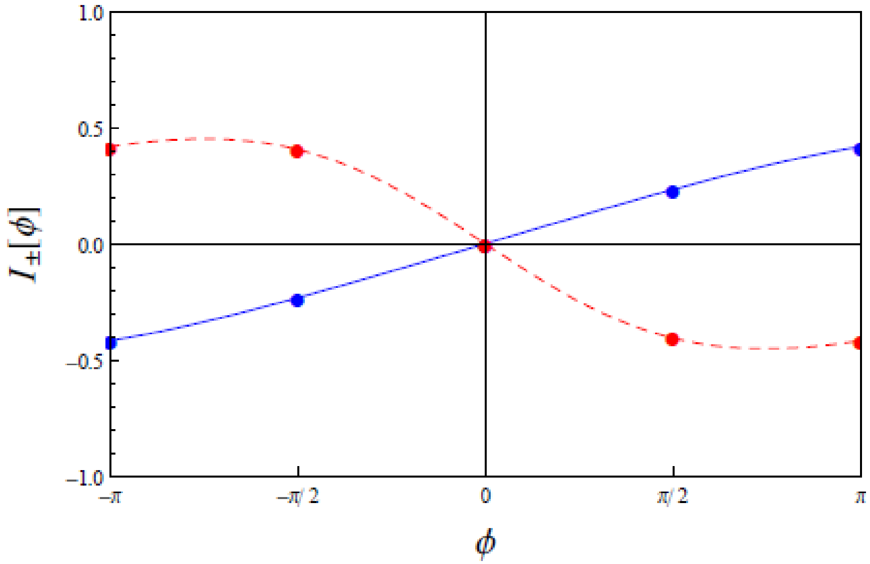


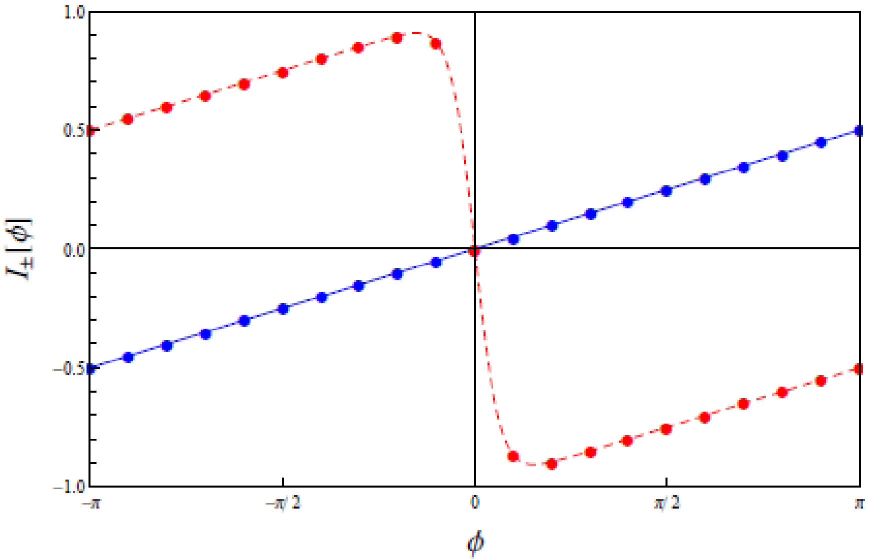
$\phi \rightarrow \phi + 2\pi$ parity pumping

Crépin & BT arXiv 2013

Josephson current

$$I_{up,\pm}\left[\phi\right] = e\frac{v_F}{L}\frac{\phi}{2\pi} - \frac{2e}{\hbar\beta}\partial_\phi \ln\theta_{3/2}\left[i\beta\frac{\hbar\pi v_F}{2L}\frac{\phi}{\pi}, e^{-2\beta\frac{\hbar\pi v_F}{L}}\right]$$





related work: *Beenakker, Pikulin, Hyart, Schomerus & Dahlhaus* PRL 2013

Crépin & BT arXiv 2013

Take home messages

- Critical current:

$$2I_c \text{ even case} \qquad I_c \text{ odd case} \qquad \text{with } I_c = \frac{ev_F}{L}$$

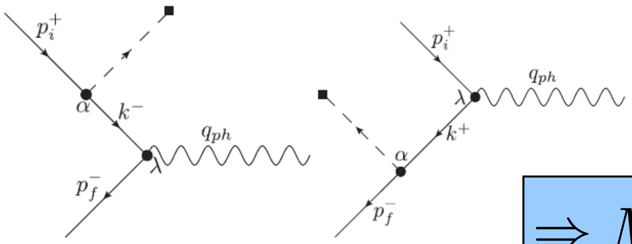
- Phase dependence:

$$J[\phi] = J[\phi + 4\pi] \text{ even case}$$
$$J[\phi] = J[\phi + 2\pi] \text{ odd case}$$

 parity detector

- Phase dependence could be modified by disorder

Summary

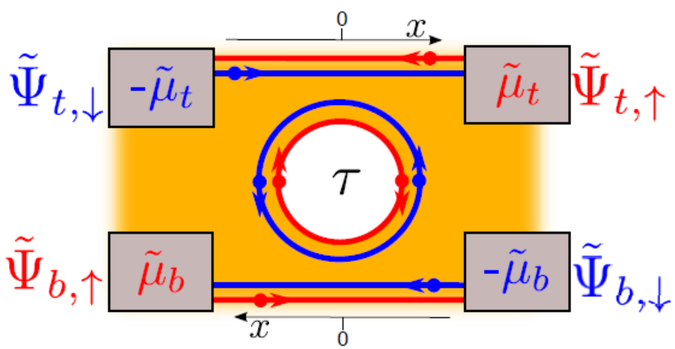


$$\Rightarrow M_{if} = 0$$

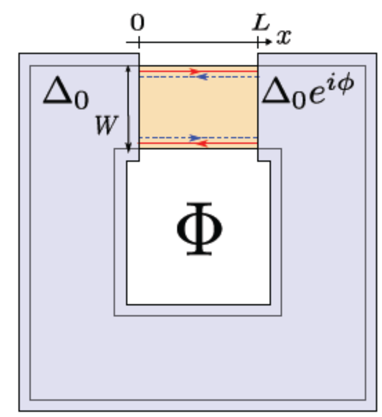
Budich, Dolcini, Recher & BT PRL 2012

$$\Rightarrow \frac{\delta G}{G_0} \sim \begin{cases} \left(a_0 T / v\right)^{4K} & \text{if } K > \frac{1}{2} \\ \left(a_0 T / v\right)^{8K-2} & \text{if } \frac{1}{4} < K < \frac{1}{2} \end{cases}$$

Crépin, Budich, Dolcini, Recher & BT PRB 2012



Posske, Liu, Budich & BT PRL 2013



Crépin & BT arXiv 2013